EVALUATION OF EFFICIENCY IN ONTARIO UNIVERSITIES USING DATA ENVELOPMENT ANALYSIS

BY

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Evaluation of Efficiency in Ontario Universities Using Data Envelopment Analysis

Abstract

Data Envelopment Analysis (DEA) is a popular operation research technique for determining the relative efficiency of non-profit organizations. The main goal of this study is to develop a unique stochastic DEA model to evaluate the efficiency of Ontario universities using some inputs and outputs. It focuses on the stochastic measure because service industries like universities are interested in qualitative outputs whose measurement through deterministic model seems non-practicable. The results of this study show that the selection of inputs and outputs plays a crucial role in determining the ranks of universities using DEA.

Keywords: Data Envelopment Analysis (DEA); Universities; Canada; Stochastic models; Optimization

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric performance evaluation model for estimation of efficiency of a certain entity with respect to the other entities having similar characteristics and objectives (Mardani et al., 2017; Koronakos et al., 2019). DEA was first introduced by Charnes, Cooper, and Rhodes (CCR) in 1978 for the estimation of the relative efficiency of Decision-Making Units (DMUs) (Cooper et al., 2007). Applying DEA is useful when DMUs are homogeneous in nature (Charnes et al., 1978; Hajiagha et al., 2019).

Earlier use of DEA was limited to the deterministic inputs and outputs and considered noise-free. However, these assumptions may fail in real-world scenarios as many inputs and outputs like quality and personal satisfaction are non-deterministic in nature. Moslemi et al. (2019) suggested that the nature of DMUs may vary widely depending upon the objective of the study, personal preference of decision-maker, and accessibility and reliability of data. Many of these inputs and outputs are qualitative or obtained by evidential reasoning which cannot be expressed quantitatively with a high degree of accuracy. DEA is found efficient to measure the uncertainties in one or more DMUs by the introduction of stochastic DEA (SDEA).

Most organizations are evaluated by revenue collection, profit maximization, and other quantifiable inputs and outputs. However, universities are non-profit organizations and it is difficult to evaluate the efficiency for them based on traditional factors. Some authors have evaluated the performances of universities by some methods. Guarino et al. (2005) applied a Bayesian latent variable analysis for the estimation of the degree of uncertainty in ranking using qualitative data, and compared old weight and sum ranking with latent variable ranking. Dobrota et al. (2015) proposed a substitutional Quacquarelli Symonds (QS) scores, referred to as the composite I-distance indicator (CIDI) procedure. There are some papers in the literature for the applications of DEA to assess the efficiency of universities which are mentioned in the next section.

Canada is a multilingual, multicultural, and diverse country with migrants from all over the world. Universities of Canada are in top ranks attracting people to have higher education from all over the world. Ontario is the most populated province with many universities within itself. In this paper, we calculate the efficiency of Ontario universities according to three groups including medical/doctoral universities, comprehensive universities, and primarily undergraduate universities. To this aim, the appropriate inputs and outputs are identified, and the related data are collected and analyzed. The proposed DEA model can handle uncertainty in this problem.

The rest of this publication is organized as follows. The related literature is provided in Section 2. Section 3 is assigned to the problem statement. Then, data collection is discussed in Section 4. In Section 5, the details of the solution approach are provided. Section 6 is related to the results. Then, the discussion is provided in Section 7. Finally, conclusions and future research are provided in Section 8.

2. Literature review

DEA is a non-parametric and useful decision-making tool based on linear programming. It is a logical method for benchmarking a group of entities capable of analyzing multiple inputs and outputs (Mardani et al., 2017). It was primarily developed by Charnes, Cooper, and Rhodes in 1978 (Cooper et al., 2007). There are many papers associated with the modelling and application of DEA. Some of the related papers are mentioned in this section.

2.1. DEA models

DEA models have several applications. Nayar and Ozcan (2008) compared the efficiency of hospitals of Virginia using the DEA method to expose the growing concern that even an inefficient hospital may be showing good performance in terms of the economy by reducing the quality. Lee and Kim (2014) utilized the DEA tool for benchmarking of service quality. They took auto repair service as an example. Iparraguirre and Ma (2015) benchmarked local authorities that provided social care services in UK using the self-reported quality life of older people using DEA. Sarkar (2017) proposed a multiplier model of BCC (Banker, Charnes, Cooper) DEA which was different from regular cost-oriented DEA. They utilized the model for measuring the inclination of DMUs along with cost by evaluation and identification of the most cost-efficient school among six schools.

2.2. DEA under uncertainty

Uncertainty in the parameters of DEA models has been considered by some authors. Sueyoshi (2000) developed a stochastic DEA model formulated in the way that it can incorporate future information which was called “DEA future analysis” to redesign a Japanese petroleum company. Shang et al. (2010) assessed the efficiency of hotels by applying a stochastic DEA. The result was found to be higher than the efficiency measures of the deterministic one. The determinants of efficiency were examined by the Tobit regression model approach. Jradi and Ruggiero (2019) suggested that DEA or quantity based on deviation from production frontier is not only the case due to the inefficiency but due to the statistical noise too. To overcome such biases, Jradi and Ruggiero (2019) extended Banker’s stochastic DEA model which focussed on most likely stochastic frontier model with constrained error analysis, and under different assumptions of distribution. Wen et al. (2018) developed stochastic spares optimization model (SSOM) built based on a stochastic DEA model to tackle the problem of optimization under the uncertainty of spare parts. In this problem, the stochastic parts were converted into deterministic using probability theory. They simplified the problem from non-linear to linear programming.

Some authors have combined DEA with fuzzy sets theory in different prospects. Azadeh et al. (2013) presented a hybrid fuzzy DEA and fuzzy simulation to optimize an operator distribution in a cellular manufacturing system. Liu (2014) proposed a fuzzy two-stage DEA model to assess the upper and lower bounds of efficiency scores. They illustrated the application considering Taiwan's non-life insurance companies. Azadi et al. (2015) proposed a combined DEA with Enhanced Russel Measure (ERM) in fuzzy setting for the selection of the best suppliers in a resin production company considering the sustainability of supplier, and efficiency and productivity in an uncertain environment. Ignatius et al. (2016) developed a DEA based fuzzy model using asymmetrical and symmetrical fuzzy numbers as input and output data to evaluate the carbon efficiency, allowing the measurement of environmental impact.

Many papers have focused on robust optimization DEA to solve the problem of uncertainty in data. Sadjadi et al. (2011) presented a super-efficiency DEA model for modeling the stochastic data with the implication of robust optimization for ranking different Iranian gas companies. Omrani (2013) introduced robust optimization to impose limits on weights flexibility of DEA with uncertainty in input and output data using goal programming technique. They showed the application for provincial gas companies in Iran. Landete et al. (2017) modelled scenario-based probabilistic DEA to calculate three robust efficiency scores i.e., the expected score, the conditionally expected score, and the unconditional expected score. These efficiency scores are useful to find solutions for many linear programming problems. Moslemi et al. (2019) proposed a new consistent performance measure which is claimed to be successful in handling uncertainties in DEA.

2.3. DEA for universities

McMillan and Datta (1998) implemented DEA to compute the comparable efficiency of Canadian universities with nine specifications of inputs and outputs including comprehensive, medical, and primarily undergraduate categories. Regression analysis was used to find the determinants of efficiency.

Avkiran (2001) developed a DEA model to evaluate the technical and scale efficiency of Australian universities. The proposed model helped in recognizing the deficiency of incompetent universities in the designated category and helped in its improvement. Abbott and Doucouliagos (2003) used DEA to assess Australian universities with different input and output combinations. McMillan and Chan (2006) compared the efficiency scores of Canadian universities through the DEA and stochastic frontier methods and found divergence among the ranks and efficiency scores between the methods. Leitner et al. (2007) developed a DEA model to study the efficiency of some Austrian universities. They examined the consistency of the results by various models that were adjusted to non-homogeneous samples. In addition, they studied the impacts of size and specialization on the performances of some departments.

Abramo et al. (2011) developed a DEA model on the basis of bibliometric data to evaluate research productivity with university research staff's academic rank as input. They considered the impact of research product on field level realized by staff, as an output. Aziz et al. (2013) showed how to apply DEA to find the comparative efficiency of academic departments of universities in Malaysia including input and output data from the research and teaching/learning elements. Altamirano-Corro and Peniche-Vera (2014) estimated the productivity of 20 public universities in terms of human resources, research activity, and research income using DEA and econometric procedures. The expectations were to restructure the administration and management of universities in Greece. Sagarra, Mar-Molinero, and Agasisti (2017) used ratios with DEA to evaluate the results of Mexico's "Educational Modernization Program".

For making a systematic evaluation approach in university ranking and measuring the target level to be attained by the institution, Aleskerov et al. (2017) applied DEA in heterogeneous samples along with Stochastic Frontier Analysis (SFA) scores and Malmquist index. They considered the consistency of results calculated from different models. The authors discussed the empirical outcomes of efficiency ideas in diverse samples in higher education institutions and universities. Visbal-Cadavid et al. (2017) presented the output of productivity study of Colombian public universities using output-oriented CCR, BCC, and Slacks Based Measure (SBM) models of DEA.

Barra et al. (2018) estimated the effectiveness of the Italian higher education system based on parametric and non-parametric methods. They concluded that the selection of method did not make a significant difference in the outcome, but the selection of input and output parameters vastly changes the ranks of DMUs. Yang et al. (2018) investigated the productivity and inadequacy of research universities in China during 2010-2013. They developed a two-stage network DEA model to evaluate the efficiency of universities and to measure the productivity change.

Ai et al. (2019) implemented DEA and Fisher Information (FI) as a complementary method to assess the sustainability of urban universities in Chicago, USA. Zhang and Shi (2019) used the principal component analysis method and the DEA technique to evaluate the university teaching performance in China. Duan (2019) applied DEA and strategic group analysis to assess the comparative efficiency of universities in Australia considering three major fields of study, i.e., overall university operations efficiency, research efficiency, and teaching efficiency. Table 1 includes a summary of DEA models to evaluate the efficiency of universities.

Table 1
A summary of recent papers about DEA models to assess the efficiency of universities

Authors	Technique	Uncer	Country	Inputs	Outputs
McMillan and Datta (1998)	CCR DEA		Canada	Total number of full-time faculty, expenditure	Full-time equivalent enrolments, total sponsored research expenditure, grants
Avkiran (2001)	BCC and CCR DEA		Australia	Academic staffs, non-academic staffs	Enrolments, student retention rate, total research grants, number of academic publications by faculty members
Abbott and Doucouliagos (2003)	DEA		Australia	Number of academic staffs, number of non-academic staffs, expenditure, the value of non-current assets	Research quantum, EFTS, research grants, research spending
Leitner et al. (2007)	DEA (BCC, CCR model)		Austria	Staffs, room space	Examination, reports, projects, thesis, presentation, publication, patents, financial funds provided by the third party
Abramo et al. (2011)	Analysis option DEA		Italy	Instructional expenditure, overhead expenditure, physical investments	Number of undergraduate and graduate students, federal research grants and contracts
Aziz et al. (2013)	CCR DEA		Malaysia	Number of non-academic staffs, number of academic staffs, yearly operating expenses	Total research grants received, number of graduates for the year, number of academic publications
Altamirano-Corro and Periche-vera (2014)	AHP and DEA		Greece	PTC Doctorado, PTC SNI	Graduate studies in PNC, CA, PE

Sagarra, Mar-Molinero and Agasisti (2017)	Traditional ratio and DEA		Mexico	Full-time equivalent faculty, total enrolment, first joining the graduation	Scopus paper, graduates
Visbal-cadavid et al. (2017)	CCR, BCC, and SBM DEA models		Colombia	Full-time equivalent admin staff, faculty, financial resources, physical resources, expenditure	Numbers of student's enrolments, indexed journal, and articles, faculty mobility
Yang et al. (2018)	Two-stage DEA		China	R & D funds, research, and teaching staffs, a government fund	Publications, total number of patents, number of other intellectual property, total number of students
Barra et al. (2018)	SFA and DEA	✓	Italy	Number of academic staffs, % of enrolments with certain criteria	Number of graduates, research grants
Duan (2019)	DEA and strategic group analysis		Australia	Expense, total staff number	Revenue, total number of graduates
Our paper	Stochastic DEA	✓	Canada	Expenses, number of faculties	Tri-council grants, satisfaction level of students, total full-time enrolment, number of publications

Note: AHP - Analytic Hierarchy Process, CA - Cuerpo Académico, DEA - Data Envelopment Analysis, EFTS - Equivalent Full Time Student, PE - Programas Educativos Acreditados, PNPC - Programa Nacional de Posgrados de Calidad, PTC Doctorado - Full time professors with a doctorate degree, PTC SNI - Full time professors that belong to Sistema Nacional de Investigadores (SNI), SBM - Slacks Based Measure, SDEA - Stochastic Data Envelopment Analysis, SFA - Stochastic Frontier Approach, Uncer - Uncertainty.

There are some research gaps according to Table 1 and our knowledge. There are just two academic journal papers in the literature that have evaluated the efficiency of Canadian universities by DEA models (McMillan and Datta, 1998; McMillan and Chan, 2006). It is noticeable that they are old, and there is a need to update the data. In addition, new universities have been established in Canada. In addition, most papers about DEA for university assessment have ignored uncertainty in the assessment of efficiency. It is noticeable that uncertainty plays a prominent role when qualitative factors such as satisfaction level are utilized.

2.4. Research contributions

In this paper, a stochastic DEA model is developed to evaluate the efficiency of universities in Ontario, Canada. The major research contributions of this paper are summarized in this subsection.

- To develop a unique stochastic DEA model to evaluate the efficiency of universities.
- To utilize real data and to apply the model for universities in Ontario, Canada. Three types of universities are investigated including medical/doctoral universities, comprehensive universities, and primarily undergraduate universities.
- To identify inputs and outputs of the model, collect, and analyze the related data.
- To provide discussions and managerial insights based on the results.

3. Problem statement

Universities produce different levels of manpower that enable countries to use the available resources and contribute to national economies. The government of Ontario (a province in Canada) has focused on improving the quality of education and believes that the economy is driven by knowledge and research. At the same time, it has cut the basic fuel of each university, which is a provincial budget (University Affairs, 2019). Currently, the budget allocation to universities is an important topic in Ontario which is influenced by the rank and performance of the universities. There are some major problems in ranking Ontario universities which are highlighted in this section.

a) Quality performance: Quality of service cannot be estimated by number and quantity. There are more important things to consider for the measurement of the quality performance of universities. The satisfaction level is an important factor among them. To our knowledge, there is no academic publication about the assessment of universities using DEA and this factor.

b) Need to update the ranks: The competition among countries in the business world has changed the required skills and technology. Universities are places where people get the right education to cope with the change in the world, and individuals can stand in the competitive market to sell their knowledge. It is necessary for universities to change and use their resources. Ontario universities are no exception in this context (Ontario's Universities, 2019). Some universities have been able to take good positions in the global market and receive good ranks. Evaluation and ranking of universities are important because of identifying areas of performance improvement. The province has traveled a long distance after rankings that were done by McMillan and Datta (1998), and McMillan and Chan (2006).

c) Selection of inputs and outputs: It is essential to consider input and output parameters rigorously to avoid misguiding policy-makers and executives. There are different organizations that rank universities in their own standard measures. Significant differences exist in the ranks of universities using different systems. For example, the University of Toronto has Rank 21 based on Times Higher Education (THE) (2019), and 28 according to Top Universities (2019). If we look at the ranks of Ontario universities in diverse ranking systems, we can see huge differences. A good selection of inputs and outputs is needed to have a reliable and useful ranking.

d) Selection of right tool for evaluation: Lesser-known universities may be doing great in some fields but may not meet the criteria imposed by ranking systems. Thus, the ranks of those universities may not be among the top ranks. An appropriate tool is required to handle this issue. On the other hand, there are some qualitative criteria (e.g., satisfaction level) in the ranking of universities. A suitable technique should be used to address this issue. To provide necessary insights for policy-makers and executives, it is necessary to investigate the efficiency of Ontario universities based on some inputs and outputs such as satisfaction levels of students after graduation and expenditures of universities. In this study, a novel DEA model is developed.

4. Data collection

In this research, inputs and outputs for evaluation of Ontario universities are identified by examining the literature and searching the available data. There are some papers and publications in the literature about the efficiency assessment of universities by DEA models in different countries. Most of them have been reviewed in the literature review part of this paper. The inputs and outputs of those DEA models have been collected. Generally, it is not difficult to measure the inputs and define quantitative metrics for them. In most cases, there are some challenges in measuring the outputs, quantitatively. Table 2 includes the selected inputs and outputs. The data include two important inputs that have high impacts on the performances of universities and four outputs prioritized according to their impacts on the efficiency of universities. Information about expenditures and the number of faculties are obtained from CUDO (Common University Data Ontario) (Cudo.ouac.on.ca, 2019). In addition, CUDO is the source of tri-council grants, student's satisfaction level, and enrolment which are the outputs of this model. The number of publications is taken from the Scopus website.

Table 2
The list of inputs and outputs

Inputs	Outputs
Expenditures	Tri-council* grants
Number of faculties	Student's Satisfaction level
	Enrolment (number of students)
	Number of publications

* Tri-council includes the Natural Sciences and Engineering Research Council (NSERC), the Canadian Institutes of Health Research (CIHR), and the Social Sciences and Humanities Research Council (SSHRC)

The data in this research cover 18 Ontario universities for 2013-2017. Ontario universities are categorized in three groups to have fair and reliable rankings based on Macleans, 2019. We consider 5 Ontario medical/doctoral universities, 8 Ontario comprehensive universities, and 5 Ontario primarily undergraduate universities. To avoid conflict of interest, the real names of the universities are not mentioned in the tables of the results.

5. Solution approach

The proposed model in this paper is based on the CCR DEA model. CCR is an important mathematical model in DEA literature. This model measures the max ratio of weighted output to input. The ratio of every DMU should be equal to or less than 1 expressed as constraints (Charnes et al., 1978). Ignatius et al. (2016) suggested that if the ratio of weighted output to input is 1, then the DMU is considered efficient. Otherwise, it is inefficient.

The proposed model in this paper is a stochastic model (scenario-based). Therefore, it can handle uncertainty in the problem. In addition, the weights of inputs and outputs are added to the mathematical model. The following sets, parameters, and decision-variables are utilized in this mathematical model.

Sets

Universities: $n = 1, \ldots, N$

Outputs: $j = 1, \ldots, J$

Inputs: $i = 1, \ldots, I$

Scenarios: $t = 1, \ldots, T$

Parameters

y_{jnt}: Output j of university n in scenario t

x_{int}: Input i for university n in scenario t

p_t: Probability of scenario t

ε: A small number

b_j: Weight of output j

c_i: Weight of input i

Decision-variables

u_{jt}: Decision variable related to output j in scenario t

v_{it}: Decision variable related to input i in scenario t

The efficiency of University a is calculated by the following formula:

$$Max\ z = \sum_t \sum_j p_t b_j y_{jat} u_{jt} \qquad (1)$$

$s.t.$

$$\sum_i c_i x_{iat} v_{it} = 1 \qquad \forall t \qquad (2)$$

$$\sum_j b_j y_{jnt} u_{jt} - \sum_i c_i x_{int} v_{it} \le 0 \qquad \forall n, t \qquad (3)$$

$$\sum_i c_i = 1 \qquad (4)$$

$$\sum_j b_j = 1 \qquad (5)$$

$$u_{jt}, v_{it} \ge \varepsilon \qquad \forall i, j, t \qquad (6)$$

The objective function (1) maximizes the output of the University a. Constraint (2) is related to the inputs of the University a. In addition, the relationships between the inputs and the outputs of universities are expressed in Constraint (3). Constraints (4) and (5) are related to the weights of outputs and inputs, respectively. Finally, Constraint (6) is devoted to the non-negative variables. ε has a very small value.

6. Results

In this study, it is supposed that the student's satisfaction level and the enrolment (number of students) are uncertain parameters. The student's satisfaction level is uncertain because the data is collected every three years, and the value may vary every year. Furthermore, the enrolment is uncertain because some students may leave the program during each academic year which leads to uncertainty.

In this section, the optimization model is solved using GAMS software. It is a high-level mathematical modelling language for optimization. First, we ignore the uncertainty in the mathematical model, and we run the GAMS files. The results are written in Tables 3, 4, 5.

Table 3
Comprehensive universities and their efficiency scores

Inputs, Weights		Outputs, Weights				Comprehensive universities and their efficiency scores							
c_1	c_2	b_1	b_2	b_3	b_4	*Uf*	*Ug*	*Uh*	*Ui*	*Uj*	*Uk*	*Ul*	*Um*
0.5	0.5	0.25	0.25	0.25	0.25	1	1	1	0.9012	1	1	0.9372	1
1	0	1	0	0	0	0.2774	0.64	0.5953	0.3226	1	0.2636	0.4368	0.4757
1	0	0	1	0	0	0.8858	0.5523	0.5494	0.3181	0.3035	1	0.5138	0.1763
1	0	0	0	1	0	1	0.9146	0.6900	0.8079	0.6730	0.8570	0.7714	0.8516
1	0	0	0	0	1	0.4618	0.8140	0.6595	0.4717	1.0000	0.3390	0.5435	0.4676
1	0	0.5	0.5	0	0	0.9191	0.9266	0.8905	0.4989	1	1	0.7420	0.5012
1	0	0.5	0	0.5	0	1	1	0.8106	0.8297	1	0.8627	0.8206	0.9045
1	0	0	0.5	0.5	0	1	0.9146	0.6900	0.8079	0.6730	1	0.7714	0.8516
1	0	0.5	0	0	0.5	0.4618	0.8140	0.7282	0.4717	1	0.3390	0.6577	0.4757
1	0	0	0.5	0	0.5	1	1	0.8873	0.5783	1	1	0.7773	0.4886
1	0	0	0	0.5	0.5	1	1	0.7844	0.8294	1	0.8570	0.8122	0.8678
1	0	0.25	0.25	0.25	0.25	1	1	0.9077	0.8297	1	1	0.8206	0.9045
0	1	1	0	0	0	0.1845	0.4530	0.6936	0.2505	1	0.2117	0.3840	0.3936
0	1	0	1	0	0	0.7335	0.4867	0.7971	0.3075	0.3779	1	0.5624	0.1817
0	1	0	0	1	0	0.8272	0.8052	1	0.7801	0.8371	0.8561	0.8434	0.8766
0	1	0	0	0	1	0.3071	0.5761	0.7684	0.3662	1	0.2723	0.4778	0.3870
0	1	0.5	0.5	0	0	0.7448	0.6337	1	0.3792	1	1	0.6648	0.4125
0	1	0.5	0	0.5	0	0.8272	0.8449	1	0.8896	1	1	0.9191	0.9573
0	1	0	0.5	0.5	0	0.8608	0.8052	1	0.7801	0.8371	1	0.8434	0.8766
0	1	0.5	0	0	0.5	0.3071	0.5761	1	0.3662	1	0.2723	0.7984	0.3936
0	1	0	0.5	0	0.5	0.7731	0.6990	1	0.4435	1	1	0.6819	0.4032
0	1	0	0	0.5	0.5	0.8272	0.8052	1	0.7801	1	0.8561	0.8434	0.8766
0	1	0.25	0.25	0.25	0.25	0.8608	1	1	0.9472	1	1	0.9324	0.9573
0.5	0.5	1	0	0	0	0.2774	0.6400	0.6936	0.3226	1	0.2636	0.4368	0.9573
0.5	0.5	0	1	0	0	0.8858	0.5523	0.7971	0.3181	0.3779	1	0.5624	0.1817
0.5	0.5	0	0	1	0	1	0.9563	1	0.9012	0.8877	0.9785	0.9359	0.9923
0.5	0.5	0	0	0	1	0.4618	0.8140	0.7684	0.4717	1	0.4717	0.5435	0.4676
0.5	0.5	0.5	0.5	0	0	0.9191	0.9266	1	0.4989	1	1	0.7420	0.5012
0.5	0.5	0.5	0	0.5	0	1	1	1	0.9012	1	0.9785	0.9371	1
0.5	0.5	0	0.5	0.5	0	1	0.9563	1	0.9012	0.8877	1	0.9359	0.9923
0.5	0.5	0.5	0	0	0.5	0.4618	0.8140	1	0.4717	1	0.3390	0.5435	0.4757
0.5	0.5	0	0.5	0	0.5	1	1	1	0.5863	1	1	0.7988	0.4886
0.5	0.5	0	0	0.5	0.5	1	1	1	0.9012	1	0.9785	0.9368	0.9923
					Av. Score	0.7656	0.8158	0.8700	0.6131	0.9047	0.7878	0.7251	0.6585
					Rank	5	3	2	8	1	4	6	7

Based on the results in Table 3, it is observed that all comprehensive universities perform almost equal and well when equal weights are assigned for the inputs and outputs. However, the performances fluctuate significantly when single input and single output are considered. The average scores and the related ranks are written at the last rows of Table 3. It is noticeable that there are small differences between the scores of the comprehensive universities which show good average efficiency.

For the higher weights assigned to Outputs b_1 and b_4, the efficiency of *Uf* is very low with the lowest score of 0.18. The administrations of this university may focus on these two outputs and improve the efficiency. Outputs b_1 and b_2 have high impacts on the reduction of efficiency in *Ug* with the least score of 0.45. Input c_1 has significant influence on the low efficiencies for *Uh* with a minimum score of 0.55. *Ui* has room for improvement in all aspects with a minimum score of 0.25. *Uj* may focus on improvement in Output b_2 for its betterment. This university is very efficient in the other aspects. Outputs b_1 and b_4 are very effective factors for the low efficiency of *Uk* and *Ul* with minimum scores of 0.21 and 0.38, respectively.

Table 4
Medical universities and their efficiency scores

Inputs, Weights		Outputs, Weights				Medical universities and their efficiency scores				
c_1	c_2	b_1	b_2	b_3	b_4	*Ua*	*Ub*	*Uc*	*Ud*	*Ue*
0.5	0.5	0.25	0.25	0.25	0.25	1	1	1	1	0.9629
1	0	1	0	0	0	0.7886	0.7460	0.8233	1	0.7237
1	0	0	1	0	0	0.6011	0.2777	1	0.1229	0.6031
1	0	0	0	1	0	0.8861	1	0.9113	0.7625	0.9217
1	0	0	0	0	1	0.6289	0.6975	1	0.7010	0.3344
1	0	0.5	0.5	0	0	0.8877	0.7826	1	1	0.8247
1	0	0.5	0	0.5	0	0.9564	1	0.9903	1	0.9416
1	0	0	0.5	0.5	0	0.9283	1	1	0.7625	0.9629
1	0	0.5	0	0	0.5	0.8204	0.8189	1	1	0.7237
1	0	0	0.5	0	0.5	0.6289	0.6975	1	0.7010	0.6031
1	0	0	0	0.5	0.5	0.8888	1	1	0.8037	0.9217
1	0	0.25	0.25	0.25	0.25	0.9611	1	1	1	0.9629
0	1	1	0	0	0	0.6826	0.5127	0.6744	1	0.4541
0	1	0	1	0	0	0.6351	0.2330	1	0.1500	0.4620
0	1	0	0	1	0	1	0.8961	0.9732	0.9941	0.7540
0	1	0	0	0	1	0.6646	0.5852	1	0.8558	0.2562
0	1	0.5	0.5	0	0	0.8756	0.5693	1	1	0.5967
0	1	0.5	0	0.5	0	0.7445	0.6255	1	1	0.4541
0	1	0	0.5	0.5	0	1	0.8961	1	0.9941	0.7540
0	1	0.5	0	0	0.5	0.7445	0.6255	1	1	0.4541
0	1	0	0.5	0	0.5	0.6646	0.5852	1	0.8558	0.4620
0	1	0	0	0.5	0.5	1	0.8961	1	1	0.7540
0	1	0.25	0.25	0.25	0.25	1	1	1	1	0.7540
0.5	0.5	1	0	0	0	0.7886	0.7460	0.8233	1	0.7237
0.5	0.5	0	1	0	0	0.6351	0.2777	1	0.1500	0.6031
0.5	0.5	0	0	1	0	1	1	1	0.9941	0.9217
0.5	0.5	0	0	0	1	0.6646	0.6975	1	0.8558	0.3344
0.5	0.5	0.5	0.5	0	0	0.8877	0.7826	1	1	0.8247
0.5	0.5	0.5	0	0.5	0	1	1	1	1	0.9416
0.5	0.5	0	0.5	0.5	0	1	1	1	0.9941	0.9629
0.5	0.5	0.5	0	0	0.5	0.8204	0.8189	1	1	0.7237
0.5	0.5	0	0.5	0	0.5	0.6646	0.6975	1	0.8558	0.6031
0.5	0.5	0	0	0.5	0.5	1	1	1	1	0.9217
					Av. Score	0.8318	0.7717	0.9756	0.8652	0.7031
					Rank	3	4	1	2	5

Table 4 includes the results of efficiency scores of Medical universities based on different weights assigned to the inputs and outputs. It is clear from the table that the efficiency scores of all universities are high, and almost equal when equal weights are assigned to the inputs and outputs. In addition, these scores do not fluctuate considerably, and they remain high and consistent with equal weights of outputs for a single input. But there are large differences in the efficiency scores considering a single output.

The average scores and the ranks are written at the end of Table 4. It is observed that the average results are high, and all universities perform well compared to the universities of this category in Ontario. The performance of *Ua* seems to be average for all criteria, and not hardly affected by any input or output with a minimum score of 0.6. *Ub* should focus on improving Output b_2. It has very low efficiency for the high weight assigned to Output b_2 with a minimum score of 0.23. *Uc* is overall good in all aspects. *Ud* is mainly affected by its low score with the high weight assigned to b_2 with very low efficiency of 0.12.

Table 5
Primary undergraduate universities and their efficiency scores

Inputs, Weights		Outputs, Weights				Primary undergraduate universities and their efficiency scores				
c_1	c_2	b_1	b_2	b_3	b_4	*Un*	*Uo*	*Up*	*Uq*	*Ur*
0.5	0.5	0.25	0.25	0.25	0.25	1.0000	1.0000	1.0000	1.0000	1.0000
1	0	1	0	0	0	0.7900	1	0.2064	0.6151	0.7299
1	0	0	1	0	0	0.2658	0.2424	1	0.3177	0.5218
1	0	0	0	1	0	0.7973	0.6734	0.7931	1	0.9571
1	0	0	0	0	1	0.7829	0.5213	0.4232	1	0.7241
1	0	0.5	0.5	0	0	0.8482	1	1	0.7471	1
1	0	0.5	0	0.5	0	0.9440	1	0.7931	1	1
1	0	0	0.5	0.5	0	0.7998	0.6790	1	1	1
1	0	0.5	0	0	0.5	1	1	0.4232	1	0.9243
1	0	0	0.5	0	0.5	0.7943	0.5725	1	1	0.9185
1	0	0	0	0.5	0.5	0.7973	0.6734	0.7931	1	0.9571
1	0	0.25	0.25	0.25	0.25	1.0000	1.0000	1	1	1
0	1	1	0	0	0	0.8385	1	0.1847	0.7995	0.9423
0	1	0	1	0	0	0.3153	0.2710	1	0.4616	0.7529
0	1	0	0	1	0	0.6510	0.5181	0.5459	1	0.9506
0	1	0	0	0	1	0.6393	0.4011	0.2913	1	0.7192
0	1	0.5	0.5	0	0	0.8487	1	1	0.8279	1
0	1	0.5	0	0.5	0	0.8655	1	0.5459	1	1
0	1	0	0.5	0.5	0	0.6534	0.5231	1	1	1
0	1	0.5	0	0	0.5	0.8897	1	0.2913	1	1
0	1	0	0.5	0	0.5	0.6528	0.4584	1	1	1
0	1	0	0	0.5	0.5	0.6510	0.5181	0.5459	1	0.9506
0	1	0.25	0.25	0.25	0.25	0.8897	1	1	1	1
0.5	0.5	1	0	0	0	0.8385	1	0.2064	0.7995	0.9423
0.5	0.5	0	1	0	0	0.3153	0.2710	1	0.4616	0.7529
0.5	0.5	0	0	1	0	0.7973	0.6734	0.7931	1	0.9571
0.5	0.5	0	0	0	1	0.7829	0.5213	0.4232	1	0.7241
0.5	0.5	0.5	0.5	0	0	0.8487	1	1	0.8279	1
0.5	0.5	0.5	0	0.5	0	0.9440	1	0.7931	1	1
0.5	0.5	0	0.5	0.5	0	0.7998	0.6790	1	1	1
0.5	0.5	0.5	0	0	0.5	1	1	0.4232	1	1
0.5	0.5	0	0.5	0	0.5	0.7943	0.5725	1	1	1
0.5	0.5	0	0	0.5	0.5	0.7973	0.6734	0.7931	1	0.9571
					Av. Score	0.7767	0.7407	0.7354	0.9048	0.9207
					Rank	3	5	4	2	1

Table 5 includes the information about the efficiency scores of primarily undergraduate universities in Ontario. All these universities have optimal efficiency considering equal weights assigned to the inputs and outputs. It can be seen that they have high scores for the equal weights assigned to the outputs with a single input, in addition to high scores for a single output. Output b_2 has a high impact to reduce the efficiency of *Un* and *Uo* with the scores of 0.27 and 0.24, respectively. b_1 and b_4 have significant impacts on *Up* with the least score which is 0.2. *Uq* and *Ur* may focus on Output b_2 because of the minimum scores (0.32 and 0.52). They have high efficiency when all combinations of inputs and outputs are considered.

The results from Tables 3, 4 and 5 highlight the fact that all the universities have acceptable efficiency in their categories. Some universities can improve the efficiency in some aspects. However, a few universities have excellent efficiency scores in all aspects, and they have the highest ranks. Moreover, we observe that the average scores calculated at the end of each table have led to nearly the same range of scores. It means that the overall performances of all universities are good compared to each other in Ontario. It is noticeable that the case may be different if they are compared to other universities of other provinces or the world.

Some analyses are completed according to the various scenarios in this section. There are nine scenarios because of the two selected sources of uncertainty i.e., enrolment (number of students) and student's satisfaction level. In this research, it is supposed that the student's satisfaction level may increase or decrease by 10% compared to the deterministic case (Scenario 5). In addition, 5% and 10% decrease in enrolment is considered. Table 6 includes information about the scenarios.

Table 6
Nine scenarios in this problem

Scenario	Enrolment (number of students), Output *e*	Student's satisfaction level, Output *f*	Probability
1	0.95 y_{ent}	1.1 y_{fnt}	0.05
2	0.95 y_{ent}	y_{fnt}	0.15
3	0.95 y_{ent}	0.9 y_{fnt}	0.05
4	y_{ent}	1.1 y_{fnt}	0.15
5	y_{ent}	y_{fnt}	0.2
6	y_{ent}	0.9 y_{fnt}	0.15
7	0.9 y_{ent}	1.1 y_{fnt}	0.05
8	0.9 y_{ent}	y_{fnt}	0.15
9	0.9 y_{ent}	0.9 y_{fnt}	0.05

The results of the DEA model when we consider uncertainty are written in Tables 7, 8, and 9. Table 7 includes the efficiency scores of comprehensive universities with uncertainty in the enrolment and student's satisfaction level. The results suggest that all universities are in good shape with high-efficiency scores in most of the cases. However, when enrolment is not considered in the model, the ranges between the efficiency scores are slightly higher than the other cases.

The efficiency scores of medical universities with uncertain parameters are presented in Table 8. With a variety of weights of inputs and outputs, the efficiency of *Uc* seems optimal (or 1). *Ud* has slightly lower scores in some cases, and have optimal values in the other cases. There are some areas of improvement for *Ue* based on the efficiency scores. It is observable that the majority of the universities in this category come up with average good efficiency scores.

Table 9 comprises the results of the efficiency scores of primary undergraduate universities. According to this table, *Uq* has the first rank with the optimal scores all over the ranges of inputs and outputs. The other universities have high average efficiency scores with some areas for improvement.

Table 7
Comprehensive universities and their efficiency scores under uncertainty

Inputs, Weights		Outputs, Weights				Comprehensive universities and the efficiency scores							
c_1	c_2	b_1	b_2	b_3	b_4	*Uf*	*Ug*	*Uh*	*Ui*	*Uj*	*Uk*	*Ul*	*Um*
0.5	0.5	0.25	0.25	0.25	0.25	1	1	1	0.9012	1	1	0.9775	1
1	0	0.33	0.33	0.33	0	1	1	0.8961	0.8297	1	1	0.8503	0.9045
1	0	0.33	0.33	0	0.33	1	1	0.9077	0.5783	1	1	0.7803	0.5012
1	0	0.33	0	0.33	0.33	1	1	0.8106	0.8297	1	0.8627	0.8503	0.9045
1	0	0	0.33	0.33	0.33	1	1	0.8872	0.8294	1	1	0.8406	0.8678
1	0	0.25	0.25	0.25	0.25	1	1	0.9077	0.8297	1	1	0.8503	0.9045
0	1	0.33	0.33	0.33	0	0.8608	0.8449	1	0.8896	1	1	0.9627	0.9573
0	1	0.33	0.33	0	0.33	0.7731	1	1	0.4435	1	1	0.8631	0.4125
0	1	0.33	0	0.33	0.33	0.8272	1	1	0.9472	1	0.8561	0.9749	0.9573
0	1	0	0.33	0.33	0.33	0.8608	0.8052	1	0.7801	1	1	0.8779	0.8766
0	1	0.25	0.25	0.25	0.25	0.8608	1	1	0.9472	1	1	0.9749	0.9573
0.5	0.5	0.33	0.33	0.33	0	1	1	1	0.9048	1	1	0.9772	1
0.5	0.5	0.33	0.33	0	0.33	1	1	1	0.5863	1	1	0.8631	0.5012
0.5	0.5	0.33	0	0.33	0.33	1	1	1	0.9472	1	0.9785	0.9775	1
0.5	0.5	0	0.33	0.33	0.33	1	1	1	0.9012	1	1	0.9747	0.9923
					Av. Score	0.9455	0.9767	0.9606	0.8097	1.0000	0.9798	0.9064	0.8491
					Rank	5	3	4	8	1	2	6	7

Table 8
Medical universities and their efficiency scores under uncertainty

Inputs, Weights		Outputs, Weights				Medical universities				
c_1	c_2	b_1	b_2	b_3	b_4	*Ua*	*Ub*	*Uc*	*Ud*	*Ue*
0.5	0.5	0.25	0.25	0.25	0.25	1	1	1	1	0.9629
1	0	0.33	0.33	0.33	0	0.9611	1	1	1	0.9629
1	0	0.33	0.33	0	0.33	0.8877	0.8189	1	1	0.8247
1	0	0.33	0	0.33	0.33	0.9564	1	1	1	0.9416
1	0	0	0.33	0.33	0.33	0.9283	1	1	0.8037	0.9629
1	0	0.25	0.25	0.25	0.25	0.9611	1	1	1	0.9629
0	1	0.33	0.33	0.33	0	1	0.8961	1	1	0.7540
0	1	0.33	0.33	0	0.33	0.8756	0.6255	1	1	0.5967
0	1	0.33	0	0.33	0.33	1	1	1	1	0.8177
0	1	0	0.33	0.33	0.33	1	0.8961	1	1	0.7540
0	1	0.25	0.25	0.25	0.25	1	1	1	1	0.7540
0.5	0.5	0.33	0.33	0.33	0	1	1	1	1	0.9629
0.5	0.5	0.33	0.33	0	0.33	0.8877	0.8189	1	1	0.8247
0.5	0.5	0.33	0	0.33	0.33	1	1	1	1	0.9416
0.5	0.5	0	0.33	0.33	0.33	1	1	1	1	0.9629
					Av. Score	0.96386	0.937033	1	0.986913	0.86576
					Rank	3	4	1	2	5

Table 9
Primary undergraduate universities and their efficiency scores under uncertainty

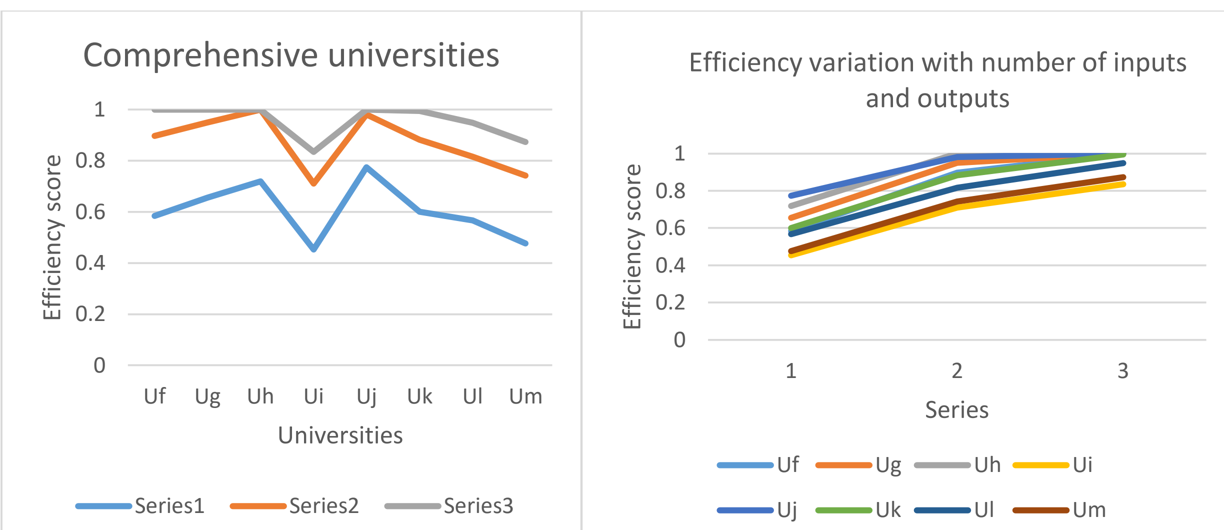
Inputs, Weights		Outputs, Weights				Primary undergraduate universities				
c_1	c_2	b_1	b_2	b_3	b_4	*Un*	*Uo*	*Up*	*Uq*	*Ur*
0.5	0.5	0.25	0.25	0.25	0.25	1	1	1	1	1
1	0	0.33	0.33	0.33	0	0.9272	1	1	1	1
1	0	0.33	0.33	0	0.33	1	1	1	1	1
1	0	0.33	0	0.33	0.33	1	1	0.7615	1	0.9849
1	0	0	0.33	0.33	0.33	0.7987	0.6577	1	1	0.9837
1	0	0.25	0.25	0.25	0.25	1	1	1	1	1
0	1	0.33	0.33	0.33	0	0.8635	1	1	1	1
0	1	0.33	0.33	0	0.33	0.8897	1	1	1	1
0	1	0.33	0	0.33	0.33	0.8897	1	0.5241	1	1
0	1	0	0.33	0.33	0.33	0.6532	0.5101	1	1	1
0	1	0.25	0.25	0.25	0.25	0.8897	1	1	1	1
0.5	0.5	0.33	0.33	0.33	0	0.9272	1	1	1	1
0.5	0.5	0.33	0.33	0	0.33	1	1	1	1	1
0.5	0.5	0.33	0	0.33	0.33	1	1	0.7615	1	1
0.5	0.5	0	0.33	0.33	0.33	0.7987	0.6577	1	1	1
					Av. Score	0.909173	0.9217	0.936473	1	0.997907
					Rank	5	4	3	1	2

7. Discussion

In this paper, we have calculated and compared the efficiency scores of different Ontario universities using diverse weights for inputs and outputs. Considering the student's satisfaction level is one of the unique features of this study. We believe that the ranking of universities without considering this factor does not reflect the true status of the quality of universities. We also have considered uncertainty in the model. The selection of inputs and outputs and non-consistent weights have been challenges in various studies. This paper is able to consider reality by taking the average of the results obtained from several weights assigned to the inputs and outputs, and to calculate the average of them.

The results of ranking universities based on DEA models may not be consistent with the traditional well-known ranking systems such as QS World University, and Financial Times rankings. Selection of inputs and outputs and the impacts of them play prominent role in DEA models which may lead to different rankings compared to the traditional methods.

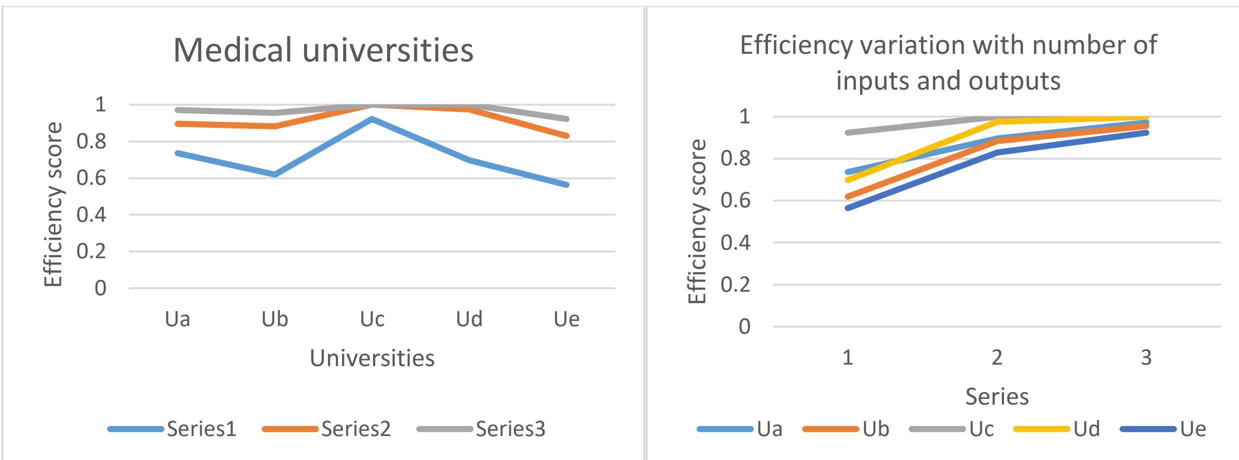
Figure 1 shows the comprehensive universities and their efficiency scores. In comprehensive universities, the efficiency ranges from 0.45 to 0.77 for single input and single output. The score of *Ui* is minimum. For two inputs and two outputs, the efficiency scores are between 0.71 and 1. For two inputs and three outputs, the range is between 0.83 to 1. Figure 1 (b) shows efficiency score of comprehensive universities at different series (number of inputs and outputs). It can be seen that *Um* has low efficiency among the universities of this category.



(a) (b)

Figure 1. Comprehensive universities and their efficiency scores
Series 1: 1 input and 1 output, Series 2: 2 inputs and 2 outputs, Series 3: 2 inputs and 3 outputs

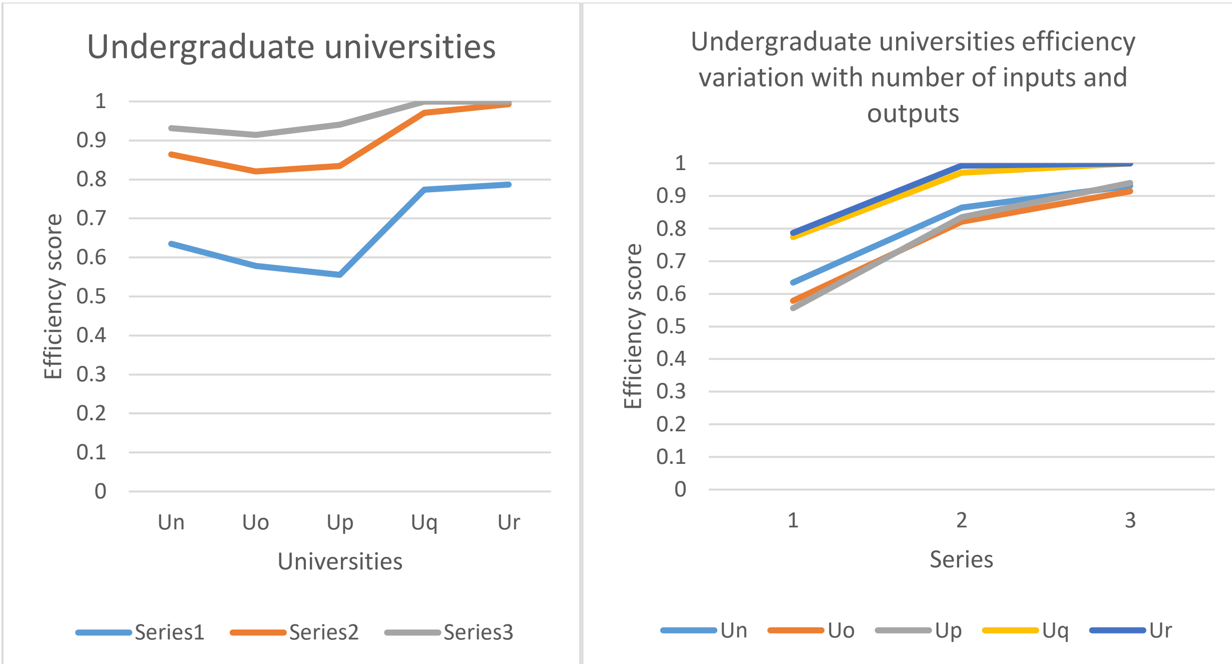
Medical universities and their efficiency scores are illustrated in Figure 2. When we compare the efficiency of medical universities, the minimum efficiency is 0.56 for single input and single output whereas it increases to 0.82 for two inputs and two outputs. The highest value is 0.92 for two inputs and three outputs. It can be seen that *Uc* has the highest efficiency score. In addition, the efficiency score of *Ue* shows rooms for improvement.



(a) (b)

Figure 2. Medical universities and their efficiency scores
Series 1: 1 input and 1 output, Series 2: 2 inputs and 2 outputs, Series 3: 2 inputs and 3 outputs

Figure 3 demonstrates primary undergraduate universities and their efficiency scores. The average efficiency ranges from 0.55 to 0.78 considering single input and single output. The efficiency score increases to the range of 0.82 and 0.99 for two inputs and two outputs. In the case of two inputs and three outputs, the efficiency of universities is between 0.91 to 1. It can be seen that with increase in the number of inputs and outputs, the range of efficiency increases. This point implies that if we consider many inputs and several outputs at the same time, all primarily undergraduate universities will have the highest efficiency score which is 1, with equal ranks.



(a) (b)

Figure 3. Primary undergraduate universities and their efficiency scores
Series 1: 1 input and 1 output, Series 2: 2 inputs and 2 outputs, Series 3: 2 inputs and 3 outputs

8. Conclusions

Evaluation of the efficiency of universities is an important topic. In this paper, a stochastic DEA model has been developed to assess the efficiency of Ontario universities in Canada. Two inputs (i.e., Expenditures and Number of faculties), and four outputs (i.e., Tri-council grants, Student's satisfaction level, Enrolment (number of students), Number of publications) have been considered in this study. Considering the satisfaction level is one of the unique features of this research.

In this study, the Ontario universities have been evaluated based on the developed scenario-based DEA model in three groups including comprehensive, medial, and primary undergraduate universities. First, the models have been solved without considering uncertainty. Then, the stochastic models have been solved, and the results have been reported.

Based on the results of this investigation, the Ontario universities have a very good performance against each other. But it is not possible to mention that there is no area for improvement. Based on the analyses of Ontario universities, the selection of inputs and outputs has significant impacts on the efficiency scores of the universities. Each university has a unique strategic plan. For instance, some universities may focus on teaching more than research. The administrations of universities may use the results to evaluate the relative efficiency and find the areas of improvement according to their preferred inputs and outputs.

Some future areas can be explored related to this research. The selection of broader inputs and outputs to evaluate efficiency may be the next area of research. The other future research direction is related to uncertainty. Robust optimization is a new technique in operations research that can handle uncertainty. It is helpful to solve this problem through robust optimization and compare the results. Calculating the efficiency of the universities located in the other provinces of Canada such as Nova Scotia and British Colombia in another future research area.

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References

Abbott, M. and Doucouliagos, C. (2003). The efficiency of Australian universities: a data envelopment analysis. *Economics of Education Review*, 22(1), pp.89-97.

Abramo, G., Cicero, T. and D'Angelo, C.A., 2011. A field-standardized application of DEA to national-scale research assessment of universities. *Journal of Informetrics*, *5*(4), pp.618-628.

Ai, N., Kjerland, M., Klein-Banai, C. and Theis, T. (2019). Sustainability assessment of universities as small-scale urban systems: A comparative analysis using Fisher Information and Data Envelopment Analysis. *Journal of Cleaner Production*, 212, pp.1357-1367.

Aleskerov, F., Belousova, V. and Petrushchenko, V. (2017). Models of data envelopment analysis and stochastic frontier analysis in the efficiency assessment of universities. *Automation and Remote Control*, 78(5), pp.902-923.

Altamirano-Corro, A. and Peniche-Vera, R. (2014). Measuring the Institutional Efficiency Using DEA and AHP: The Case of a Mexican University. *Journal of Applied Research and Technology*, 12(1), pp.63-71.

Avkiran, N. (2001). Investigating technical and scale efficiencies of Australian Universities through data envelopment analysis. *Socio-Economic Planning Sciences*, 35(1), pp.57-80.

Azadeh, A., Sheikhalishahi, M., and Koushan, M. (2013). An integrated fuzzy DEA–Fuzzy simulation approach for optimization of operator allocation with learning effects in multi products CMS. *Applied Mathematical Modelling*, 37(24), pp.9922-9933.

Azadi, M., Jafarian, M., Farzipoor Saen, R. and Mirhedayatian, S. (2015). A new fuzzy DEA model for evaluation of efficiency and effectiveness of suppliers in sustainable supply chain management context. *Computers & Operations Research*, 54, pp.274-285.

Aziz, N., Janor, R. and Mahadi, R. (2013). Comparative Departmental Efficiency Analysis within a University: A DEA Approach. *Procedia - Social and Behavioral Sciences*, 90, pp.540-548.

Barra, C., Lagravinese, R. and Zotti, R. (2018). Does econometric methodology matter to rank universities? An analysis of Italian higher education system. *Socio-Economic Planning Sciences*, 62, pp.104-120.

Charnes, A., Cooper, W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2(6), pp.429-444.

Cooper, W., Seiford, L., Tone, K. and Zhu, J. (2007). Some models and measures for evaluating performances with DEA: past accomplishments and future prospects. *Journal of Productivity Analysis*, 28(3), pp.151-163.

Cudo.ouac.on.ca. (2019). *CUDO - Homepage*. [online] Available at: https://cudo.ouac.on.ca/, Accessed Jan. 14, 2020.

Dobrota, M., Bulajic, M., Bornmann, L. and Jeremic, V. (2015). A new approach to the QS university ranking using the composite I-distance indicator: Uncertainty and sensitivity analyses. *Journal of the Association for Information Science and Technology*, 67(1), pp.200-211.

Duan, S. (2019). Measuring university efficiency. Benchmarking: *An International Journal*, 26(4), pp.1161-1173.

Guarino, C., Ridgeway, G., Chun, M., Buddin, R. (2005). Latent Variable Analysis: A New Approach to University Ranking. *Higher Education in Europe*, 30(2), pp.147-165.

Hajiagha, S. H. R., Mahdiraji, H. A., & Tavana, M. (2019). A new bi-level data envelopment analysis model for efficiency measurement and target setting. *Measurement*, *147*, 106877.

Ignatius, J., Ghasemi, M., Zhang, F., Emrouznejad, A. and Hatami-Marbini, A. (2016). Carbon efficiency evaluation: An analytical framework using fuzzy DEA. *European Journal of Operational Research*, 253(2), pp.428-440.

Iparraguirre, J. and Ma, R. (2015). Efficiency in the provision of social care for older people. A three-stage Data Envelopment Analysis using self-reported quality of life. *Socio-Economic Planning Sciences*, 49, pp.33-46.

Jradi, S. and Ruggiero, J., 2019. Stochastic data envelopment analysis: A quantile regression approach to estimate the production frontier. *European Journal of Operational Research*, 278(2), pp.385-393.

Koronakos, G., Sotiros, D. and Despotis, D.K., 2019. Reformulation of Network Data Envelopment Analysis models using a common modeling framework. *European Journal of Operational Research*, 278(2), pp.472-480.

Landete, M., Monge, J. and Ruiz, J. (2017). Robust DEA efficiency scores: A probabilistic/combinatorial approach. *Expert Systems with Applications*, 86, pp.145-154.

Lee, H. and Kim, C. (2014). Benchmarking of service quality with data envelopment analysis. *Expert Systems with Applications*, 41(8), pp.3761-3768.

Leitner, K., Prikoszovits, J., Schaffhauser-Linzatti, M., Stowasser, R. and Wagner, K. (2007). The impact of size and specialization on universities' department performance: A DEA analysis applied to Austrian universities. *Higher Education*, 53(4), pp.517-538.

Liu, S. (2014). Restricting weight flexibility in fuzzy two-stage DEA. *Computers & Industrial Engineering*, 74, pp.149-160.

Macleans. (2019). https://www.macleans.ca/education/university-rankings/university-rankings-2019-canadas-top-comprehensive-schools/, Accessed Jan. 14, 2020.

Mardani, A., Zavadskas, E.K., Streimikiene, D., Jusoh, A. and Khoshnoudi, M., 2017. A comprehensive review of data envelopment analysis (DEA) approach in energy efficiency. *Renewable and Sustainable Energy Reviews*, 70, pp.1298-1322.

McMillan, M. and Chan, W. (2006). University Efficiency: A Comparison and Consolidation of Results from Stochastic and Non-stochastic Methods. *Education Economics*, 14(1), pp.1-30.

McMillan, M.L. and Datta, D., 1998. The relative efficiencies of Canadian universities: a DEA perspective. *Canadian Public Policy/Analyse de Politiques*, pp.485-511.

Moslemi, S., Izadbakhsh, H. and Zarinbal, M. (2019). A new reliable performance evaluation model: IFB-IER–DEA. OPSEARCH, 56(1), pp.14-31.

Nayar, P. and Ozcan, Y. (2008). Data Envelopment Analysis Comparison of Hospital Efficiency and Quality. *Journal of Medical Systems*, 32(3), pp.193-199.

Omrani, H. (2013). Common weights data envelopment analysis with uncertain data: A robust optimization approach. *Computers & Industrial Engineering*, 66(4), pp.1163-1170.

Ontario's Universities. (2019). Jobs and Skills - Ontario's Universities. [online] Available at: https://ontariosuniversities.ca/issues-priorities/jobs-and-skills, Accessed Jan. 14, 2020.

Sadjadi, S., Omrani, H., Abdollahzadeh, S., Alinaghian, M. and Mohammadi, H. (2011). A robust super-efficiency data envelopment analysis model for ranking of provincial gas companies in Iran. *Expert Systems with Applications*, 38(9), pp.10875-10881.

Sagarra, M., Mar-Molinero, C. and Agasisti, T., 2017. Exploring the efficiency of Mexican universities: Integrating data envelopment analysis and multidimensional scaling. *Omega*, *67*, pp.123-133.

Sarkar, S. (2017). A modified multiplier model of BCC DEA to determine cost-based efficiency. Benchmarking: *An International Journal*, 24(6), pp.1508-1522.

Shang, J., Wang, F. and Hung, W. (2010). A stochastic DEA study of hotel efficiency. *Applied Economics*, 42(19), pp.2505-2518.

Sueyoshi, T. (2000). Stochastic DEA for restructures strategy: an application to a Japanese petroleum company. *Omega*, 28(4), pp.385-398.

Times Higher Education (THE). (2019). World University Rankings. [online] Available at: https://www.timeshighereducation.com/world-university-rankings, Accessed Jan. 14, 2020.

Top Universities. (2019). QS Stars University Ratings. [online] Available at: https://www.topuniversities.com/qs, Accessed Jan. 14, 2020.

University Affairs. (2019). Five challenges facing Canada's universities | University Affairs. [online] Available at: https://www.universityaffairs.ca/opinion/margin-notes/five-challenges-facing-canadas-universities, Accessed Jan. 14, 2020.

Visbal-Cadavid, D., Martínez-Gómez, M. and Guijarro, F. (2017). Assessing the Efficiency of Public Universities through DEA. A Case Study. *Sustainability*, 9(8), p.1416.

Wen, M., Zu, T., Guo, M., Kang, R. and Yang, Y. (2018). Optimization of Spare Parts Varieties Based on Stochastic DEA Model. IEEE Access, 6, pp.22174-22183.

Yang, G., Fukuyama, H., and Song, Y. (2018). Measuring the inefficiency of Chinese research universities based on a two-stage network DEA model. *Journal of Informetrics*, 12(1), pp.10-30.

Zhang, X. and Shi, W. (2019). Research about the university teaching performance evaluation under the data envelopment method. *Cognitive Systems Research*, 56, pp.108-115.