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An analytical investigation of the effectiveness of customer loyalty programs

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AN ANALYTICAL INVESTIGATION ON THE EFFECTIVENESS OF CUSTOMER LOYALTY PROGRAMS

BY

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Abstract

An Analytical Investigation on the Effectiveness of Customer Loyalty Programs

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This dissertation investigates the effectiveness of customer loyalty programs. Loyalty programs, as a prevalent marketing strategy, aim to enhance customers' loyalty and thereby increase a firm's long-term profitability. Despite the ubiquity of loyalty programs, the empirical research shows contradictory findings on their effectiveness. Analytical studies, on the other hand, leave notable gaps in the research on different aspects of loyalty programs. In this study, taking an analytical approach, some of the research gaps in the adoption as well as the design of loyalty programs are addressed. Specifically, in evaluating the profitability of loyalty programs, this study incorporates customers' valuation for the product and their post-purchase satisfaction level, two factors that are not considered in previous studies. The formulations result in stochastic models that are analyzed to gain insights into the profitability of loyalty programs. Based on the model assumptions, it is observed that if a firm maintains satisfaction among customers, not adopting a loyalty program yields optimal profits. It is also shown that this result holds even when the uncertainty in customers' satisfaction levels is incorporated in the model.

From the design perspective, the reward structure is discussed as one of the main drivers of loyalty programs' effectiveness. Particularly, the multitier reward scheme, a commonly-used yet underexplored reward structure, is studied. A model is developed to compare the effectiveness of multitier reward schemes with that of single-tier rewards. In the framework of the model assumptions, it is shown that the multitier scheme is more effective than the single-tier scheme in increasing a firm's profitability and driving repeat purchase behavior. Also, a model is developed to characterize the conditions under which a multitier reward scheme is the preferred choice. The conditions are formulated based on the sensitivity of customers' demand to the offered price, rewards and the distance from the next tier, and also based on the price offered by the competitor. It is shown that low levels of sensitivity to rewards and distance in customers can result in suboptimal revenue from a multitier reward scheme. Also, it is observed that if the competitor offers deeper discounts, the multitier reward structure becomes optimal in more cases.

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I am deeply grateful to my parents for giving me a wonderful childhood, instilling in me the value of higher education and inspiring me throughout my life. I would also like to thank my in-laws for their unconditional love and endless encouragement.

Lastly, I would like to thank my beloved wife, without whose love, patience and support, this journey would have been unbearable. Farzaneh, I am blessed to have you as my best friend and I love you dearly!

Dedication

To Farzaneh, my beautiful, devoted and patient wife

Content

List of Tables

List of Figures

Nomenclature

α_d: Customers' sensitivity to the distance from the next reward tier

$\alpha_d^{'}$: Relative sensitivity of utility to the distance

α_p: Marginal sensitivity of the utility to the price

α_r: Marginal sensitivity of demand to reward

$\alpha_r^{'}$: Relative sensitivity of utility to rewards

γ: Probability that a first-period buyer proceeds to the second period

δ: Customers' satisfactions level

ϵ: Customers' anticipated change in their valuations for the product in period 2

θ: Size of heavy-user segment in each period

μ_s: Mean of δ

$\xi_{A,i}$: Random component of the customers' utility at firm A

$\xi_{B,i}$: Random component of the customers' utility at firm B

ρ_l: Lower bound of the optimal loyalty reward

σ^2: Variance of ϵ

σ_s: Standard deviation of δ

$\overline{\sigma}_s$: Upper bound for σ_s below which $r = 0$ is optimal

ω_i: Sum of the rewards of the current tier and the next tier

Π_2^l: Probability that a light user makes a purchase in period 2

Π_i: Probability that a customer makes a purchase in period i

$\Pi_i^{h(k)}$: Probability that a heavy user makes a purchase in period i, k represents the purchase history index

$\Phi(.)$: Standard normal CDF

A: Transition probability matrix whose $(j+1,k+1)^{th}$ element is the probability that the customer's total number of purchases at Firm A reaches k given that he/she has bought j units of the product at Firm A

b_1: Break point between tier 1 and tier 2

b_2: Break point between tier 2 and tier 3

C_r: Expected cost of reward that Firm A incurs during the selling horizon

d_i: Distance to the next loyalty level

D: Price-cut offered by Firm B

$F(.)$: Cumulative distribution function of customers' valuation in the first period

HR: Firm's expected revenue from the heavy-user segment

LR: Firm's expected revenue from the light-user segment

n: Number of discrete time periods in the selling horizon

p: Price offered by Firm A offers Firm B charges $p - D$ in each period

p_i: Offered price in period i

$p_{i,A}$: Probability that the customer buys at Firm A in period i

$p_{tier\ i}$: Probability that a customer achieves tier i

P: A vector containing the elements of the first row of $A^{(n)}$ whose $(k+1)^{th}$ entry represents the probability that a customer buys k products over the entire selling cycle

r: Loyalty reward

r_1: Loyalty reward for two-period buyers

r_2: Loyalty reward for three-period buyers

r_i: Loyalty reward in tier i

R: Firm's expected total revenue function

R_A: Firm A's expected revenue function

s_i: Total money spent by the customer at Firm A up to period i

S_1^h: Heavy-users' surplus from buying the product in period 1

$S_2^{h(k)}$: Heavy-users' surplus from buying the product in period 2. k indicates the past purchase history: $k=1$ denotes the case where customer has made a purchase in period 1 and $k=2$ *corresponds* to customers who have failed to buy in period 1

$S_3^{h(k)}$: Heavy-users' surplus from buying the product in period 3, k is the purchase history index: $k=1$ for buying in both first and second periods, $k=2$ for buying only in one of the periods and $k=3$ failing to buy in either period

S_i: Customers' surplus from buying a unit of product in period i

S_i^l: Light users' surplus from buying a unit of the product in period i

$u_{A,i}$: Customers' utility from buying at Firm A in period i

$u_{B,i}$: Customers' utility from buying at Firm B in period i

v_2^l: Light-users' valuation for the product

$v_{A,i}$: Customers' valuation for the product offered by Firm A in period i

$v_{B,i}$: Customers' valuation for the product offered by Firm B in period i

v_i: Customers' valuation for the product in the first period i

v_i^h: Heavy users' valuation for the product in period i

v_i^l: Light-users' valuation for the product in period i

x_i: Customers' total number of purchases at Firm A up to period i

Chapter 1

Introduction

1.1 Introduction

Loyalty programs are one of the marketing strategies to build and enhance customer loyalty and thereby increase a firm's long-term profitability. Since loyal customers are thought to be more profitable to a firm (O'Brien & Jones 1995; Reichheld & Teal 2001), loyalty reward programs are presumed to increase a firm's long-term profitability. Since the time American Airlines lunched AAdvantage, the first contemporary loyalty program (Liu 2007), loyalty programs have proliferated in various industries including airlines, credit card companies, retail, hotel chains and the non-profit sector (Kim et al. 2001; Pauler & Dick 2006; Kumar 2008; Bijmolt et al. 2010). According to Gartner Analyst Adam Sarner, U.S. companies collectively spent more than $1.2 billion to manage their loyalty programs in 2003 (Kumar 2008, p.16). In Europe, the top 16 retailers spent more than $1 billion in 2000 on their loyalty programs (Reinartz & Kumar 2002).

Customers' participation in loyalty programs, on the other hand, has also substantially increased over the past decade (Dekay et al. 2009; Smith & Sparks 2009; McCall & Voorhees 2010). More than 60% of European and American customers belonged to at least one grocery store loyalty program in 2005, and memberships have experienced double-digit annual growth rates (11%) (Meyer-Waarden & Benavent 2009). More recently, the COLLOQUY Loyalty Census (2011) revealed that the number of memberships in the US exceeds 2 billion, which shows an average of more than 18 memberships per household. This indicates a 114.7% increase in memberships in the US since 2000. COLLOQUY also reports 120.7 million memberships in Canada, which shows a 3.9% increase from their report in 2009. Furthermore, Air Miles, Canada's largest loyalty program, counts nearly half of Canadian households as members (Cigliano et al. 2000).

Given the widespread use of loyalty programs, academics have also shown interest in this area. There are mainly two streams of research on loyalty programs. Some researchers have taken the consumer's perspective and explored the effects of loyalty programs on customers' buying behavior using empirical studies (e.g., Sharp & A. Sharp 1997). The second stream of research uses mathematical modeling to analyze the effects of loyalty programs on a firm's profitability and/or market competitiveness (e.g., Singh et al. 2008). As discussed in Chapter 2, despite loyalty programs' wide-spread use, empirical researchers have not reached consensus on whether they are effective in all cases. In other words, it is not apparent that loyalty programs are effective in influencing the established buying patterns of customers and increase a firm's long-term profitability. Analytical studies on loyalty programs, on the other hand, are in their early stages. There are only a few published articles on loyalty programs, which are mainly focused on the programs' overall effectiveness. Specifically, we are unaware of any study on the design of loyalty programs.

1.2 Thesis Objective

In this research, taking an analytical approach[1], some new insights are gained into the effectiveness of loyalty programs by addressing the identified gaps in the literature. More specifically, building upon the existing literature on the profitability of loyalty programs, new models are formulated by incorporating customers' valuation and their satisfaction level. Valuation and satisfaction are incorporated both as deterministic and probabilistic variables. Through sensitivity analyses, the effects of customers' valuation and satisfaction level on the optimal profits of the firm are investigated. Thus, this research investigates the relationship between customer valuation/satisfaction level and the profitability of loyalty programs.

Moreover, this research contributes to the literature of loyalty programs by studying multitier reward structures. To the best of our knowledge, this is the first attempt that analyzes multitier reward structures in the context of loyalty programs. A model is developed to evaluate the effectiveness of a two-tier scheme. Also, a model is proposed to

[1] The term "analytical approach" is used to contrast the "empirical approach". Specifically, the term is not referring to the approach used to solve optimization models, as in some cases the optimal solution is obtained using numerical techniques.

investigate the conditions under which a three-tier scheme is not the optimal reward structure.

1.3 Thesis Outline

The organization of the thesis is as follows: Chapter 2 presents a brief literature review on loyalty programs and identifies the gaps in the literature. Chapter 3 evaluates the effects of customer satisfaction, where valuation is incorporated as a random variable and satisfaction level is treated as a parameter. Chapter 4 extends the model developed in Chapter 2 by modeling the satisfaction level as a random variable. Chapter 5 and Chapter 6 are focused on multitier reward schemes. In Chapter 5, the effectiveness of multitier reward schemes is evaluated by analyzing the profitability of a two-tier reward system in a three-period model. In Chapter 6, a model is developed to characterize the conditions under which a three-tier reward is not the optimal choice. Chapter 7 summarizes the findings and contributions and proposes some avenues for future research.

Chapter 2

Literature Review

2.1 Customer Loyalty

Broadly, there exist two distinct definitions for customers' loyalty in the marketing literature: behavioral loyalty and attitudinal loyalty. Many researchers (e.g., Blattberg & Sen 1974; Kahn et al. 1988; Ehrenberg et al. 1990; Lewis 2004; Liu 2007) have defined loyalty from a behavioral perspective. That is, the loyalty of a customer is evaluated based on the customer's purchase behavior. Various measures of behavioral loyalty have been proposed based on recency, frequency, and monetary value of a customer's past purchases (Zhang et al. 2010).

On the other hand, some researchers argue that behavioral loyalty measures are insufficient to capture the multidimensionality of the concept of loyalty (e.g., Jacoby & Kyner 1973; Dick & Basu 1994; Kumar & Shah 2004). As a result, some studies emphasize the importance of considering the attitudinal aspect of loyalty along with the behavioral side (e.g., Oliver 1999; Pritchard et al. 1992; Keh & Lee 2006). Attitudinal loyalty, in fact, incorporates psychological factors implicit in the customers' decision to buy (Oliver 1999; Butcher et al. 2001; Bustos-Reyes & González-Benito 2008). In accordance with this approach, Oliver (1999) defines loyalty as "a deeply held commitment to rebuy or repatronize a preferred product or service consistently in the future, despite situational influences and marketing efforts having the potential to cause switching behavior" (p. 392).

Although attitudinal loyalty is conceptually richer than behavioral loyalty (Uncles et al. 2003), it is more difficult to measure. Consequently, attitudinal loyalty, compared to behavioral loyalty, is less frequently used in the literature. Specifically, we are unaware of any analytical study in the marketing literature that has incorporated attitudinal loyalty. In this research, the behavioral aspect of loyalty is considered and a loyal customer will be identified based on the frequency of their past purchases.

2.2 Loyalty Reward Programs

Various researchers have claimed that loyal customers are more profitable to a firm (e.g., (Reichheld & Sasser 1990; O'Brien & Jones 1995; Johnson 1998; Edvardsson et al. 2000; Reichheld & Teal 2001; Van den Poel & Lariviere 2004; Buckinx & Van den Poel 2005; Li et al. 2006). This profitability is thought to be generated by loyal customers' reduced servicing costs, their lower price sensitivity and higher spending levels, and their favourable recommendations to other potential customers (Dowling & Uncles 1997).

Thus, establishing customer loyalty is one of the potential means to achieve profitability (Reinartz & Kumar 2002). Loyalty programs are structured marketing efforts (Sharp & Sharp 1997) that aim to enhance customers' loyalty by rewarding their repeat purchase behavior. As mentioned earlier, loyalty programs have increasingly grown in popularity since the time American Airlines launched AAdvantage, its frequent-flyer program, in 1981. Kumar (2008) describes the advancement of loyalty programs in different industries in the timeline depicted in Figure 2.1.

Given the widespread use of loyalty programs, academics have also shown interest in this topic. As mentioned earlier, the literature on loyalty programs can be categorized into two groups: empirical studies and analytical studies. Empirical studies (e.g., Sharp & Sharp 1997; Kivetz & Simonson 2002; Yi & Jeon 2003; Lewis 2004; Liu 2007; Meyer-Waarden 2007; Wirtz et al. 2007; Meyer-Waarden 2008) mainly investigate the effects of

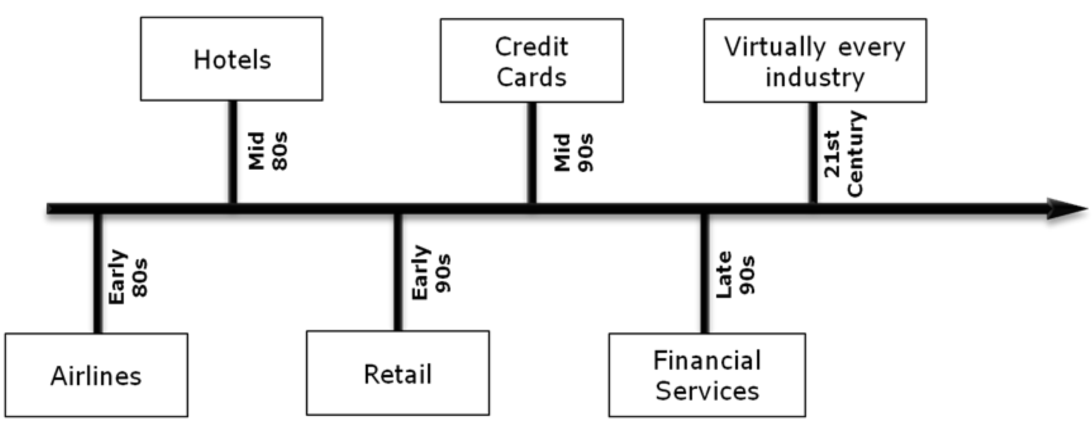


Figure 2.1: Evolution of loyalty programs in different industries (according to Kumar 2008, p.13)

loyalty programs from the customers' perspective. This type of research examines the efficacy of loyalty programs in changing the customers' buying behavior. Analytical studies (e.g., Caminal & Matutes 1990; Kim et al. 2001; Singh et al. 2008), on the other hand, evaluate the effectiveness of loyalty programs from the firm's perspective. This stream of studies analyzes the impact of loyalty programs on the firm's profitability and market competiveness. In the next two sections, a summary of the results of the recent empirical and analytical studies on this topic is presented.

2.3 Empirical Studies

While most of the literature in this category focuses on behavioral loyalty (i.e., repeat purchase) (e.g., Sharp & Sharp 1997; Lewis 2004; Liu 2007; Meyer-Waarden 2008), Pritchard et al. (1992) emphasize the importance of considering the attitudinal aspect of loyalty along with the behavioral side. Similarly, Keh & Lee (2006) state that researchers are increasingly considering the psychological aspect of loyalty in order to distinguish it from the behavioral loyalty.

Empirical research, regardless of the definition of loyalty, shows mixed findings on efficacy of loyalty programs (Liu 2007; Meyer-Waarden 2007, 2008; McCall & Voorhees 2010). Meyer-Waarden (2008) has compared the results of different empirical studies on loyalty programs. Based on this comparative investigation, Meyer-Waarden (2008) has found contradictory empirical evidence on the impact of loyalty programs. While some researchers have reported significant impact (e.g., Nako 1992; Passingham 1998; Lal & Bell 2003; Smith et al. 2003; Taylor & Neslin 2005; Leenheer et al. 2007; Meyer-Waarden 2008;), others assert that loyalty schemes that are prevalent today are unlikely to change the established buying pattern of customers (e.g. Dowling & Uncles 1997; Sharp & Sharp 1997; Mägi 2003).

2.4 Analytical Studies

While empirical studies show contradictory findings on the effectiveness of loyalty programs, there are only a few analytical studies on this important marketing field. This fact is evident from Kim et al. (2001) (the first published analytical study on loyalty programs), who view their work "as an initial step, and clearly far removed from the ideal model in which the implications directly translate into managerial practice" (p. 113).

These studies have mainly considered a duopoly market structure and, using a game-theoretic approach, have analyzed the impact of loyalty programs on market equilibrium conditions. For instance, Kim et al. (2001) have found that competitive market prices generally increase by adopting loyalty programs. Furthermore, they have shown that, depending on light users' relative price sensitivity, firms may gain less profit compared to the case without loyalty programs. While almost all of the studies have focused on a symmetric market, the market with both firms offering loyalty rewards, in a recent work by Singh et al. (2008), the asymmetric case is examined. By exploring the existence of asymmetric equilibrium, Singh et al. (2008) have addressed the question that whether it is always optimal for a firm to adopt a loyalty program in response to the competitor's similar program. Using a game theoretic approach, they have found certain conditions under which it would be profitable for the firm to not offer a loyalty program but to resort to lower-price strategy.

Loyalty programs, by offering rewards based on cumulative buying, create *switching cost* for customers. Thus, literature on the effects of switching cost is also relevant to loyalty programs. The effect of switching cost on market competitiveness was first studied by Von Weizsacker (1984). Using a Hotelling model, Von Weizsacker (1984) shows that the price sensitivity of customers and market competitiveness rise with the switching cost. These findings, however, depend on the underlying assumptions that firms are committed to maintaining the same price over time, and also customers randomly change their preferences for the competing products within a market. However, by relaxing the constancy of price assumption and adding a segment of customers with fixed preferences over time, Klemperer (1987a) shows that the market may be less competitive than a market with no switching costs. Furthermore, Klemperer (1987a) concludes that the effect of switching cost on a firm's profitability, similar to market competiveness, depends on the size of the segment with constant preference. Namely, firms may be either better off or worse off with switching costs than without them, depending on the proportion of customers whose preferences remain constant over time. In contrast to Von Weizsacker (1984) and Klemperer (1987a,b,c, 1988), who take the switching costs as exogenous, Caminal & Matutes (1990) treat them as endogenous. In essence, switching costs created by loyalty programs are endogenous, because firms directly decide upon the amount of reward. As Singh et al. (2008) state, a common finding of all these studies is that loyalty programs sustain in equilibrium since they create switching costs for customers.

2.5 Gaps in the Literature

2.5.1 Customer Valuation

One of the gaps in the existing analytical literature on loyalty programs is the lack of research on the effect of customers' valuation (or reservation price) on the programs' effectiveness. Breidert (2006) defines valuation as "the highest price an individual is willing to accept to pay for some good or service" (p. 27). Except for Kopalle & Neslin (2003), we are unaware of any study that considers customers' valuation in the context of loyalty programs. Kopalle & Neslin (2003), using numerical simulation, examine the viability of loyalty programs in a strategic competitive game by incorporating customers' valuation as a logistically distributed random variable. Other previous studies on loyalty programs have either not considered the valuation as a factor in customers' decision-making (e.g., Singh et al. 2008) or have assumed that customers' valuation for the product is sufficiently high that it exceeds the product price (e.g., Kim et al. 2001). Similarly, to the best of our knowledge, none of the studies on the switching cost includes a random valuation. For instance, Klemperer (1987a,b,c, 1988) incorporates valuation as a parameter. Caminal & Matutes (1990), who study the endogenous switching cost, assume that customers' valuation is higher than the offered prices. Farrell & Shapiro (1988) assume that buyers' valuation is so high that it is not binding and state that their results are affected if the valuation binds.

2.5.2 Customer Satisfaction

Another gap in the literature is the lack of research on the effects of customers' satisfaction on the profitability of loyalty programs. In previous studies on loyalty programs, the customers' purchase experience is not considered as a factor in their decision to buy. In other words, it is presumed that, no matter whether they are satisfied or dissatisfied with their past purchase, customers return to the firm with either the same valuation or a valuation independent from their past valuations. For instance, Kopalle & Neslin (2003) assume that a customer's valuation in each period is a random draw from the logistic distribution. This implies the assumption that a customer's valuation changes independently from his/her previous valuations.

As pointed out earlier, some studies on switching costs allow for customers to randomly change their preferences over time. For instance, Klemperer (1987c) assumes that a fraction of customers change their position on the Hotelling line segment. Since customers' taste for the product is represented by their position on the line segment, the shift in a buyer's position can be interpreted as his/her satisfaction level. However, in these studies, customers' locations change randomly and independently from their location in the previous periods. As a result, the spatial distribution of customers in the market will not change in favor of a specific firm in a subsequent period.

In contrast to these underlying assumptions, previous studies show that the customers' perception of the product quality and their level of satisfaction can alter after a purchase experience (see Morley 2004). On the other hand, empirical studies have found that satisfied and dissatisfied customers perceive loyalty programs in different ways (Keh & Lee 2006). Moreover, based on the expectation disconfirmation theory (EDT) proposed by Oliver (1980), customers' satisfaction with the actual product or service experience is the primary determinant of their post-purchase intentions (Premkumar & Bhattacherjee 2008). Au et al. (2002) also state that customer satisfaction results in repeat business and increases the firm's profitability. Likewise, Nie (2000) posits that customer dissatisfaction hurts repeat business and, as a result, jeopardizes the company's long-term profitability. Thus, the customer satisfaction level is an important factor that may affect the performance of a loyalty program in driving repeat purchase behavior and increasing the firm's profitability.

2.5.3 Multitier Reward Scheme

As noted earlier, another area that merits further research is the design of loyalty programs. Previous studies show that the effectiveness of a loyalty program is a function of the program's design (Kivetz & Simonson 2002; Roehm et al. 2002; Yi & Jeon 2003). Thus, the reward structure, as one of the main components of loyalty programs' design (Kumar & Reinartz 2006, p.172), is one of the key drivers of their effectiveness. Reward structure has several components such as reward type, rate of reward and timing of reward. By manipulating the configuration of these components, various reward structures have been designed and employed in real world markets. A particular reward structure is the multitier reward scheme. Based on a multitier scheme, customers' loyalty reward is determined based on their level of loyalty, where the loyalty level is evaluated based on a measure of customers' past purchase behavior. Examples of such a measure are cumulative spending level, accumulated points balance, or purchase frequency. Intuitively, a multitier reward scheme offers disproportionately higher rewards to more loyal customers. In other words, the reward per dollar spent increases with a customer's loyalty level. Some examples of such reward systems are Microsoft's Xbox Live® reward in the US, Shopper's Optimum Rewards Program® in Canada and British Airways Executive Club® in the UK.

Although multitier reward schemes are common in practice, the analytical studies on loyalty programs have either assumed a fixed percentage-off discount (Caminal & Claici 2007; Singh et al. 2008) or a constant reward value (Von Weizsacker 1984; Kim et al. 2001) for loyal customers. Similarly, multitier reward schemes have remained underexplored in empirical studies on loyalty programs. Apart from Liu (2007), who analyzes data from a multilevel point system of a convenience store, we are unaware of any empirical research on multitier reward schemes. Thus, the lack of research on multitier reward structure is a significant gap in the literature on evaluating loyalty programs' effectiveness.

Chapter 3

The Effect of Customer Satisfaction on Loyalty Programs' Profitability: Parametric Satisfaction

3.1 Introduction

This study builds on previous analytical studies on loyalty programs' profitability, notably Kim et al. (2001) and Singh et al. (2008), by treating customers' valuation as a random variable. As discussed in Section 2.5, previous studies on loyalty programs have either not incorporated the valuation as a factor in customers' decision making or have assumed that the valuation is sufficiently high that is not binding. In this chapter, however, valuation is incorporated both as a deterministic and as a probabilistic variable.

Moreover, the impact of customers' satisfaction with their past purchases on the profitability of loyalty programs is studied. As noted in Chapter 2, previous studies on loyalty programs have not considered customer satisfaction as a factor in their decision-making. That is, regardless of their past purchase experience, customers always return to the firm with the same valuation for the product being offered. In contrast to these assumptions, empirical studies show that customer satisfaction level is an important factor that may affect the effectiveness of a loyalty program in driving repeat purchase behavior and increasing the firm's profitability (e.g., Keh & Lee 2006).

In Section 2.4, it is discussed that this gap also exists in the literature on switching cost. As pointed out, some studies on switching costs allow for customers to randomly change their location on a Hotelling line (e.g., Klemperer 1987c). Since customers' taste for the product is represented by their position on the line segment, the shift in a buyer's position can be interpreted as his/her satisfaction level. However, in these studies, customers' preferences change randomly and independently from their previous periods' preferences. As a result, the distribution of customers' preferences in the market will not change in favor of a specific firm in a subsequent period. Thus, none of the firms realizes a change in the market share and profits that stem from the shift in preferences. In contrast, here the satisfaction level is incorporated as a parameter in the model that can take on any value. In a Hotelling framework, this would imply that the customers' distribution might become asymmetric, depending upon the firms' relative performance in driving customer satisfaction in previous periods.

In summary, this study contributes to the existing literature on loyalty programs by incorporating a random valuation and by analyzing the effects of customers' satisfaction levels on the profitability of loyalty programs. A unique feature of this study is that the models are solved analytically.

The objective of the model is to maximize the firm's revenue function in term of its decision variables. Assuming a deterministic valuation, the optimization problem turns out to be linear programming. Probabilistic valuation, on the other hand, results in nonlinear programming. The models consist of two parameters, and despite their complexities, the analytical optimal solutions are found in terms of the parameters. Structural properties of the optimal solutions yield valuable insight into the profitability of loyalty programs.

The rest of this chapter is structured as follows. The model formulation and its underlying assumptions are described in Section 3.2. Section 3.3 focuses on solving the model under three different valuation distributions. In Section 3.4, the model is extended to a case where customers' anticipated change in their future valuation is taken into account. The obtained results are discussed in Section 3.5. Section 3.6 concludes the chapter with a summary of the findings.

3.2 Model Formulation

Consider a firm selling a good or service through two periods. The firm adopts a loyalty reward program based on which customers gain a reward for repeating their purchase. More specifically, if a customer makes a purchase in both periods, he/she earns the loyalty reward of r.

Similar to Singh et al. (2008), it is assumed that only a certain proportion of first-period buyers proceeds to the second period. This proportion is modeled as a parameter,

γ. So, γ is the probability that a first-period buyer becomes a potential buyer in the second period. As a result, $(1-\gamma)$ fraction of buyers in period 1 fails to proceed to period 2. This is to capture the effect of those who join the loyalty program of the firm, but decide not to return to buy the product in the second period. This fraction does not show up in period 2 mainly because of a low consumption rate.

Customers' decision to buy in the first period is a function of the following factors: their valuation for the product, current and future prices, loyalty reward value and possibility of returning in the second period. These factors are derived based on the assumption that customers are forward-looking in their decision, that is, they consider future (period 2) gains or losses when deciding to buy in the current period (period 1). These factors are summarized in a variable called *surplus.* A customer's surplus from purchasing a product is, in fact, the value he/she will gain by obtaining the product subtracted by the amount he/she has to pay for it. Similar to Biyalogorsky et al. (2001), it is assumed that a customer buys one unit of the product, if the offer yields a nonnegative surplus. Customers' surplus from buying a unit of product in period 1 is

$$S_1 = [v_1 - p_1] + \gamma[v_1 - (p_2 - r)], \tag{3.1}$$

where v_1 is the valuation of the customer for the product in the first period, p_i $(i = 1, 2)$ is the offered price in period i and r is the loyalty reward value. As can be seen, since customers are assumed to be strategic, S_1 is the sum of the first period surplus and the expected surplus from buying the product in period 2. The expected surplus in period 2 is the surplus value (i.e., $v_1 - (p_2 - r)$) multiplied by the probability of proceeding to the second period (γ). Without loss of generality, it is assumed that the discount factor for costumers is equal to one. That is, customers are not considering the time value of rewards.

One of the assumptions underlying the surplus formulation in Equation (3.1) is that first-period buyers do not anticipate a change in their future valuations. In other words, the shift in customers' valuation is unknown to them at the time of purchase in period 1. This assumption is in line with Klemperer (1987c), where it is assumed that whether a first-period buyer has fixed tastes for the product, has changing tastes, or leaves the market is independent of his/her position on the line segment, not influenced by his/her decision in the first period, and unknown to him/her until after the purchase in period 1. In Section 3.4, this assumption is relaxed and it is investigated how customers' anticipation of future valuation affects the results.

A customer buys if the firm's offer gives him/her a nonnegative surplus. Thus, the probability that a customer makes a purchase in period 1 is

$$\Pi_1 = Pr(S_1 \geq 0) = 1 - F\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}\right), \tag{3.2}$$

where $F(.)$ denotes the CDF of the customers' valuation in the first period.

Customers who buy in period 1 will proceed to the second period with the probability of γ. For a given customer in period 2, the utility derived from making a purchase is

$$S_2 = v_2 - (p_2 - r), \tag{3.3}$$

where v_2 is the valuation of the customer in the second period. Here, unlike the previous studies, v_1 and v_2 are neither equal nor independent. Intuitively, customers' valuation in the second period is affected by their past experience of buying and using the good or service. To model this dependency, one can assume $v_2 = v_1 + \delta$ for each customer. This implies that the past purchase experience leads to a shift of size δ in the valuation of the customer. This assumption is consistent with the findings of Homburg et al. (2005) who have explored the effect of customers' satisfaction on their valuation using empirical research. Hence, for a satisfied customer δ is nonnegative and for a dissatisfied customer δ is negative. Here, δ is incorporated as a parameter in the model. The next chapter investigates the case where δ is probabilistic.

Considering the relationship between v_1 and v_2, one can infer that S_1 and S_2 are not independent. Thus, the probability that a Period 1 buyer makes a purchase in period 2 is conditioned on the fact that his/her surplus in period 1 has been nonnegative. That is,

$$\Pi_2 = \Pr(S_2 \geq 0 \mid S_1 \geq 0) = \Pr\left(v_2 \geq p_2 - r \mid v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right). \tag{3.4}$$

But $v_2 = v_1 + \delta$. Thus, from Equation (3.4) it follows that

$$\Pi_2 = \Pr\left(v_1 \geq p_2 - r - \delta \mid v_1 \geq \frac{p_1 - \gamma(p_2 - r)}{1+\gamma}\right). \tag{3.5}$$

Applying the conditional probability theorem, Equation (3.5) can be expressed as the following piecewise function:

$$\Pi_2 = \begin{cases} \dfrac{1 - F(p_2 - r - \delta)}{1 - F\left(\dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right)}, & \text{if } p_2 - r - \delta \geq \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \\ 1, & \text{if } p_2 - r - \delta \leq \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \end{cases}. \tag{3.6}$$

The market size in the first period is normalized to one. Similar to Singh et al. (2008), it is assumed that a new group of customers joins the market in the second period with the size $(1 - \gamma)$. This new group of customers are called light users, because they are potential buyers of only a single period. Clearly, light users are not the subject of the loyalty program. Light users, in fact, make up for the "missing" potential first-period buyers who fail to proceed to period 2 because of their low consumption rate. This assumption is mainly made to avoid market expansion or contraction due to customers' consumption rate affecting the results (Singh et al. 2008).

Similarly, one can find the surplus of a light user in period 2 and the corresponding probability of making a purchase. Light users only consider their valuation and the offered price when making the decision to buy in period 2. Hence, their surplus from buying a product in period 2 is

$$S_2^l = v_2^l - p_2, \tag{3.7}$$

where v_2^l is the valuation of a light user. Light users are homogeneous to customers in period 1 in the sense that they are first-time product buyers. Thus, without loss of generality, one can assume that v_2^l follows the same distribution as v_1. Hence, a light user makes a purchase with the probability of

$$\Pi_2^l = \Pr\left(S_2^l \geq 0\right) = 1 - F(p_2). \tag{3.8}$$

Now, based on the firm's revenue from selling a product and customers' probabilities of buying in each period, the firm's expected total revenue function can be found as follows:

$$R = p_1 \Pi_1 + \gamma(p_2 - r)\Pi_1 \Pi_2 + (1-\gamma) p_2 \Pi_2^l. \tag{3.9}$$

The objective of the model is to optimize the revenue function with respect to the firm's decision variables (i.e., p_1, p_2 and r). However, decision variables are nonnegative and also, the reward, r, cannot exceed the second period price, p_2. These impose some constraints on the model. The resulting problem is formulated as follows:

$$\underset{p_1,p_2,r}{\text{maximize}}\ R = p_1\Pi_1 + \gamma(p_2 - r)\Pi_1\Pi_2 + (1-\gamma)p_2\Pi_2^l \tag{3.10}$$

Subject to:

$$r \le p_2 \tag{3.10a}$$

$$p_1, p_2 \text{ and } r \ge 0 \tag{3.10b}$$

3.3 Solving the Model

Based on the model in Equation (3.10), the optimal values of p_1, p_2 and r can be found and evaluated under different distributions for customers' valuation. This study considers uniform and normal distributions, which are frequently used in the literature. Also, the deterministic valuation case is examined. By comparing the results, one can draw conclusions on how customers' satisfaction level affects the optimal structure of a loyalty reward program.

3.3.1 Uniform Valuation

Assume v_1, first-period buyers' valuation, is uniformly distributed and normalized and rescaled within the range [0,1]. Note that prices, rewards and satisfaction parameter must be normalized with the same factor. Hence, from Equation (3.2), it follows that:

$$\Pi_1 = \begin{cases} 1 - \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma}, & \text{if } \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \le 1 \\ 0, & \text{if } \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \ge 1 \end{cases}. \tag{3.11}$$

As mentioned earlier, Π_1 denotes the probability that a customer makes a purchase in period 1. Similarly, one can find Π_2 from Equation (3.6) as follows:

$$\Pi_2 = \begin{cases} \dfrac{1-(p_2-r-\delta)}{1-\left(\dfrac{p_1+\gamma(p_2-r)}{1+\gamma}\right)}, & \text{if } p_2-r-\delta \geq \dfrac{p_1+\gamma(p_2-r)}{1+\gamma} \text{ and } p_2-r-\delta \leq 1 \\ 0, & \text{if } p_2-r-\delta \geq \dfrac{p_1+\gamma(p_2-r)}{1+\gamma} \text{ and } p_2-r-\delta \geq 1 \\ 1, & \text{if } p_2-r-\delta \leq \dfrac{p_1+\gamma(p_2-r)}{1+\gamma} \end{cases} \quad (3.12)$$

Note that when $p_2 - r - \delta \geq \frac{p_1+\gamma(p_2-r)}{1+\gamma}$, since $\frac{p_1+\gamma(p_2-r)}{1+\gamma} \geq 0$, it follows that $p_2 - r - \delta \geq 0$.

The distribution of v_2^l, the light-users' valuation, is assumed to be identical to that of v_1. Thus, Equation (3.8) can be rewritten as follows:

$$\Pi_2^l = \begin{cases} 1 - p_2, & 0 \leq p_2 \leq 1 \\ 0, & p_2 \geq 1 \end{cases}. \quad (3.13)$$

Now, the firm's expected revenue function (R) can be derived based on Equation (3.9). However, Π_1, Π_2 and Π_2^l are piecewise functions. Thus, R's expression is not unique and depends on the values of p_1, p_2, r, γ and δ. In fact, there are 12 different combinations of Π_1, Π_2 and Π_2^l functions. For instance, consider the following set of conditions:

$$\left\{\frac{p_1+\gamma(p_2-r)}{1+\gamma} \leq 1, \frac{p_1+\gamma(p_2-r)}{1+\gamma} \leq p_2-r-\delta, p_2-r-\delta \leq 1, p_2 \leq 1\right\} \quad (3.14)$$

Finding Π_1, Π_2 and Π_2^l based on the above conditions and substituting them in Equation (3.9), the following R function is obtained:

$$R_1 = p_1\left(1 - \frac{p_1+\gamma(p_2-r)}{1+\gamma}\right) + \gamma(p_2-r)(1-p_2+r+\delta) + (1-\gamma)p_2(1-p_2) \quad (3.15)$$

The subscript of R_1 denotes the combination number. To find the optimal R_1, the following nonlinear programming must be solved:

$$\underset{p_1,p_2,r}{\text{maximize}}\, R_1 = $$

$$p_1\left(1-\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)+\gamma(p_2-r)(1-p_2+r+\delta)+(1-\gamma)p_2(1-p_2) \tag{3.16}$$

Subject to:

$$\frac{p_1+\gamma(p_2-r)}{1+\gamma}\leq 1 \tag{3.16a}$$

$$\frac{p_1+\gamma(p_2-r)}{1+\gamma}\leq p_2-r-\delta \tag{3.16b}$$

$$p_2-r-\delta\leq 1 \tag{3.16c}$$

$$0\leq p_2\leq 1 \tag{3.16d}$$

$$r\leq p_2 \tag{3.16e}$$

$$p_1, p_2 \text{ and } r\geq 0 \tag{3.16f}$$

Constraints of the above NLP are obtained by combining the model-specific constraints (i.e., equations (3.10a) and (3.10b)) and combination-specific constraints (i.e., Equation (3.14)).

In order to find the optimal solution, following the same procedure, all R_k's ($k=$ 1, ...,12) must be found and corresponding NLP's must be formulated. By exploring the objective functions of the resulting 12 models, however, one can see that all of them are dominated by R_1, R_2 or R_3 (depending on p_1, p_2, r, γ and δ values). So, one can disregard the other nine combinations. The next step is to solve the resulting models and compare the optimums in order to obtain the global maximum. Note that γ and δ are modeled as parameters. Hence, the optimal solution is expected to be a function of γ and δ.

As indicated above, R_1 is the total revenue function resulting from the set of constraints given in Equation (3.14). R_2 and R_3, however, are obtained from the conditions given in equations (3.17) and (3.18), respectively:

$$\left\{\frac{p_1+\gamma(p_2-r)}{1+\gamma} \leq 1, p_2-r-\delta \leq \frac{p_1+\gamma(p_2-r)}{1+\gamma}, p_2 \leq 1\right\}, \tag{3.17}$$

$$\left\{\frac{p_1+\gamma(p_2-r)}{1+\gamma} \leq 1, \frac{p_1+\gamma(p_2-r)}{1+\gamma} \leq\ p_2-r-\delta, p_2-r-\delta \geq 1, p_2 \leq 1\right\}. \tag{3.18}$$

Based on the above sets of constraints, it follows that:

$$R_2 = p_1\left(1-\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)+\gamma(p_2-r)\left(1-\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)+(1-\gamma)p_2(1-p_2), \tag{3.19}$$

and

$$R_3 = p_1\left(1-\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)+(1-\gamma)p_2(1-p_2). \tag{3.20}$$

Now, one can find the optimums of R_1, R_2 and R_3 subject to their corresponding constraints. In Appendix 1, it is shown that R_1, R_2 and R_3 are concave over their feasible regions. Moreover, all of their constraints are linear in terms of p_1, p_2 and r. Hence, Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for their optimality. By applying KKT conditions and solving the resulting system for p_1, p_2 and r, optimal solutions of R_1, R_2 and R_3 models are obtained which are presented in Appendix 2 (equations (A2.1), (A2.2) and (A2.4). Comparing these solutions, it can be seen that the global optimum of the firm's total revenue, R^*, is $1/2$ and the corresponding optimal values of decision variables are as follows:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ \forall r \in [-\delta, \frac{1}{2}] \end{pmatrix} & \text{when } \delta \geq -\dfrac{1}{2} \\ \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} & \text{when } \delta \leq -\dfrac{1}{2} \end{cases}. \tag{3.21}$$

From Equation (3.21), it can be seen that when $\delta \geq -1/2$, any reward value in the range $[-\delta,\ 1/2]$ can be optimal given that p_1 is adjusted according to $\gamma r^* + 1/2$.

3.3.2 Normal Valuation

Suppose v_1 follows a standard normal variable. Similar to above, Π_1, Π_2 and Π_2^l can be found based on equations (3.2), (3.6) and (3.8), respectively. This yields

$$\Pi_1 = 1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right), \tag{3.22}$$

$$\Pi_2 = \begin{cases} \dfrac{1 - \Phi(p_2 - r - \delta)}{1 - \Phi\left(\dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right)}, & \text{if } p_2 - r - \delta \geq \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \\ 1, & \text{if } p_2 - r - \delta \leq \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \end{cases} \tag{3.23}$$

and

$$\Pi_2^l = \Pi_1 = 1 - \Phi(p_2), \tag{3.24}$$

where $\Phi(.)$ denotes the standard normal CDF. From Equation (3.9), the firm's revenue functions can be formulated as follows:

$$R_1 = p_1\left(1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right)\right) + \gamma(p_2 - r)\big(1 - \Phi(p_2 - r - \delta)\big) + (1-\gamma)p_2\big(1 - \Phi(p_2)\big) \tag{3.25}$$

and

$$R_2 = p_1\left(1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}\right)\right) + \gamma(p_2 - r)\left(1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}\right)\right) + (1 - \gamma)p_2\big(1 - \Phi(p_2)\big). \tag{3.26}$$

R_1 and R_2 are in fact the two possible combinations of the revenue function which are derived based on the two different sub-domains of Π_2 in Equation (3.23). Thus, there are two NLPs that must be solved to obtain the optimal values of decision variables. To illustrate it, one can see that the NLP corresponding to R_1 is

$$\underset{p_1, p_2, r}{\text{maximize}} \; R_1 = p_1\left(1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}\right)\right) + \gamma(p_2 - r)\big(1 - \Phi(p_2 - r - \delta)\big) + (1 - \gamma)p_2\big(1 - \Phi(p_2)\big) \tag{3.27}$$

Subject to:

$$p_2 - r - \delta \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \tag{3.27a}$$

$$r \leq p_2 \tag{3.27b}$$

$$p_1, p_2 \text{ and } r \geq 0 \tag{3.27c}$$

R_1 and R_2 are both highly nonlinear functions and it can be observed that their Hessian matrices (H_1 and H_2) are not everywhere negative semidefinite. So, R_1 and R_2 are not concave over the entire domains of variables (p_1, p_2 and r) and parameters (γ and δ). However, by checking the conditions under which the Cholesky factorization of $-H_1$ and $-H_2$ exists, it was found that R_1 and R_2 are concave when p_1 and $p_2 \leq 1.4$. Thus, any local maximum is also global in the range of p_1 and $p_2 \in [0,1.4]$. Given this fact, the interior point algorithm of Matlab©, which is based on Byrd et al. (2000), is applied to solve R_1 and R_2 models. As anticipated, any initial solution that satisfies p_1 and $p_2 \in$

[0,1.4] yields the same optimum. In other words, when p_1 and $p_2 \leq 1.4$, the algorithm converges to an identical optimal. The optimal solution is found by running the algorithm for different values of γ and δ and analyzing the structure of results. More specifically, the problem was solved for ten γ values in the range [0, 1] and twenty δ values in the range [-2, 2]. Evaluating the relationship between parameters and the obtained results, the following solution was derived:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma r^* + 0.75 \\ 0.75 \\ \forall r \in [-\delta, 0.75] \end{pmatrix} & \text{when } \delta \geq -0.75 \\ \begin{pmatrix} \gamma r^* + 0.75 \\ 0.75 \\ 0.75 \end{pmatrix} & \text{when } \delta \leq -0.75 \end{cases}, \tag{3.28}$$

which produces $R^* = 0.34$. Note that $p = 0.75$ (rounded to two decimals) is the revenue maximizing price for a single-period pricing problem when customers' valuation follows standard normal distribution.

In order to evaluate local maxima when p_1 or $p_2 > 1.4$, the multi-start procedure proposed by Ugray et al. (2007) is applied. This procedure is claimed to be one of the most effective global optimization algorithms (see Lasdon et al. 2010). In more detail, the multi-start method was applied for different values of parameters and the resulting maxima were compared to $R^* = 0.34$. Accordingly, no better solution was achieved, which suggests that the solution in Equation (3.28) can be the global maximum.

3.3.3 Deterministic Valuation

Here, the case where customers' valuations are equal and deterministic is evaluated. Thus, v_1 is a fixed value which can be modeled as a parameter, v. Consequently, Π_1, which was originally defined as the probability of making a purchase in period 1, becomes an indicator function as follows:

$$\Pi_1 = \begin{cases} 1 & \text{if } v \geq \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \\ 0 & \text{if } v < \dfrac{p_1 + \gamma(p_2 - r)}{1+\gamma} \end{cases} \tag{3.29}$$

Equation (3.29) implies that customers in period 1 will *all* buy or reject the firm's offer. Similarly, one can derive the indicator functions corresponding to Period 1 buyers and light users in period 2 as follows:

$$\Pi_2 = \begin{cases} 1 & \text{if } v \geq p_2 - r - \delta \\ 0 & \text{if } v < p_2 - r - \delta \end{cases} \tag{3.30}$$

and

$$\Pi_2^l = \begin{cases} 1 & \text{if } v \geq p_2 \\ 0 & \text{if } v < p_2 \end{cases}. \tag{3.31}$$

Substituting Π_1, Π_2 and Π_2^l in Equation (3.9), eight distinct revenue functions are obtained. It can be seen that all of them are dominated by R_1, the revenue function resulting from $\Pi_1 = \Pi_2 = \Pi_2^l = 1$. Thus, based on the model in Equation (3.10), the optimal values of the firm's decision factors are determined by solving the following linear programming:

$$\underset{p_1,p_2,r}{\text{maximize}}\ R_1 = p_1 + \gamma(p_2 - r) + (1-\gamma)p_2 \tag{3.32}$$

Subject to:

$$\frac{p_1 + \gamma(p_2 - r)}{1+\gamma} \leq v \tag{3.32a}$$

$$p_2 - r - \delta \leq v \tag{3.32b}$$

$$p_2 \leq v \tag{3.32c}$$

$$r \leq p_2 \tag{3.32d}$$

$$p_1, p_2 \text{ and } r \geq 0 \tag{3.32e}$$

As in the uniform valuation case, the first three constraints of the above model are enforced by the domain of the customers' decision indicator functions.

Exploring the constraints of the model in Equation (3.32), one can see that the model is infeasible when $v + \delta < 0$. In fact, $v + \delta < 0$ violates the second constraint, knowing that $p_2 - r \geq 0$. In other words, when customers' dissatisfaction level is less than $-v$, the model in Equation (3.32) is no longer applicable. In this case (i.e., when $v + \delta < 0$), based on the domains of Equation (3.30), it follows that $\Pi_2 = 0$. This forms another combination of the revenue function, R_2, which is obtained from $\Pi_1 = \Pi_2^l = 1$ and $\Pi_2 = 0$. The corresponding LP model is

$$\underset{p_1, p_2, r}{\text{maximize}}\ R_2 = p_1 + (1 - \gamma)p_2 \tag{3.33}$$

Subject to:

$$\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \leq v \tag{3.33a}$$

$$p_2 - r - \delta \geq v \tag{3.33b}$$

$$p_2 \leq v \tag{3.33c}$$

$$r \leq p_2 \tag{3.33d}$$

$$p_1, p_2 \text{ and } r \geq 0 \tag{3.33e}$$

Now, one can find the optimums by solving Equations (3.32) and (3.33). In order to find the optimal solution in terms of parameters δ and γ, the Lagrange method was employed along with the complementary slackness theorem, which resulted in a linear system of size seven for each model. Each system was solved by considering the primal and dual feasibility conditions. By comparing the optimal solutions of the mentioned two models, the following global maximum was obtained for the case valuation is constant:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma r^* + v \\ v \\ \forall r \in [-\delta, v] \end{pmatrix} & \text{when } \delta > -v \\ \begin{pmatrix} \gamma r^* + v \\ v \\ v \end{pmatrix} & \text{when } \delta \leq -v \end{cases}, \tag{3.34}$$

which yields $R^* = 2v$.

3.4 Model Extension

In this section, the basic model is extended to the case where customers anticipate a change in their future valuation while making decision in period 1. As discussed earlier, the customers' surplus in period 1 is derived based on the assumption that they expect no changes in their valuation in period 2. This section analyzes the effect of customers' uncertainty about their future valuation on the framework of the results. In fact, it is more realistic to incorporate this uncertainty in the customers' decision-making. Now, the customers' surplus from buying in period 1 is:

$$S_1 = [v_1 - p_1] + \gamma[v_1 + \epsilon - (p_2 - r)]. \tag{3.35}$$

where ϵ captures the customers' anticipated change in their valuations for the product in period 2. Note that ϵ is the anticipated change in the *future* valuations when customers are in period 1 and it differs from δ which is the shift in valuation that customers *realize* in period 2. It is assumed that ϵ is a random variable independent from v_1 with mean zero and a known variance, σ^2. Here, we focus on the normal valuation case. More specifically,

v_1 follows a standard normal distribution and ϵ is normally distributed with a mean of zero and a variance of σ^2. Thus, the probability that a customer makes a purchase in period 1 can be found as:

$$\Pi_1 = \Pr(S_1 \geq 0) = \Pr\left(v_1 + \frac{\gamma}{1+\gamma}\epsilon \geq \frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right). \tag{3.36}$$

For notational simplicity let

$$v_1' = v_1 + \frac{\gamma}{1+\gamma}\epsilon. \tag{3.37}$$

v_1' follows a normal distribution with mean 0 and variance $1 + (\gamma\sigma/(1+\gamma))^2$. Now, Equation (3.36) can be rewritten as:

$$\Pi_1 = 1 - F_{v_1'}\left(\frac{p_1 + \gamma(p_2 - r)}{1+\gamma}\right) \tag{3.38}$$

where $F_{v_1'}(.)$ denotes the CDF of v_1'.

To formulate a customer's purchase probability in period 2, Equation (3.5) can be rewritten as follows:

$$\Pi_2 = \Pr\left(v_1 \geq p_2 - r - \delta \,|v_1' \geq \frac{p_1 - \gamma(p_2 - r)}{1+\gamma}\right). \tag{3.39}$$

To find the explicit expression of Π_2 one needs to find the joint distribution of v_1 and v_1'. Note that v_1 and v_1' are both a linear combination of v_1 and ϵ. Since v_1 and ϵ are independent and normally distributed, it can be shown that (v_1, v_1') follows a bivariate normal distribution (see, e.g., Bertsekas & Tsitsiklis 2002) with the mean vector and covariance matrix of:

$$M = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1 & 1 \\ 1 & 1 + \left(\frac{\gamma\sigma}{1+\gamma}\right)^2 \end{bmatrix}. \tag{3.40}$$

Thus, the joint probability density function of (v_1, v_1') is:

$$f_{v_1,v_1'}(V_1,V_1') = \frac{1}{2\pi\left(\frac{\gamma\sigma}{1+\gamma}\right)} \exp\left(\frac{-(\gamma^2\sigma^2+(1+\gamma)^2)}{2\gamma^2\sigma^2}\left(V_1^2 + \frac{{V_1'}^2}{1+\left(\frac{\gamma\sigma}{1+\gamma}\right)^2} - \frac{2V_1V_1'}{1+\left(\frac{\gamma\sigma}{1+\gamma}\right)^2}\right)\right). \tag{3.41}$$

Now, based on Equation (3.39), Π_2 can be explicitly expressed in terms of model variables and parameters:

$$\Pi_2 = \frac{1 - \int_{\frac{p_1+\gamma(p_2-r)}{1+\gamma}}^{\infty} \int_{p_2-r-\delta}^{\infty} f_{v_1,v_1'}(V_1,V_1')\ dV_1 dV_1'}{1 - F_{v_1'}\left(\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)}. \tag{3.42}$$

The probability that a light user buys in period 2 is not influenced by the model extension and follows Equation (3.24). The optimization problem is formulated and solved in Appendix 3. As discussed in Appendix 3, the model yields the following optimal solution:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{pmatrix} \gamma r^* + 0.75 \\ 0.75 \\ \forall r \in [\rho_l, 0.75] \end{pmatrix}. \tag{3.43}$$

where ρ_l is the lower limit of optimal reward values and depends on γ, δ and σ^2, the model parameters. While the analysis of relationship between ρ_l and parameters is insightful, here the focus is only on the effect of σ^2, the variance in the customers' anticipated valuation shift, on the framework of previous findings. In other words, the objective is to evaluate whether inclusion of customers' anticipated valuation alters the structure of previous results. Comparing the solutions in equations (3.28) and (3.43), it is evident that the structure of optimal solution is consistent across both models. More specifically, it can be seen that p_1^* and p_2^* remain unaffected. r^* also falls in a range under both models, however, in the extended model the lower bound of optimal reward range, depends not only on δ but also on γ and σ^2**.** In conclusion, it is observed that whether or not one incorporates customers' anticipation of their future tastes in the model, the framework of the results remains unchanged.

3.5 Discussion

Comparing the optimal solutions under different valuation distributions (i.e., equations (3.21), (3.28) and (3.34)), it can be observed that they follow the same structure. For instance, no matter the distribution of v, there is a negative δ threshold (or a *dissatisfaction* threshold) below which the firm must offer a free product to loyal buyers in period 2 to maximize its revenue. For example, when δ is uniformly distributed, Equation (3.21) shows that for $\delta \leq -\frac{1}{2}$, $p_2^* = r^* = \frac{1}{2}$, which implies a free product for loyal buyers in period 2. Offering a free product in the second period is optimal because it motivates the customers to pay higher prices in period 1. This is evident from $p_1^* = \gamma r^* + \frac{1}{2}$, based on which the first period's price increases with reward. The same inference holds for the normal and deterministic valuation.

Equations (3.21), (3.28) and (3.24) also show that p_1^* is directly proportional to γr^*. This implies that when repurchase intention is higher among customers, at a given value of reward, the firm is better off charging higher prices in earlier periods. The intuition is that when customers are more likely to repurchase, the loyalty program imposes more expenses on the firm, because more customers will take advantage of the firm's *promise* to reward repurchase. Thus, the firm must offer higher prices in the first period to offset the reward cost. The increased p_1 deflects some customers who would have otherwise made a purchase; however, the total revenue increases as the firm charges higher prices to those who are willing to pay more (i.e., who have a higher valuation for the product).

Moreover, it can be inferred that under optimal conditions the firm exploits the second period price to capture the highest possible revenue from light users. This is evident from the fact that $p_2^* = \frac{1}{2}$, $p_2^* = .7518$ and $p_2^* = v$ will maximize the revenue from the light-user segment in period 2 when customers' valuation is uniform, normal and deterministic, respectively. The first period price and the loyalty reward are then adjusted to maximize the revenue from potential two-period buyers in periods 1 and 2. This confirms the intuitive fact that the added degree of freedom provided by the loyalty reward program is used to provoke loyalty among "heavy users".

The relationship between optimal loyalty reward (i.e., r^*) and the post-purchase shift in customers' valuation (i.e., δ) is of great importance, because this relationship helps to evaluate the effect of customers' satisfaction level on the optimal structure of the loyalty program. Here, the uniformly distributed valuation is considered. It can be seen that the conclusions hold for other cases as well. According to Equation (3.17), when $\delta \le -\frac{1}{2}$, $r^* = p_2^* = \frac{1}{2}$. That is, with the *prior* knowledge that customers will experience a drastic drop in their valuation, the firm should offer a free product to loyal customers in the second period. As mentioned above, this offer motivates customers to make a purchase in the first period to become qualified for the loyalty reward in the second period. Thus, the revenue that the firm makes in the first period offsets the loss of offering the free product in the second period.

However, when $\delta \geq -\frac{1}{2}$ (i.e., when customers are either satisfied with their purchase in the first period ($\delta \geq 0$) or they experience a moderate dissatisfaction ($-\frac{1}{2} \leq \delta < 0$), any loyalty reward value in the range of $[-\delta, \frac{1}{2})$ is optimal. Note that $p_1^* = \gamma r^* + \frac{1}{2}$. Thus, if higher loyalty rewards are offered to loyal customers, the firm must charge a higher price in the first period in order to gain the optimal revenue. When $\delta \geq 0$, based on Equation (3.17), the interval $[0, \frac{1}{2})$ is optimal for r. Since the fixed costs associated with the loyalty program is not incorporated in the model, one can conclude that $r = 0$ is the optimal value of reward. This is because, by not offering loyalty rewards, the firm will not incur the mentioned fixed expenses. In other words, if the firm preserves satisfaction among customers, not offering a loyalty reward is optimal.

On the other hand, for $-\frac{1}{2} \leq \delta < 0$, $r = 0$ is no longer optimal. In fact, as δ tends from 0 to $-\frac{1}{2}$, the optimal range of reward values becomes smaller. At the marginal value of $\delta = -\frac{1}{2}$, the reward must be equal to the offered price in period 2. Hence, with the prior knowledge that customers' valuation will decrease in the second period, it is optimal for the firm to offer a loyalty reward.

Based on the above discussion, one may argue that, regardless of the value of δ, the firm is able to adjust p_1, p_2 and r so that it obtains the maximum total revenue (e.g., $R^* = \frac{1}{2}$ when valuation is uniformly distributed). This conclusion holds provided that the firm precisely estimates the post-purchase shift in customers' valuation. However, there are various endogenous and exogenous factors affecting customers' valuations in the second period. Thus, it is likely that the firm underestimates or overestimates the δ value.

Consequently, the firm may obtain a suboptimal revenue. For instance, consider the case in which the firm predicts that customers will become satisfied with their purchase in the first period and, hence, does not offer the loyalty reward. If customers turn out to be dissatisfied (i.e., $\delta < 0$), $r = 0$ is suboptimal. Since p_1^* is also a function of r^*, a suboptimal r results in a suboptimal p_1. Therefore, overestimating the value of δ reduces the firm's total revenue. This special case shows how the effectiveness of loyalty reward programs is tied to customers' overall satisfaction level.

3.6 Summary and Conclusion

In this chapter, an analytical model is developed to evaluate the effect of customers' satisfaction on the loyalty programs' profitability. The objective of the model was to maximize the revenue of a firm selling a good or service through two periods in terms of its decision variables, that is, offered prices in each period and loyalty reward value. It was assumed that the firm pre-commits to the first and second period prices and loyalty reward. Customers who repeat their purchase in the second period earn the loyalty reward in the form of an absolute value. However, it can be seen that using a percent discount for reward will not affect the findings.

One of the distinctive aspects of the developed model is that the satisfaction level is incorporated as a factor in customers' decision to buy. The satisfaction level was modeled as a parameter, δ, which is added to/subtracted from customers' valuation in the first period to form their valuation in the second period. Another factor in forming customers' decision was their probability of repatronizing the firm in the second period. This was also modeled as a parameter, γ. The conclusions were drawn based on how these two parameters affect optimal settings of the loyalty program.

Optimal values of the firm's decision variables were found under three different valuation settings: uniformly distributed, normally distributed and deterministic valuation. The uniform and normal cases resulted in nonlinear models. Despite the complexity of the models, analytical solutions were derived in terms of the parameters. For deterministic valuation, a linear programming was obtained which was also solved in terms of γ and δ. It was observed that regardless of the valuation distribution, the optimal solutions follow the same structure. Moreover, the effect of customers' anticipation of their future valuation on the results was investigated. It was observed that inclusion of foreseen future valuation would not affect the framework of results.

Based on the obtained optimal solutions, the relationship between customers' satisfaction level and profitability of loyalty programs was established. It was shown that if the firm manages to maintain satisfaction among customers, not offering a loyalty reward is optimal from the profitability perspective. However, if the firm decides not to implement a loyalty program and customers experience dissatisfaction with their purchase in early periods, the firm will obtain suboptimal revenue. It was also seen that by offering a loyalty reward, the firm must increase the first period price proportional to the reward value and to the degree to which customers intend to return in the second period.

Chapter 4

The Effect of Customer Satisfaction on Loyalty Programs' Profitability: Probabilistic Satisfaction

4.1 Introduction

As discussed in Chapter 2, previous analytical studies on loyalty programs have not considered the customers' purchase experience as a factor in their decision to buy. This indicates the implicit assumption that customers always return to the firm with the same valuation for the product being offered, no matter whether they are satisfied or dissatisfied with their past purchases. In Chapter 3, a model was developed to evaluate the effect of customer satisfaction level on a loyalty programs' profitability. Customer satisfaction was captured by the shift in customers' valuation for the product through two periods, which was denoted by δ. In Chapter 3, δ was treated as a parameter. In this chapter, the analysis is extended by incorporating the satisfaction level as a random variable. The objective of this study is to evaluate how introducing a random satisfaction level affects the framework of the findings in Chapter 3.

Similar to the previous chapter, to capture the uncertainty in customers' preferences, the customers' valuation is incorporated into the model as a random variable. Specifically, it is assumed that customers' valuation for the product is normally distributed.

The objective of the model is to maximize the firm's expected revenue function in terms of its decision variables. The model is formulated as a stochastic programming problem with a nonlinear non-convex objective function, which must be solved in terms of three parameters. Here, the Matlab©'s interior-point algorithm is employed to find the local optimums. More specifically, the model is solved for some pre-determined sets of parameter values. Subsequently, the functional form of the optimal solution will be derived by analyzing the relationship between optimal solutions and the corresponding parameters' values. To ensure the obtained optimums are global maxima, the multi-start procedure of Ugray et al. (2007) is applied.

The results reveal remarkable insights into the effectiveness of loyalty programs. It will be shown that, depending on the mean and variance of customers' satisfaction levels, the firm may be better off not to offer a loyalty reward. More specifically, the firm will maximize its revenue if it maintains a positive satisfaction level among customers with a variance less than a certain threshold. Moreover, it will be observed that the optimal prices increase with the loyalty reward and the customers' intentions to repurchase the product.

The rest of this chapter is organized as follows: The model is formulated in Section 4.2. Section 4.3 describes the procedure to solve the model. The obtained results are discussed in Section 4.4. Section 4.5 concludes the chapter with a summary of findings.

4.2 Model Formulation

Similar to the model developed in Section 3.2, the model consists of a revenue-maximizing firm selling a good or service through two periods. The firm pre-commits to the price of the product in each period (i.e., p_1 and p_2) and the loyalty reward (i.e., r). Customers earn the loyalty reward if they make purchases in both periods. The reward is offered in the form of an absolute value in the second period. The objective is to maximize the firm's total revenue in terms of its decision variables, that is, p_1, p_2 and r.

Customers' decision-making is modeled using the surplus they obtain from their purchases. The surplus is the difference between a customer's valuation for the product and the amount he/she must pay to buy it. If a customer gets a nonnegative surplus, he/she will make a purchase.

The surplus from the purchase in period 1 (S_1), can be expressed as the sum of two terms: the surplus resulting from buying in the first period and the expected surplus from the purchase in the second period, that is,

$$S_1 = [v_1 - p_1] + \gamma[v_1 - (p_2 - r)], \tag{4.1}$$

where v_1 denotes the customer's valuation in the first period and γ denotes the probability of a first-period buyer to repatronize the firm in the second period. Thus, γ represents the buyer's intention to return to the firm in period 2. As stated before, this is to capture the effect of customers who sign up for the loyalty program, but fail to show up in the next period, mainly because of a low consumption rate. γ will be treated as a parameter in the model. Similar to the earlier models, the discount factor is assumed to be one.

Customers may differ in their valuations, but it is assumed that they have enough information to determine the product's utility. The firm, on the other hand, is unaware of each individual's valuation; however, the aggregate-level valuation distribution is known to the firm. Here, the specific, but commonly studied, case is considered where v_1 is normally distributed.

As mentioned above, a customer buys if he/she earns a nonnegative surplus from the purchase. Assuming that v_1 is normalized to follow a standard normal distribution[1], from Equation (4.1), one can find the probability of a customer making a purchase as follows:

$$\Pi_1 = \Pr(S_1 \geq 0) = 1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}\right), \tag{4.2}$$

where $\Phi(.)$ denotes the standard normal CDF.

$1 - \Pi_1$ fraction of customers who have not accepted the offer in the first period exit the firm's market. γ fraction of buyers, on the other hand, proceed to the second period. Since these customers have made a purchase in period 1, they are eligible for the loyalty reward. Thus, the first-period buyers should pay $p_2 - r$ for the product in period 2. Subsequently, a customer's surplus from buying a unit of the product in period 2 is:

$$S_2 = v_2 - (p_2 - r) \tag{4.3}$$

[1] p_1, p_2, r and δ must be normalized with the same factor used to normalize v_1.

where v_2 is the customer's valuation for the product in period 2.

Similar to Chapter 3, here v_2 is neither equal to nor independent from v_1. To model this dependency, it is assumed that $v_2 = v_1 + \delta$, where δ represents the shift in each individual customer's valuation. Essentially, δ serves as the customer's satisfaction level with the purchase in period 1. A negative δ signifies dissatisfaction with the product itself and/or with the firm's quality of service. A positive δ, on the contrary, indicates that the customer has been satisfied with the past purchase..

Here, variability in customers' satisfaction levels is allowed. In other words, it is assumed that customers' satisfaction may vary independently from their initial valuation. Specifically, it is assumed that, at the aggregate level, δ is normally distributed with parameters (μ_s, σ_s). Since v_1 and δ are added up to form v_2, they must have been normalized with the same factor. Thus, without loss of generality, it is assumed that μ_s and σ_s are obtained by normalizing the satisfaction level with the same factor used to normalize v_1.

There are many endogenous and exogenous factors influencing customers' perceived satisfaction. As a result, it is not plausible to assume μ_s and σ_s are primarily known to the firm. So, μ_s and σ_s are modeled as parameters, and through a sensitivity analysis, the effect of these parameters on the performance of loyalty programs is evaluated.

A customer in the second period makes a purchase if $S_2 \geq 0$. Since $v_2 = v_1 + \delta$, v_1 and v_2 are clearly dependent, and thus, so are S_1 and S_2. Hence, a customer's probability of earning a nonnegative surplus in the second period is conditioned on the fact that his/her surplus has been nonnegative in the first period. That is,

$$\Pi_2 = \Pr(S_2 \geq 0 \mid S_1 \geq 0), \tag{4.4}$$

where Π_2 is the probability of making a purchase in period 2. Substituting S_1 and S_2 from equations (4.1) and (4.3) into the above expression and rearranging them, it follows that:

$$\Pi_2 = \Pr\left(v_2 \geq p_2 - r \mid v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right). \tag{4.5}$$

Applying conditional probability theory, the above equation can be rewritten as:

$$\Pi_2 = \frac{\Pr\left(v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma},\ v_2 \geq p_2 - r \right)}{\Pr\left(v_1 \geq \frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right)}. \tag{4.6}$$

In order to find the explicit expression of Π_2 as a function of the model variables and parameters, one must find the joint distribution of v_1 and v_2. In Appendix 4, it is shown that v_1 and v_2 follow a bivariate normal distribution with the following mean vector and covariance matrix:

$$M = \begin{bmatrix} 0 \\ \mu_s \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1 & 1 \\ 1 & 1 + \sigma_s^2 \end{bmatrix}. \tag{4.7}$$

The resulting pdf can be found as:

$$f_{v_1, v_2}(V_1, V_2) = \frac{1}{2\pi\sigma_s} \exp\left(-\frac{(1 + \sigma_s^2)}{2\sigma_s^2} \left(V_1^2 + \frac{(V_2 - \mu_s)^2}{1 + \sigma_s^2} - \frac{2V_1(V_2 - \mu_s)}{1 + \sigma_s^2} \right) \right). \tag{4.8}$$

Now, based on the above pdf, Π_2 in Equation (4.6) can be restated as:

$$\Pi_2 = \frac{\int_{p_2 - r}^{\infty} \int_{\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma}}^{\infty} f_{v_1, v_2}(V_1, V_2)\ dV_1 dV_2}{1 - \Phi\left(\frac{p_1 + \gamma(p_2 - r)}{1 + \gamma} \right)}. \tag{4.9}$$

As mentioned earlier, Π_2 denotes the probability of a first-period buyer to repurchase in the second period.

The market size in the first period is normalized to one. Similar to the model formulated in Section 3.2, it is assumed that a group of light users joins the firm's market in the second period with the size of $(1-\gamma)$. This assumption is mainly made to avoid market expansion or contraction due to customers' consumption rate affecting the results (Singh et al. 2008). Light users are one-time buyers, so they do not take the loyalty reward into account. Thus, their surplus from making a purchase is:

$$S_2^l = v_2^l - p_2. \tag{4.10}$$

v_2^l is the light-users' valuation for the product. It is reasonable to assume v_2^l follows the same distribution as v_1, since light users are similar to customers in period 1 in the sense that they have not yet experienced the product. So, a light-user makes a purchase with the probability of:

$$\Pi_2^l = Pr\left(S_2^l \geq 0\right) = 1 - \Phi(p_2), \tag{4.11}$$

Now, one can formulate the firm's expected revenue function based on the customers' probabilities of buying the product in each period, as follows:

$$R = p_1\Pi_1 + \gamma(p_2 - r)\Pi_1\Pi_2 + (1-\gamma)p_2\Pi_2^l. \tag{4.12}$$

R is a function of the firm's decision variables (i.e., p_1, p_2 and r) and the model parameters (i.e., γ, μ_s and σ_s). The purpose is to maximize the revenue function with respect to the decision variables. The optimal solution is expected to depend on the parameters. Such a solution yields valuable insights into how customers' satisfaction influences the optimal structure of a loyalty program. The optimization is, of course, subject to the non-negativity of the prices and reward. Moreover, the offered reward cannot exceed the product price in period 2. The resulting maximization problem is:

$$\underset{p_1,p_2,r}{\text{maximize}}\ R =$$

$$p_1\left(1-\Phi\left(\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)\right)+$$

$$\gamma(p_2-r)\left(\int_{p_2-r}^{\infty}\int_{\frac{p_1+\gamma(p_2-r)}{1+\gamma}}^{\infty}\frac{1}{2\pi\sigma_s}\exp\left(-\frac{(1+\sigma_s^2)}{2\sigma_s^2}\left(V_1^2+\frac{(V_2-\mu_s)^2}{1+\sigma_s^2}-\frac{2V_1(V_2-\mu_s)}{1+\sigma_s^2}\right)\right)\ dV_1dV_2\right)+$$

$$(1-\gamma)p_2\big(1-\Phi(p_2)\big) \tag{4.13}$$

Subject to:

$$r \le p_2 \tag{4.13a}$$

$$p_1, p_2 \text{and } r \ge 0 \tag{4.13b}$$

4.3 Results

The revenue function in Equation (3.13) is highly nonlinear. This can be evaluated using the eigenvalues of its Hessian matrix. Evaluating the eigenvalues of its Hessian matrix, it can be seen that R is not a concave function. In more detail, there are some points at which some eigenvalues of the Hessian of R are positive. Hence, it can be inferred that the Hessian of R is not a negative semidefinite matrix, and thus, R is not a concave function. This conclusion holds based on the fact that a function is concave if and only if its Hessian matrix is negative semidefinite. Moreover, a symmetric matrix is negative semidefinite if and only if all its eigenvalues are non-positive (Searle 1982, p.309). Therefore, since the Hessian is a symmetric matrix, a positive eigenvalue violates the concavity condition. For instance given $p_1 = 1.5,\ p_2 = 1.5,\ r = 1,\ \gamma = 0.5,\ \mu_s = 1, \sigma_s = 1$, the Hessian matrix is

$$\mathrm{H} = \begin{bmatrix} -0.09 & -0.04 & 0.04 \\ -0.04 & -0.01 & 0.03 \\ 0.04 & 0.03 & -0.03 \end{bmatrix}. \tag{4.14}$$

Eigenvalues of the above matrix are

$$\{-0.12, -0.01, 0.01\}. \tag{4.15}$$

As can be seen, two eigenvalues are negative and one is positive. This example shows that R is not generally a concave (nor a convex) function. As a result, standard optimization algorithms are not guaranteed to converge to the global optimum. To overcome this challenge, first the Matlab©'s interior-point algorithm is employed to find the local maxima. Subsequently, a global search method will be invoked to investigate if any better solutions exist.

As mentioned earlier, the optimal solution will be a function of the parameters, γ, μ_s and σ_s. In order to derive the optimum in terms of the parameters, the model is solved for pre-determined sets of parameters' values. Then, the function is extracted by evaluating the relationship between the obtained optimums and the corresponding values of parameters. More specifically, ten values in the range [0, 1], ten points in the range [-2, 2] and five values in the interval [0.1, 2] are chosen for γ, μ_s and σ_s, respectively. The values are all equally spaced in their respective ranges. Next, the model should be solved for all 500 combinations of parameter values. To illustrate it, some of the solutions obtained from the interior-point algorithm are presented in Table 4.1. Evaluating the relationship between the optimal values of the decision variables (i.e., p_1^*, p_2^* and r^*) and the parameters' values, it can be seen that:

$$p_1^* = \gamma r^* + 0.75 \tag{4.16}$$

$$p_2^* = 0.75 \tag{4.17}$$

Table 4.1: Results of the interior point algorithm*

μ_s	σ_s	γ	p_1^*	p_2^*	r^*	R^*
-1	0.1	0.1	0.83	0.75	0.75	0.34
-1	0.1	0.9	1.43	0.75	0.75	0.34
-1	0.3	0.1	0.83	0.75	0.75	0.34
-1	0.3	0.9	1.43	0.75	0.75	0.34
-0.5	0.1	0.1	0.83	0.75	0.75	0.34
-0.5	0.1	0.9	1.43	0.75	0.75	0.34
-0.5	0.3	0.1	0.83	0.75	0.75	0.34
-0.5	0.3	0.9	1.43	0.75	0.75	0.34
-0.2	0.1	0.1	0.81	0.75	0.64	0.34
-0.2	0.1	0.9	1.36	0.75	0.67	0.34
-0.2	0.3	0.1	0.83	0.75	0.75	0.34
-0.2	0.3	0.9	1.43	0.75	0.75	0.34
0	0.1	0.1	0.80	0.75	0.48	0.34
0	0.1	0.9	1.21	0.75	0.51	0.34
0	0.3	0.1	0.83	0.75	0.75	0.34
0	0.3	0.9	1.43	0.75	0.75	0.34
0.2	0.1	0.1	0.79	0.75	0.38	0.34
0.2	0.1	0.9	1.09	0.75	0.38	0.34
0.2	0.3	0.1	0.83	0.75	0.75	0.34
0.2	0.3	0.9	1.43	0.75	0.75	0.34
0.5	0.1	0.1	0.79	0.75	0.38	0.34
0.5	0.1	0.9	1.09	0.75	0.38	0.34
0.5	0.3	0.1	0.82	0.75	0.74	0.34
0.5	0.3	0.9	1.42	0.75	0.75	0.34
1	0.1	0.1	0.79	0.75	0.38	0.34
1	0.1	0.9	1.09	0.75	0.38	0.34
1	0.3	0.1	0.80	0.75	0.46	0.34
1	0.3	0.9	1.25	0.75	0.56	0.34

* Values are calculated to 15 decimal places, but for simplicity they are displayed with two decimals.

Note that $p = 0.75$ is, in fact, the revenue-optimizing price in a single-period pricing problem where customers' valuation follows the standard normal distribution. This solution shows that under optimal conditions, p_2 is set to maximize the revenue from the light users. On the other hand, p_1^* increases with the offered reward and the customers' level of intention to rebuy the product. Moreover, as discussed in the previous chapter, if the firm precisely estimates the distribution of δ, it will able to adjust p_1, p_2 and r so that it obtains the optimal total revenue (i.e., $R^* = 0.34$). Since estimating the true distribution of δ is practically impossible, the firm may obtain suboptimal revenues.

From the obtained optimal solutions, it can be inferred that r^*, the optimal reward value, depends on γ, μ_s and σ_s. In order to derive r^* in terms of parameters, one can substitute p_1^* and p_2^* from equations (4.16) and (4.17) into the original model (Equation (4.13)), and find the maximum with respect to r. The resulting optimization model is:

$$\underset{r}{\text{maximize}}\ R_2 =$$

$$0.2261(\gamma r + 0.75) +$$

$$\gamma(0.75 - r)\left(\int_{0.75-r}^{\infty}\int_{0.75}^{\infty}\frac{1}{2\pi\sigma_s}exp\left(-\frac{(1+\sigma_s^2)}{2\sigma_s^2}\left(V_1^2 + \frac{(V_2-\mu_s)^2}{1+\sigma_s^2} - \frac{2V_1(V_2-\mu_s)}{1+\sigma_s^2}\right)\right)\ dV_1 dV_2\right) +$$

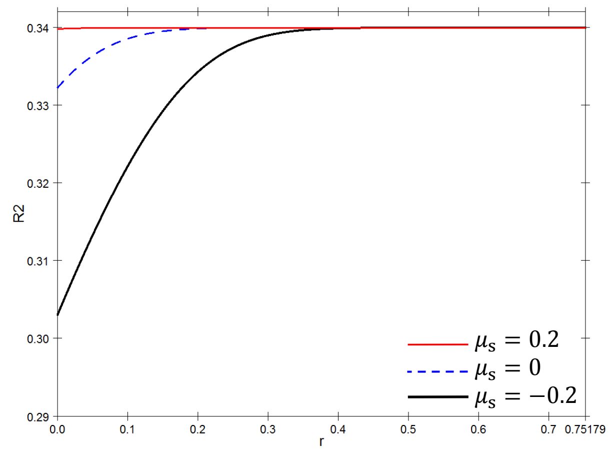
$$0.2261 \tag{4.18}$$

Subject to:

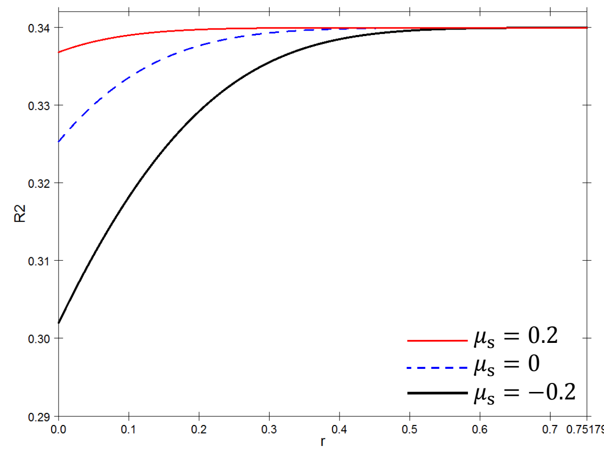
$$0 \leq r\ \ 0.75 \tag{4.18a}$$

Figure 4.1 illustrates R_2 in the above model versus r for some specific values of γ, μ_s and σ_s. Based on the Constraint (4.18a), the domain of R_2 is restricted to $r \in [0,0.75]$. From this figure, it can be seen that $r = p_2^* = 0.75$ is the revenue-maximizing reward. This conclusion can be verified by substituting $r^* = 0.75$ into the revenue function in Equation (4.18) which yields $R^* = 0.34$. However, depending on the parameters' values, lower rewards may also lead to the maximum revenue. That is, there might be an optimal *range* of reward values. For example, Figure 4.1(a) shows that when $\mu_s = 0.2$, $\gamma = 0.9$ and $\sigma_s = 0.1$, roughly any r in the range [0,0.75] is optimal. Note that, based on Equation

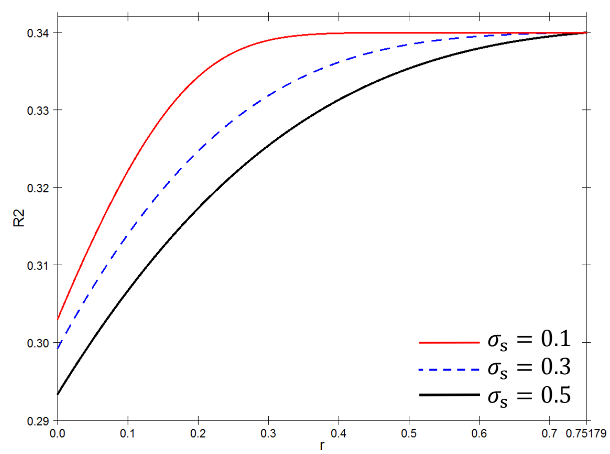
(4.16), p_1^* increases with r^*. That is, the price of the product in period 1 must be adjusted based on the reward in the second period. In other words, depending on the parameters'



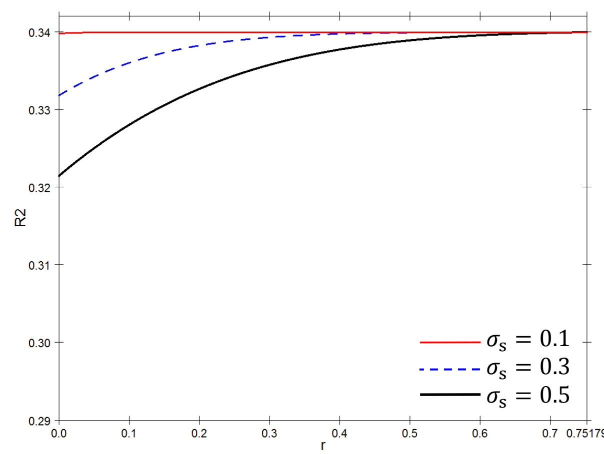
(a) $\gamma = 0.9$, $\sigma_s = 0.1$



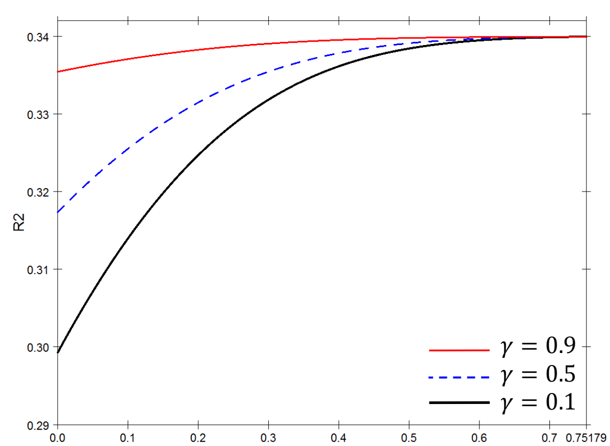
(b) $\gamma = 0.9$, $\sigma_s = 0.2$



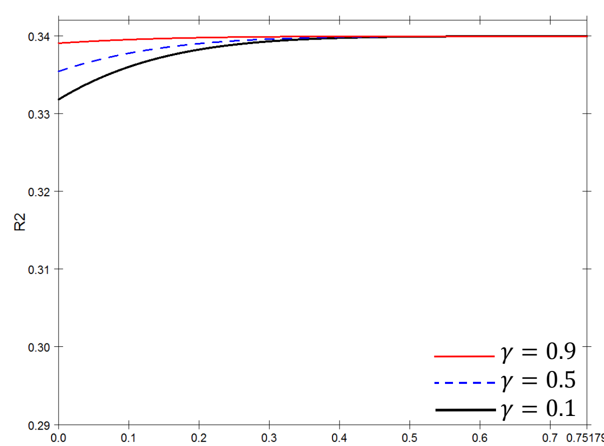
(c) $\gamma = 0.9$, $\mu_s = -0.2$



(d) $\gamma = 0.9$, $\mu_s = 0.2$



(e) $\mu_s = -0.2$, $\sigma_s = 0.2$



(f) $\mu_s = 0.2$, $\sigma_s = 0.2$

Figure 4.1: R_2 versus r for different values of parameters

values, the model may yield alternate optimal solutions. The implications of this conclusion in terms of each parameter will be discussed in the next section.

In summary, the optimal solution to the model in Equation (4.13) can be formulated as follows:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{pmatrix} \gamma r^* + 0.75 \\ 0.75 \\ \forall r \in [\rho_l, 0.75] \end{pmatrix}, \tag{4.19}$$

where ρ_l is the lower bound of the optimal loyalty reward. ρ_l depends on γ, μ_s and σ_s. This relationship will be addressed later and it will be shown that ρ_l ranges from 0 to 0.75. The solution in Equation (4.19) yields $R^* = 0.34$ which can be used as a basis to verify whether it is a global optimum. More specifically, one can employ the multi-start procedure proposed by Ugray et al. (2007) to check if any better solution exists. The result of this analysis for some certain values of parameters is presented in Table 4.2, which indicates that the solution in Equation (4.19) can be the global maximum.

Table 4.2: Results of the global search algorithm

μ_s	σ_s	γ	p_1^*	p_2^*	r^*	R^*
-0.2	0.1	0.1	0.83	0.75	0.75	0.34
-0.2	0.1	0.9	1.43	0.75	0.75	0.34
-0.2	0.3	0.1	0.83	0.75	0.75	0.34
-0.2	0.3	0.9	1.43	0.75	0.75	0.34
0	0.1	0.1	0.83	0.75	0.75	0.34
0	0.1	0.9	1.43	0.75	0.75	0.34
0	0.3	0.1	0.83	0.75	0.75	0.34
0	0.3	0.9	1.43	0.75	0.75	0.34
0.2	0.1	0.1	0.83	0.75	0.75	0.34
0.2	0.1	0.9	1.43	0.75	0.75	0.34
0.2	0.3	0.1	0.83	0.75	0.75	0.34
0.2	0.3	0.9	1.43	0.75	0.75	0.34

4.4 Discussion

The effect of each parameter on the optimal reward can be analyzed using Figure 4.1. Figure 4.1(a) illustrates how the mean of customers' satisfaction level, μ_s, affects the optimal reward value. From this figure, it can be inferred that ρ_l decreases as μ_s increases. That is, as the average of the individuals' satisfaction levels increase, a broader range of reward values becomes optimal. For example for $\mu_s = -0.2$ (which implies an overall dissatisfaction with the purchase in the first period), the optimal range of reward is approximately [0.4,0.75], while for $\mu_s = 0$ this range is [0.2,0.75]. Figure 4.1(b) shows that this conclusion is still valid at a different level of σ_s.

By comparing Figure 4.1(a) and Figure 4.1(b), it can be seen that a higher variability in customers' satisfaction level results in a higher ρ_l. That is, ρ_l increases with σ_s. This fact is also evident from Figure 4.1(c) which displays the revenue function versus reward for three different σ_s values. The top curve in this figure, which belongs to $\sigma_s = 0.1$, converges faster to the maximum revenue of 0.34 when compared to the revenue functions for $\sigma_s = 0.3$ and $\sigma_s = 0.5$. This leads to a wider range of optimal rewards when $\sigma_s = 0.1$ compared to the other two cases. Figure 4.1(d) demonstrates that this conclusion is true when $\mu_s = 0.2$.

The effect of γ on the optimal range of r can be evaluated using Figure 4.1(e) and Figure 4.1(f). As can be seen, revenue functions on Figure 4.1(e) reach the maximum level nearly at the same r value. This indicates that γ does not have a significant effect on ρ_l. Figure 4.1(f) illustrates that this result holds at a different level of μ_s.

Hence, ρ_l decreases with μ_s and increases with σ_s while it remains nearly constant at different levels of γ. This implies that, regardless of the customers' repurchase intentions, the firm will benefit from a broader range of optimal loyalty reward values if it manages to *monotonically* increase satisfaction among customers. This is because $[\rho_l, 0.75]$ is the optimal range of the loyalty rewards and lower ρ_l results in a wider range.

Moreover, the top curve in Figure 4.1(a) suggests that ρ_l can be equal to zero. $\rho_l = 0$ signifies the optimal reward range of $[0, 0.75]$, which includes the particular point $r^* = 0$. Thus, if there exists $\rho_l = 0$, the firm will achieve the maximum revenue even if it does not offer any reward to loyal customers. In fact, $r = 0$ generates a higher profit compared to other r values in the optimal range. This is because, by not offering loyalty rewards, the firm will not incur the fixed expenses associated with adopting a loyalty program.

In order to investigate the existence of $\rho_l = 0$, one can examine whether $r = 0$ generates the optimal revenue ($R^* = 0.34$) at different levels of μ_s and σ_s. Here, one can exploit the fact that ρ_l increases with σ_s. As a result, at any given μ_s level, if a *very small* σ_s value does not lead to $\rho_l = 0$, it can be inferred that $\rho_l > 0$ at that level of μ_s (the reason for considering a very small σ_s is that σ_s cannot be equal to zero). Algorithm 4.1 summarizes the procedure to perform this analysis. Following this procedure an interesting result was obtained. When $\mu_s \geq 0$ and $\sigma_s = 0.1\times10^{-2}$, regardless of γ, $\rho_l = 0$. This implies that if the firm maintains satisfaction among customers with a low variation in the satisfaction levels, not offering a loyalty reward is optimal. On the other hand, when $\mu_s < 0$, $r = 0$ is not an optimal reward value. So, it can be deduced that when $\mu_s < 0$, $\rho_l > 0$.

Algorithm 4.1: To determine whether there exists $\rho_l = 0$

$\sigma_s = 0.1 \times 10^{-2}$

for $\gamma = 0.1$ to 0.9 step 0.1 do

for $\mu_s = -2$ to 2 step 0.1 do

Substitute $r = 0$ and γ, μ_s and σ_s values into the revenue function in Equation (4.18)

if $R < R^* = 0.34$ then $\rho_l \neq 0$ else $\rho_l = 0$

For a given μ_s value, it is also useful to find the maximum σ_s that yields $\rho_l = 0$. Such a σ_s value is an upper bound for σ_s, denoted by $\overline{\sigma}_s$, below which $r = 0$ is optimal. Thus, at a given μ_s level, if the standard deviation of customers' satisfaction levels turns out to be less than $\overline{\sigma}_s$, the firm is able to gain the optimal revenue without adopting a loyalty program.

As mentioned earlier, when $\mu_s < 0$, $\rho_l > 0$. Consequently, for $\mu_s < 0$, $\overline{\sigma}_s = 0$. In order to compute $\overline{\sigma}_s$ for a given nonnegative μ_s, one can gradually increase σ_s until $r = 0$ results in a suboptimal revenue ($R < 0.34$) in Equation (4.18). $\overline{\sigma}_s$ will then be the greatest σ_s that yields the optimal revenue. One can explore the relationship between μ_s and $\overline{\sigma}_s$ by finding $\overline{\sigma}_s$ at some certain μ_s levels. The detailed procedure is presented in Algorithm 4.2. Figure 4.2 illustrates the resulting $\overline{\sigma}_s$ values versus μ_s. As can be seen, $\overline{\sigma}_s$ linearly increases with μ_s. Least-square regression lines (with $R^2 = 1$), as shown in Figure 4.3, possess a zero y-intercept and a slope of approximately 0.13. In accordance with the previous findings, this figure indicates that γ has a negligible effect on $\overline{\sigma}_s$.

Algorithm 4.2: To compute $\overline{\sigma}_s$ for $\mu_s \geq 0$

$\sigma_s = 0.1 \times 10^{-2}$

for $\gamma = 0.1$ to 0.9 step 0.4 do

for $\mu_s = 0$ to 2 step 0.05 do

Substitute $r = 0$ and γ, μ_s and σ_s values into the revenue function in Equation (4.18)

if $R < R^* = 0.34$, then $\overline{\sigma}_s = \sigma_s - 0.10 \times 10^{-3}$ and break,

else $\sigma_s = \sigma_s + 0.10 \times 10^{-3}$

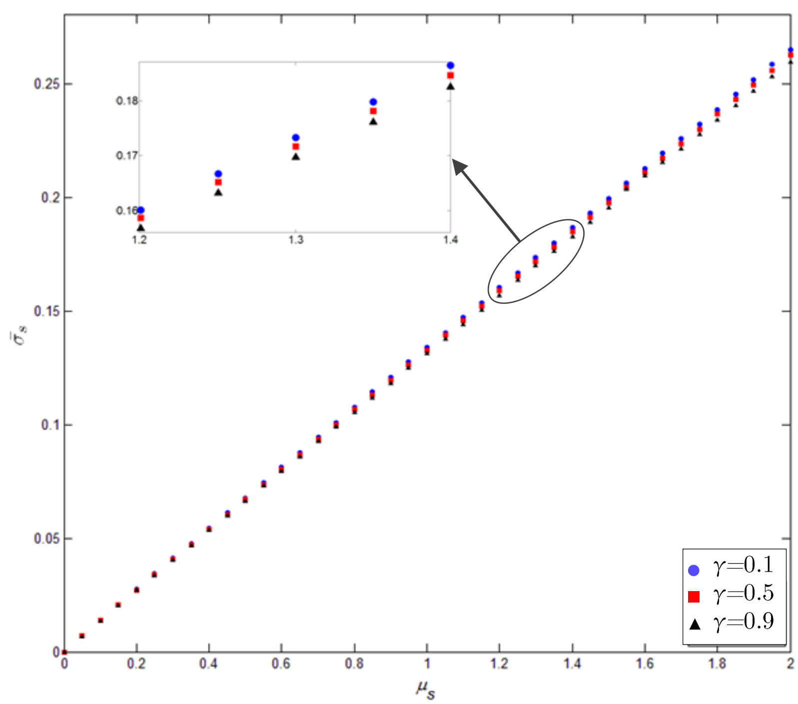


Figure 4.2: μ_s versus $\overline{\sigma}_s$ for different γ values

This analysis reinforces the conclusion that a higher average satisfaction level leads to a broader range of σ_s for which not offering a loyalty reward is optimal. Specifically, when the average of customers' satisfaction/dissatisfaction levels turns out to be positive and

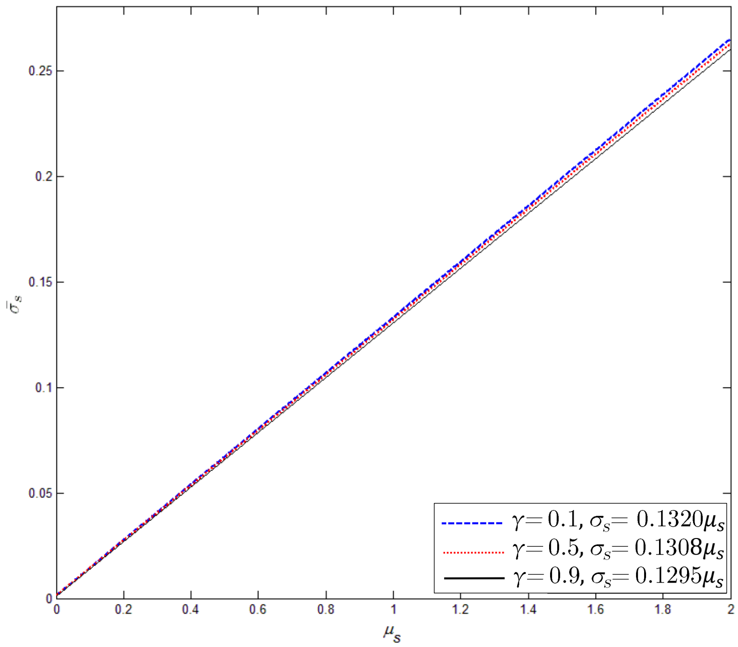


Figure 4.3: $\overline{\sigma}_s$ as a function of μ_s for different γ levels

coefficient of variation of satisfaction levels (σ_s / μ_s) is less than 0.13, $r = 0$ is optimal. As a result, under the mentioned conditions, the firm is better off to stick to the lower price strategy instead of adopting a loyalty program (based on the optimal solution in Equation (4.19), the price in period 1 decreases as the loyalty reward decreases).

4.5 Summary and Conclusion

In this chapter, an analytical model was developed to evaluate the profitability of loyalty programs. The model consists of a revenue-maximizing firm selling a good or service through two periods. Customers earn a loyalty reward in the form of an absolute discount on the product in the second period if they purchase in both periods. Customers who reject the offer in the first period will leave the firm's market. Those who buy in period 1 may also leave the market with a certain probability denoted by γ. γ, in fact, represents customers' intention to rebuy the product. The value of γ depends on different factors like the product category and the overall consumption level. γ is incorporated as a parameter in the model.

One of the features of the model is that customers' satisfaction level is incorporated as a factor in their decision making in the second period. The satisfaction level is modeled as a normally distributed random variable which is summed up with the customers' valuation in period 1 to form their valuation in period 2. The mean and standard deviation of the satisfaction level are modeled as parameters. Customers' valuation in the first period is assumed to follow the standard normal distribution. Thus, this model captures the uncertainty in customers' preferences as well as in buyers' perceived quality.

The objective of the model is to maximize the firm's revenue in terms of its decision variables, that is, the price of the product in the first and second period and the loyalty reward amount. As mentioned above, the model consists of three parameters on which the optimal solution depends. To derive the optimal solution in terms of parameters, the model was solved for some pre-specified values of parameters. Subsequently, by evaluating the obtained results, the optimal solution was formulated as a function of parameters.

The obtained results yielded useful insights into the profitability of loyalty programs. Specifically, it was observed that under certain conditions the firm may obtain the maximum revenue without adopting a loyalty program. These conditions refer to the mean and variance of customers' satisfaction levels. Particularly, if the mean of satisfaction levels turns out to be positive with a standard deviation less than a certain threshold, not offering reward yields the optimal revenue. Thus, if the firm homogenously maintains satisfaction among all customers, it is optimal not to offer a loyalty reward. It was shown that the so-called standard deviation threshold linearly increases with average satisfaction level.

Moreover, consistent with previous analytical studies (e.g., Kim et al. 2001), it was shown that the optimal price in period 1 generally increases with loyalty reward. The increased price, in fact, offsets the reward cost. It was also observed that the price in period 1 increases as customers' intention to repurchase (i.e., γ) rises. This is also an intuitive finding, because the higher the γ, the larger the number of customers who earn the loyalty reward. Thus, the firm must charge higher prices to offset this increased expense. Note that the customers will still be willing to pay higher prices in order to become eligible for the loyalty reward.

Chapter 5

Effectiveness of Multitier Reward Structures

5.1 Introduction

As stated in Chapter 2, previous studies show that the effectiveness of loyalty programs is contingent upon the program's design (Kivetz & Simonson 2002; Roehm et al. 2002; Yi & Jeon 2003). Reward structure is known as one of the two dimensions of loyalty programs' design (Kumar & Reinartz 2006, p.172). Therefore, reward structure is one of the key drivers of loyalty programs' effectiveness. This chapter seeks to shed light on the effectiveness of a particular type of reward structure known as the multitier reward scheme. In a multitier reward structure, the amount of reward per dollar depends on some kind of a measure of customers' past purchase behavior. Examples of such a measure are cumulative spending level, accumulated points balance, or purchase frequency.

As an example, a multitier reward structure is depicted in Figure 5.1. As can be seen, the reward rate in this structure is not constant and it is a non-decreasing stepwise function of points balance. Therefore, customers' reward of loyalty increases as they progress through point levels. Consequently, customers have an incentive to remain loyal to the firm, because they become qualified for higher reward levels. At the same time, the firm benefits from customers' "lock-in" due to their higher switching costs at higher tiers.

As discussed in Chapter 2, despite the ubiquity of multitier reward schemes in practice, they have remained largely underexplored in the OR/Marketing literature. In this chapter, a mathematical model is developed to address this gap in the literature. Specifically, a two-tier loyalty reward scheme is modeled and analyzed in which customers earn rewards based on their purchase frequency. That is, the reward for loyalty is not equal for all frequent buyers and customers with higher repatronage rates gain higher reward values. Similar to the models presented in Chapters 3 and 4, the customers' valuation is incorporated as a random variable.

Findings are based on comparative analyses. The profitability of a firm that adopts the multitier reward scheme is compared with two other cases: the case without a loyalty program and the case with constant reward value, that is, single-tier reward scheme. Also,

Points Required	100% Reward Value
7,000	$10
15,000	$25
30,000	$55
40,000	$75
75,000	$150

Figure 5.1: An example of multitier reward structures
(adopted from Shopper's Optimum Rewards Program®)

the efficacies of multitier and single-tier reward schemes in motivating customers to repurchase at the firm are compared. Results show that a multitier scheme, compared to a single-tier scheme, is more profitable and also more effective in driving repeat purchase.

The rest of this chapter is structured as follows. Section 5.2 describes the model. Section 5.3 focuses on solving the model. Section 5.4 presents the comparative analysis. Section 5.5 concludes with a summary of the findings.

5.2 Model Formulation

In order to examine the effectiveness of a multitier reward scheme, a firm's problem of selling a product in a three-period model is studied. Customers who purchase in two periods receive the loyalty reward of r_1 and those who remain loyal from the first period to the third period gain the loyalty reward of r_2. Thus, r_1 is the reward for showing repeat purchase behavior over any two periods and r_2 is the reward for being loyal over all three periods. In contrast to single-tier reward schemes, here r_1 and r_2 are not necessarily equal.

It is assumed that the market consists of two segments: light user and heavy user. Heavy users are potential customers of each single period and thus their decision to buy is affected by the loyalty program. However, light users purchase only in one period. Therefore, they do not bother about loyalty program and decide solely based on the offered price in the current period, that is, p_i $(i = 1, 2, 3)$.

Similar to Singh et al. (2008), a market of unit size is assumed in each period to ensure that the firm's profit is not affected by market contraction or expansion and only depends on the firm's decision variables, that is, price of product in each period, $p_i\,(i = 1, 2, 3)$, and reward values, $r_j\,(j = 1, 2)$. The size of each market segment is modeled through the parameter θ, which is defined as the size of heavy-user segment in each period. Thus, $(1-\theta)$ fraction of customers in each period are light users. Heavy users patronize the firm in each period and if the offer yields them a nonnegative surplus, they make a purchase at the firm. Otherwise, they will satisfy their demand from other sellers in the market. Light users, on the other hand, leave the market after a given period. In the subsequent period, a new group of light users with size $(1-\theta)$ joins the market to preserve the unit-size assumption. Customer flow through three periods is depicted in Figure 5.2. θ is assumed to be exogenous, implying that the firm does not have control over the size of customer segments.

As mentioned above, customers' decision to buy in each period is based on the surplus they get from their purchase. Surplus is the difference between what the customer is willing to pay for a unit of product and the amount he/she actually has to pay for it. If the offer gives the customer a nonnegative surplus, he/she buys one unit of the product.

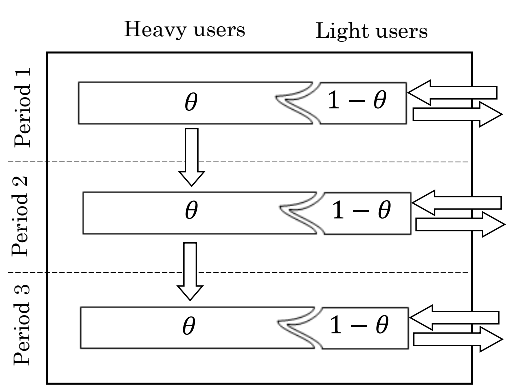


Figure 5.2: Customers' flow diagram

Customers' valuation is modeled as a random variable and is assumed to be uniformly distributed and normalized to the range $[0,1]$. The amount of money a customer pays is generally a function of both offered prices and loyalty rewards. However, it is different over market segments; light users are one-time buyers and as a result, they do not take the loyalty reward into account. Thus, light-users' utility derived from a unit of the product purchased for price p_i is:

$$S_i^l = v_i^l - p_i \qquad i = 1, 2, 3, \tag{5.1}$$

where $v_i^l \, (i = 1, 2, 3)$ is the light-users' valuation in period i and $p_i \, (i = 1, 2, 3)$ is the price offered by the firm in period i.

v_i^l's are assumed to be i.i.d uniform random variables between zero and one. Thus, probability of a light user making a purchase in period i is:

$$\Pi_i^l = Pr\left(S_i^l \geq 0\right) = \begin{cases} 1 - p_i, & 0 \leq p_i < 1 \\ 0, & p_i \geq 1 \end{cases} \qquad i = 1, 2, 3. \tag{5.2}$$

Heavy users are assumed to be forward-looking in the sense that they consider future prices and potential rewards in their decision to buy. Namely, a heavy user will buy if the sum of current period surplus and surpluses from buying in subsequent periods results in a nonnegative value. Particularly, for a heavy user in period 1, the net surplus is given by

$$S_1^h = S_1 + S_2 + S_3 = \left[v_1^h - p_1\right] + \left[v_1^h - (p_2 - r_1)\right] + \left[v_1^h - (p_3 - r_2)\right], \tag{5.3}$$

where v_1^h is the heavy users' valuation for the product in period 1. Thus, probability of a heavy user making a purchase in period 1 is:

$$\Pi_1^h = Pr\left(S_1^h \geq 0\right) = \begin{cases} 1 - \dfrac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}, & 0 \leq \dfrac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} \leq 1 \\ 0, & \dfrac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} > 1 \end{cases}. \tag{5.4}$$

Note that $\frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} \geq 0$, because $p_i \geq 0$ ($i = 1, 2, 3$), $p_2 \geq r_1$ and $p_3 \geq r_2$. The last two conditions hold because, clearly, the loyalty reward would not exceed the offered price in the second and third periods.

Similarly, one can find the probability that a heavy user makes a purchase in period 2. However, surplus in this case depends on the customer's decision in the previous period. For those heavy users who have already made a purchase in period 1, surplus from getting a product in period 2 is:

$$S_2^{h^{(1)}} = S_2 + S_3 = \left[v_2^h - (p_2 - r_1)\right] + \left[v_2^h - (p_3 - r_2)\right]. \tag{5.5}$$

The superscript k in $S_i^{h^{(k)}}$ indicates the past purchase history. Here, $S_i^{h^{(1)}}$ denotes the surplus of buying in period 2 for heavy users who have made a purchase in period 1. Those heavy users who have failed to buy in period 1 ($k = 2$) gain the surplus of:

$$S_2^{h^{(2)}} = S_2 + S_3 = \left[v_2^h - p_2\right] + \left[v_2^h - (p_3 - r_1)\right]. \tag{5.6}$$

Corresponding probabilities are (Similar to $S_i^{h^{(k)}}$, superscript k in $\Pi_i^{h^{(k)}}$ is the purchase history index):

$$\Pi_2^{h^{(1)}} = Pr\left(S_2^{h^{(1)}} \geq 0\right) = \begin{cases} 1 - \dfrac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2}, & 0 \leq \dfrac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2} \leq 1 \\ 0, & \dfrac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2} > 1 \end{cases} \tag{5.7}$$

and

$$\Pi_2^{h^{(2)}} = Pr\left(S_2^{h^{(2)}} \geq 0\right) = \begin{cases} 1 - \dfrac{\sum_{i=2}^{3} p_i - r_1}{2}, & 0 \leq \dfrac{\sum_{i=2}^{3} p_i - r_1}{2} \leq 1 \\ 0, & \dfrac{\sum_{i=2}^{3} p_i - r_1}{2} > 1 \end{cases}. \tag{5.8}$$

Heavy users' surplus in period 3 depends on their previous purchase history. There are three possibilities: buying in both first and second periods ($k = 1$), buying only in one of the periods ($k = 2$) and failing to buy in either period ($k = 3$). If two-period buyers repeat their purchase in period 3, they receive a loyalty reward of r_2. Thus, their surplus from buying in period 3 is:

$$S_3^{h^{(1)}} = v_3^h - (p_3 - r_2). \tag{5.9}$$

Consequently, the probability of making a purchase equals:

$$\Pi_3^{h^{(1)}} = Pr\left(S_3^{h^{(1)}} \geq 0\right) = \begin{cases} 1 - (p_3 - r_2), & 0 \leq p_3 - r_2 \leq 1 \\ 0, & p_3 - r_2 > 1 \end{cases}. \tag{5.10}$$

Heavy users who have bought one unit up to the end of period 2 would get a reward of r_1 by making a purchase in period 3. Thus, their probability of making a purchase is:

$$\Pi_3^{h^{(2)}} = \begin{cases} 1 - (p_3 - r_1), & 0 \leq p_3 - r_1 \leq 1 \\ 0, & p_3 - r_1 > 1 \end{cases}. \tag{5.11}$$

Finally, heavy users who have failed to make any purchase until period 3 buy with the probability of:

$$\Pi_3^{h^{(3)}} = \begin{cases} 1 - p_3, & 0 \leq p_3 \leq 1 \\ 0, & p_3 > 1 \end{cases}. \tag{5.12}$$

The purpose is to find values of $p_i\,(i = 1, 2, 3)$ and $r_j\,(j = 1, 2)$ that maximize the firm's revenue. In order to formulate the firm's optimization problem, the expected revenue in each period must be found.

Table 5.1: Firm's revenue in each period

Period	Customer type	Purchase history*		Probability of making a purchase	Firm's revenue
		Period 1	Period 2		
1	Heavy-user	-	-	$\theta \Pi_1^h$	p_1
1	Light-user	-	-	$(1-\theta)\Pi_1^l$	p_1
2	Heavy-user	B	-	$\theta \Pi_1^h \Pi_2^{h^{(1)}}$	$p_2 - r_1$
2	Heavy-user	F	-	$\theta\left(1-\Pi_1^h\right)\Pi_2^{h^{(2)}}$	p_2
2	Light-user	-	-	$(1-\theta)\Pi_2^l$	p_2
3	Heavy-user	B	B	$\theta \Pi_1^h \Pi_2^{h^{(1)}} \Pi_3^{h^{(1)}}$	$p_3 - r_2$
3	Heavy-user	F	B	$\theta\left(1-\Pi_1^h\right)\Pi_2^{h^{(2)}}\Pi_3^{h^{(2)}}$	$p_3 - r_1$
3	Heavy-user	B	F	$\theta \Pi_1^h \left(1-\Pi_2^{h^{(1)}}\right)\Pi_3^{h^{(2)}}$	$p_3 - r_1$
3	Heavy-user	F	F	$\theta\left(1-\Pi_1^h\right)\left(1-\Pi_2^{h^{(2)}}\right)\Pi_3^{h^{(3)}}$	p_3
3	Light-user	-	-	$(1-\theta)\Pi_3^l$	p_3

* B: Bought a unit of product, F: Failed to buy

Table 5.1 summarizes revenues from different types of customers and corresponding probabilities in each period. Probabilities are derived based on the assumption that customers' valuation in each period is independent of their valuation in previous period(s), that is, v_i^h's ($i = 1, 2, 3$) are independent. Based on Table 5.1, the total three-period revenue function is as follows:

$$\begin{aligned} R = {} & [\text{Revenue in period 1}] + [\text{Revenue in period 2}] + [\text{Revenue in period 3}] = \\ & \left[\theta \Pi_1^h p_1 + (1-\theta)\Pi_1^l p_1\right] + \\ & \left[\theta \Pi_1^h \Pi_2^{h^{(1)}}(p_2 - r_1) + \theta\left(1-\Pi_1^h\right)\Pi_2^{h^{(2)}} p_2 + (1-\theta)\Pi_2^l p_2\right] + \\ & \Big[\theta \Pi_1^h \Pi_2^{h^{(1)}} \Pi_3^{h^{(1)}}(p_3 - r_2) + \theta\left(1-\Pi_1^h\right)\Pi_2^{h^{(2)}}\Pi_3^{h^{(2)}}(p_3 - r_1) + \\ & \theta \Pi_1^h \left(1-\Pi_2^{h^{(1)}}\right)\Pi_3^{h^{(2)}}(p_3 - r_1) + \theta\left(1-\Pi_1^h\right)\left(1-\Pi_2^{h^{(2)}}\right)\Pi_3^{h^{(3)}} p_3 + (1-\theta)\Pi_3^l p_3\Big] \end{aligned} \tag{5.13}$$

5.3 Solving the Model

At any given value of θ, the optimal value of the firm's total revenue is obtained by solving the following NLP:

$$R^* = \underset{p_1, p_2, p_3, r_1, r_2}{\text{maximize}} \ R \tag{5.14}$$

Subject to:

$$p_2 \geq r_1 \tag{5.14a}$$

$$p_3 \geq r_1 \tag{5.14b}$$

$$p_3 \geq r_2 \tag{5.14c}$$

$$p_i \geq 0 \ (i = 1,2,3) \tag{5.14d}$$

$$r_j \geq 0 \ (j = 1,2) \tag{5.14d}$$

Note that Π_i^l's and Π_i^h's are piecewise functions. Consequently, the objective function depends on values of the firm's decision variables. Since there are eight different piecewise functions (in fact, there are 9 probability functions, namely Π_1^h, $\Pi_2^{h^{(1)}}$, $\Pi_2^{h^{(2)}}$, $\Pi_3^{h^{(1)}}$, $\Pi_3^{h^{(2)}}$, $\Pi_3^{h^{(3)}}$, Π_1^l, Π_2^l and Π_3^l, however, one can see that $\Pi_3^{h^{(3)}} = \Pi_3^l$, which leaves eight unequal functions) each with two possible values, there exist $2^8 = 256$ NLP's with different objective functions and sets of constraints. As an example, one of the combinations with corresponding NLP model is presented in Appendix 5. Some of the combinations may result in conflicting constraints, and hence lead to an infeasible problem. Each model must be solved at given values of θ over the range of [0,1]. After finding R_k^* ($k = 1, \ldots, 256$), they must be compared to obtain the global maximum. Optimal total revenues versus θ are shown in Figure 5.3. Each R_k^* curve is obtained by solving the corresponding NLP at 100 equally spaced θ values between 0 and 1. This figure shows that R_1^* yields the highest revenue when $\theta < 0.6$, and R_{65}^* dominates other optimal revenues when $\theta \geq 0.6$ (the subscript of R only denotes a combination number and following the order by which the possible values of piecewise functions were changed, the 65th combination is the one which results in highest revenue for $\theta \geq 0.6$).

R_1 is, in fact, the total revenue function associated with the following set of constraints:

$$\left\{\frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} \leq 1, \frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2} \leq 1, \frac{\sum_{i=2}^{3} p_i - r_1}{2} \leq 1, \right.$$
$$\left. p_3 - r_2 \leq 1, p_3 - r_1 \leq 1, p_i \leq 1 \;\; (i = 1,2,3)\right\} \tag{5.15}$$

R_{65}, however, is the revenue function resulting from the same constraints as above with the exception that $p_1 \geq 1$.

Optimal values of p_i ($i = 1, 2, 3$) and r_j ($j = 1, 2$) versus θ are shown in Figure 5.4. As can be seen, all optimal values are discontinuous at $\theta = 0.6$. This is because optimums are derived from different models in the vicinity of this point. For $\theta \leq 0.6$, optimal prices and rewards are obtained from R_1^* , while for $\theta > 0.6$, they result from R_{65}^*.

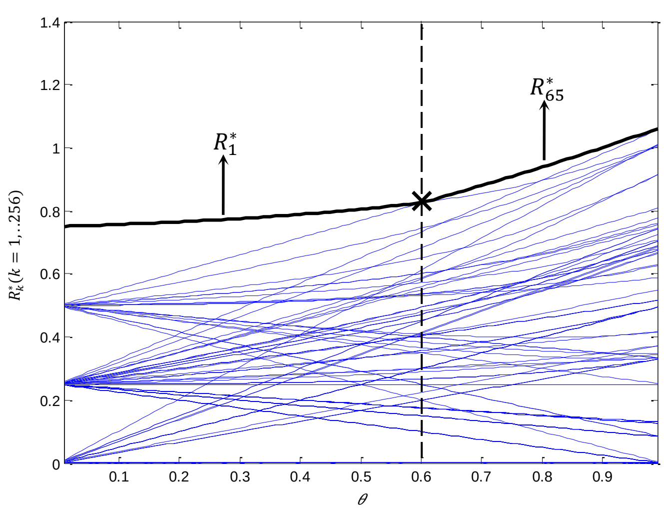
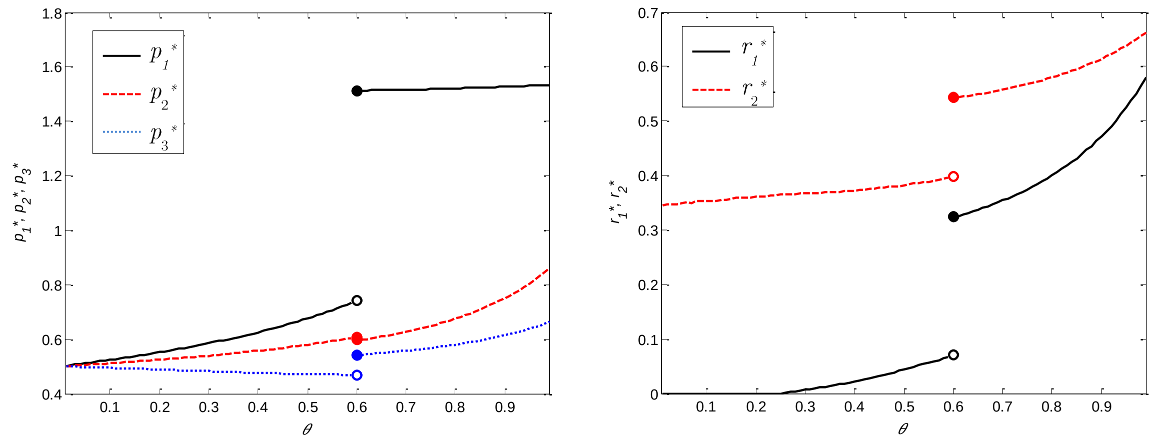


Figure 5.3: Optimal total revenues versus θ



(a) (b)

Figure 5.4: Optimal values of (a) p_1, p_2 and p_3 and (b) r_1 and r_2

Figure 5.4(a) shows that $p_1 \geq p_2 \geq p_3$ over the whole range of θ. This finding is intuitive, because the firm charges higher prices in the current period by making the promise of lower prices and higher rewards in subsequent period(s). Furthermore, from Figure 5.4(b) it is evident that $r_2^* > r_1^*$, which is consistent with real world multitier reward scheme practices in which loyal customers get higher benefits as they buy more often.

5.4 Comparative Analysis

In this section, the profitability of a multitier reward scheme is compared with that of two other cases: the case without a loyalty program and the case in which loyalty rewards are equal regardless of the number of periods in which a customer has made a purchase, that is, the single-tier reward scheme. These comparisons provide a benchmark to evaluate the effectiveness of a multitier reward system. First, the case without a loyalty program is evaluated.

5.4.1 Comparison with the Case without a Loyalty Program

Eliminating the loyalty reward program implies that $r_1 = r_2 = 0$. Substituting $r_1 = r_2 = 0$ in Equation (5.14), the following optimal prices are obtained:

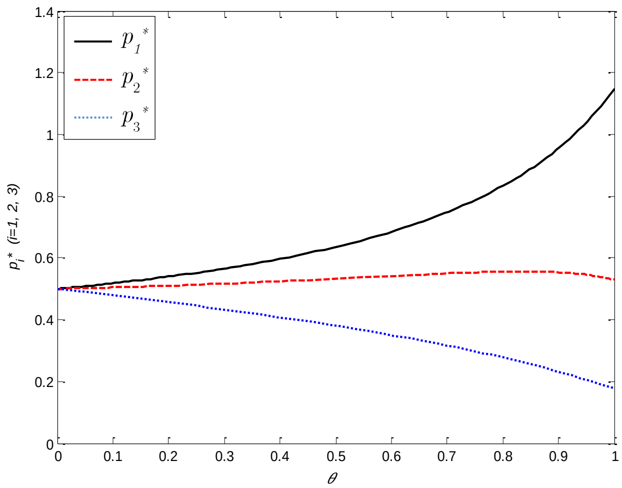
$$p_1^* = \frac{3(5\theta^2 + -40\theta + 48)}{2(10\theta^3 + 31\theta^2 - 168\theta + 144)}, \tag{5.16}$$

$$p_2^* = \frac{3(5\theta - 6)(\theta - 4)}{(10\theta^3 + 31\theta^2 - 168\theta + 144)} \text{ and} \tag{5.17}$$

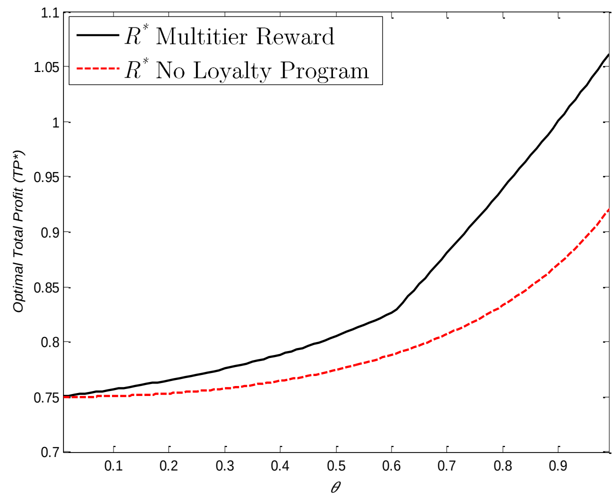
$$p_3^* = \frac{3(5\theta - 6)(3\theta - 4)}{(10\theta^3 + 31\theta^2 - 168\theta + 144)}. \tag{5.18}$$

The optimal p_i's versus θ are plotted in Figure 5.5(a). As can be seen at $\theta = 0$, $p_1^* = p_2^* = p_3^* = 0.5$. This result is consistent with results of the simple dynamic pricing problem when customers are myopic and their valuation is uniformly distributed in the range of [0,1].

Furthermore, Figure 5.5(a) shows that in the first and third periods the deviation from the light-users' revenue-maximizing price (i.e., $|p_i^* - 0.5|$) increases with θ, while period 2 prices are almost constant. In fact, $p = 0.5$ is the revenue-maximizing price for the light-user segment in each period, and the greater the distance from this price, the greater the deviation from optimal revenue from the light-user segment. Since deviation from $p_i = 0.5$ leads to lower revenues from light-user segment, the total revenue from light users decreases with θ. However the dashed curve in Figure 5.5(b) reveals that the optimal total



(a)



(b)

Figure 5.5: (a) Optimal values of p_1, p_2 and p_3 when $r_1 = r_2 = 0$ (b) Comparing optimal revenue function of multitier reward scheme with that of no loyalty program

revenue increases with θ. Thus, revenue loss from the light-user segment has been offset by the rise in revenue from heavy users.

Additionally, Figure 5.5(b) shows that the multitier reward scheme yields higher revenues compared to the case $r_j = 0$ ($j = 1, 2$). More specifically, the difference between the two functions increases with θ. This finding is intuitive, since reward systems are normally implemented to derive benefits from the heavy-user segment and a higher θ implies a greater potential for this purpose.

In Figure 5.6, optimal prices under the multitier reward system are compared to optimal prices when no loyalty program is implemented. From this figure it is evident that optimal prices increase by introducing the loyalty program. This finding is consistent with Kim et al. (2004), who have found that reward programs increase prices.

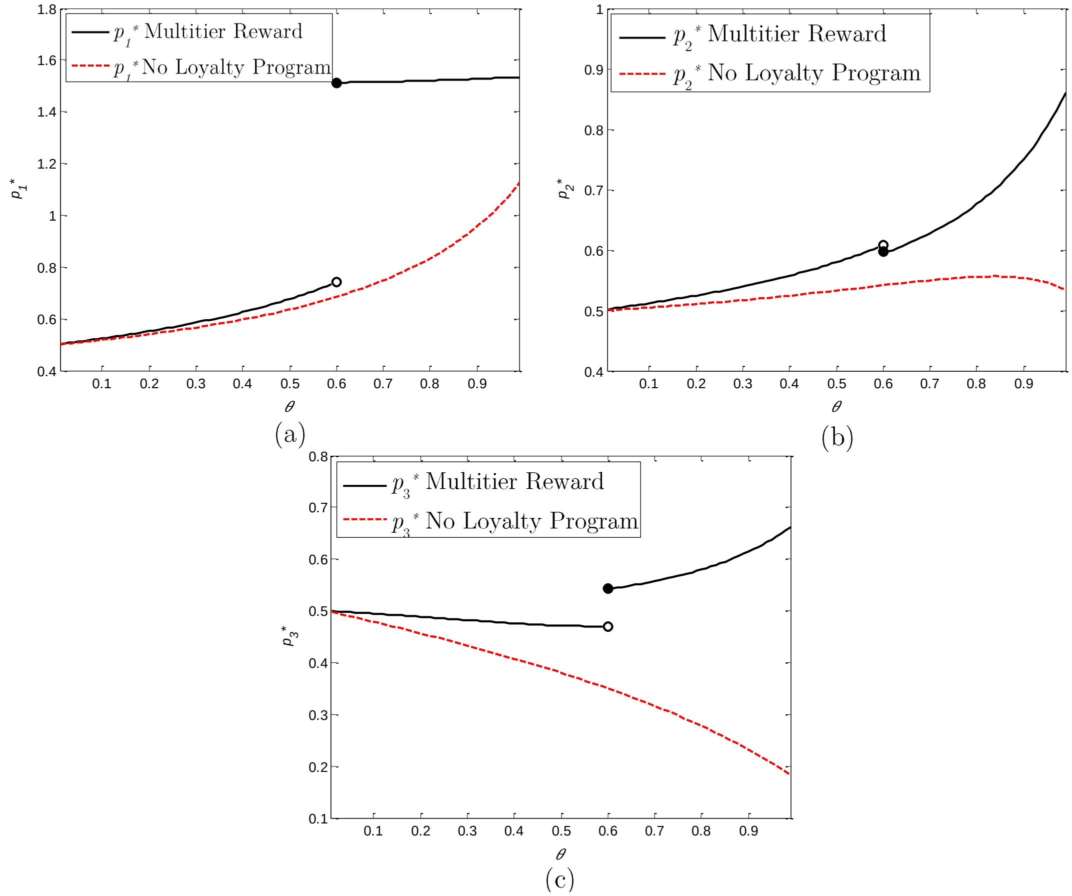


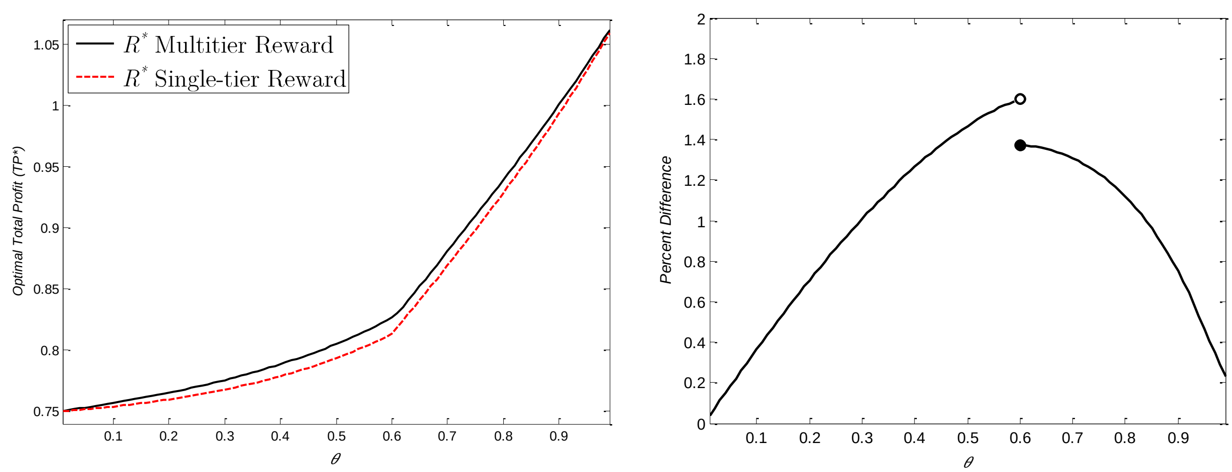
Figure 5.6: Comparing optimal prices ((a) p_1^*, (b) p_2^* and (c) p_3^*) under multitier reward scheme with optimal prices when no loyalty program is implemented

5.4.2 Comparison with the Single-tier Reward Scheme

Here, the effectiveness of the multitier reward system is compared with that of the single-tier reward scheme in which loyal customers get the same loyalty reward, regardless of the "magnitude" of their loyalty, that is, $r_1 = r_2$. This is analogous to the one-tier point systems in which the monetary value of each point is fixed, regardless of the number of points accumulated.

The optimal revenue function of the single-tier reward system is obtained by replacing $r_1 = r_2$ in the Equation (5.14). The result is plotted in Figure 5.7(a) (dashed curve). The solid curve in this figure corresponds to the optimal total revenue when the multitier scheme is implemented. Evident from this figure, the revenue of both schemes is an increasing function of θ; however, the multitier scheme surpasses the single-tier reward system over the whole range of θ values. The percentage difference between these two R^*'s (i.e., $(R^*_{r_1 \neq r_2} - R^*_{r_1 = r_2})/R^*_{r_1 = r_2} \times 100$) is shown in Figure 5.7(b). As this figure illustrates, the degree to which the multitier scheme outperforms the single-tier scheme depends on the proportion of the heavy users in the market, that is, θ. Specifically, the percentage difference is higher for intermediate values of θ, which suggests that the multitier reward is more preferable when neither of the segments is dominant in the market.

It is useful to evaluate how multitier and single-tier reward schemes affect different segments of customers, that is, light user and heavy user. The evaluation can be performed by isolating the contribution of each segment to the total revenue. Results are



(a) (b)

Figure 5.7: (a) Optimal total revenue from multitier and single-tier reward schemes (b) Percent difference between optimal total revenue derived from multitier and single-tier reward schemes

shown in Figure 5.8. The solid curve in Figure 5.8(a) corresponds to the optimal total revenue when multitier reward scheme is adopted. Total revenue is the sum of revenues gained from heavy users and light users, which are plotted as dashed and dotted lines respectively in Figure 5.8(a). As can be seen, revenue gained from light users decreases as θ increases. This observation is intuitive, because higher θ means fewer light users and, therefore, less potential revenue from their segment. Moreover, in a market mainly comprised of heavy users, prices and rewards are more biased in favor of the heavy-user segment. As a result, one expects lower revenues from the light-user segment compared to the case where θ is lower.

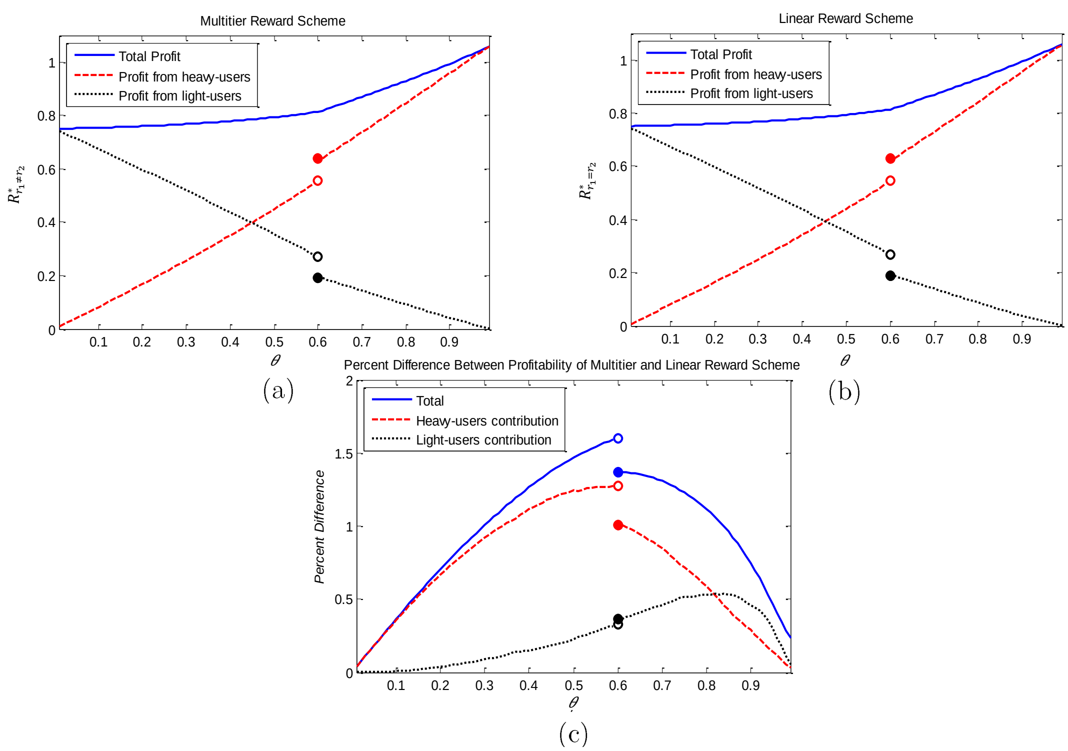


Figure 5.8: Revenue obtained by adopting (a) Multitier reward scheme and (b) Single-tier reward scheme, (c) Percent difference between profitability of multitier and single-tier reward scheme

Figure 5.8(b) shows the total revenue and revenues from each segment when the single-tier reward is adopted. Although Figure 5.8(b) suggests that the single-tier reward scheme performs equivalently to the multitier one (Figure 5.8(a)), their effect on two market segments is slightly different. To illustrate it, the percent difference between revenues obtained from multitier and single-tier reward systems is plotted versus θ in Figure 5.8(c). The solid curve in this figure, same as Figure 5.7(b), corresponds to the percent difference between the profitability of multitier and single-tier reward schemes. The dashed curve in Figure 5.8(c) shows the contribution of heavy users to the difference that the multitier scheme makes in total revenues (i.e., $(HR^*_{r_1 \neq r_2} - HR^*_{r_1=r_2})/R^*_{r_1=r_2} \times 100$, where HR^* is the optimal revenue from heavy-user segment). Similarly, the dotted curve corresponds to the percent difference between revenues gained from the light-user segment by implementing multitier versus single-tier reward (i.e., $(LR^*_{r_1 \neq r_2} - LR^*_{r_1=r_2})/LR^*_{r_1=r_2} \times 100$, where LR^* is the optimal revenue from the light-user segment). Clearly, at any given θ, the percent difference in total revenue is the sum of the percent difference made by heavy users and light users.

Since the percent difference is always positive in Figure 5.8(c), it is evident that the multitier reward scheme, compared to the single-tier scheme, yields higher revenues from both segments of customers. However, the degree to which the multitier scheme outperforms the single-tier reward system depends on θ. As can be seen, when $\theta < 0.6$, the excess revenue generated by implementing the multitier system is mostly derived from the heavy-user segment. On the other hand, for $\theta \leq 0.9$, the percent difference in revenues from the light-user segment increases with θ. Also, when the heavy-user segment is dominant in the market, that is, when $\theta > 0.8$, the difference in revenue from light users is greater than that of heavy users.

Another key measure in evaluating the performance of loyalty programs is their effectiveness in driving repurchase behavior. To determine which reward scheme is more effective in this context, one can compare the fraction of heavy users who have made a purchase in all of the three periods (i.e., $\Pi_3^{h^{(1)}}$). Figure 5.9 shows the proportions of three-period buyers under multitier and single-tier reward schemes. From this figure, it is evident that when $\theta < 0.93$, the multitier reward scheme is more effective in developing behavioral loyalty, since the fraction of customers with three purchases is greater when a multitier reward is adopted. When $\theta > 0.93$, multitier and single-tier systems perform equivalently in terms of driving repeat purchase behavior.

To justify the above findings, one must evaluate the effect of different reward schemes on customers' decision factors, that is, $p_i\,(i = 1, 2, 3)$ and $r_j\,(j = 1, 2)$. Figure 5.10 compares the optimal $p_i\,(i = 1, 2, 3)$ obtained from multitier and single-tier systems. This figure interestingly reveals that, $p^*_{i\,(multitier)} \leq p^*_{i\,(single\text{-}tier)}\,(i = 1, 2, 3)$, that is, the multitier

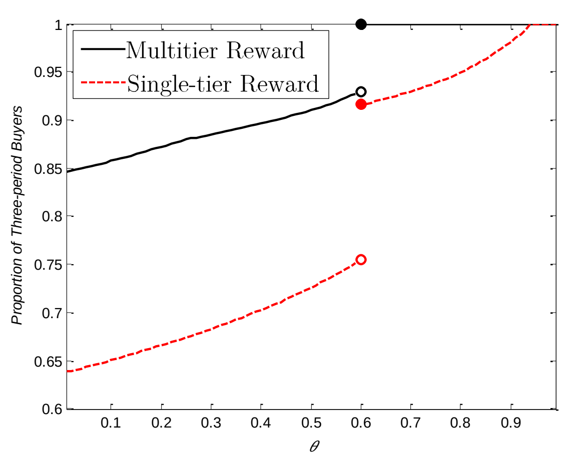


Figure 5.9: Proportion of three-period buyers

reward yields lower prices compared to the single-tier reward. Since the fraction of the light-user segment making a purchase is a decreasing function of p_i, it is concluded that multitier reward increases the sales in the light-user segment. Furthermore, it can be seen that $\left|p^*_{i\,(multitier)} - 0.5\right| \leq \left|p^*_{i\,(single\text{-}tier)} - 0.5\right|$ $(i = 1, 2, 3)$, that is, a deviation from $p_i = 0.5$ is less when the multitier reward is adopted. As mentioned earlier, a deviation from $p = 0.5$ results in suboptimal revenues from the light-user segment in each period. Thus, the multitier reward generates more revenues from the light-user segment compared to the

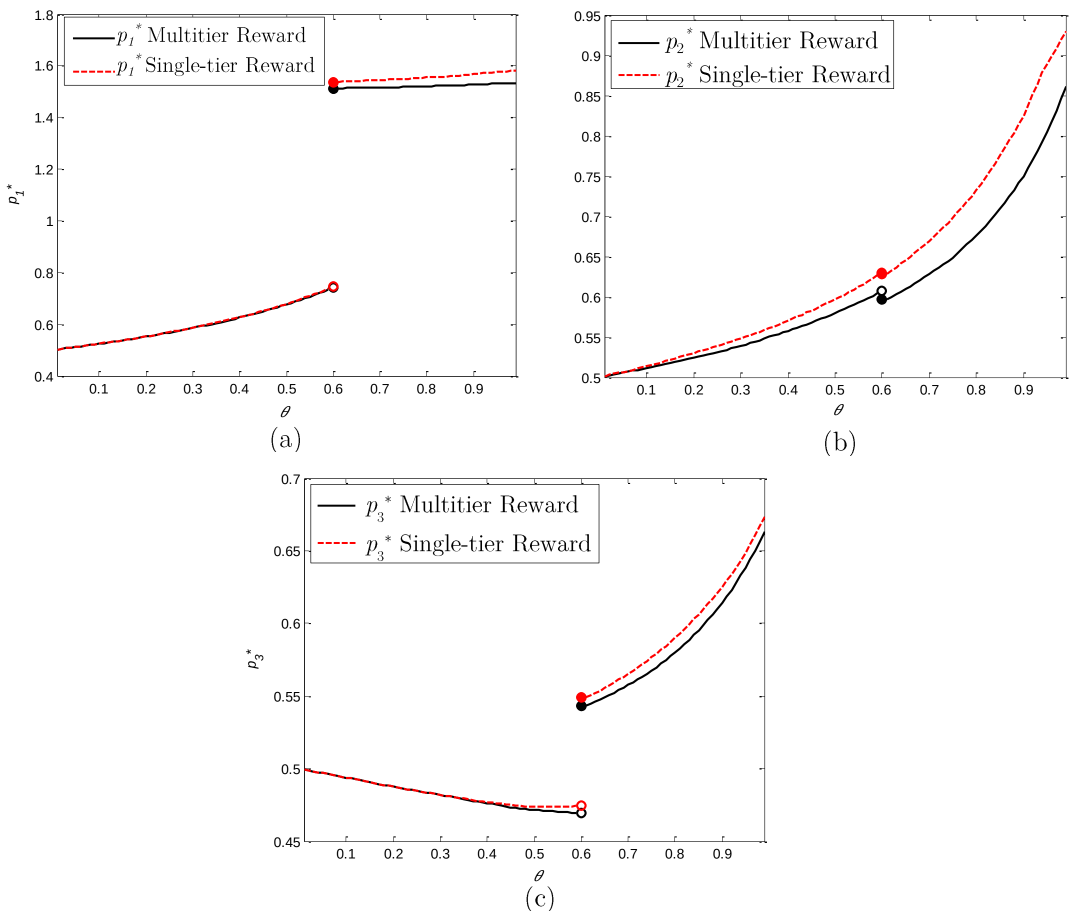
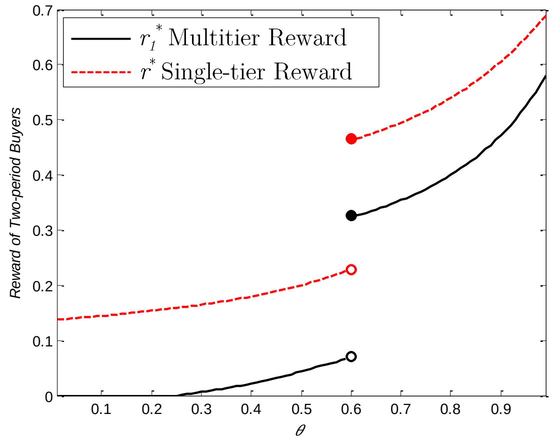


Figure 5.10: Comparing the effect of different reward systems on optimal prices ((a) p_1^*, (b) p_2^* and (c) p_3^*)

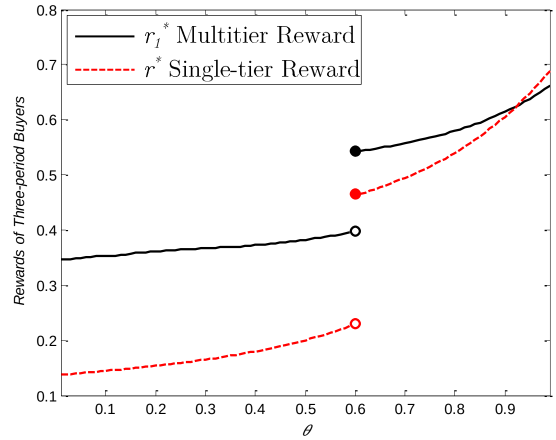
single-tier reward scheme. This explains the finding that multitier reward schemes improve the revenue from the light-user segment.

It was also found that the multitier reward structure, compared to the single-tier reward, brings higher revenues from the heavy-user segment. It is difficult to explain this result by analyzing the effect of different reward schemes on a customer's decision factors, because heavy users are forward-looking and their decision to buy in each period is a function of current and future prices and rewards. Consequently, it is not possible to isolate the effects of one or two variables on the revenues from the heavy-user segment. However, it was found that the multitier reward is more effective than the single-tier reward in driving repurchase behavior. Since this behavior is potentially of benefit to the firm (Sharp & Sharp 1997), one expects higher revenues from the heavy-user segment by adopting the multitier reward scheme (which is consistent with the results of this study).

Why is the multitier reward more effective than the single-tier reward in promoting behavioral loyalty? To find the answer, loyalty reward values under multitier and single-tier schemes are compared in Figure 5.11. Figure 5.11(a) shows the reward of two-period buyers (those who buy either in periods 1 and 2, 2 and 3 or 1 and 3). From this figure, it is inferred that the loyalty benefit of two-period buyers is strictly smaller when the multitier reward is implemented. However, as Figure 5.11(b) indicates, three-period buyers gain higher rewards when the multitier system is adopted (with the exception of $\theta > 0.93$). In fact, for $\theta \geq 0.6$, three-period buyers get the highest possible reward, that is, a free product, when the multitier reward scheme is adopted. This can be inferred from



(a)



(b)

Figure 5.11: Loyalty reward of (a) two-period buyers, (b) three-period buyers under multitier and single-tier reward schemes

Figure 5.9 where $\Pi_3^{h(1)} = 1$, that is, $1 - (p_3 - r_2) = 1$, thus $p_3 = r_2$. The same conclusion holds for the single-tier reward scheme when $\theta > 0.93$. Hence, the reward of three-period buyers is greater under the single-tier scheme (when $\theta > 0.93$) because third-period prices are higher when this scheme is implemented. So, the multitier scheme, when compared to the single-tier reward, gives lower rewards to two-period buyers, but higher rewards to three-period buyers. This finding elucidates the mechanism by which the multitier reward outperforms the single-tier reward in motivating repurchase behavior: offering lower loyalty rewards for earlier repurchases and, at the same time, higher rewards for later repurchases.

5.5 Summary and Conclusion

In this chapter, the effectiveness of multitier loyalty reward schemes was studied. It was assumed that the market is composed of two customer segments: heavy-user and light-user. Results were based on a three-period model in which heavy users' decision to buy is a function of their valuation, current and future prices and loyalty rewards. Light users, however, decide solely based on their valuation and offered price in the current period. In the framework of the assumptions, it was found that the multitier scheme is more effective than the single-tier scheme in increasing the profitability of a firm and developing behavioral loyalty (i.e., repeat purchase) among customers. It was found that multitier schemes, compared to single-tier systems, yield higher revenues from both segments of customers. The magnitude of this outperformance, however, depends on the size of each market segment which was modeled as a parameter.

Chapter 6

Three-tier Loyalty Reward Schemes: Conditions under Which They Yield Suboptimal Revenues

6.1 Introduction

As discussed earlier, the multitier reward scheme has remained understudied in the OR/marketing literature. Kumar & Reinartz (2006) refer to the reward structure as one of the main components of loyalty program's design. Thus, the reward structure is a key driver of the loyalty programs' overall effectiveness. So the lack of research on multitier reward schemes is a significant gap in the loyalty programs' literature. In this chapter, a mathematical model is developed to investigate the conditions under which a three-tier reward structure is not the optimal reward stucture. The three-tier scheme is considered because it is one of the most common types of multi-level rewards systems. A model is developed to characterize conditions that justify adopting a three-tier reward scheme. To the best of our knowledge, this is the first attempt to develop a framework that enables firms to evaluate optimality of the design of their loyalty reward schemes.

A duopoly market is modeled where one firm offers loyalty rewards and the other firm competes with a discounted-price strategy. Customers choose one of the firms and buy one unit of the product in each period. Their choice behavior is modeled using the binary-logit model where the utility is a function of the offered price, the reward value and the distance to the reward. By including the distance component in the utility, this study addresses the fact that customers accelerate their purchasing process as they progress toward earning a particular reward (Kivetz et al. 2006). The distance, in fact, captures the effort that customers should make to achieve a reward. It will be shown that the customers' accumulated purchase evolves according to a Markov chain. The transition probabilities of the Markov chain are obtained in terms of the decision variables and are used to formulate the firm's expected revenue function. The revenue function is optimized in terms of the reward scheme's design elements (i.e., reward values and the tiers' break points). The optimal solution yields useful insights into the optimality of three-tier reward systems. Specifically, it will be shown that customers' low levels of sensitivity to rewards can make the three-tier scheme an undesirable reward structure. Similarly, if the effect of distance factor on customers' decision to buy is insignificant, offering a three-tier reward yields suboptimal revenues.

The rest of the chapter is organized as follows: Section 6.2 describes the underlying assumptions and the model formulation. Section 6.3 presents results and discussions. Section 6.4 concludes the chapter with a summary of findings.

6.2 Model Formulation

6.2.1 The market

An asymmetric duopoly market is studied where one firm offers a loyalty program (Firm A) and the other firm sticks to the lower-price strategy (Firm B). Firms sell the same good/service through the selling horizon which is divided into n discrete time periods. Market size is normalized to one and remains constant across periods. Suppose that Firm A offers a constant price, p, throughout the selling horizon and Firm B charges $p - D$ in each period, where D denotes the price-cut offered by Firm B. p and D are treated as parameters in the model.

Firm A adopts a three-tier reward scheme. Customers progress through reward tiers based on their accumulated purchase at Firm A. The reward is determined based on a customer's loyalty status at the time of the purchase. A customer in Tier 1 earns r_1 for his/her purchase. Customers who have made enough purchases to achieve the Tier 2 status earn a reward of r_2 and those who have made it to the Tier 3 win a reward of r_3. To illustrate, the reward scheme is shown in Figure 6.1. b_1 and b_2 are the breakpoints that specify the limits of each loyalty level. Customers whose total purchase exceeds b_2 earn the Tier 3 rewards. The Tier 2 status is granted to accumulated purchases falling in the range $[b_1, b_2)$. If the total purchase is less than b_1, a reward of r_1 is offered to the customer. Note that b_1 and b_2 are the monetary values of the breakpoints. If the loyalty program adopts a different promotional currency, b_1 and b_2 can be translated into the program's currency. For example, if every dollar spent is worth 10 points, a lower limit of

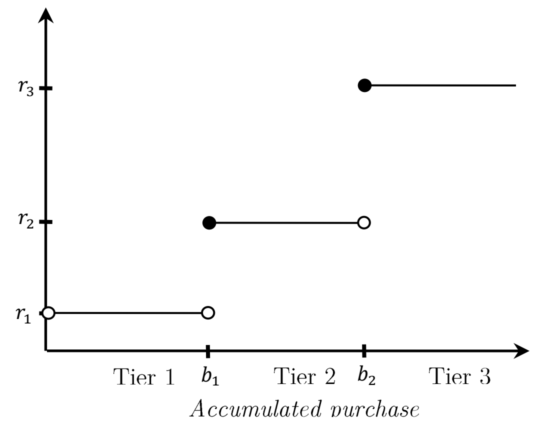


Figure 6.1: Firm A's loyalty reward scheme

8000 points for the second tier signifies $b_1 = \$800$. r_1, r_2, b_1 and b_2 are incorporated as Firm A's decision variables in the model.

6.2.2 The Demand

Customers buy one unit of the product in each period, either at Firm A or at Firm B. Here, the binary-logit model is used to formulate the customers' choice behavior. First, let

$$u_{B,i} = v_{B,i} - \alpha_p(p - D) + \xi_{B,i}, \quad i = 1, \ldots, n, \tag{6.1}$$

be a customer's utility from buying at Firm B in period i. $v_{B,i}$ denotes the customers' valuation for the product offered by Firm B in period i, α_p is the marginal sensitivity of the utility to the price and $\xi_{B,i}$ is the random component of the utility. $\xi_{B,i}$ captures the heterogeneity in customers' preferences as well as the noise due to the unobserved components of customers' utility.

Similarly, one can model customers' utility for Firm A's product. As mentioned above, Firm A adopts a multitier reward scheme. Here, it is assumed that each individual's utility depends upon two additional factors: the reward in the current and the next tier and the distance to the next tier. The values of both factors in period i depend on a customer's accumulated purchase up to period i. Let ω_i represent the sum of the rewards of the current tier and the next tier. Based on Firm A's reward scheme depicted in Figure 6.1, it follows that:

$$\omega_i = \begin{cases} r_1 + r_2 & \text{if} \quad s_i < b_1, \\ r_2 + r_3 & \text{if } b_1 \le s_i < b_2, \\ r_3 & \text{if} \quad s_i \ge b_2 \end{cases} \quad i = 1, \dots, n, \tag{6.2}$$

where s_i denotes the total money spent by the customer at Firm A up to the period i. It is assumed $s_0 = 0$, that is, customers initiate accumulating points at the beginning of the selling horizon. This can be observed in many real-world applications, where the accumulated points expire after a certain period of time.

Moreover, let d_i denote the distance to the next loyalty level. d_i can be written as a function of s_i as follows:

$$d_i = \begin{cases} b_1 - s_i & \text{if} \quad s_i < b_1, \\ b_2 - s_i & \text{if } b_1 \le s_i < b_2, \\ 0 & \text{if} \quad s_i \ge b_2, \end{cases} \quad i = 1, \dots, n. \tag{6.3}$$

d_i is, in fact, the minimum amount of money that the customer must spend at Firm A to attain the next reward tier. Note that if a customer reaches the highest tier, $d_i = 0$.

Having defined ω_i and d_i, one can now formulate the utility derived from making a purchase at Firm A in period i, $u_{A,i}$, as follows:

$$u_{A,i} = v_{A,i} - \alpha_p p + \alpha_r \omega_i - \alpha_d d_i + \xi_{A,i}, \; i = 1, \dots, n. \tag{6.4}$$

In the above equation, $v_{A,i}$ is the valuation for Firm A's product in period i, α_p denotes the utility sensitivity to the price, α_r denotes the sensitivity of demand to the reward, α_d

denotes the customers' sensitivity to the distance from the next reward tier and $\xi_{A,i}$ is the random disturbance term.

The utility formulation in Equation (6.4) implicitly incorporates the findings of Kivetz et al. (2006), which show that customers accelerate their purchasing process as they approach a reward. Based on Equation (6.4), the farther the distance to the breakpoint, the lower the utility. In other words, customers experience a higher utility as they approach a reward level, which is in line with of Kivetz et al. (2006)'s findings.

One of the underlying assumptions in Equation (6.4) is that customers are forward-looking while making the where-to-buy decision, since they consider the possibility of earning the higher tier rewards in the future. Furthermore, through the utility formulation in Equation (6.4), this intuitive fact is incorporated that customers may treat rewards and discounts differently. In most real applications, customers will not consider the reward as a fixed discount on the product's price. In other words, the actual value of a reward for a customer might be less or more than the reward's monetary value. This fact is captured by assigning different sensitivity coefficients to the price and to the rewards, α_p and α_d respectively.

In every period, customers choose between Firm A and Firm B. Assuming that utility maximization accurately predicts the customers' choice behavior, one can derive the choice probabilities based on equations (6.1) and (6.4). More specifically, a customer chooses Firm A over Firm B in period i if $u_{A,i} \geq u_{B,i}$. Hence, the probability that the customer buys at Firm A in period i, $p_{i,A}$, is given by

$$p_{i,A} = \Pr(u_{A,i} \geq u_{B,i})$$
$$= \Pr(\xi_{B,i} - \xi_{A,i} \leq v_{A,i} - v_{B,i} + \alpha_r \omega_i - \alpha_d d_i - \alpha_p D), \; i = 1, \ldots, n. \tag{6.5}$$

Since firms are selling identical products, without loss of generality, one can assume $v_{A,i} = v_{B,i}$ (i=1,...,n). Also, for analytical simplicity, one can divide the above inequality by α_p. Thus, from Equation (6.5) it follows that

$$p_{i,A} = \Pr(\xi'_{B,i} - \xi'_{A,i} \leq \alpha'_r \omega_i - \alpha'_d d_i - D), \; i = 1, \ldots, n. \tag{6.6}$$

Note that $\alpha'_r = \alpha_r/\alpha_p$. So, α'_r can be described as the *relative* sensitivity to rewards (i.e., relative to the sensitivity to cost). So, for instance, $\alpha'_r = 0.5$ implies that the effect of price on the customer's utility is twice the effect of rewards. Similarly, α'_d is the relative sensitivity of utility to the distance. Now, one can adopt the commonly used assumption that $\xi'_{B,i}$ and $\xi'_{A,i}$ ($i = 1$,...,n) are i.i.d random variables following the type-1 extreme value (Gumbel) distribution with location parameter η and scale parameter $\mu > 0$ (or equivalently variance $\pi^2\mu^2/6$) (see e.g., Ben-Akiva & Lerman 1985). As a result, $\xi_{B,i} - \xi_{A,i}$ ($i = 1$,...,n) has a logistic distribution with mean zero and the same scale parameter (or variance $\pi^2\mu^2/3$) (see e.g., Talluri & Van Ryzin 2004). Without loss of generality, the scale parameter μ is conventionally set to 1, as its value is unidentifiable and will not affect the choice probabilities (see Ben-Akiva & Lerman 1985). Thus, $\xi_{B,i} - \xi_{A,i}$ ($i = 1$,...,n) is logistically distributed with mean zero and scale parameter 1. Let $F(.)$ denote the CDF of the distribution. From Equation (6.6), it follows that

$$p_{i,A} = F(\alpha_p(\omega_i - D) - \alpha_d d_i) = \frac{1}{1 + \exp(\alpha'_r \omega_i - \alpha'_d d_i - D)} \;\; i = 1, \ldots, n. \tag{6.7}$$

$p_{i,A}$ is a function of variables r_1, r_2, b_1 and b_2 and parameters α'_r, α'_d and D. Note that ω_i, the sum of current and future rewards, and d_i, the distance to the next tier, are piecewise functions and depend upon s_i, the customer's accumulated purchase in period i. Therefore, $p_{i,A}$ is also a piecewise function with discontinuities at b_1 and b_2. More specifically, from equations (6.2), (6.3) and (6.7), $p_{i,A}$ can be rewritten as

$$p_{i,A} = \begin{cases} \dfrac{1}{1+\exp(\alpha_r'(r_1+r_2)-\alpha_d'(b_1-s_i)-D)} & \text{if} \quad s_i < b_1, \\ \dfrac{1}{1+\exp(\alpha_r'(r_2+r_3)-\alpha_d'(b_2-s_i)-D)} & \text{if } b_1 \le s_i < b_2, \quad i = 1,\dots,n. \\ \dfrac{1}{1+\exp(\alpha_r' r_3 - D)} & \text{if} \quad s_i \ge b_2, \end{cases} \tag{6.8}$$

Since in each period customers make a purchase either at Firm A or at Firm B, it follows that $p_{i,B} = 1 - p_{i,A}$.

6.2.3 The Revenue Function

To formulate Firm A's expected revenue function, one needs to derive the probability distribution of accumulated purchase at the end of the selling horizon. That is, one must find $\Pr(s_n = kp)$ for any $k = \{0,\ldots,n\}$. It can be shown that $\{s_i | i = 1,..,n\}$ satisfies the conditional independence property and hence evolves according to a Markov chain. Specifically, according to Equation (6.8), a customer's response probability only depends on his/her *state* (accumulated purchase) at the current period, not his/her entire previous history of purchases. In other words,

$$Pr(s_i = k_i p | s_{i-1} = k_{i-1}p, s_{i-2} = k_{i-2}p, \dots, s_0 = k_0 p) = Pr(s_i = k_i p | s_{i-1} = s_{i-1}p), \quad i = 1, \dots n. \tag{6.9}$$

For notational simplicity, let

$$x_i = s_i/p\,,\; i = 1,\dots,n. \tag{6.10}$$

x_i, in fact, represents a customer's total *number* of purchases at Firm A up to period i. Clearly, $\{x_i | i = 1,..,n\}$ is also a Markov chain with the state space $\Omega = \{0,1,\dots, n\}$ that increments by 1 when the customer chooses Firm A in any period. Let $\boldsymbol{A}_{(n+1 \times n+1)}$ be the transition probability matrix whose $(j+1,\, k+1)^{\text{th}}$ element is

$$A_{j+1,k+1} = \Pr(x_i = k | x_{i-1} = j),\; i = 1,\dots,n,\;\; j \text{ and } k = 0,\dots,n. \tag{6.11}$$

In other words, $A_{j+1,k+1}$ is the probability that the customer's total number of purchases at Firm A reaches k given that he/she has bought j units of the product at Firm A. Considering the fact that each customer buys exactly one unit of the product in each period either at Firm A or at Firm B, $A_{j+1,k+1}$ in the above equation can be expressed as

$$A_{j+1,k+1} = \begin{cases} \bar{p}_{j,A} & \text{for } \; k = j+1, \\ 1 - \bar{p}_{j,A} & \text{for } \; k = j, \\ 0 & \text{else,} \end{cases} \qquad j \textit{ and } k = 0,\dots,n, \tag{6.12}$$

where $\bar{p}_{j,A}$ can be derived based on equations (6.8) and (6.10) as

$$\bar{p}_{j,A} = \begin{cases} \dfrac{1}{1 + \exp(\alpha_r'(r_1 + r_2) - \alpha_d'(b_1 - jp) - D)} & \text{if} \qquad jp < b_1, \\ \dfrac{1}{1 + \exp(\alpha_r'(r_2 + r_3) - \alpha_d'(b_2 - jp) - D)} & \text{if } b_1 \le jp < b_2, \\ \dfrac{1}{1 + \exp(\alpha_r' r_3 - D)} & \text{if} \qquad jp \ge b_2, \end{cases} \quad j = 0,\dots,n. \tag{6.13}$$

As mentioned earlier, one needs the distribution of customers' accumulated purchase at the end of the selling horizon (s_n) to form Firm A's expected revenue function. It is clear that

$$Pr(s_n = kp) = Pr(x_n = k), \quad k = 0,\dots,n. \tag{6.14}$$

Now, $\Pr(x_n = k)$ can be found using the n-step transition probability matrix of x_i, $\boldsymbol{A}^{(n)}$. More specifically, since $x_0 = 0$, the $(1, k+1)^{\text{th}}$ entry of matrix $\boldsymbol{A}^{(n)}$ represents the probability that a customer buys k products at Firm A throughout the selling cycle. To simplify the notation, let

$$\boldsymbol{P} = \left[A_{1,j}^{(n)}\right], \quad j = 1, \dots, n+1 \tag{6.15}$$

be a vector containing the elements of the first row of $\boldsymbol{A}^{(n)}$. Thus, the $(k+1)^{\text{th}}$ entry of $\boldsymbol{P}$ represents the probability that a customer buys k products over the entire selling cycle.

Having found the total purchase probabilities, one can formulate Firm A's expected revenue function, R_A, as follows:

$$R_A = \sum_{k=0}^{n} kpPr(s_n = kp) - C_r = \sum_{k=0}^{n} kpP_{k+1} - C_r, \tag{6.16}$$

Where C_r denotes the expected cost of reward that Firm A incurs during the selling horizon. Based on Firm A's reward structure,

$$C_r = p_{tier\ 1}r_1 + p_{tier\ 2}r_2 + p_{tier\ 3}r_3 \tag{6.17}$$

where $p_{\text{tier i}}$ is the probability that a customer achieves tier i. $p_{\text{tier i}}$'s can be expressed in terms of the state variable, x_i. Specifically, define

$$l_1 = \text{Min}\,\{k: kp \geq b_1, k = 0, \dots, n\} \text{ and } l_2 = \text{Max}\,\{k: kp < b_2, k = 0, \dots, n\}. \tag{6.18}$$

Now, from the definition of $\boldsymbol{P}$ in Equation (6.15), it follows that

$$p_{\text{tier 2}} = \sum_{k=0}^{l_1-1} P_{k+1}, \quad p_{\text{tier 2}} = \sum_{k=l_1}^{l_2} P_{k+1} \text{ and } \quad p_{\text{tier 3}} = \sum_{k=l_2+1}^{n} P_{k+1}. \tag{6.19}$$

Thus, the revenue function in Equation (6.16) can be rewritten as

$$R_A = \sum_{k=0}^{n} kpP_{k+1} - r_1 \sum_{k=0}^{l_1-1} P_{k+1} - r_2 \sum_{k=l_1}^{l_2} P_{k+1} + r_3 \sum_{k=l_2+1}^{n} P_{k+1}. \tag{6.20}$$

6.2.4 The Optimization Model

The purpose is to study the structural properties of Firm A's optimal reward scheme. The reward scheme is characterized by reward values (i.e., r_1, r_2 and r_3) and breakpoints (i.e., b_1 and b_2). We are looking for the scheme that maximizes profitability of Firm A. Thus, one can optimize Firm A's revenue function, R_A, in terms of r_1, r_2, b_1 and b_2. For analytical convenience, the price (p) is normalized to 1 and α_r' and α_d' are changed correspondingly, so that the choice probabilities remain the same. Subsequently, after obtaining the optimal solution, the values of decision variables and revenue functions must be scaled back with the same factor. For example, if the original price is \$100, p and D are divided and α_p and α_d are multiplied by 100. Then, an optimal r_1 value of 0.25 in the resulting model indicates $r_1 = \$25$ in the original scale. From Equation (6.13), it is evident that such normalization will not affect the choice probabilities. However, Equation (6.20) shows that the revenue function will change by the same ratio used for normalization. The optimization model can be written as follows:

$$\underset{r_1,r_2,r_3,b_1,b_2}{\text{maximize}}\ R_A = \sum_{k=0}^{n} kpP_{k+1} - r_1 \sum_{k=0}^{l_1-1} P_{k+1} - r_2 \sum_{k=l_1}^{l_2} P_{k+1} + r_3 \sum_{k=l_2+1}^{n} P_{k+1}, \tag{6.21}$$

Subject to:

$$b_1 \geq 1, \tag{6.21a}$$

$$b_2 \geq 2, \tag{6.21b}$$

$$b_2 \leq n, \tag{6.21c}$$

$$r_2 \leq b_1 - (l_1 - 1), \tag{6.21d}$$

$$r_3 \leq b_2 - l_2, \tag{6.21e}$$

$$r_1 \leq r_2 \tag{6.21f}$$

$$r_2 \leq r_3 \tag{6.21g}$$

$$r_1, r_2, r_3 \leq 1 \tag{6.21h}$$

$$r_1, r_2, r_3, b_1, b_2 \geq 0. \tag{6.21i}$$

The first two constraints state that at least two purchases are needed to achieve the second tier and minimum three purchases are required to earn the Tier-3 loyalty status. Constraint (6.21c) guarantees that achieving the last-tier reward is possible. Constraints (6.21d) and (6.21e) refer to the fact that under optimal conditions, the value of the next reward must be less than or equal to the amount of money a customer must pay to gain it. In other words, the distance to the next reward must be less than the reward itself. Otherwise, a customer may *earn* money by making an additional purchase. Intuitively, customers earn higher rewards in higher loyalty levels. This is captured through Constraints (6.21f) and (6.21g). Constraint (6.21h) guarantees that the reward will not exceed the offered price in any period.

6.3 Results

Various analyses can be performed using the model. The objective of this study is to find the conditions under which adopting a three-level reward is not optimal. The main characteristic of such a reward system is the inequality of rewards at different tiers. Thus, if an optimal solution satisfies $r_1^* \neq r_2^* \neq r_3^*$, one can argue that a three-level reward is the preferred configuration. As noted earlier, the objective function in Equation (6.21) consists of three parameters, namely, D, α_r' and α_d'. So, one can solve the model for different sets of parameter values and specify the sets that satisfy the rewards inequality condition. This analysis characterizes some market conditions that justify adopting a three-tier reward scheme. The conditions that are incorporated in the model are the price offered by the firm's competitor, the customers' sensitivity to loyalty reward and their sensitivity to distance to attain a reward.

It can be shown that R_A in Equation (6.21) is a non-convex nonlinear function. Here, the Matlab©'s interior-point algorithm is employed to find the optimal solution (i.e., r_1^*, r_2^*, r_3^*, b_1^* and b_2^*) for different values of D, α_r' and α_d'. Also, by expanding P_{k+1} based on equations (6.12), (6.13) and (6.15), one can see that R_A is a piecewise function whose expression depends on b_1 and b_2. Defining different ranges for b_1 and b_2, the model can be broken into sub-models with unique expressions. It can be proved that the number of sub-models equals $(n-1)(n-2)/2$, where n is the number of periods in a selling horizon. In this study, the case $n=4$ is considered. Because, while $n=4$ keeps the problem at a manageable level, it has practical implications in the real world (e.g., buying once a week in a month or once a season in a year).

As mentioned above, the model should be solved for different sets of parameter values. α_r' and α_d' values were set in the range [0,3] with a step size of 0.1, resulting in 31 values for each parameter. The value of D was set at {0, 0.1, 0.2, 0.3, 0.4, 0.5}. Note that since the price is normalized to one, as an instance, $D=0.1$ indicates that the price offered by Firm B is 10% less than Firm A's price. Thus, in total 5,766 ($31 \cdot 31 \cdot 6$) combinations of parameter values were generated. Given each set of parameter values, the three sub-models were formulated and optimized. Comparing the optimal R_A values obtained from the three sub-models, the overall optimal solution of the problem was specified for any given set of parameters. To illustrate the optimization process, some sample results are presented in Table 6.1. As can be seen, for $\alpha_r' = 1.7, \alpha_d' = 1.5$, $D = 0.2$, the optimal revenue from Sub-model 3 is greater than optimal values of the other sub-models. Thus, optimal solution is obtained from Sub-model 3.

To find the conditions under which a three-level reward is the optimal choice in the framework of the model, first the model was solved under all 5,766 parameter combinations. The resulting optimal solutions were tested against the rewards inequality condition (i.e., $r_1^* \neq r_2^* \neq r_3^*$). For instance, Table 6.1 shows that when $\alpha_r' = 1.7$, $\alpha_d' = 1.5$, $D = 0.2$, the optimal reward values are $r_1^* = 0$, $r_2^* = 0.05$ and $r_3^* = 0.08$. Since rewards are unequal, this solution indicates that, given the parameter values, a three-level loyalty reward scheme optimizes the profitability. Whereas, when $\alpha_r' = 0.8$, $\alpha_d' = 0.6$, $D = 0.2$, not offering a loyalty reward is the optimal decision. The result of this analysis is depicted in Figure 6.2. Each graph in this figure represents a specific D value. The shaded area shows the combinations of α_r' and α_d' for which the optimal solution characterizes a three-tier reward scheme. Thus, using this figure, a firm can judge whether implementing a three-tier reward scheme is justified, given the state of the market in which they are competing. Here, we focus on the structural properties of three-tier reward optimality conditions to gain insights into the profitability of three-tier reward schemes.

Table 6.1: Sample optimization results

			Sub-model 1						Sub-model 2						Sub-model 3					
α_r'	α_d'	D	r_1^*	r_2^*	r_3^*	b_1^*	b_2^*	$\boldsymbol{R_A^*}$	r_1^*	r_2^*	r_3^*	b_1^*	b_2^*	$\boldsymbol{R_A^*}$	r_1^*	r_2^*	r_3^*	b_1^*	b_2^*	$\boldsymbol{R_A^*}$
1.7	1.5	0.2	0.30	0.32	0.32	2.32	3.32	**0.98**	0.00	0.25	1.00	1.25	4.00	**1.37**	0.00	0.05	0.08	1.05	2.08	**1.80**(*)
0.8	0.6	0.2	0.00	0.31	0.31	2.31	3.31	**1.46**	0.00	0.00	1.00	1.00	4.00	**1.65**	0.00	0.00	0.00	1.00	2.00	**1.80**(*)
1.1	0.2	0.1	0.00	0.56	0.56	2.56	3.56	**2.04**	0.00	0.00	1.00	1.00	4.00	**2.14**(*)	0.00	0.08	0.21	1.08	2.21	**1.92**

—(*) Optimal sub-model

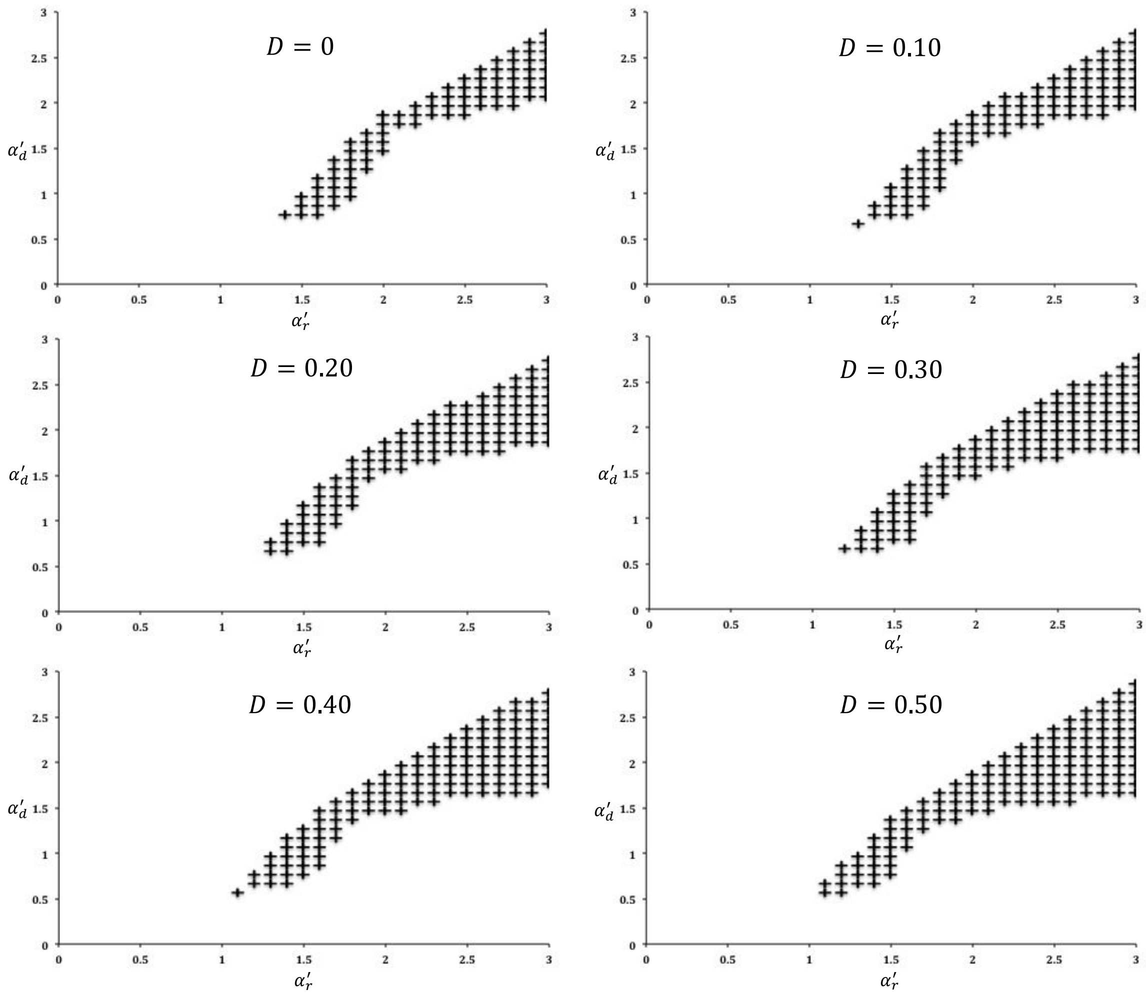


Figure 6.2: Three-tier rewards' optimality conditions

From Figure 6.2, it is evident that, regardless of $\alpha_d^{'}$ value, a three-tier reward is not optimal when $\alpha_r^{'} < 1$. As pointed out earlier, $\alpha_r^{'} = \alpha_r/\alpha_p$. So, $\alpha_r^{'} < 1$ specifies $\alpha_r < \alpha_p$. Thus, when the effect of price on customers' decision to buy is greater than the effect of rewards, adopting a three-tier scheme is not optimal. In other words, if the loyalty program's role in customers' decision is not as important as price, implementing a three-tier loyalty program yields suboptimal revenue. Comparing all six graphs, one can see that the finding is valid regardless of the competitors' offered price. The intuition is that the multitier reward structure, in such circumstances, only adds complexity to the customers' decision-making, without making a significant contribution to the outcome of the decision-making process.

Similarly, an interesting insight is gained by evaluating the three-tier schemes' "optimality areas" from $\alpha_d^{'}$ perspective. $\alpha_d^{'}$ represents the utility's relative sensitivity to the distance. The distance, in fact, captures the effort that a customer should expend to achieve a reward. As discussed earlier, the distance has a negative effect on the utility. More specifically, a fraction of the reward's utility is offset by the distance's disutility. Figure 6.2 shows that when $\alpha_d^{'} < 0.5$, the three-tier rewards system is not optimal, regardless of $\alpha_r^{'}$ and D. Since $\alpha_d^{'} = \alpha_d/\alpha_p$, $a_d^{'} < 0.5$ signifies $\alpha_d < 0.5\,\alpha_p$. In other words, three-tier rewards can be optimal only when customer's sensitivity to the distance is greater than half of their sensitivity to the price. In fact, low distance sensitivities result in higher fraction of customers in the top loyalty tiers. This will impose higher reward costs on the firm, which leads to suboptimal revenue. Note that the effect of α_r should also be considered while analyzing the effect of α_d on the three-tier rewards' optimality conditions. From Figure 6.2, it can be seen that as α_d increases, generally higher α_r values will justify the three-tier reward scheme. That is, there exists a specific balance between the utility of reward and disutility of distance at which three-tier rewards becomes optimal.

Comparing the size of the shaded areas in different graphs, one can evaluate the effect of D on the optimality conditions. More specifically, it can be seen that the size of area increases with D. That is, when the competitor offers lower prices, greater combinations of

$\alpha_r^{'}$ and $\alpha_d^{'}$ values justify the three-tier loyalty scheme. In other words, higher price cuts by the competitor increases the chances of the optimality of the multi-tier reward. Although, here the issue of whether loyalty program strategy is more effective than lower price strategy is not addressed. One can perform this analysis by comparing the optimal revenues of Firm A and Firm B under different conditions.

6.4 Summary and Conclusion

In this chapter, an analytical model was developed to analyze the optimality conditions of a three-tier loyalty reward scheme. The decision variables were different tiers' breakpoints and reward values. It was assumed that the market is served by two firms, one offering loyalty rewards and one offering a lower price. The binary-logit model was employed to model customers' demand. Using stochastic processes' theories the distribution of customers' demand was identified. The objective was to optimize the expected revenue function of the firm offering loyalty rewards. The formulation resulted in a nonlinear programming. The optimal solution of the model was found at some arbitrary values of $\alpha_r^{'}$, $\alpha_d^{'}$, the relative utility sensitivity coefficients and, D, the price cut offered by the competitor.

The focus of this study was to formulate the conditions under which three-tier reward schemes yield optimal revenues. In other words, the objective was to address this important question in the context of loyalty programs: "to adopt or not to adopt a multitier reward system?". Optimality conditions were derived and discussed with respect to four factors: the effect of price, rewards and distance on customers' decision to buy and the price offered by the competitor. Based on the model settings, it was found that a low level of sensitivity to rewards and distance in customers may lead to suboptimal revenues from a three-tier reward scheme. Also, it was observed that higher discounts from the competitor make three-tier reward structure a viable alternative in more cases.

Chapter 7

Conclusion, Contributions and Future Research

7.1 Conclusion and Summary of Contributions

In Chapter 3, a model was developed to analyze the effects of customer satisfaction on the profitability of loyalty programs. Also, through a stochastic model, the lack of research on the effects of customer valuation for the product being offered was addressed. However, satisfaction level was incorporated as a parameter. As a result, the uncertainty in customers' satisfaction levels was not captured in the model. Based on the obtained results, the relationship between customer satisfaction and profitability of a loyalty program was characterized. In the framework of the assumptions, it was shown that if the firm maintains satisfaction among customers, not adopting a loyalty program yields optimal profits. However, if the firm decides not to offer a loyalty reward and customers turn out to be dissatisfied, the firm will obtain suboptimal profits. It was also observed that offering a loyalty reward causes the first period price to increase with a factor proportional to customers' repurchase intentions, an intuitive result which is consistent with earlier studies on loyalty programs.

In Chapter 4, the model formulation was extended to the case where uncertainty in customers' satisfaction levels is also accounted for. That is, instead of assuming a parametric satisfaction, satisfaction was incorporated as a random variable. The results were in line with the previous findings. Again, it was observed that under certain conditions the firm may be better off not offering a loyalty reward. Particularly, it was seen that if the mean of satisfaction levels turns out to be positive with a standard deviation less than a certain threshold, not offering loyalty rewards yields the optimal revenue. Thus, if the firm homogenously maintains satisfaction among customers, it is optimal not to offer a loyalty reward.

In Chapter 5, the focus was turned to the reward structure as one of the key determinants of a loyalty program's effectiveness. Loyalty reward structures are categorized in two schemes with respect to the reward rate: single-tier rewards and multitier rewards. In Chapter 5, a model was developed to evaluate the effectiveness of multitier reward schemes. Based on the framework of the model assumptions, it was found that the multitier scheme is more effective than the single-tier scheme in increasing a firm's profitability and driving repeat purchase behavior. It was also found that multitier schemes, compared to single-tier systems, yield higher revenues from both light users and heavy users.

In Chapter 6, a more realistic, and hence a more sophisticated, model was developed to analyze the effectiveness of multitier reward structures. A new feature of this model was to explicitly incorporate the "outside firm" which, in the other models in this study, would satisfy the customers' demand in case they would decide not to buy at the focal firm. The objective of the study was to characterize the conditions under which a three-tier reward scheme is not the optimal choice. The conditions were formulated based on the sensitivity of customers' demand to the offered price, rewards and distance and also based on the price offered by the competitor. It was found that low levels of sensitivity to rewards and distance in customers can result in suboptimal revenues from a three-tier reward scheme. Also, it was observed that if the competitor offers deeper discounts, the three-tier reward structure becomes optimal in more cases.

This study contributed to the literature on loyalty programs by developing new analytical models to evaluate the effectiveness of such programs. From one viewpoint, the contributions can be categorized in two areas. In part, this research can be viewed as a step forward toward realistic models to gain managerial insights into the adoption of loyalty programs. In this line, some of the restrictive assumptions in earlier models were relaxed. Specifically, customers' valuation for the product and their post-purchase satisfaction were incorporated in the models. Customer valuation and satisfaction have not been considered in the existing literature on loyalty programs. Chapters 3 and 4 of the thesis are primarily concerned with this direction of this research.

Secondly, some new models were developed to analyze the loyalty reward structure, a design element that has been overlooked in the loyalty program literature. In more detail, a model was formulated to investigate the effectiveness of multitier rewards to that of single-tier rewards. Also, a model was proposed to decide which reward scheme is the optimal choice depending on the market conditions. Although the main focus was to evaluate the effectiveness of single-tier and multitier reward schemes, a framework is developed that can be used to optimize the reward structure. Chapters 5 and 6 of the thesis refer to this aspect of the work.

7.2 Future Research

As discussed earlier, in many cases the firms' decision to adopt a loyalty program is mainly a response to their competitors' similar programs. On the other hand, frequent changes in the design of some loyalty programs can be attributed to the lack of adequate theoretical frameworks to support the design decisions. These observations suggest a strong need for research in different aspects of loyalty programs. This study is an attempt to address part of this gap. Yet, a lot remains to be discovered. The title of an article by Nsakanda (2011) portrays this significant potential of research on loyalty programs: "Reward Loyalty Programs Industry – a mine for OR/MS applications". The followings are some direction for future "excavations":

In chapters 3 and 4, customers' satisfaction level was treated as an exogenous factor. One direction for future research is to endogenize the satisfaction level by making it a function of some endogenous variables. Such a model would provide a framework that enables a firm to balance the trade-off between investing in loyalty reward programs and implementing customer satisfaction programs. In both models, by allowing a customer not to buy from the firm, it was implicitly assumed that there are other sellers in the market that will meet the customer's demand. However, the effect of other seller's actions was not considered in the firm's pricing decisions. Thus, the models can be extended by explicitly incorporating competition.

In Chapter 5, a particular multitier reward scheme was analyzed in which tiers are defined based on purchase frequency. Future research may assess the robustness of the findings to other loyalty measures like cumulative spending or accumulated points balance.

The model developed in Chapter 6 can be used to perform various other analyses. For instance, it is insightful to study the effects of the model parameters on the optimal values of rewards and each firm's overall profitability. Also, despite its complexity, the model can be used to develop a practical decision-making framework for the design of loyalty programs. In Chapter 6, it was assumed that the offered prices by the firms remain constant in the selling horizon. This assumption ensures that customers' accumulated purchases follow the Markov chain. One can expand the model by removing this assumption. The stochastic dynamic programming approach can be used to formulate and optimize the revenue function under the dynamic pricing assumption. In this study, the optimality of the commonly used three-tier reward scheme was evaluated. A direction for further studies is to develop a framework to study a general k-tier reward scheme. Finally, incorporating game theoretic models, one can study the equilibrium conditions in the framework of the model.

Empirical studies on loyalty programs are increasingly emphasizing the importance of considering both behavioral and attitudinal aspects of loyalty. To capture the effect of loyalty programs on attitudinal loyalty in analytical models, one can incorporate the measure that are used to define brand loyalty. Raju et al. (1990) define loyalty as “the minimum difference between the prices of the two competing brands necessary to induce the loyal consumers of one brand to switch to the competing brand”. By redefining this measure in the context of loyalty programs, future studies may examine the effect of loyalty programs on both behavioral and attitudinal loyalty.

The loyalty point promotions is another area that merits further research. Through the point promotions, price discounts for products are replaced by higher rates of reward points. In this line, it would be interesting to compare the effectiveness of price promotions and point promotions. One can also incorporate the literature on price promotions to devise frameworks for point-promotion decisions.

Nowadays, loyalty rewards tend to be accumulated through partners as well as through principal operators. For example, customers may gain reward points through an airline's partners (e.g., other airlines, car rentals and hotels) and even through non-travel purchases (e.g., grocery or gas). From the firm's perspective, this makes the assessment of the program’s profitability much more complex. It remains for the future studies to evaluate the effects of alliance in the context of loyalty programs. Since the alliance is a design decision, such studies will contribute to the literature on the design of loyalty programs.

The reward type and reward expiry are some other design elements that demand further research. There has been an extensive behavioral literature on the effectiveness of monetary and non-monetary rewards that can be incorporated to guide the reward-type decisions. Also, one may analyze the effects of reward expiry on the customers' purchase behavior and the loyalty programs' profitability. The findings of studies on design elements such as reward rate, reward type, reward timing and partnership can be integrated to develop a decision-making framework to optimize the design of a loyalty program in different industries and markets.

Appendix 1 – Concavity of Revenue Function When Valuation Is Uniformly Distributed

Here, it is proved that firm's revenue functions are concave when customers' valuation is uniformly distributed. R_1 is given in Equation (3.15). The Hessian of R_1 is:

$$H_1 = \begin{pmatrix} -\frac{2}{1+\gamma} & -\frac{\gamma}{1+\gamma} & \frac{\gamma}{1+\gamma} \\ -\frac{\gamma}{1+\gamma} & -2 & 2\gamma \\ \frac{\gamma}{1+\gamma} & 2\gamma & -2\gamma \end{pmatrix}. \tag{A1.1}$$

Similarly, Hessian matrices of the two other dominating revenue functions, that is R_2 and R_3, can be found as follows:

$$H_2 = \begin{pmatrix} -\frac{2}{1+\gamma} & -\frac{2\gamma}{1+\gamma} & \frac{2\gamma}{1+\gamma} \\ -\frac{2\gamma}{1+\gamma} & 2\gamma - \frac{2\gamma^2}{1+\gamma} - 2 & \frac{2\gamma^2}{1+\gamma} \\ \frac{2\gamma}{1+\gamma} & \frac{2\gamma^2}{1+\gamma} & -\frac{2\gamma^2}{1+\gamma} \end{pmatrix} \tag{A1.2}$$

and

$$H_3 = \begin{pmatrix} -\frac{2}{1+\gamma} & -\frac{\gamma}{1+\gamma} & \frac{\gamma}{1+\gamma} \\ -\frac{\gamma}{1+\gamma} & 2\gamma - 2 & 0 \\ \frac{\gamma}{1+\gamma} & 0 & 0 \end{pmatrix}. \tag{A1.3}$$

A symmetric matrix is negative semi-definite if and only if all its eigenvalues are non-positive [30]. Thus, since the Hessian matrix is symmetric, one can check the convexity of R functions by evaluating the eigenvalues of their Hessians. Eigenvalues of H_1, H_2 and H_3 versus γ are depicted in Figure A1.1. As can be seen, all three eigenvalues of both H_1 and H_2 are non-positive over the whole range of γ values. Hence, R_1 and R_2 are concave.

One of the eigenvalues of H_3 is positive. Thus, R_3 is not concave; however, it can be seen that it is concave over its feasible region. To prove the concavity of R_3 over its feasible region, one can check if $X^T H_3 X$ is non-positive in the region, where X is the vector of the decision variables (p_1, p_2 and r). $X^T H_3 X$ is obtained as follows:

$$X^T H_3 X = \frac{2}{1+\gamma}\left[-p_1^2 + (\gamma^2 - 1)p_2^2 - \gamma p_1 p_2 + \gamma r p_1\right]. \tag{A1.4}$$

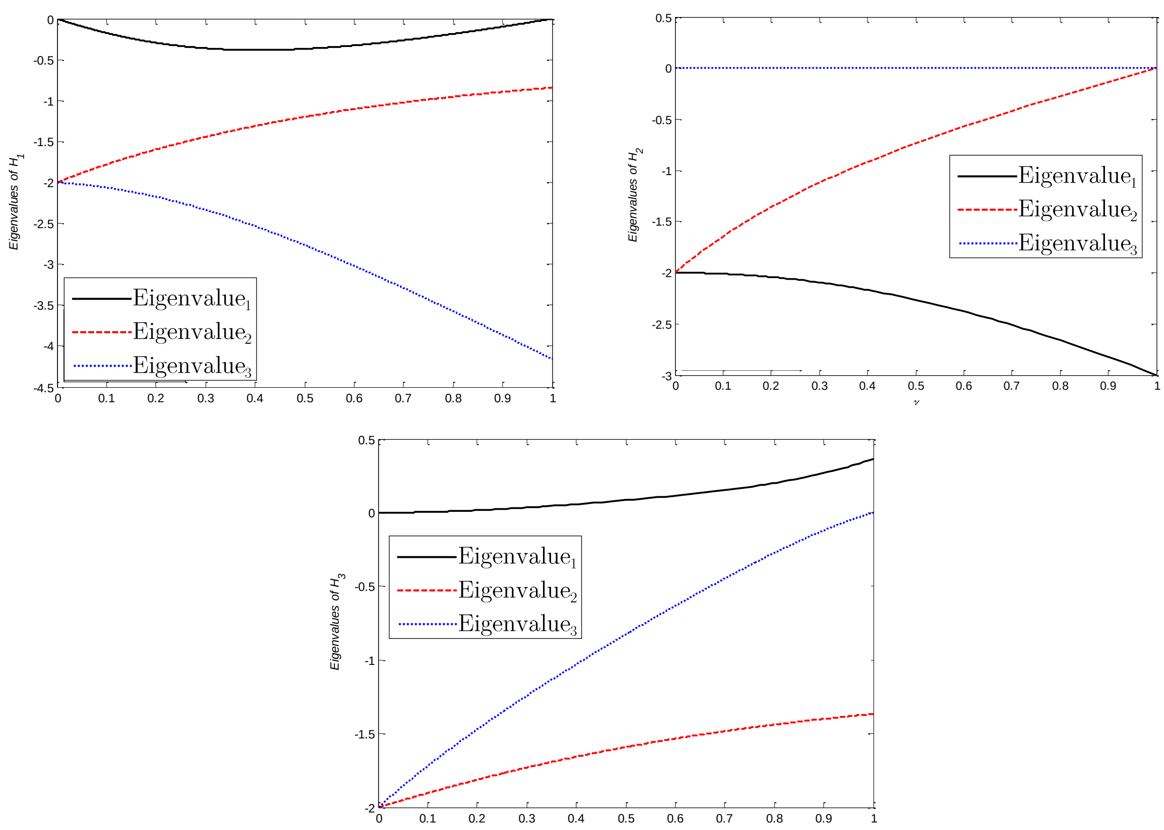


Figure A1.1: Eigenvalues of H_1, H_2 and H_3

On the other hand, from Equation (3.18), it is evident that $p_2 \leq 1$ is one of the conditions based on which R_3 is derived. Considering the general constraint of $r \leq p_2$, it is concluded that $r \leq 1$. Consequently, one must evaluate the sign of the function in the Equation (A1.4) when p_2 and r are in the range [0, 1] and $r \leq p_2$.

The evaluation was performed by plotting $X^T H_3 X$ versus p_2 and r at different values of γ and p_1. More specifically, we held γ constant and visualized how p_1 affects $X^T H_3 X$ as a function of p_2 and r. In other words, at each arbitrarily chosen γ value, an animation graph was obtained showing the changes of the $X^T H_3 X$ surface with respect to p_1. Snapshots of the animation graphs at three p_1 values, namely $p_1 = 0$, 0.5 and 1.5, are displayed in Figure A1.2. From this figure, it is evident that $X^T H_3 X$ turns positive for some values of p_2 and r when γ increases over .6. Thus, R_3 is not concave for some values of p_2 and r. However, it can be observed that these values of p_2 and r violate the $r \leq p_2$ constraint, meaning that non-concavity falls outside the feasible region of R_3.

To illustrate it, consider the case $\gamma = 1$ in Figure A1.2. As can been seen, the middle $X^T H_3 X$ surface (which belongs to $p_1 = .5$) is positive for r in the range [.6,.8] and p_2 in the range [0,.4]. These values of p_2 and r, however, are not inside the half-space specified by $r \leq p_2$. Hence, $X^T H_3 X$ is negative and R_3 is concave inside its feasible region.

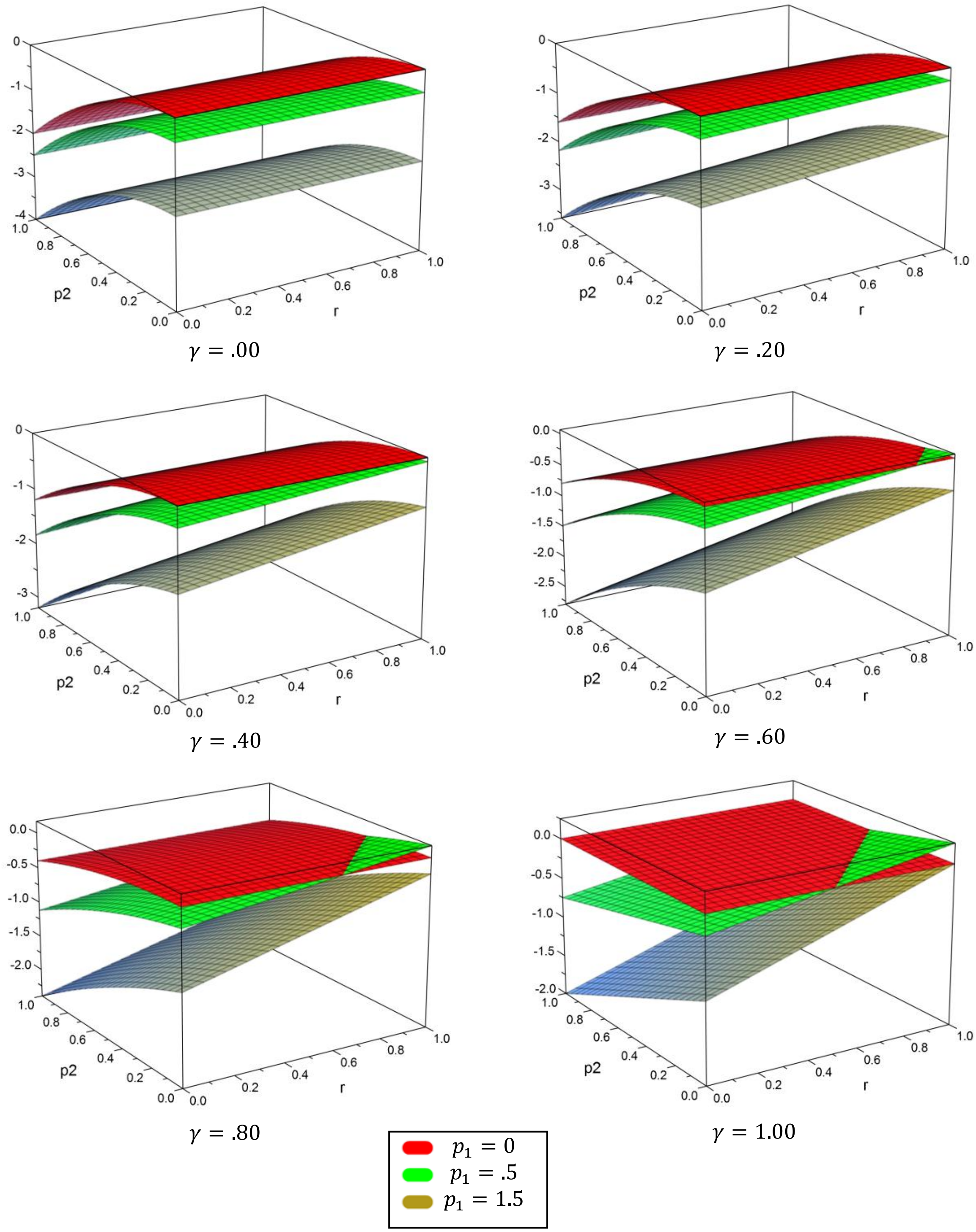


Figure A1.2: $X^T H_3 X$ versus p_2 and r at different values of γ and p_1

Appendix 2 – Optimal Solution of the Revenue Maximization Problem under Uniform Valuation

Optimal solution of R_1 model (Equation (3.16)) is obtained as follows:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ -\delta \end{pmatrix} & \text{when } 0 \leq \delta \leq -\frac{1}{2} \\ \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} & \text{when } -\frac{1}{2} \leq \delta \leq -1 \end{cases} . \tag{A2.1}$$

The above solution yields in $R_1^* = \frac{1}{2}$.

Similarly, based on KKT optimality conditions, the analytical optimal solution of R_2 model is derived as follows:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ \forall r \in \left[-\delta, \frac{1}{2}\right] \end{pmatrix} & \text{when } \delta \geq -\frac{1}{2} \\ \begin{pmatrix} \gamma + \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} & \text{when } \delta \leq -\frac{1}{2} \end{cases} . \tag{A2.2}$$

The above solution results in:

$$R_2^* = \begin{cases} \dfrac{1}{2} & \text{when } \delta \geq -\dfrac{1}{2} \\ \left(\dfrac{1-\gamma}{4}\right) - \delta(1+\delta)(1+\gamma) & \text{when } \delta \leq -\dfrac{1}{2} \end{cases}. \tag{A2.3}$$

Optimal solution of R_3 model is:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ r^* \end{pmatrix} = \begin{cases} \begin{pmatrix} \frac{1-\gamma\delta}{2} \\ \frac{1}{2} \\ -\delta-\frac{1}{2} \end{pmatrix} & \text{when } -1 \leq \delta \leq -\dfrac{1}{2} \\ \begin{pmatrix} \gamma r^* + \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} & \text{when } \delta \leq -1 \end{cases}, \tag{A2.4}$$

which produces the following revenue:

$$R_3^* = \begin{cases} \dfrac{\gamma^2\delta^2 - \gamma^2 - 2\gamma\delta + 2}{4(1+\gamma)} & \text{when } -1 \leq \delta \leq -\dfrac{1}{2} \\ \dfrac{1}{2} & \text{when } \delta \leq -1 \end{cases}. \tag{A2.5}$$

The global maximum revenue can be obtained by finding the maximum envelope of R_1^*, R_2^* and R_3^* considering that $\left(\frac{1-\gamma}{4}\right) - \delta(1+\delta)(1+\gamma) \leq \frac{1}{2}$ and $\frac{\gamma^2\delta^2-\gamma^2-2\gamma\delta+2}{4(1+\gamma)} \leq \frac{1}{2}$ in their defined domains, the global optimum is determined as in Equation (3.21).

Appendix 3 – Optimizing the Revenue Function When Customers Anticipate a Change in Their Future Valuations

Given the purchase probabilities in equations (3.36), (3.42) and (3.24), one can formulate the revenue maximization problem based on Equation (3.10) as follows:

$$\underset{p_1,p_2,r}{\text{maximize}} \quad R =$$

$$p_1\left(1 - F_{v_1'}\left(\frac{p_1+\gamma(p_2-r)}{1+\gamma}\right)\right) +$$

$$\gamma(p_2-r)\left(1-\int_{\frac{p_1+\gamma(p_2-r)}{1+\gamma}}^{\infty}\int_{p_2-r-\delta}^{\infty} f_{v_1,v_1'}(V_1,V_1')\ dV_1 dV_1'\right)+ \tag{A3.1}$$

$$(1-\gamma)p_2\big(1-\Phi(p_2)\big)$$

Subject to:

$$r \le p_2 \tag{A3.1a}$$

$$p_1, p_2 \text{ and } r \ge 0 \tag{A3.1b}$$

In the above model, $F_{v_1'}(.)$ denotes the CDF of v_1' which is normal with mean 0 and variance $1 + (\gamma\sigma/(1+\gamma))^2$, $f_{v_1,v_1'}(V_1, V_1')$ represents the joint distribution of (v_1, v_1') given in Equation (3.41) and $\Phi(.)$ denotes the CDF of the standard normal distribution.

The above model was solved using the Matlab©'s interior point algorithm for 500 sets of parameters' (γ, δ and σ) values. The obtained solutions were verified by incorporating the global search algorithm proposed by Ugray et al. (2007). Table A3.1 shows a sample of results. By examining the relationship between parameters and optimal values of variables, one can easily observe that:

$$p_1^* = \gamma r^* + 0.75 \text{ and } p_2^* = 0.75. \tag{A3.2}$$

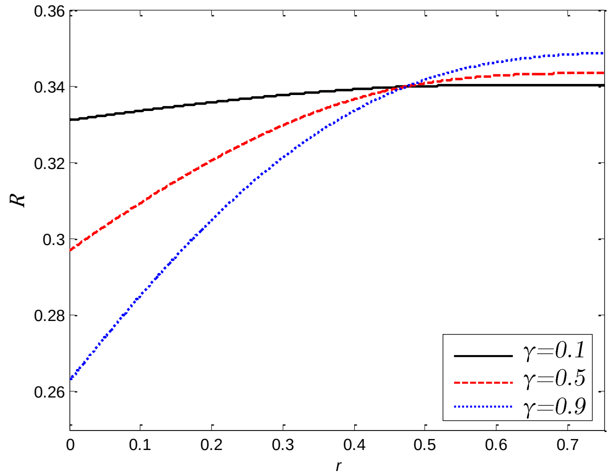
Now, to analyze the relationship between parameters and r^*, one can substitute the above p_1^* and p_2^* in the optimization model and plot R versus r. Figure A3.1 shows the resulting graph for some values of parameters. As can be seen, depending on the parameters' values, there can be an optimal *range* for r. Specifically,

$$r^* = \forall r \in [\rho, 0.75]. \tag{A3.3}$$

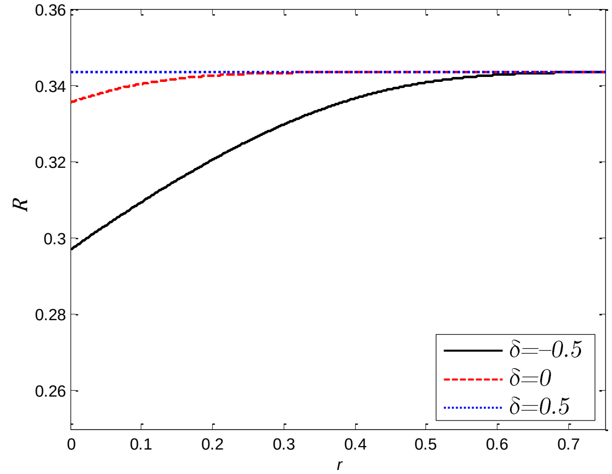
where ρ denotes the lower bound of the optimal range of r.

Table A3.1: Sample results from optimization model in Equation (A3.1)

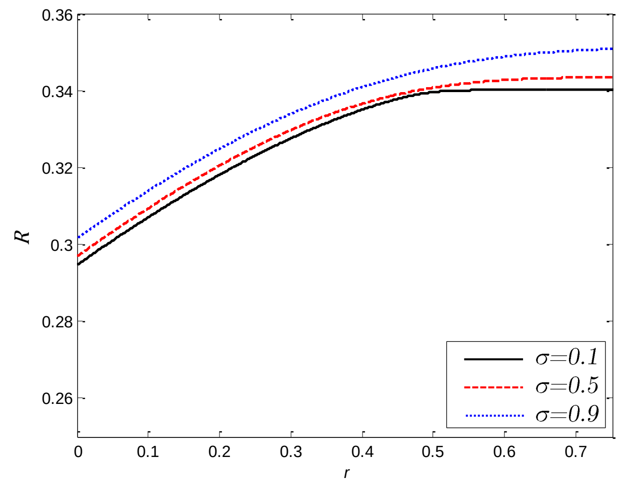
δ	σ	γ	p_1^*	p_2^*	r^*
-0.5	0.1	0.1	0.81	0.75	0.53
-0.5	0.1	0.2	0.86	0.75	0.56
-0.5	0.5	0.1	0.82	0.75	0.69
-0.5	0.5	0.2	0.90	0.75	0.73
0	0.1	0.1	0.79	0.75	0.38
0	0.1	0.2	0.83	0.75	0.38
0	0.5	0.1	0.79	0.75	0.38
0	0.5	0.2	0.83	0.75	0.38
0.5	0.1	0.1	0.79	0.75	0.38
0.5	0.1	0.2	0.83	0.75	0.38
0.5	0.5	0.1	0.79	0.75	0.38
0.5	0.5	0.2	0.83	0.75	0.38



(a) $\delta\ =-0.5, \sigma\ =\ 0.5$



(b) $\gamma\ =0.5,\ \sigma\ =\ 0.5$



(c) $\gamma\ =0.5,\ \delta\ =-0.5$

Figure A3.1: Revenue function versus r at different values of parameters

Appendix 4 – Joint Distribution of v_1 and v_2

v_1 is normally distributed with zero mean and unit variance. v_2, on the other hand, is the sum of two independent normally distributed variables: v_1 and δ with parameters (0,1) and (μ_s,σ_s), respectively. Thus, v_1 and v_2 can both be expressed as linear functions of the same normal variables, namely,

$$v_1 = 1 \times v_1 + 0 \times \delta, \tag{A4..1}$$

$$v_2 = 1 \times v_1 + 1 \times \delta. \tag{A4.2}$$

Therefore, from Bertsekas & Tsitsiklis (2002), it follows that v_1 and v_2 are jointly normal. Parameters of the resulting bivariate normal distribution are

$$\mu_{v_1} = 0 \tag{A4.3}$$

$$\mu_{v_2} = \mu_{v_1} + \mu_s = \mu_s, \tag{A4.4}$$

$$\sigma_{v_1}^2 = 1, \tag{A4.5}$$

$$\sigma_{v_2}^2 = \sigma_{v_1}^2 + \sigma_s^2 = 1 + \sigma_s^2, \tag{A4.6}$$

$$\begin{aligned} \mathrm{cov}(v_1, v_2) &= E[v_1 v_2] - E[v_1]E[v_2] \\ &= E[v_1(v_1 + s)] - 0 = E[v_1^2] + E[v_1\, s] \\ &= \sigma_{v_1}^2 + \mu_{v_1}^2 + \mu_{v_1}\mu_s = 1. \end{aligned} \tag{A4.7}$$

The above parameters form the mean vector and covariance matrix presented in Equation (4.7).

Appendix 5 – An Example of Revenue Optimization Formulation

For instance, given the following conditions:

$$\left\{\frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} \le 1, \frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2} \le 1, \frac{\sum_{i=2}^{3} p_i - r_1}{2} \le 1, p_3 - r_2 \le 1,\right.$$

$$\left. p_3 - r_1 \le 1 \text{ and } p_i \le 1 \ (i = 1,2,3)\right\}, \tag{A5.1}$$

the total revenue function is:

$$\begin{aligned}
R_1 = &\left[\gamma\left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)p_1 + (1-\gamma)(1 - p_1)p_1\right] + \\
&\left[\gamma\left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\left(1 - \frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2}\right)(p_2 - r_1) + \right. \\
&\left.\gamma\left(1 - \left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\right)\left(1 - \frac{\sum_{i=2}^{3} p_i - r_1}{2}\right)p_2 + (1-\gamma)(1 - p_2)p_2\right] + \\
&\left[\gamma\left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\left(1 - \frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2}\right)\big(1 - (p_3 - r_2)\big)(p_3 - r_2) + \right. \\
&\gamma\left(1 - \left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\right)\left(1 - \frac{\sum_{i=2}^{3} p_i - r_1}{2}\right)\big(1 - (p_3 - r_1)\big)(p_3 - r_1) + \\
&\gamma\left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\left(1 - \left(1 - \frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2}\right)\right)\big(1 - (p_3 - r_1)\big)(p_3 - r_1) + \\
&\left.\gamma\left(1 - \left(1 - \frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3}\right)\right)\left(1 - \left(1 - \frac{\sum_{i=2}^{3} p_i - r_1}{2}\right)\right)(1 - p_3)p_3 + (1-\gamma)(1 - p_3)p_3\right]
\end{aligned} \tag{A5.2}$$

In order to find the optimal revenue under the above-mentioned conditions, the following NLP must be solved:

$$\underset{p_1, p_2, p_3, r_1, r_2}{\text{maximize}} \; R_1 \tag{A5.3}$$

Subject to:

$$\frac{\sum_{i=1}^{3} p_i - \sum_{j=1}^{2} r_j}{3} \leq 1 \tag{A5.2a}$$

$$\frac{\sum_{i=2}^{3} p_i - \sum_{j=1}^{2} r_j}{2} \leq 1 \tag{A5.2b}$$

$$\frac{\sum_{i=2}^{3} p_i - r_1}{2} \leq 1 \tag{A5.2c}$$

$$p_3 - r_2 \leq 1 \tag{A5.2d}$$

$$p_3 - r_1 \leq 1 \tag{A5.2e}$$

$$p_i \leq 1 \;\; (i = 1,2,3) \tag{A5.2f}$$

$$p_2 \geq r_1 \tag{A5.2g}$$

$$p_3 \geq r_1 \tag{A5.2h}$$

$$p_3 \geq r_2 \tag{A5.2i}$$

$$p_i \geq 0 \; (i = 1,2,3) \tag{A5.2j}$$

References

Au, N., Ngai, E.W.T. & Cheng, T.C.E., 2002. A critical review of end-user information system satisfaction research and a new research framework. *Omega : The International Journal of Management Science*, 30(6), pp.451–478.

Ben-Akiva, M. & Lerman, S.R., 1985. *Discrete choice analysis: Theory and application to travel demand* 1st ed., The MIT Press.

Bertsekas, D.P. & Tsitsiklis, J.N., 2002. *Introduction to probability* 1st ed., Nashua, NH: Athena Scientific.

Bijmolt, T.H.A., Dorotic, M. & Verhoef, P.C., 2010. Loyalty programs: Generalizations on their Adoption, effectiveness and design. *Foundations and Trends in Marketing*, 5(4), pp.197–258.

Biyalogorsky, E., Gerstner, E. & Libai, B., 2001. Customer referral management: Optimal reward programs. *Marketing Science*, 20(1), pp.82–95.

Blattberg, R.C. & Sen, S.K., 1974. Market Segmentation Using Models of Multidimensional Purchasing Behavior. *Journal of Marketing*, 38(4), pp.17–28.

Breidert, C., 2006. *Estimation of willingness-to-pay: theory, measurement, application*, Wiesbaden: Deutscher Universitäts-Verlag.

Buckinx, W. & Van den Poel, D., 2005. Customer base analysis: partial defection of behaviourally loyal clients in a non-contractual FMCG retail setting. *European Journal of Operational Research*, 164(1), pp.252–268.

Bustos-Reyes, C.A. & González-Benito, Ó., 2008. Store and store format loyalty measures based on budget allocation. *Journal of Business Research*, 61(9), pp.1015–1025.

Butcher, K., Sparks, B. & O'Callaghan, F., 2001. Evaluative and relational influences on service loyalty. *International Journal of Service Industry Management*, 12(4), pp.310–327.

Byrd, R.H., Gilbert, J.C. & Nocedal, J., 2000. A trust region method based on interior point techniques for nonlinear programming. *Mathematical Programming*, 89(1), pp.149–185.

Caminal, R. & Claici, A., 2007. Are loyalty-rewarding pricing schemes anti-competitive? *International Journal of Industrial Organization*, 25(4), pp.657–674.

Caminal, R. & Matutes, C., 1990. Endogenous switching costs in a duopoly model. *International Journal of Industrial Organization*, 8(3), pp.353–373.

Cigliano, J., Georgiadis, M., Pleasance, D., & Whalley, S., 2000. The price of loyalty. *McKinsey Quarterly*, (4), pp.68–77.

COLLOQUY, 2011. The billion member march: The 2011 COLLOQUY loyalty census. Available at: http://www.colloquy.com/files/2011-COLLOQUY-Census-Talk-White-Paper.pdf [Accessed July 15, 2011].

Dekay, F., Toh, R.S. & Raven, P., 2009. Loyalty programs: Airlines outdo hotels. *Cornell Hotel and Restaurant Administration Quarterly*, 50(3), pp.371–382.

Dick, A. & Basu, K., 1994. Customer loyalty: Toward an integrated conceptual framework. *Journal of the Academy of Marketing Science*, 22(2), pp.99–113.

Dowling, G.R. & Uncles, M., 1997. Do customer loyalty programs really work? *Sloan Management Review*, 38(4), pp.71–82.

Edvardsson, B., Johnson, M. D., Gustafsson, A., & Strandvik, T., 2000. The effects of satisfaction and loyalty on profits and growth: products versus services. *Total Quality Management*, 11(7), p.917.

Ehrenberg, A.S.C., Goodhardt, G.J. & Barwise, T.P., 1990. Double jeopardy revisited. *The Journal of Marketing*, 54(3), pp.82–91.

Farrell, J. & Shapiro, C., 1988. Dynamic competition with switching costs. *The RAND Journal of Economics*, pp.123–137.

Homburg, C., Koschate, N. & Hoyer, W.D., 2005. Do satisfied customers really pay more? A Study of the relationship between customer satisfaction and willingness to Pay. *Journal of Marketing*, 69(2), pp.84–96.

Jacoby, J. & Kyner, D.B., 1973. Brand loyalty vs. repeat purchasing behavior. *Journal of Marketing Research*, 10(1), pp.1–9.

Johnson, M.D., 1998. *Customer orientation and market action*, Englewood Cliffs, NJ: Prentice Hall.

Kahn, B.E., Kalwani, M.U. & Morrison, D.G., 1988. Niching versus change-of-pace brands: using purchase frequencies and penetration rates to infer brand positionings. *Journal of Marketing Research*, 25(4), pp.384–390.

Keh, H.T. & Lee, Y.H., 2006. Do reward programs build loyalty for services?: The moderating effect of satisfaction on type and timing of rewards. *Journal of Retailing*, 82(2), pp.127–136.

Kim, B.-D., Shi, M. & Srinivasan, K., 2001. Reward programs and tacit collusion. *Marketing Science*, 20(2), pp.99–120.

Kim, B.-D., Shi, M. & Srinivasan, K., 2004. Managing capacity through reward programs. *Management Science*, 50(4), pp.503–520.

Kivetz, R. & Simonson, I., 2002. Earning the right to indulge: Effort as a determinant of customer preferences toward frequency program rewards. *Journal of Marketing Research*, 39(2), pp.155–170.

Kivetz, R., Urminsky, O. & Zheng, Y., 2006. The goal-gradient hypothesis resurrected: Purchase acceleration, illusionary goal progress, and customer retention. *Journal of Marketing Research*, 43(1), pp.39–58.

Klemperer, P., 1987a. Entry deterrence in markets with consumer switching costs. *The Economic Journal*, 97, pp.99–117.

Klemperer, P., 1987b. Markets with consumer switching costs. *The Quarterly Journal of Economics*, 102(2), pp.375–394.

Klemperer, P., 1987c. The competitiveness of markets with switching costs. *The RAND Journal of Economics*, 18(1), pp.138–150.

Klemperer, P., 1988. Welfare effects of entry into markets with switching costs. *The Journal of Industrial Economics*, pp.159–165.

Kopalle, P.K. & Neslin, S.A., 2003. The economic viability of frequency reward programs in a strategic competitive environment. *Review of Marketing Science*, 1(1), pp.1–42.

Kumar, V., 2008. *Managing customers for profit: Strategies to increase profits and build loyalty*, Indianapolis, IN: Wharton School Publishing.

Kumar, V. & Reinartz, W.J., 2006. *Customer relationship management: A database approach*, Hoboken, N.J: John Wiley & Sons.

Kumar, V. & Shah, D., 2004. Building and sustaining profitable customer loyalty for the 21st century. *Journal of Retailing*, 80(4), pp.317–329.

Lal, R. & Bell, D.E., 2003. The Impact of Frequent Shopper Programs in Grocery Retailing. *Quantitative Marketing and Economics*, 1(2), pp.179–202.

Lasdon, L., Duarte, A., Glover, F., Laguna, M., & Martí, R., 2010. Adaptive memory programming for constrained global optimization. *Computers and Operations Research*, 37(8), pp.1500–1509.

Leenheer, J., van Heerde, H. J., Bijmolt, T. H. A., & Smidts, A., 2007. Do loyalty programs really enhance behavioral loyalty? An empirical analysis accounting for self-selecting members. *International Journal of Research in Marketing*, 24(1), pp.31–47.

Lewis, M., 2004. The Influence of Loyalty Programs and Short-Term Promotions on Customer Retention. *Journal of Marketing Research*, 41(3), pp.281–292.

Li, S., Ragu-Nathan, B., Ragu-Nathan, T. S., & Subba Rao, S., 2006. The impact of supply chain management practices on competitive advantage and organizational performance. *Omega : The International Journal of Management Science*, 34(2), pp.107–124.

Liu, Y., 2007. The long-term impact of loyalty programs on consumer purchase behavior and loyalty. *Journal of Marketing*, 71(4), pp.19–35.

Mägi, A.W., 2003. Share of wallet in retailing: the effects of customer satisfaction, loyalty cards and shopper characteristics. *Journal of Retailing*, 79(2), pp.97–106.

McCall, M. & Voorhees, C., 2010. The drivers of loyalty program success: An organizing framework and research agenda. *Cornell Hospitality Quarterly*, 51(1), pp.35–52.

Meyer-Waarden, L., 2007. The effects of loyalty programs on customer lifetime duration and share of wallet. *Journal of Retailing*, 83(2), pp.223–236.

Meyer-Waarden, L., 2008. The influence of loyalty programme membership on customer purchase behaviour. *European Journal of Marketing*, 42(1/2), pp.87–114.

Meyer-Waarden, L. & Benavent, C., 2009. Grocery retail loyalty program effects: self-selection or purchase behavior change? *Journal of the Academy of Marketing Science*, 37(3), pp.345–358.

Morley, R., 2004. *Customer-based antecedents of satisfaction and dissatisfaction in business-to-business services*. London: The University of Western Ontario.

Nako, S.M., 1992. Frequent flyer programs and business travellers: An empirical investigation. *Logistics and Transportation Review*, 28(4), pp.395–414.

Nie, W., 2000. Waiting: integrating social and psychological perspectives in operations management. *Omega : The International Journal of Management Science*, 28(6), pp.611–629.

Nsakanda, A.L., 2011. Reward Loyalty Programs Industry – a mine for OR/MS applications. *CORS Bulletin*, 45(2), pp.18–19.

O'Brien, L. & Jones, C., 1995. Do rewards really create loyalty? *Long range planning*, 28(4), pp.130–130.

Oliver, R.L., 1980. A cognitive model of the antecedents and consequences of satisfaction decisions. *Journal of Marketing Research*, 17(4), pp.460–469.

Oliver, R.L., 1999. Whence consumer loyalty? *Journal of Marketing*, 63(Special Issue), pp.33–44.

Passingham, J., 1998. Grocery retailing and the loyalty card. *Journal of the Market Research Society*, 40(1), pp.55–64.

Pauler, G. & Dick, A., 2006. Maximizing profit of a food retailing chain by targeting and promoting valuable customers using Loyalty Card and Scanner Data. *European Journal of Operational Research*, 174(2), pp.1260–1280.

Van den Poel, D. & Lariviere, B., 2004. Customer attrition analysis for financial services using proportional hazard models. *European journal of operational research*, 157(1), pp.196–217.

Premkumar, G. & Bhattacherjee, A., 2008. Explaining information technology usage: A test of competing models. *Omega : The International Journal of Management Science*, 36(1), pp.64–75.

Pritchard, M.P., Howard, D.R. & Havitz, M.E., 1992. Loyalty measurement: A critical examination and theoretical extension. *Leisure Sciences*, 14(2), pp.155–164.

Raju, J.S., Srinivasan, V. & Lal, R., 1990. The effects of brand loyalty on competitive price promotional strategies. *Management Science*, 36(3), pp.276–304.

Reichheld, F.F. & Sasser, W.E., 1990. Zero defections: Quality comes to services. *Harvard Business Review*, 68(5), pp.105–11.

Reichheld, F.F. & Teal, T., 2001. *The loyalty effect: the hidden force behind growth, profits, and lasting value*, Boston, MA: Harvard Business Press.

Reinartz, W. & Kumar, V., 2002. The mismanagement of customer loyalty. *Harvard business review*, 80(7), pp.86–94.

Roehm, M.L., Pullins, E.B. & Roehm Jr., H.A., 2002. Designing loyalty-building programs for packaged goods brands. *Journal of Marketing Research*, 39(2), pp.202–213.

Searle, S.R., 1982. *Matrix Algebra Useful for Statistics*, New York: Wiley.

Sharp, B. & Sharp, A., 1997. Loyalty programs and their impact on repeat-purchase loyalty patterns. *International Journal of Research in Marketing*, 14(5), pp.473–486.

Singh, S.S., Jain, D.C. & Krishnan, T.V., 2008. Customer loyalty programs: are they profitable? *Management Science*, 54(6), pp.1205–1211.

Smith, A., Sparks, L., Hart, S., & Tzokas, N, 2003. Retail loyalty schemes: results from a consumer diary study. *Journal of Retailing and Consumer Services*, 10(2), pp.109–119.

Smith, A. & Sparks, L., 2009. “It’s nice to get a wee treat if you’ve had a bad week”: Consumer motivations in retail loyalty scheme points redemption. *Journal of Business Research*, 62(5), pp.542–547.

Talluri, K.T. & Van Ryzin, G., 2004. *The theory and practice of revenue management*, Boston, MA: Kluwer Academic Publishers.

Taylor, G.A. & Neslin, S.A., 2005. The current and future sales impact of a retail frequency reward program. *Journal of Retailing*, 81(4), pp.293–305.

Ugray, Z., Lasdon, L., Plummer, J., Glover, F., Kelly, J., & Martí, R, 2007. Scatter search and local NLP solvers: A multistart framework for global optimization. *INFORMS Journal on Computing*, 19(3), pp.328–340.

Uncles, M.D., Dowling, G.R. & Hammond, K., 2003. Customer loyalty and customer loyalty programs. *Journal of Consumer Marketing*, 20(4), pp.294–316.

Von Weizsacker, C., 1984. The costs of substitution. *Econometrica*, 52(5), pp.1085–1116.

Wirtz, J., Mattila, A.S. & Lwin, M.O., 2007. How effective are loyalty reward programs in driving share of wallet? *Journal of Service Research*, 9(4), pp.327–334.

Yi, Y. & Jeon, H., 2003. Effects of loyalty programs on value perception, program loyalty, and brand loyalty. *Journal of the Academy of Marketing Science*, 31(3), pp.229–240.

Zhang, J.Q., Dixit, A. & Friedmann, R., 2010. Customer Loyalty and Lifetime Value: An Empirical Investigation of Consumer Packaged Goods. *The Journal of Marketing Theory and Practice*, 18(2), pp.127–140.