

11-27-2009

# Introducing CGE Models to the Classroom Using EXCEL

Amy Peng  
*Ryerson University*

Follow this and additional works at: <http://digitalcommons.ryerson.ca/economics>



Part of the [Economics Commons](#)

---

## Recommended Citation

Peng, Amy, "Introducing CGE Models to the Classroom Using EXCEL" (2009). *Economics Publications and Research*. Paper 37.  
<http://digitalcommons.ryerson.ca/economics/37>

This Working Paper is brought to you for free and open access by the Economics at Digital Commons @ Ryerson. It has been accepted for inclusion in Economics Publications and Research by an authorized administrator of Digital Commons @ Ryerson. For more information, please contact [bcameron@ryerson.ca](mailto:bcameron@ryerson.ca).

# Introducing CGE Models to the Classroom Using EXCEL

Amy Peng<sup>1</sup>

Ryerson University

November 27, 2009

<sup>1</sup>*Contact:* Department of Economics, Ryerson University, 350 Victoria Street, Toronto, Ontario, Canada, M5B 2K3. Phone: (416)979-5000 ext. 4795. Email: [apeng@ryerson.ca](mailto:apeng@ryerson.ca)

## **Abstract**

This paper demonstrates how simple general equilibrium models can be solved with the help of Microsoft Excel. Two different general equilibrium models for tax incidence analysis are used as illustrative examples. The methods presented here are intended to be beneficial to both students and teachers working with general equilibrium theory in the classroom and can easily be extended to various policy analysis term projects. The techniques presented here are simple and effective tools for inclusion in any student's toolkit.

Key words: Excel, Solver, General Equilibrium, Optimization, Newton's Method

JEL classification: A22, A23, C61, D58

The history of general equilibrium (GE) theory can be traced back more than 100 years to the work of Leon Walras (1874). However, only recently have computable general equilibrium (CGE) models become widely used in policy applications<sup>1</sup>. Since GE models are often thought to be too complicated to implement in a classroom setting and GE related computing packages are relatively expensive with significant learning curves, only a handful of instructors have taken CGE models into the classroom (for example, Professor Peter Wilcoxon at Syracuse University). This paper demonstrates how to build and solve simple CGE models using standard features in EXCEL.

It is important to point out that there is growing need to expose students to applied GE analysis. These models are not only an important part of applied economic research, they have been widely adopted by different levels of federal and provincial/state governments and the private sector as a useful tool for tax and environmental policy analysis. I have surveyed a number of economics instructors in terms of their teaching methods related to GE models. For those who have incorporated some kind of general equilibrium theory into their intermediate/advanced Micro (mostly undergraduate) courses, none of them have considered showing applications of the theory in simplified but practical settings. It is understandable that students who lack analytical math skills cannot easily understand the complicated structure of such models, the calibration process and the solution techniques used in applied GE models. However, teaching the abstract form of GE models, and existence proofs, involves even deeper mathematical sophistication and is of little practical relevance to non-specialists.

---

<sup>1</sup>A brief historical survey of developments in GE theory and CGE applications including Jones (1965), Scarf (1969), Shoven and Whalley (1984, 1992) may be of interest to students.

When faced with students with little programming experience (in software such as Matlab or SAS), instructors feel that it is necessary to avoid introducing computable solutions to economic problems, especially in the case of general equilibrium analysis, which typically only receives two or three weeks of coverage and usually at the end of the semester. If we could adopt an easy-to-use platform, such as EXCEL, to show how to construct simple GE models for policy analysis, it would give students an opportunity to fill in the gap between the formal theory and numerical reality of practical general equilibrium analysis.

Teaching economics with EXCEL has received growing attention. For example, studies such as Barreto (2001), Cheng and Fan (2003), Tohamy and Mixon (2003) and Naevdal (2003) are important examples of this trend<sup>2</sup>. The models I choose to demonstrate in this paper are 1) an example in the review by Shoven and Whalley (Journal of Economics Literature, 1984) and 2) a teaching example by Wilcoxon on the implementation of the Harberger's (1962) Model <sup>3</sup>. The first example is very straightforward and students (readers) who are familiar with basic spreadsheet calculations and the EXCEL tool "Solver" can easily implement it. The second example demonstrates the use of Newton's method as a solution technique in computable general equilibrium models which involve some simple Macro/VBA programming. I choose these two studies based on two considerations<sup>4</sup>: 1) both studies are well-known and provide benchmarks upon which our results can easily be compared; 2) I choose the tax policy area since it is easy for students to relate to and we can draw important

---

<sup>2</sup>A list of papers using EXCEL to teach economics can be found at <http://econltsn.ilrt.bris.ac.uk/advice/spreadsheets.htm>

<sup>3</sup><http://wilcoxon.cp.maxwell.syr.edu/pages/371.html>

<sup>4</sup>Although, this framework can be easily extended to more complicated examples of computable general equilibrium models, such as the examples given in Kehoe and Kehoe (1994)

policy implications from the results.

The methods I present here are used in teaching a math review class for incoming master's students majoring in economics. The math review course is a preliminary course that is intended to acquaint students with some mathematical and economic methodologies typically used in graduate teaching. I usually spend a 3-hour lecture (in a computer lab) showing students the general framework of the CGE model and provide some spreadsheet examples. At the end of the class, quite a few of the students have indicated that this method is very effective and greatly improved their understanding of GE theory since they were given an opportunity for hands-on work with the model changing different parameter values and finding numerical solutions. For instructors who are interested in teaching GE theory and its applications, I also suggest a term paper be assigned to the students following the lectures. This will offer students an opportunity to improve their analytical skills.

## 1 CES Technology Model

The CES technology model corresponds to the original Arrow-Debreu model, elaborated on in Arrow and Hahn (1971) and then introduced as an applied study by Shoven and Whalley in their publication "Applied General - Equilibrium Models of Taxation and International Trade: an Introduction and Survey" in the **Journal of Economics Literature** (Vol. XXII, 1984). In this economy, we have two final goods (manufacturing and nonmanufacturing), two factors of production (capital and labor) and two types of consumer (rich, capital owners, and poor laborers). Consumers maximize their CES utility function

$$U^c = \left[ \sum_{i=1}^2 a_i^{\frac{1}{\sigma_c}} \cdot X_i^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}} \quad (1)$$

subject to household budget constraints

$$P_1 X_1^c + P_2 X_2^c \leq P_L L^c + P_K K^c = I^c \quad (2)$$

and the firms minimize their production costs subject to their CES production technology

$$Q_i = \phi_i \left[ \delta_i L_i^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \delta_i) K_i^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \quad (3)$$

A list of parameters is given by

$\alpha_i^c$  share parameters  $i = 1, 2$

$\sigma_c$  substitution elasticity

$\phi_i$  scale parameter,  $i = 1, 2$

$\delta_i$  distribution parameter,  $i = 1, 2$

$\sigma_i$  elasticity of factor substitution,  $i = 1, 2$

$I^c$  income of consumer  $c$

$P_i$  price of good  $i$

$P_{L,K}$  price of labor and capital respectively

$L^c, K^c$  consumer's endowment of labor and capital

Utility maximization subject to budget constraints given the following product demand functions

$$X_i^c = \frac{\alpha_i^c I^c}{P_i^{\sigma_c} (\alpha_1^c P_1^{(1-\sigma_c)} + \alpha_2^c P_2^{(1-\sigma_c)})} \quad (4)$$

similarly, factor demands can be derived from cost minimization:

$$L_i = \phi_i^{-1} Q_i \left[ \delta_i + (1 - \delta_i) \left[ \frac{\delta_i P_K}{(1 - \delta_i) P_L} \right]^{(1-\sigma_i)} \right]^{\frac{\sigma_i}{1-\sigma_i}} \quad (5)$$

and

$$K_i = \phi_i^{-1} Q_i \left[ (1 - \delta_i) + \delta_i \left[ \frac{(1 - \delta_i) P_L}{\delta_i P_K} \right]^{(1-\sigma_i)} \right]^{\frac{\sigma_i}{1-\sigma_i}} \quad (6)$$

The equilibrium conditions in the models are found by choosing all prices  $\{P_1, P_2, P_k, P_L\}$  and quantities  $\{X_1^1, X_2^1, X_1^2, X_2^2, K_1, K_2, L_1, L_2\}$  so that (1) Factor Markets clear

$$K_1 + K_2 = \overline{K} \quad (7)$$

$$L_1 + L_2 = \overline{L} \quad (8)$$

(2) Goods Markets clear

$$X_1^1 + X_1^2 = Q_1 \quad (9)$$

$$X_2^1 + X_2^2 = Q_2 \quad (10)$$

and (3) Zero Profit Conditions are met:

$$P_K K_1 + P_L L_1 = P_1 Q_1 \quad (11)$$

$$P_K K_2 + P_L L_2 = P_2 Q_2 \quad (12)$$

In actuality, only the relative price of capital needs to be defined as the product prices come from the zero profit conditions. In order to use EXCEL's "Solver" to find an equilibrium solution to the model we consider this problem as an optimization program. We can minimize the sum of three market clearing conditions (Factor and Goods market demand minus supply equals to zero as well as zero profit condition) by changing our choices of prices and quantities. The first step is to set up the appropriate parameter values. In order to compare my results, I use the same parameter values as in Shoven and Whalley (1984, table I, p1011). The household and production parameters are set up in the spreadsheet as Table 1

**Insert Table 1 here**

The second step is to set up the minimization problem as in Table 2:

**Insert Table 2 here**



I have organized the minimization problem into three sections: solution variables (A26:B31), market clearing conditions (A33: D45) and the minimization cell (D46). The problem can be interpreted as minimizing the sum of square errors of three market-clearing conditions by choosing solution variables  $\{P_k, Q_1, Q_2\}$ . I choose to select optimal values of  $Q_1$  and  $Q_2$  instead of factor demand and production demand variables specified in equation (1) to (12) to simplify the setting.  $P_L$  is normalized to 1 as in Shoven and Whalley (1984). Since there are 3 variables to solve, I have specified them into three market-clearing equations so that the system is just identified. The supply (or TR) and demand (or TC) are calculated as follows:

**Insert Table 3 here**

Table 3 shows the spreadsheet section which defines the calculation process for the general equilibrium conditions. Formulas are input based on equation (1) to (12). For example, cell 28 specifies the capital demand in equation (5) using all the parameters and variables that I have already set up in Table 1 and 2. (Cell B22, C22 and D22 are specified in Table 1; Cell B29 and B30 are the solution variables specified in Table 2). Factor supply is simply the capital and labor endowments from the households. Product demands are calculated based on equation (4) and the variable  $PI$ ,  $I^c$ ,  $P_1^{\sigma^c}$ , and  $P_2^{\sigma^c}$  are price indices and income functions (specified in equation (2) and (4)) used in the calculation of product demands. Product supplies are given by production amounts  $\{Q_1, Q_2\}$  and Total Revenues and Total Costs are given by equation (11) and (12). In Column I, I sum up the factor demand and product supply for both manufactured and non-manufactured goods, factor supply and product demand for both rich and poor households and total income from the production process.

The final step is to use "Solver" to solve the problem:

**Insert Figure 1 here**

The Target Cell reference is set to cell D26 which represents the minimization equation and the Changing Cells are the solution variables (B27:B31). In Table 2, I have set up the starting values, I choose to set all price variables to 1 as the starting values. If we used up all the capital (25 units) and labor (60 units) in the economy to only produce manufactured goods (based on equation (3)), we can produce approximately 66 units, so I have set the starting value for  $Q_1$  as 66 units and  $Q_2$  equal to 0<sup>5</sup>. I have further specified a constraint in the "Solver" utility so that all the solution variables are greater than or equal to 0.

**Insert Table 4 here**

The exact values after the Solver process finished are given in Table 4. As can be seen, at the optimal solution, all the market-clearing conditions are met. The equilibrium solutions calculated from the "Solver" application are exactly the same as reported in Table 2 in Shoven and Whalley (page1012, 1984). The manufactured goods are selling at a higher price than the non-manufactured goods and the economy will produce more non-manufactured goods. All capital and labor is used up. The "poor" household will receive a higher income than the "rich" household.

To illustrate how a general-equilibrium model can be adapted for policy evaluation, Shoven and Whalley further incorporate a tax policy regime into their baseline model. For a given tax program (it could be a tax imposed on capital, labor or manufactured/non-manufactured product), I need to modify the baseline model to incorporate these changes.

---

<sup>5</sup>We could choose other starting values for the above specified equations. However, the starting values need to be reasonable, otherwise, the system may converge to corner solutions with no production or not coverage at all.

The example shown in Shoven and Whalley's paper is based on a 50 percent tax rate on capital income generated in the manufactured goods sector. I would like to present a more generalized tax-policy model which includes seven tax parameters.

**Insert table 5 here**

$t_1$  and  $t_2$  are taxes imposed on manufactured and non-manufactured products respectively,  $t_L$  and  $t_K$  are taxes imposed on labor and capital,  $\tau$  is a output tax and  $\tau_{k1}$  and  $\tau_{k2}$  are taxes imposed on capital income generated in manufactured and non-manufactured product sectors. I also add a new parameter  $\theta$  in the household parameter section to indicate the distribution of tax revenue<sup>6</sup>. As in Shoven and Whalley, I assume that the rich household receives 40 percent of the tax revenue with the remaining 60 percent going to the poor household.

In order to incorporate the tax policy parameters, I need to modify equations (4) to (6) as follows:

$$X_i^c = \frac{\alpha_i^c I^c}{P_i^{\sigma_c} [\alpha_1^c (t_1 + P_1)^{(1-\sigma_c)} + \alpha_2^c (t_2 + P_2)^{(1-\sigma_c)}]} \quad (4a)$$

and

$$I^c = (1 - t_L) P_L L^c + (1 - t_K) P_K K^c \quad (2a)$$

$$L_i = \phi_i^{-1} Q_i \left[ \delta_i + (1 - \delta_i) \left[ \frac{\delta_i (P_K + \tau_{K_i})}{(1 - \delta_i) P_L} \right]^{(1-\sigma_i)} \right]^{\frac{\sigma_i}{1-\sigma_i}} \quad (5a)$$

and

$$K_i = \phi_i^{-1} Q_i \left[ (1 - \delta_i) + \delta_i \left[ \frac{(1 - \delta_i) P_L}{\delta_i (P_K + \tau_{K_i})} \right]^{(1-\sigma_i)} \right]^{\frac{\sigma_i}{1-\sigma_i}} \quad (6a)$$

---

<sup>6</sup>In this case, the government simply collects and redistributes revenue. In other models, the government may also consume goods and services.

Total revenue and total cost functions change to

$$(1 + \tau_{K_1})P_K K_1 + P_L L_1 = (1 - \tau)P_1 Q_1 \quad (11a)$$

$$(1 + \tau_{K_2})P_K K_2 + P_L L_2 = (1 - \tau)P_2 Q_2 \quad (12a)$$

Given the above parameters, the total tax revenue the government collects is given by the sum of the tax from the consumer  $T^c = \sum(t_i P_i X_i^c) + t_L P_L L^c + t_K P_K K^c$  and from the producer of manufactured and non-manufactured products  $T^s = \sum(\tau P_i Q_i + \tau_{k_i} P_k K_i)$ .

I assume the same values of the parameters and exogenous variables as given in the baseline model. In order to solve the model including the tax policy parameters, I add a new solution variable T to the minimization problem and an additional constraint that T is equal to the total revenue specified above. Table 6 illustrates the modified minimization program.

**Insert Table 6 here**

As can be seen, the tax variables are added in cell B32 and the new constraint is added in row 48. Then we simply run the "Solver" utility again. The new equilibrium solution is shown in Table 7.

**Insert Table 7 here**

Comparing Table 4 and Table 7<sup>7</sup>, one can illustrate how a simple general equilibrium framework can be used in tax policy analysis. After a 50 percent tax on manufacturing capital is imposed, the marginal cost of capital increases, leading to a higher price for manufactured products (1.474 vs. 1.399) and a lower price for non-manufactured products (1.093 vs. 1.105)

---

<sup>7</sup>Our results in Table 7 slightly differ from the numbers reported in Table 3 of Shoven and Whalley (1984, p1013). Some form of calculation or transcription error is present in the Shoven and Whalley results such that the market clearing conditions are violated.

and the price of capital after tax falls (1.153 vs. 1.373). In the household sector, the rich household, which owns capital, receives lower income so its demand for both manufactured and non-manufactured goods decreases. The poor household receives higher transfers of tax revenue from the government, its income rises and demand for both manufactured and non-manufactured goods increases. Due to a lower price and a lower demand for manufactured goods, total revenue from manufactured goods also falls and only 2.428 worth of taxes are collected<sup>8</sup>.

A frequent question policy makers ask is whether or not a proposed policy is welfare improving. We can utilize widely employed measures to follow up on the above analysis. The measures used here are Hicksian compensating variation and equivalent variation (CV and EV) associated with the household utilities and incomes before and after the tax policy at their equilibrium solutions. CV is given by  $\frac{U^A - U^B}{U^A} I^A$ , superscripts  $A$  and  $B$  denote "After" and "Before" the policy changes. CV tells how much income is lost or gained compared to utility level before the change. EV is given by  $\frac{U^A - U^B}{U^B} I^B$ , so we use the income and utility level before the change as the base value. For a tax policy to be welfare improving, we would observe CV and EV having a positive sign. The result of the simple welfare analysis is reported in Table 8. As can be seen, although the capital tax has increased the income of the poor household, the total effect on welfare is a loss. This is due to the fact that this tax policy cannot raise enough tax revenue to distribute to the poor at the same time, it has also lowered total production of manufactured goods in the economy, which in turn, substantially affects income and the utility of the rich.

---

<sup>8</sup>Naively, a student may think that a 50 percent capital tax will generate half of the capital income in the baseline case ( $0.5 \times 1.373 \times 6.212 = 4.265$ ). However, only 2.428 is collected as tax revenue. This is an opportunity for the class to discuss distortions introduced by the tax system.

### **Insert Table 8 here**

After introducing the model, we can further ask students to experiment with the model by changing parameters and initiate group discussions<sup>9</sup>. For example, if we change the endowment of capital and labor for the poor and the rich, or the elasticity of substitution of the two products, the corresponding equilibrium solution will change and will result in different welfare values. We also can ask students to change the tax parameters to conduct further policy analysis. For example, instead of a 50 percent capital tax on the manufactured product, students can simply impose the 50 percent tax on capital used in both manufactured and non-manufactured products, or on output or on the manufactured product itself. They can see easily if taxes are imposed on capital used to produce the non-manufactured product, it will increase tax revenue, but still lead to welfare losses. However, a 50 percent output tax can increase the price of both products substantially, raise tax revenue substantially and through the transfer program, increase the income of both rich and poor. Although there is no welfare loss at the society level, poor households can afford smaller amounts of manufactured and non-manufactured products and their utilities fall. We can easily separate students into different groups, and ask them to discuss the pros and cons of different tax policies and ask them to provide comments and suggestions to improve the design of the tax system. Therefore, a simple CGE model is a very effective teaching tool. If we allow students to engage in constructing and experimenting with the model, instead of lecturing based on mathematical equations, it will enhance their analytical abilities especially as it relates what they have learned to a more practical setting.

---

<sup>9</sup>All of the results for the following discussion questions are available upon request.

## 2 The Harberger Model

The Harberger model was originally developed by Arnold C. Harberger (1962, the Journal of Political Economy). The specification I am using here is based on the computable version developed by Peter J. Wilcoxon (2005, Syracuse University). The documentation for this model is available at <http://wilcoxon.cp.maxwell.syr.edu/pages/2141/harberger-doc.pdf> as maintained by Professor Wilcoxon. The Harberger model is very similar in setting as the CES model in the previous section. I still use a CES production technology in two business sectors X and Y, X is the capital intensive sector and Y is the labor intensive sector. The cost of capital may vary by sector ( $r$ , the rental rate of the capital, is the same for both sectors apart from the fact that the capital tax rates may be different for different sectors). Households also have CES utility functions, but instead of two, we have four types of households A, B, C and D. A and B are "rich" households which own both labor and capital. C and D are "poor" households which own only labor. Four types of households have different preferences: A and C have preferences for the capital intensive product X, while B and D have preferences for the labor intensive product Y. Government, the fifth household "G" can impose different taxes : a tax on capital, output or income with the total revenue given by the sum of the taxes collected from different sectors. In this model, the government uses tax revenue to purchase goods for its own consumption. In equilibrium, we choose appropriate prices and quantities so that factor markets and product markets clear.

Since the basic set up of the model is very similar to the CES model above, I will not waste space and the reader's time to explain how to fill in the spreadsheet with all the parameters, variables and formulas again. The detailed spreadsheet and its explanation can be found in Appendix 1. It demonstrates that, we can choose to solve for the price of

capital (rental rate,  $r$ ) and the quantity of production from sector X ( $q_x$ ) and sector Y ( $q_y$ ) by minimizing the sum of square errors of three market conditions: capital market equilibrium and two-product market equilibrium. This model can also be easily solved using "Solver" as in the previous case. However, for a more advanced level class, students may be interested to see how the equilibrium can be reached through alternative methods. This model can be used as a perfect example to demonstrate how to use Newton's method to compute the equilibrium solutions.

The general concept of Newton's method is easy to illustrate:

**Insert Figure 2 here**

Assume a function  $f(x)$  actually has a root. In order to find the root of  $f(x)$ , construct a tangent line through the point  $(X_0, f(X_0))$  with the slope  $f'(X_0)$ . This line intercepts the  $x$ -axis at the point  $X_1$  which may be a better approximation to the root than  $X_0$ . We iterate according to the relation  $X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$  until convergence occurs. In the case of the Harberger model, there are three variables to solve for, the rental rate of capital,  $r$ , and the outputs  $q_x$  and  $q_y$ . Unlike the univariate case above,  $x$  is the vector of  $(r, q_x, q_y)'$  and  $f(x)$  is a vector valued function of equilibrium conditions. The derivative is replaced by its multivariate analogy, the Jacobian. The set up of the model is illustrated in Table 9. I used a centered difference formula to approximate the partial derivatives (cell B51:H53) with a step size of 0.02. The partial derivative estimates (H51:H53) are copied into the corresponding columns of the Jacobian matrix (B57:D59). I then use an EXCEL matrix function to invert the Jacobian (B64:D66). The vector  $dx$  is calculated based on the rule  $dx = -J^{-1}(X_n)f(X_n)$  (see cells F64:F66). Finally  $X_{n+1}$  is updated using  $X_{n+1} = X_n + \phi dx$



(see cells H64:H66)<sup>10</sup>. The above procedure completes one Jacobian iteration step. In order to automate the iteration step, a simple VBA Macro program is used to construct the Jacobian matrix and shuffle numbers between designated cells. The detailed program can be found in Appendix 2.

### **Insert Table 9 here**

An iteration step starts with a Macro key combination of CTRL-j, students will be able to observe how the solutions are reached through each iteration. With the starting value set to  $(1, 100, 100)'$ , the program converges within 10-15 iteration steps. After setting up the model, we can conduct different kinds of policy analysis to compare any baseline model and alternative tax schemes (please refer to appendix 1, cells B6:D11). I chose a similar tax policy as in section 2 - imposing a 50 percent capital tax on the capital intensive industry X, however in this case, the government will keep all of the tax revenue and use it for its own consumption. The solution is obtained by taking Jacobian steps, by pressing CTRL-j, until convergence occurs followed by CTRL-s to copy selected statistics to the analysis page. Once the base and alternative case have been solved (and statistics copied to the analysis page) using CTRL-r will calculate and report various welfare measures. The summary of the experimental results are calculated by VBA Macro and reported in Table 10<sup>11</sup>.

### **Insert Table 10 here**

The baseline case (no capital tax) is compared to the alternative case (50 percent capital tax). The first section of the results shows the price and quantity variables before and after imposing the tax. As can be seen, the price of the capital-intensive product X is increased while the price of the labor-intensive product Y is decreased, in the meantime, output of X

---

<sup>10</sup>  $\phi$  is used to smooth convergence by taking smaller steps than would otherwise be called for.

<sup>11</sup> The detailed program can be found in Appendix 3.

is falling while that of Y is rising. Row 8 to Row 10 show three different kinds of price index measures, Paasche, Laspeyres and Fisher price indices. All of them indicate rising inflation after the tax change. Based on the total output and price index, we can calculate how real GDP is affected by the tax policy change. Row 19 to 21 report that although nominal GDP is increased from the baseline model to the alternative case, there is a loss of real GDP due to the rising price of capital intensive product X. Lastly, I also compute the welfare change for both cases. In order to take into consideration the effect of the price and output changes, I choose a different approach from the previous section. Equivalent Variation (EV) is calculated as

$$EV_i = [\alpha(P_x^0)^{1-\sigma} + (1-\alpha)(P_y^0)^{1-\sigma}]^{-\frac{1}{\sigma-1}} \Delta u_i, \text{ superscript 0 refers to the base case prices,}$$

and Compensating Variation (CV) is calculated as

$$CV_i = [\alpha(P_x^1)^{1-\sigma} + (1-\alpha)(P_y^1)^{1-\sigma}]^{-\frac{1}{\sigma-1}} \Delta u_i, \text{ superscript 1 refers to the alternate case prices.}$$

Column B and C report the utility changes from the baseline case to the alternative case. Imposing the capital tax has lowered the utility level of households A, B and C. The utility loss of A and B comes from their income loss resulting from the capital tax and an additional utility loss for A and C comes from their consumption loss on the capital intensive product X. It is not hard to show that both EV and CV have a negative sign for households A, B and C but A has the biggest welfare loss. The only household type that benefits from the tax change is household D (as well as the government sector.) However, a deadweight loss incurs since the sum of the EVs or CVs is negative which indicates that there is an overall deadweight loss resulting from this tax change.

To further this analysis, one could ask students to plot out the utility maximization

problem of the households from this excise. Figures 3 and 4 show an example for this question. Given the amount of good X and Y consumed, we can easily calculate the utility and budget constraint based on the parameters chosen above. The solid curves represent the baseline case while the dotted curves represent the alternative case. Since household A and B have the same endowment, their utility functions are subject to the same budget constraint. Given the budget constraint and household preferences, household A will always consume more of X than Y and household B will always consume more Y than X. Similarly, households C and D follow the same pattern. Imposing a capital tax of 50 percent is equivalent to a price increase so the budget constraints rotate down. We can observe that the indifference curves of households A, B and C all shift down while D's shifts upward moderately, illustrating the fact that it is the only household that is better off as a result of this tax change. This is primarily due to the fact that household D does not care as much about the taxable good as the other households do. We can also draw the conclusion that the tax burden does not only affect households which own the taxable factor, it also affects households who have preferences for the goods that depend most on that factor. Tax incidence is more complicated than the statutory incidence would otherwise suggest. This provides another excellent opportunity for class discussion.

### 3 Conclusion

Computable general equilibrium models have received growing attention from policy makers and academic researchers. However, in the past, teaching general equilibrium theory often focussed on a more abstract approach based on difficult mathematical equations and reason-

ing. Such an approach affords students lacking programming skills little chance to experiment with a computable general equilibrium framework and develop improved understanding of what is being taught. This paper provides a simple method to illustrate the use of general equilibrium models in tax policy analysis with the help of EXCEL. Two classic models are introduced and solved with two alternative methods: optimization based on the "Solver" and an iterative solution using a multidimensional Newton's method. Since EXCEL is one of the most widely used applications in the world, and it has an user friendly interface and tools for great graphic presentations, students or researchers will definitely benefit from adopting the method presented here to further their understanding of general equilibrium theory and to conduct simple policy analysis.

## References

- [1] Barreto, Humberto (2001), "Teaching Comparative Statics with Microsoft Excel", Journal of Economic Education, Vol. 32, No. 4, p397.
- [2] Cheng, Wai-yan and C. Fan (2001) "Comparison Study of Different Implementations of Derivative Pricing Models", Journal of Economic Education, Vol. 32, No. 2, p192.
- [3] Harberger, Arnold C. (1962), "The incidence of Corporation Income Tax", Journal of Political Economy, Vol. 70, No. 3, p215-240.
- [4] Jones, Ronald W. (1965), "The Structure of Simple General Equilibrium Models," Journal of Political Economy, Vol. 73, No. 6, p 557-72.

- [5] Kehoe, Patrick J. and Timothy J. Kehoe (1994), "A Primer on Static Applied General Equilibrium Models," Federal Reserve Bank of Minneapolis Quarterly Review, Spring, Vol. 18, No. 1.
- [6] Naevdal, Eric (2003), "Solving Continuous-time Optimal-Control Problems with a Spreadsheet", Journal of Economic Education, Vol. 34, No. 2, p99-121.
- [7] Scarf, Herbert (1969), "An Example of an Algorithm for Calculating General Equilibrium Prices," American Economic Review, September, Vol. 59, No. 4, part 1, p 669-77.
- [8] Shoven, John B. and J. Whalley (1984), "Applied General-Equilibrium Models of Taxation and International Trade", Journal of Economic Literature, Vol. XXI, p1007-1051.
- [9] Shoven, John B. and John Whalley (1992), Applying General Equilibrium, Cambridge University Press.
- [10] Tohamy, Soumaya M. and J. W. Mixon, Jr. (2003), "Lessons from the Specific Factors Model of International trade", Journal of Economic Education, p139-150.
- [11] Wilcoxon, Peter J. (2005), "A Numerical Implementation of the Harberger Model", <http://wilcoxon.cp.maxwell.syr.edu/pages/2141/harberger-doc.pdf>.

Table 1: Household and Production Parameters

|    | A                            | B            | C            | D          | E     | F     |
|----|------------------------------|--------------|--------------|------------|-------|-------|
| 14 | <b>Household Parameters</b>  |              |              |            |       |       |
| 15 | <b>c</b>                     | $\alpha_1^c$ | $\alpha_2^c$ | $\sigma^c$ | $K^c$ | $L^c$ |
| 16 | <b>Rich Households</b>       | 0.5          | 0.5          | 1.5        | 25    | 0     |
| 17 | <b>Poor Households</b>       | 0.3          | 0.7          | 0.75       | 0     | 60    |
| 18 |                              |              |              |            |       |       |
| 19 |                              |              |              |            |       |       |
| 20 | <b>Production Parameters</b> |              |              |            |       |       |
| 21 | <b>i</b>                     | $\phi_i$     | $\delta_i$   | $\sigma_i$ |       |       |
| 22 | <b>Manufacturing</b>         | 1.5          | 0.6          | 2          |       |       |
| 23 | <b>Nonmanufacturing</b>      | 2            | 0.7          | 0.5        |       |       |

Table 2: The Minimization Problem

|    | A                            | B             | C                           | D                            |
|----|------------------------------|---------------|-----------------------------|------------------------------|
| 26 | <b>Sol'n Variables</b>       | <b>Values</b> |                             |                              |
| 27 | $P_1$                        | 1             |                             |                              |
| 28 | $P_2$                        | 1             |                             |                              |
| 29 | $P_K$                        | 1             |                             |                              |
| 30 | $Q_1$                        | 66            |                             |                              |
| 31 | $Q_2$                        | 0             |                             |                              |
| 32 |                              |               |                             |                              |
| 33 | <b>Goods Market Eq'm</b>     |               |                             |                              |
| 34 | <b>i</b>                     | <b>Supply</b> | <b>Demand</b>               | <b>Minimization</b>          |
| 35 | 1                            | =G37          | =I34                        | =B35-C35                     |
| 36 | 2                            | =H37          | =I35                        | =B36-C36                     |
| 37 |                              |               |                             |                              |
| 38 | <b>Factor Market Eq'm</b>    |               |                             |                              |
| 39 | <b>i</b>                     | <b>Supply</b> | <b>Demand</b>               |                              |
| 40 | <b>K</b>                     | =I31          | =I28                        | =B40-C40                     |
| 41 | <b>L</b>                     | =I32          | =I29                        | =B41-C41                     |
| 42 |                              |               |                             |                              |
| 43 | <b>Zero Profit Condition</b> |               |                             |                              |
| 44 | <b>i</b>                     | <b>TR</b>     | <b>TC</b>                   |                              |
| 45 | 1                            | =G39          | =G40                        | =B45-C45                     |
| 46 | <b>Minimization</b>          |               | <b>SSE</b>                  | =SUMPRODUCT(D35:D45,D35:D45) |
| 47 |                              |               | <b>log<sub>10</sub> SSE</b> | =LN(D46)/LN(10)              |

Table 3: Calculating the Market Clearing Conditions

|    | F                              | G                                        | H                                        | I          |
|----|--------------------------------|------------------------------------------|------------------------------------------|------------|
| 26 | <b>Variable</b>                |                                          |                                          |            |
| 27 | <b>Factor Demand</b>           | Manufacturing                            | Nonmanufacturing                         | <b>Sum</b> |
| 28 | $K_i$                          | $=B22^{(-1)}*B30*(C22*((1-C22)*1/C22/B$  | $=B23^{(-1)}*B31*(C23*((1-C23)*1/C23/E$  | $=G28+H28$ |
| 29 | $L_i$                          | $=B22^{(-1)}*B30*(C22+(1-C22)*(C22*B2$   | $=B23^{(-1)}*B31*(C23+(1-C23)*(C23*B2$   | $=G29+H29$ |
| 30 | <b>Factor Supply</b>           | Rich                                     | Poor                                     |            |
| 31 | $K^c$                          | $=E16$                                   | $=E17$                                   | $=G31+H31$ |
| 32 | $L^c$                          | $=E17$                                   | $=F17$                                   | $=G32+H32$ |
| 33 | <b>Product Demand</b>          | Rich                                     | Poor                                     |            |
| 34 | $X_1^c$                        | $=B16*G43/G44/G42$                       | $=B17*H43/H44/H42$                       | $=G34+H34$ |
| 35 | $X_2^c$                        | $=C16*G43/G45/G42$                       | $=C17*H43/H45/H42$                       | $=G35+H35$ |
| 36 | <b>Product Supply</b>          | Manufacturing                            | Nonmanufacturing                         |            |
| 37 | $Q_i$                          | $=B30$                                   | $=B31$                                   |            |
| 38 | <b>Profit Maximization</b>     |                                          |                                          |            |
| 39 | TR                             | $=B27*B30$                               | $=B28*B31$                               |            |
| 40 | TC                             | $=B29*G28+1*G29$                         | $=B29*H28+1*H29$                         |            |
| 41 | <b>Other related variables</b> |                                          |                                          |            |
| 42 | PI                             | $=B16*B27^{(1-D16)}+C16*B28^{(1-D16)}$   | $=B17*B27^{(1-D17)}+C17*B28^{(1-D17)}$   |            |
| 43 | $I^c$                          | $=1*F16+B29*E16$                         | $=1*F17+B29*E17$                         | $=G43+H43$ |
| 44 | $P_1^{\sigma_c}$               | $=B27^{D16}$                             | $=B27^{D17}$                             |            |
| 45 | $P_2^{\sigma_c}$               | $=B28^{D16}$                             | $=B28^{D17}$                             |            |
| 46 | $U^c$                          | $=(B16^{(1/D16)}*G34^{((D16-1)/D16)}+C1$ | $=(B17^{(1/D17)}*H34^{((D17-1)/D17)}+C1$ |            |
| 47 |                                |                                          |                                          |            |



Table 4: Equilibrium Solution for the CES Technology Model

|    | A                             | B             | C                           | D                   | E | F                              | G             | H                | I          |
|----|-------------------------------|---------------|-----------------------------|---------------------|---|--------------------------------|---------------|------------------|------------|
| 26 | <b>Sol'n Variables</b>        | <b>Values</b> |                             |                     |   | <b>Variable</b>                |               |                  |            |
| 27 | $P_1$                         | 1.399         |                             |                     |   | <b>Factor Demand</b>           | Manufacturing | Nonmanufacturing | <b>Sum</b> |
| 28 | $P_2$                         | 1.093         |                             |                     |   | $K_i$                          | 6.212         | 18.788           | 25.000     |
| 29 | $P_K$                         | 1.373         |                             |                     |   | $L_i$                          | 26.366        | 33.634           | 60.000     |
| 30 | $Q_1$                         | 24.943        |                             |                     |   | <b>Factor Supply</b>           | Rich          | Poor             |            |
| 31 | $Q_2$                         | 54.378        |                             |                     |   | $K^c$                          | 25.000        | 0.000            | 25.000     |
| 32 |                               |               |                             |                     |   | $L^c$                          | 0.000         | 60.000           | 60.000     |
| 33 | <b>Goods Market Eq'm</b>      |               |                             |                     |   | <b>Product Demand</b>          | Rich          | Poor             |            |
| 34 | $i$                           | <b>Supply</b> | <b>Demand</b>               | <b>Minimization</b> |   | $X_1^c$                        | 11.515        | 13.428           | 24.942     |
| 35 | 1                             | 24.943        | 24.942                      | 0.000               |   | $X_2^c$                        | 16.674        | 37.704           | 54.378     |
| 36 | 2                             | 54.378        | 54.378                      | 0.000               |   | <b>Product Supply</b>          | Manufacturing | Nonmanufacturing |            |
| 37 |                               |               |                             |                     |   | $Q_i$                          | 24.943        | 54.378           |            |
| 38 | <b>Factor Market Eq'm</b>     |               |                             |                     |   | <b>Profit Maximization</b>     |               |                  |            |
| 39 | $i$                           | <b>Supply</b> | <b>Demand</b>               |                     |   | <b>TR</b>                      | 34.897        | 59.439           |            |
| 40 | $K$                           | 25.000        | 25.000                      | 0.000               |   | <b>TC</b>                      | 34.897        | 59.439           |            |
| 41 | $L$                           | 60.000        | 60.000                      | 0.000               |   | <b>Other related variables</b> |               |                  |            |
| 42 |                               |               |                             |                     |   | $PI$                           | 0.901         | 1.042            |            |
| 43 | <b>Zero Profit Conditions</b> |               |                             |                     |   | $I^c$                          | 34.337        | 60.000           | 94.337     |
| 44 | $i$                           | <b>TR</b>     | <b>TC</b>                   |                     |   | $P_1^{\sigma c}$               | 1.655         | 1.286            |            |
| 45 | 1                             | 34.897        | 34.897                      | 0.000               |   | $P_2^{\sigma c}$               | 1.143         | 1.069            |            |
| 46 | <b>Minimization</b>           |               | <b>SSE</b>                  | 2.30391E-09         |   | $U^c$                          | 27.872        | 50.891           |            |
| 47 |                               |               | <b>log<sub>10</sub> SSE</b> | -8.637534191        |   |                                |               |                  |            |

Table 5: Tax Parameters

|    | A                           | B            | C            | D        | E          | F     | G     |
|----|-----------------------------|--------------|--------------|----------|------------|-------|-------|
| 4  | <b>Tax Parameters</b>       | <b>Value</b> |              |          |            |       |       |
| 5  | $t_1$                       | 0            |              |          |            |       |       |
| 6  | $t_2$                       | 0            |              |          |            |       |       |
| 7  | $t_L$                       | 0            |              |          |            |       |       |
| 8  | $t_K$                       | 0            |              |          |            |       |       |
| 9  | $\tau$                      | 0            |              |          |            |       |       |
| 10 | $\tau_{K1}$                 | 0.5          |              |          |            |       |       |
| 11 | $\tau_{K2}$                 | 0            |              |          |            |       |       |
| 12 |                             |              |              |          |            |       |       |
| 13 |                             |              |              |          |            |       |       |
| 14 | <b>Household Parameters</b> |              |              |          |            |       |       |
| 15 | <b>c</b>                    | $\alpha_1^c$ | $\alpha_2^c$ | $\theta$ | $\sigma^c$ | $K^c$ | $L^c$ |
| 16 | <b>Rich Households</b>      | 0.5          | 0.5          | 0.4      | 1.5        | 25    | 0     |
| 17 | <b>Poor Households</b>      | 0.3          | 0.7          | 0.6      | 0.75       | 0     | 60    |

Table 6: Minimization Problem with 50 percent Capital Tax

|    | A                             | B             | C                           | D                            |
|----|-------------------------------|---------------|-----------------------------|------------------------------|
| 26 | <b>Sol'n Variables</b>        | <b>Values</b> |                             |                              |
| 27 | $P_1$                         | 1             |                             |                              |
| 28 | $P_2$                         | 1             |                             |                              |
| 29 | $P_K$                         | 1             |                             |                              |
| 30 | $Q_1$                         | 66            |                             |                              |
| 31 | $Q_2$                         | 0             |                             |                              |
| 32 | $T$                           | 1             |                             |                              |
| 33 |                               |               |                             |                              |
| 34 | <b>Goods Market Eq'm</b>      |               |                             |                              |
| 35 | <b>i</b>                      | <b>Supply</b> | <b>Demand</b>               | <b>Minimization</b>          |
| 36 | 1                             | =G37          | =I34                        | =B36-C36                     |
| 37 | 2                             | =H37          | =I35                        | =B37-C37                     |
| 38 |                               |               |                             |                              |
| 39 | <b>Factor Market Eq'm</b>     |               |                             |                              |
| 40 | <b>i</b>                      | <b>Supply</b> | <b>Demand</b>               |                              |
| 41 | $K$                           | =I31          | =I28                        | =B41-C41                     |
| 42 | $L$                           | =I32          | =I29                        | =B42-C42                     |
| 43 |                               |               |                             |                              |
| 44 | <b>Zero Profit Conditions</b> |               |                             |                              |
| 45 | <b>i</b>                      | <b>TR</b>     | <b>TC</b>                   |                              |
| 46 | 1                             | =G39          | =G40                        | =B46-C46                     |
| 47 |                               |               |                             |                              |
| 48 | <b>Tax</b>                    | =B32          | =I50                        | =B48-C48                     |
| 49 | <b>Minimization</b>           |               | <b>SSE</b>                  | =SUMPRODUCT(D36:D48,D36:D48) |
| 50 |                               |               | <b>log<sub>10</sub> SSE</b> | =LN(D49)/LN(10)              |

Table 7: Equilibrium Solution with 50 percent Capital Tax

|    | A                             | B             | C                           | D                   | E | F                              | G             | H                | I          |
|----|-------------------------------|---------------|-----------------------------|---------------------|---|--------------------------------|---------------|------------------|------------|
| 26 | <b>Sol'n Variables</b>        | <b>Values</b> |                             |                     |   | <b>Variable</b>                |               |                  |            |
| 27 | $P_1$                         | 1.474         |                             |                     |   | <b>Factor Demand</b>           | Manufacturing | Nonmanufacturing | <b>Sum</b> |
| 28 | $P_2$                         | 1.015         |                             |                     |   | $K_i$                          | 4.210         | 20.790           | 25.000     |
| 29 | $P_K$                         | 1.153         |                             |                     |   | $L_i$                          | 25.895        | 34.105           | 60.000     |
| 30 | $Q_1$                         | 22.512        |                             |                     |   | <b>Factor Supply</b>           | Rich          | Poor             |            |
| 31 | $Q_2$                         | 57.216        |                             |                     |   | $K^c$                          | 25.000        | 0.000            | 25.000     |
| 32 | $T$                           | 2.428         |                             |                     |   | $L^c$                          | 0.000         | 60.000           | 60.000     |
| 33 |                               |               |                             |                     |   | <b>Product Demand</b>          | Rich          | Poor             |            |
| 34 | <b>Goods Market Eq'm</b>      |               |                             |                     |   | $X_1^c$                        | 9.171         | 13.341           | 22.512     |
| 35 | $i$                           | <b>Supply</b> | <b>Demand</b>               | <b>Minimization</b> |   | $X_2^c$                        | 16.044        | 41.172           | 57.216     |
| 36 | 1                             | 22.512        | 22.512                      | 0.000               |   | <b>Product Supply</b>          | Manufacturing | Nonmanufacturing |            |
| 37 | 2                             | 57.216        | 57.216                      | 0.000               |   | $Q_i$                          | 22.512        | 57.216           |            |
| 38 |                               |               |                             |                     |   | <b>Profit Maximization</b>     |               |                  |            |
| 39 | <b>Factor Market Eq'm</b>     |               |                             |                     |   | $TR$                           | 33.179        | 58.082           |            |
| 40 | $i$                           | <b>Supply</b> | <b>Demand</b>               |                     |   | $TC$                           | 33.179        | 58.082           |            |
| 41 | $K$                           | 25.000        | 25.000                      | 0.000               |   | <b>Other related variables</b> | Rich          | Poor             |            |
| 42 | $L$                           | 60.000        | 60.000                      | 0.000               |   | $PI$                           | 0.908         | 1.033            |            |
| 43 |                               |               |                             |                     |   | $I^c$                          | 29.804        | 61.457           | 91.261     |
| 44 | <b>Zero Profit Conditions</b> |               |                             |                     |   | $P_1^{\sigma c}$               | 1.789         | 1.338            |            |
| 45 | $i$                           | <b>TR</b>     | <b>TC</b>                   |                     |   | $P_2^{\sigma c}$               | 1.023         | 1.011            |            |
| 46 | 1                             | 33.179        | 33.179                      | 0.000               |   | $U^c$                          | 24.579        | 53.934           |            |
| 47 |                               |               |                             |                     |   | <b>Tax</b>                     |               |                  |            |
| 48 | <b>Tax</b>                    | 2.428         | 2.428                       | 0.000               |   | $T_i$                          | 2.428         | 0.000            | 2.42797    |
| 49 | <b>Minimization</b>           |               | <b>SSE</b>                  | 3.83721E-09         |   | $T^c$                          | 0.000         | 0.000            | 0          |
| 50 |                               |               | <b>log<sub>10</sub> SSE</b> | -8.415984203        |   |                                |               |                  | 2.42797    |

Table 8: Welfare Analysis on the Tax Policy

|    | A                       | B           | C                | D                 |
|----|-------------------------|-------------|------------------|-------------------|
| 1  | <b>Welfare Analysis</b> |             |                  |                   |
| 2  |                         |             |                  |                   |
| 3  |                         | <b>Base</b> | <b>Alternate</b> | <b>Difference</b> |
| 4  | URich                   | 27.872      | 24.579           | <b>-3.293</b>     |
| 5  | UPoor                   | 50.891      | 53.934           | <b>3.043</b>      |
| 6  | IRich                   | 34.337      | 29.804           | <b>-4.532</b>     |
| 7  | IPoor                   | 60.000      | 61.457           | <b>1.457</b>      |
| 8  |                         |             |                  |                   |
| 9  |                         | <b>CV</b>   | <b>EV</b>        |                   |
| 10 | Rich                    | -3.993      | -4.056           |                   |
| 11 | Poor                    | 3.467       | 3.588            |                   |
| 12 |                         | -0.525      | -0.469           |                   |

Table 9: Newton's Method

|    | A               | B                  | C          | D              | E        | F                       | G                        | H                |
|----|-----------------|--------------------|------------|----------------|----------|-------------------------|--------------------------|------------------|
| 43 | Step            | 0.02               |            |                |          |                         |                          |                  |
| 44 |                 |                    |            |                |          |                         |                          |                  |
| 45 | Name            | x                  |            | Condition      | f(x)     |                         |                          |                  |
| 46 | r               | 1                  |            | k-mkt          | =F17-C30 |                         | SSE                      | =SUMPRODUCT(B    |
| 47 | q <sub>x</sub>  | 100                |            | l-mkt          | =G17-B30 |                         | log10 SSE                | =LN(H46)/LN(10)  |
| 48 | q <sub>y</sub>  | 100                |            | q <sub>y</sub> | =B48-H30 |                         |                          |                  |
| 49 |                 |                    |            |                |          |                         |                          |                  |
| 50 | Iteration       | x                  | dx         | x+dx           | f(x+dx)  | x-dx                    | f(x-dx)                  | df               |
| 51 | r               | 1                  | 0          | =B51+C51       |          | =B51-C51                |                          |                  |
| 52 | q <sub>x</sub>  | 100                | 0          | =B52+C52       |          | =B52-C52                |                          |                  |
| 53 | q <sub>y</sub>  | 100                | 0.01       | =B53+C53       |          | =B53-C53                |                          |                  |
| 54 |                 |                    |            |                |          |                         |                          |                  |
| 55 | Jacobian,       |                    |            |                |          |                         |                          |                  |
| 56 |                 | 1                  | 2          | 3              |          | CTRL-j                  | Run a jacobian iteration |                  |
| 57 | 1               |                    |            |                |          | CTRL-s                  | Copy stats               |                  |
| 58 | 2               |                    |            |                |          | CTRL-r                  | Report results           |                  |
| 59 | 3               |                    |            |                |          |                         |                          |                  |
| 60 |                 |                    |            |                |          |                         |                          |                  |
| 61 |                 |                    |            |                |          |                         | $\phi$                   | 0.75             |
| 62 | J <sup>-1</sup> |                    |            |                |          |                         |                          |                  |
| 63 |                 | 1                  | 2          | 3              |          | dx                      |                          | x <sub>n+1</sub> |
| 64 | 1               | =MINVERSE(B57:D59) | =MINVERSE( | =MINVERSE(     |          | =MMULT(B64:D66,E46:E48) |                          | =B51-\$H\$61*F64 |
| 65 | 2               | =MINVERSE(B57:D59) | =MINVERSE( | =MINVERSE(     |          | =MMULT(B64:D66,E46:E48) |                          | =B52-\$H\$61*F65 |
| 66 | 3               | =MINVERSE(B57:D59) | =MINVERSE( | =MINVERSE(     |          | =MMULT(B64:D66,E46:E48) |                          | =B53-\$H\$61*F66 |

Table 10: Equilibrium Solutions for the Harberger Model

|    | A                                      | B           | C                | D         | E         |
|----|----------------------------------------|-------------|------------------|-----------|-----------|
| 1  | <b>Summary of Experimental Results</b> |             |                  |           |           |
| 2  |                                        |             |                  |           |           |
| 3  |                                        | <b>Base</b> | <b>Alternate</b> |           |           |
| 4  | $p_X$                                  | 1.439       | 1.524            |           |           |
| 5  | $p_Y$                                  | 1.710       | 1.628            |           |           |
| 6  | $q_X$                                  | 599.133     | 571.692          |           |           |
| 7  | $q_Y$                                  | 518.989     | 541.012          |           |           |
| 8  | $p_P$                                  | 1.000       | 1.002            |           |           |
| 9  | $p_L$                                  | 1.000       | 1.004            |           |           |
| 10 | $p_F$                                  | 1.000       | 1.003            |           |           |
| 11 |                                        |             |                  |           |           |
| 12 | <b>Household</b>                       | <b>Base</b> | <b>Alternate</b> | <b>EV</b> | <b>CV</b> |
| 13 | <b>A</b>                               | 445.045     | 411.579          | -50.760   | -67.482   |
| 14 | <b>B</b>                               | 415.359     | 400.792          | -23.675   | -11.631   |
| 15 | <b>C</b>                               | 131.860     | 128.673          | -4.835    | -9.789    |
| 16 | <b>D</b>                               | 123.064     | 125.300          | 3.633     | 7.201     |
| 17 | <b>G</b>                               | 0.000       | 45.945           | 72.143    | 72.143    |
| 18 |                                        |             |                  |           |           |
| 19 | <b>Output</b>                          | <b>Base</b> | <b>Alternate</b> |           |           |
| 20 | <b>GDP</b>                             | 1750.053    | 1751.832         |           |           |
| 21 | <b>RGDP</b>                            | 1750.053    | 1746.176         |           |           |
| 22 |                                        |             |                  |           |           |

Figure 1: Set up “Solver”

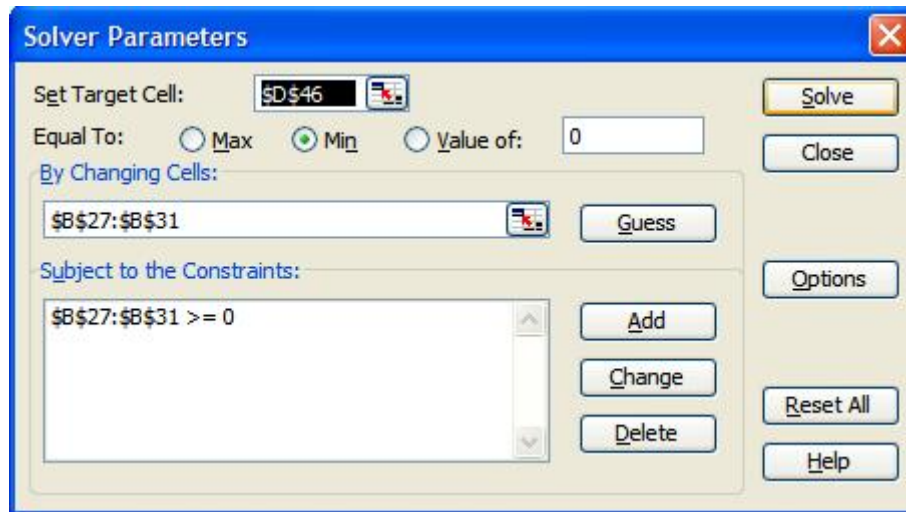


Figure 2: Illustration of Newton's Method

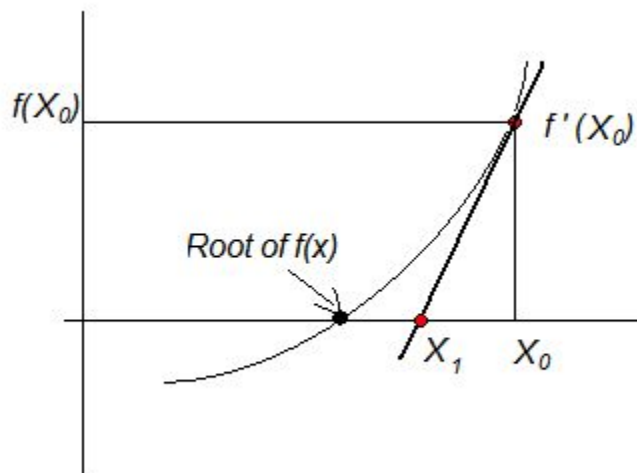




Figure 3: Impact of Capital Tax Applied in Sector X

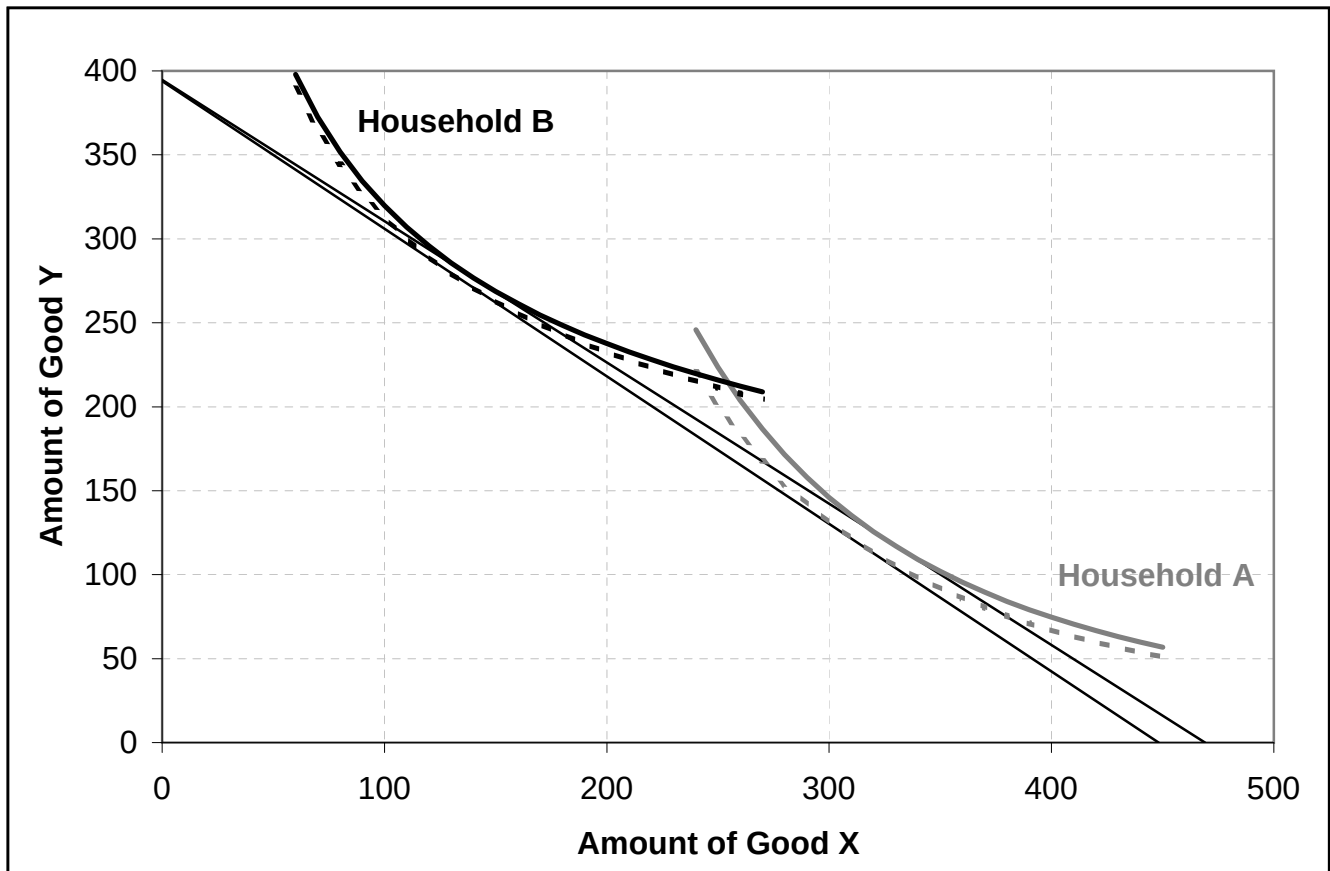
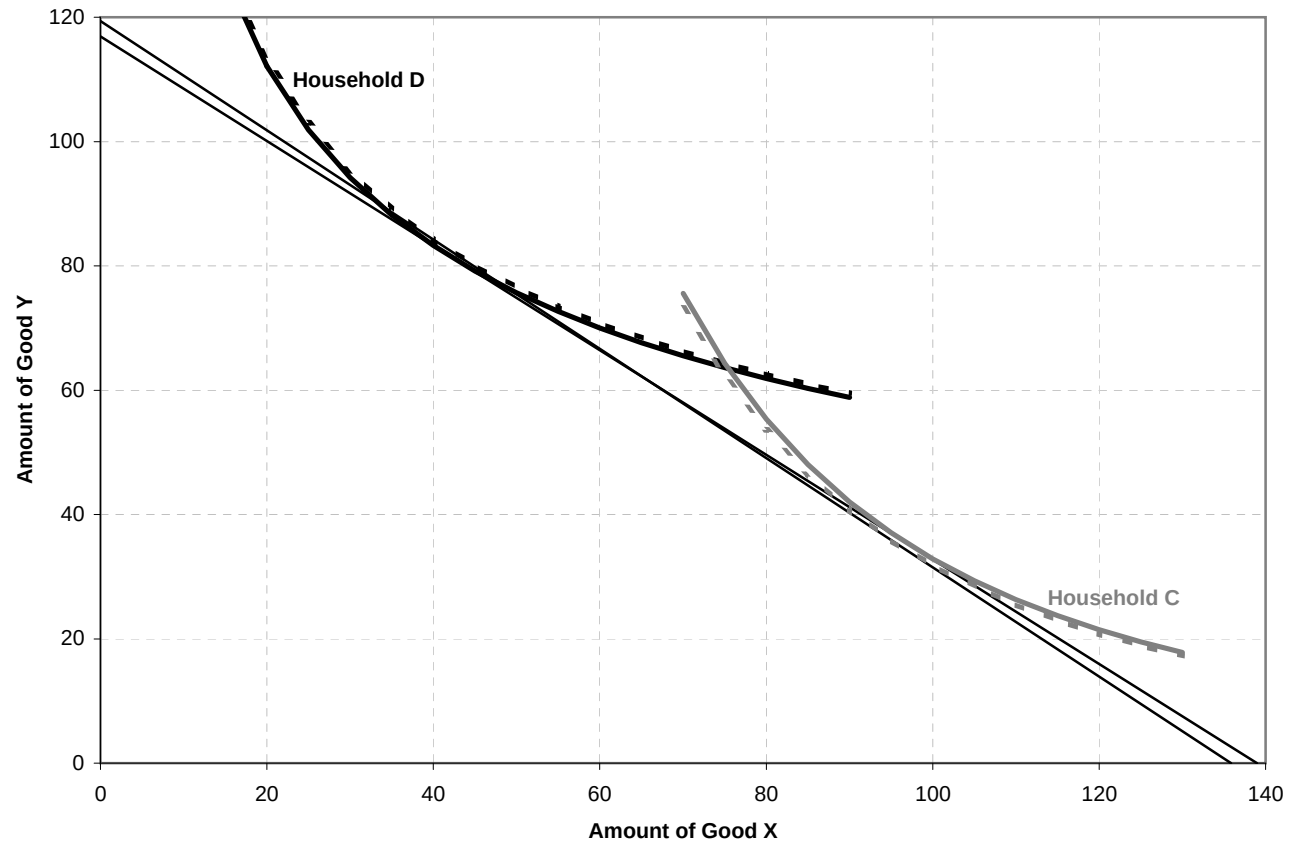


Figure 4: Impact of Capital Tax Applied in Sector X



# Appendix 1

## Parameter and Variable Explanation (The Harberger Model)

### Parameter specifications

|             |                                                                          |
|-------------|--------------------------------------------------------------------------|
| $\tau_{kX}$ | Capital Tax on product in Sector X                                       |
| $\tau_{kY}$ | Capital Tax on product in Sector Y                                       |
| $\tau_w$    | Tax on labour income                                                     |
| $\tau_X$    | Tax on production in Sector X                                            |
| $\tau_Y$    | Tax on production in Sector Y                                            |
| $\tau_m$    | Tax on Capital Income                                                    |
| $\alpha_h$  | Share parameter in household utility function                            |
| $\sigma$    | Substitution elasticity in utility function, identical across households |
| $w$         | wage rate, normalize to 1                                                |
| $k_h$       | Capital endowment for the households                                     |
| $l_h$       | Labour endowment for the households                                      |
| $\sigma_i$  | Substitution elasticity in production function                           |
| $\delta_i$  | Share parameter in production function                                   |

### Variable specifications

- Business Sector ( $i = X, Y$ )

CES Cost function  $C_i = (\delta_i r_i^{1-\sigma_i} + (1 - \delta_i) w_i^{1-\sigma_i})^{\frac{1}{1-\sigma_i}}$

Cost of Capital  $r_i = r + \tau_{ki}$ ,  $r$ , capital rental rate, *solution variable*

Capital Demand  $k_i = \delta_i \left( \frac{C_i}{r_i} \right)^{\sigma_i} q_i$ ,  $q_i$ , output in section  $i$ , *solution variables*

Labour Demand  $l_i = (1 - \delta_i) \left( \frac{C_i}{r_i} \right)^{\sigma_i} q_i$

Price  $p_i = C_i + \tau_i$

- Household Sector ( $i = A, B, C$  and  $D$ )

CES Utility function  $u_i = (\alpha_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}} + (1 - \alpha_i)^{\frac{1}{\sigma}} y_i^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$

Budget Constraint  $m_h = r k_h + w(1 - \tau_w) l_h$

Demand for product in sector X  $x_i = \frac{\alpha_i m_i}{p_{ch}} \left( \frac{p_{ch}}{p_x} \right)^\sigma$

Demand for product in sector Y  $y_i = \frac{(1-\alpha_i) m_i}{p_{ch}} \left( \frac{p_{ch}}{p_y} \right)^\sigma$

Price index  $p_{ch} = (\alpha_i p_x^{1-\sigma} + (1-\alpha_i) p_y^{1-\sigma})^{\frac{1}{1-\sigma}}$

Government Budget  $m_g = \tau_{kx} k_x + \tau_{ky} k_y + \tau_w w \sum l_i + \tau_x q_x + \tau_y q_y$

Appendix 1 (Table): Set up the Harburger Model

|    | A                                                                      | B                               | C                              | D                             | E                       | F                          | G                       | H                       | I                       | J                                 |
|----|------------------------------------------------------------------------|---------------------------------|--------------------------------|-------------------------------|-------------------------|----------------------------|-------------------------|-------------------------|-------------------------|-----------------------------------|
| 1  | <b>Harberger Model</b>                                                 |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 2  | Based on specification provided by Peter Wilcoxon, Syracuse University |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 3  |                                                                        |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 4  | <b>Policy Parameters</b>                                               |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 5  |                                                                        | <b>Used</b>                     | <b>Base</b>                    | <b>Alternate</b>              |                         | <b>Switch</b>              |                         |                         |                         |                                   |
| 6  | $\tau_{kX}$                                                            | =C6+\$F\$6*(D6-C6)              | 0                              | 0.5                           |                         | 1                          |                         |                         |                         |                                   |
| 7  | $\tau_{kY}$                                                            | =C7+\$F\$6*(D7-C7)              | 0                              | 0                             |                         |                            |                         |                         |                         |                                   |
| 8  | $\tau_w$                                                               | =C8+\$F\$6*(D8-C8)              | 0                              | 0                             |                         |                            |                         |                         |                         |                                   |
| 9  | $\tau_x$                                                               | =C9+\$F\$6*(D9-C9)              | 0                              | 0                             |                         |                            |                         |                         |                         |                                   |
| 10 | $\tau_y$                                                               | =C10+\$F\$6*(D10-C10)           | 0                              | 0                             |                         |                            |                         |                         |                         |                                   |
| 11 | $\tau_m$                                                               | =C11+\$F\$6*(D11-C11)           | 0                              | 0                             |                         |                            |                         |                         |                         |                                   |
| 12 |                                                                        |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 13 | <b>Sectors</b>                                                         |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 14 | <b>i</b>                                                               | <b><math>\sigma_i</math></b>    | <b><math>\delta_i</math></b>   | <b><math>c_i</math></b>       | <b><math>r_i</math></b> | <b><math>k_i</math></b>    | <b><math>l_i</math></b> | <b><math>q_i</math></b> | <b><math>p_i</math></b> |                                   |
| 15 | X                                                                      | 0.8                             | 0.4                            | =C15*E15^(1-B15)              | =B\$46+B6               | =C15*(D15/E15)^            | =(1-C15)*(D15/E15)      | =B47                    |                         | =D15+B9                           |
| 16 | Y                                                                      | 0.8                             | 0.6                            | =C16*E16^(1-B16)              | =B\$46+B7               | =C16*(D16/E16)^            | =(1-C16)*(D16/E16)      | =B48                    |                         | =D16+B10                          |
| 17 | <b>Sum</b>                                                             |                                 |                                |                               |                         | =F15+F16                   | =G15+G16                |                         |                         |                                   |
| 18 |                                                                        |                                 |                                | <b>TR</b>                     | <b>TC</b>               | <b>rK</b>                  | <b>wl</b>               | <b>tax</b>              | <b>Fact.Pay.</b>        |                                   |
| 19 | X                                                                      |                                 |                                | =I15*H15                      | =D15*H15                | =B46*F15                   | =B41*G15                | =B6*F15                 |                         | =F19+G19+H19                      |
| 20 | Y                                                                      |                                 |                                | =I16*H16                      | =D16*H16                | =B46*F16                   | =B41*G16                | =B7*F16                 |                         | =F20+G20+H20                      |
| 21 | <b>Sum</b>                                                             |                                 |                                | =D19+D20                      | =E19+E20                | =F19+F20                   | =G19+G20                | =H19+H20                |                         | =I19+I20                          |
| 22 |                                                                        |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 23 | <b>Households and Government</b>                                       |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 24 | <b>h</b>                                                               | <b><math>l_h</math></b>         | <b><math>k_h</math></b>        | <b><math>\alpha_h</math></b>  | <b><math>m_h</math></b> | <b><math>p_{ch}</math></b> | <b><math>x_h</math></b> | <b><math>y_h</math></b> |                         |                                   |
| 25 | A                                                                      | 200                             | 200                            | 0.7                           | =(1-\$B\$11)*\$B\$46    | =(D25*\$I\$15^(1-\$        | =D25*E25/F25*(F         | =(1-D25)*E25/F25*(      |                         |                                   |
| 26 | B                                                                      | 200                             | 200                            | 0.3                           | =(1-\$B\$11)*\$B\$46    | =(D26*\$I\$15^(1-\$        | =D26*E26/F26*(F         | =(1-D26)*E26/F26*(      |                         |                                   |
| 27 | C                                                                      | 200                             | 0                              | 0.7                           | =(1-\$B\$11)*\$B\$46    | =(D27*\$I\$15^(1-\$        | =D27*E27/F27*(F         | =(1-D27)*E27/F27*(      |                         |                                   |
| 28 | D                                                                      | 200                             | 0                              | 0.3                           | =(1-\$B\$11)*\$B\$46    | =(D28*\$I\$15^(1-\$        | =D28*E28/F28*(F         | =(1-D28)*E28/F28*(      |                         |                                   |
| 29 | G                                                                      | 0                               | 0                              | 0.5                           | =B6*F15+B7*F16          | =(D29*\$I\$15^(1-\$        | =D29*E29/F29*(F         | =(1-D29)*E29/F29*(      |                         |                                   |
| 30 | <b>Sum</b>                                                             | =SUM(B25:B29)                   | =SUM(C25:C29)                  |                               | =SUM(E25:E29)           |                            | =SUM(G25:G29)           | =SUM(H25:H29)           |                         |                                   |
| 31 |                                                                        |                                 |                                |                               |                         |                            |                         |                         |                         |                                   |
| 32 | <b>h</b>                                                               | <b><math>m_{capital}</math></b> | <b><math>m_{labour}</math></b> | <b><math>m_{total}</math></b> | <b><math>k_X</math></b> | <b><math>k_Y</math></b>    | <b><math>t_w</math></b> | <b><math>t_x</math></b> | <b><math>t_y</math></b> | <b><math>t-m_{capital}</math></b> |
| 33 | A                                                                      | =B\$46*C25                      | =(1-\$B\$8)*\$B\$41*           | =B33+C33                      | =B\$6*\$F\$15*C25       | =B\$7*\$F\$16*C2           | =B\$8*(G\$15+\$         | =B\$9*G25               | =B\$10*H25              | =B\$11*\$B\$46*C25                |
| 34 | B                                                                      | =B\$46*C26                      | =(1-\$B\$8)*\$B\$41*           | =B34+C34                      | =B\$6*\$F\$15*C26       | =B\$7*\$F\$16*C2           | =B\$8*(G\$15+\$         | =B\$9*G26               | =B\$10*H26              | =B\$11*\$B\$46*C26                |
| 35 | C                                                                      | =B\$46*C27                      | =(1-\$B\$8)*\$B\$41*           | =B35+C35                      | =B\$6*\$F\$15*C27       | =B\$7*\$F\$16*C2           | =B\$8*(G\$15+\$         | =B\$9*G27               | =B\$10*H27              | =B\$11*\$B\$46*C27                |
| 36 | D                                                                      | =B\$46*C28                      | =(1-\$B\$8)*\$B\$41*           | =B36+C36                      | =B\$6*\$F\$15*C28       | =B\$7*\$F\$16*C2           | =B\$8*(G\$15+\$         | =B\$9*G28               | =B\$10*H28              | =B\$11*\$B\$46*C28                |
| 37 | G                                                                      | =B\$46*C29                      | =(1-\$B\$8)*\$B\$41*           | =SUM(E38:I38)                 | 0                       | 0                          | 0                       | 0                       | 0                       | 0                                 |
| 38 | <b>Sum</b>                                                             | =SUM(B33:B37)                   | =SUM(C33:C37)                  | =SUM(D33:D37)                 | =SUM(E33:E37)           | =SUM(F33:F37)              | =SUM(G33:G37)           | =SUM(H33:H37)           | =SUM(I33:I37)           | =SUM(J33:J37)                     |

## Appendix 2

### VBA Macro Code

Option Explicit

```
Public Sub CalcJacobian()  
Dim src As Range  
Dim i As Integer, j As Integer, k As Integer, iter As Integer  
Dim sum As Double  
  
Set src = Range("Sheet1!$A$1")  
  
For i = 46 To 48  
    'set increment  
    For j = 46 To 48  
        If i = j Then  
            src.Cells(j + 5, 3).Value = src.Cells(43, 2).Value / 2  
        Else  
            src.Cells(j + 5, 3).Value = 0  
        End If  
    Next j  
  
    'copy x+dx to x  
    For j = 46 To 48  
        src.Cells(j, 2).Value = src.Cells(j + 5, 4).Value  
    Next j  
  
    'copy f(x) to f(x+dx)  
    For j = 46 To 48  
        src.Cells(j + 5, 5).Value = src.Cells(j, 5).Value  
    Next j  
  
    'copy x-dx to x  
    For j = 46 To 48  
        src.Cells(j, 2).Value = src.Cells(j + 5, 6).Value  
    Next j  
  
    'copy f(x) to f(x-dx)  
    For j = 46 To 48  
        src.Cells(j + 5, 7).Value = src.Cells(j, 5).Value  
    Next j  
  
    'calculate df  
    For j = 46 To 48  
        src.Cells(j + 5, 8).Value = (src.Cells(j + 5, 5).Value - src.Cells(j + 5,  
7).Value) / src.Cells(43, 2).Value  
    Next j  
  
    'copy df to jacobian  
    For j = 46 To 48  
        src.Cells(j + 11, i - 44).Value = src.Cells(j + 5, 8).Value  
    Next j  
Next i  
  
    'copy x back  
    For i = 46 To 48  
        src.Cells(i, 2).Value = src.Cells(i + 5, 2).Value  
    Next i  
  
    'copy update to x  
    For i = 46 To 48  
        src.Cells(i + 5, 2).Value = src.Cells(i + 18, 8).Value  
    Next i  
  
End Sub
```

```

Public Sub CopyStats()
Dim src As Range, dst As Range
Dim sw As Integer, i As Integer
Dim alpha As Double, sigma As Double, x As Double, y As Double
Dim signv As Double, sm1os As Double, sosm1 As Double

Set src = Range("Sheet1!$A$1")
Set dst = Range("Sheet2!$A$1")

If src.Cells(6, 6).Value > 0.5 Then sw = 1 Else sw = 0

'copy px, py, qx and qy to summary area
dst.Cells(4, 2 + sw).Value = src.Cells(15, 9).Value
dst.Cells(5, 2 + sw).Value = src.Cells(16, 9).Value
dst.Cells(6, 2 + sw).Value = src.Cells(15, 8).Value
dst.Cells(7, 2 + sw).Value = src.Cells(16, 8).Value

'calculate utility for each household
sigma = src.Cells(40, 2).Value
signv = 1 / sigma
sm1os = (sigma - 1) / sigma
sosm1 = 1 / sm1os
For i = 25 To 29
    alpha = src.Cells(i, 4).Value
    x = src.Cells(i, 7).Value
    y = src.Cells(i, 8).Value
    If (x > 0) And (y > 0) Then
        dst.Cells(i - 12, 2 + sw).Value = (alpha ^ signv * x ^ sm1os + (1 - alpha) ^
signv * y ^ sm1os) ^ sosm1
    Else
        dst.Cells(i - 12, 2 + sw).Value = 0
    End If
Next i

End Sub

Public Sub CalcResults()
Dim src As Range, dst As Range
Dim alpha As Double, sigma As Double, s1 As Double, s2 As Double, U0 As Double, U1 As
Double
Dim px0 As Double, px1 As Double, py0 As Double, py1 As Double
Dim qx0 As Double, qx1 As Double, qy0 As Double, qy1 As Double
Dim PNum As Double, PDen As Double, LNum As Double, LDen As Double
Dim eP0U0 As Double, eP0U1 As Double, eP1U0 As Double, eP1U1 As Double

Dim i As Integer

Set src = Range("Sheet1!$A$1")
Set dst = Range("Sheet2!$A$1")

px0 = dst.Cells(4, 2).Value: px1 = dst.Cells(4, 3).Value
py0 = dst.Cells(5, 2).Value: py1 = dst.Cells(5, 3).Value
qx0 = dst.Cells(6, 2).Value: qx1 = dst.Cells(6, 3).Value
qy0 = dst.Cells(7, 2).Value: qy1 = dst.Cells(7, 3).Value

PNum = px1 * qx1 + py1 * qy1: PDen = px0 * qx1 + py0 * qy1
LNum = px1 * qx0 + py1 * qy0: LDen = px0 * qx0 + py0 * qy0

'Report price indices
dst.Cells(8, 2).Value = 1
dst.Cells(9, 2).Value = 1
dst.Cells(10, 2).Value = 1

'Paasche index
If PDen > 0.0001 Then
    dst.Cells(8, 3).Value = PNum / PDen
Else
    dst.Cells(8, 3).Value = 0
End If

```

```

'Laspeyres index
If LDen > 0.0001 Then
    dst.Cells(9, 3).Value = LNum / LDen
Else
    dst.Cells(9, 3).Value = 0
End If

'Fisher index
dst.Cells(10, 3).Value = Sqr(dst.Cells(8, 3).Value * dst.Cells(9, 3).Value)

'Calculate GDP & RGDP
dst.Cells(20, 2).Value = px0 * qx0 + py0 * qy0
dst.Cells(21, 2).Value = dst.Cells(20, 2).Value

dst.Cells(20, 3).Value = px1 * qx1 + py1 * qy1
dst.Cells(21, 3).Value = dst.Cells(20, 3).Value / dst.Cells(10, 3).Value

'Calculate EV & CV
sigma = src.Cells(40, 2).Value
s1 = 1 - sigma
s2 = 1 / s1
For i = 1 To 5
    alpha = src.Cells(i + 24, 4).Value
    U1 = dst.Cells(i + 12, 3).Value
    U0 = dst.Cells(i + 12, 2).Value

    eP0U0 = (alpha * px0 ^ s1 + (1 - alpha) * py0 ^ s1) ^ s1 * U0
    eP0U1 = (alpha * px0 ^ s1 + (1 - alpha) * py0 ^ s1) ^ s1 * U1
    eP1U0 = (alpha * px1 ^ s1 + (1 - alpha) * py1 ^ s1) ^ s1 * U0
    eP1U1 = (alpha * px1 ^ s1 + (1 - alpha) * py1 ^ s1) ^ s1 * U1

    dst.Cells(i + 12, 4).Value = eP0U1 - eP0U0
    dst.Cells(i + 12, 5).Value = eP1U1 - eP1U0
Next i

End Sub

```