PRODUCTION DECISION ANALYSIS UNDER EXCHANGE RATE, DEMAND, AND CARBON PRICES UNCERTAINTIES

by

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Production Decision Analysis Under Exchange Rate, Demand, And Carbon Prices Uncertainties

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ABSTRACT

This thesis presents an optimal production decision analysis for a multinational firm under exchange rate, carbon allowance prices, and demand uncertainties. Firms having production and sales in two different countries experience both demand and exchange rate uncertainties. When exchange rates move unfavorably, multinational firms face financial losses because of falling profits. Demand uncertainties may result in underage cost when production quantities are less than the demand, or overage cost when production quantities are more than the demand. Additionally, recent environmental regulations on emissions of green house gases, particularly carbon dioxide emissions, also pose risk on firms's profitability. It is thus important for a risk-averse manager to decide how to mitigate these uncertainties to protect the firm's financial losses.

In order to address these issues, mathematical models that capture firm's production allocation problem under different scenarios of exchange rate, carbon emissions, and demand uncertainties have been developed. The risk attitude of the firm manager is assumed to be risk averse and is modeled by a mean-variance (MV) utility function. In order to hedge downside risk of exchange rates and upside risk of carbon allowance prices, the firm takes long positions in currency put and carbon call options, respectively. The objective is to maximize the MV function of the firm subject to various capacity and demand constraints and determine the optimal number of currency put and carbon call options. The firm possesses real options capability in the form of capacity flexibility represented by a vector of discrete capacity levels to meet uncertainties of demand. Demand uncertainties are assumed to follow regime-switching behaviors – considering both one-state and two-state probability distributions. The stochastic behavior of exchange rate is modeled by a geometric Brownian motion and its limiting case as a random walk. Functioning under a cap-and-trade emission trading scheme, the firm is obliged to buy carbon allowances for its carbon emissions. Carbon allowance prices are modeled as both geometric Brownian motion and geometric Brownian motion with jump processes. Results demonstrate that integration of real options and financial options increases the utility of the firm, while financial options reduce the variance of the profit.

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Nomenclature

χ	Total demand
μ	Drift of a geometric Brownian motion
ϕ	Step size in a lattice
ψ	Uncorrelated stochastic process
ρ	Correlation coefficient
σ	Volatility of a geometric Brownian motion
θ	Stochastic demand
α	Lagrangian coefficient
Δ	Number of forward contracts
γ	Mean–variance (MV) ratio
$\mathbb{E}[\cdot]$	Expectation of inside expression
$\mathbb{V}[\cdot]$	Variance of inside expression
$\mathbf{M}_t$	Capacity vector with discrete capacity levels
$\mathrm{cov}[\cdot]$	Covariance of inside expression
c_0	Initial capacity installment cost
c_1	Unit production cost
c_3	Percent of initial installment cost
c_4	Percent of unit cost
c_5	Fixed cost

D_1 Amount of local demand

D_2 Amount of foreign demand

dz Increment in Wiener process

e Stochastic exchange rate

e^f Forward exchange rate

$g(\cdot)$ Product cost function

K_c Strike or exercise price for carbon options

K_e Strike or exercise price for currency options

$S(\mathbf{M}_{t-1}, \mathbf{M}_t)$ Switching cost

s_1 Sales price in local market

s_2 Sales price in foreign market

X Number of carbon options

x_{11} Capacity allocation from the local production facility to local markets

x_{12} Capacity allocation from the local production facility to foreign markets

x_{21} Capacity allocation from the foreign production facility to local markets

x_{22} Capacity allocation from the foreign production facility to foreign markets

Y Number of currency put options

Chapter 1

Introduction

Many firms have worldwide production and sales. For example, Bombardier, a Canadian multinational aerospace and transportation company, has sales in the USA. The Delta Air Lines Incorporated, an US carrier, put order for new CSeries aircrafts from Bombardier. According to a report by the Financial Post, Bombardier would gain from the recent decline in the exchange rate, as the exchange rate went to US\$0.85/CAD ($\approx$ CAD1.176/USD) (Ratner 2015). Fluctuation in exchange rate, therefore, plays a major role to the profitability of global firms. In 1982, Laker Airways filed bankruptcy when the US dollar got stronger than the British pound, because Laker Airways had expenses in US dollars and revenues in British pounds (Dornier et al. 1998). According to the US Department of Commerce, in 2013, when one US dollar was, on an average, equal to 0.9711 Canadian dollar, Canadian visitors spent US\$23.4 million in the U.S. However, the spending went down 2% in 2015, because of the unfavourable exchange rate at 0.7830 CAD/USD (Sorensen 2015). These are few among many examples where exchange rates play an important factor in the decision making processes and firms need to take preventive measures against the uncertainty of exchange rates.

Stricter environmental regulatory rules have also compelled firm managers to consider the cost of green house gas emissions, particularly, carbon dioxide emissions. The advent of carbon emissions market adds more risk to business companies and has a significant impact on investment planning and management (McKinsey 2007). A PricewaterhouseCoopers (PwC) report (Pricewaterhouse-Coopers, 2012) also shows that investors' attentions in climate-related risk has risen significantly in recent years. Analyzing the carbon emission data that are voluntarily exposed to the Carbon Disclosure Project (CDP) by S&P 500 firms, Matsumura et al. (2014) find that for each additional 1000 tons of carbon emission, the firm value drops by approximately two hundred thousand dollars. In order to counterbalance the decrease in the firm value, an effective way could be using the financial options on carbon allowance prices under a cap-and-trade market based mechanism.

The following section exhibits a simple example how financial derivatives could mitigate the risks associated with exchange rate and carbon allowance prices uncertainties.

1.1 Impact of global warming in supply chains

A supply chain is a coordinated activities of organizations, people, information, and resources in moving a product and service from producers to end-users. The supply chain management is the management of this chain that facilitates the flow of products and services. It involves managing storage of materials, work-in-process inventory, and finished goods from producers to consumers. Disruptions in the chain adversely affect both producers and customers. Extreme weather conditions are one of major factors that pose potential risks to supply chains. Increasing temperatures due to climate change manifest high levels of uncertainty in supply chains. In other words, climate change has an impact on supply chains. Supplies in commodities such as oil, natural gas, and electricity are subject to extreme weather conditions. Supply chains in agricultural products, for instance, rice and wheat are also influenced by weather behaviors. Scientists reason out global warming as a major cause of such uncertain weather whims. Emissions of carbon dioxide from various human activities are responsible for global warming and climate changes.

1.2 Risks associated with a supply chain

Uncertainty involves a situation in which there is imperfect or little information or no information at all. It tends to produce an unpredictable and uncontrollable outcome. Uncertainty arises in various ways in many fields, for instance, in finance, economics, insurance, engineering. Risk is a consequence of uncertainties. Risk is the possibility of attaining or losing the value of something. Following example illustrates a phenomenon of risk associated with exchange rates. The profit from foreign sales varies with uncertain exchange rates. Let the index 1 be the local and 2 be the foreign. hence, the sales quantity of a local production to a foreign market is assumed to be x_{12} units. Similarly, x_{22} denotes the sales from the foreign production to the foreign market. Let the sales price in the foreign market be s_2. For a fixed value of s_2, revenue from sales in foreign market would be $s_2 x_{12} e$, where e is the exchange rate between the two countries. The fluctuation of exchange rate, e, would make the revenue uncertain. As an example, let $x_{12} = 30$ units sold from Canada to the US market, and $s_2 = 5$ USD. When e, for instance, varies linearly from 0.80 to 1.20 CAD/USD, from Figure 1.1 it is observed that an increase in the exchange rate increases the profit in CAD. However, if the exchange rate drops, the profit (in CAD) also drops. Assume that the firm functions under environmental restrictions and is obliged to buy carbon allowances for its carbon emissions. Let β be the amount of carbon emission per unit of products, and δ be the carbon allowance prices. The carbon dioxide emissions are assumed to be the total amount of emissions per product from all stages of production and shipments. For producing x_{12} units of product, the firm then experiences a carbon cost of $\delta\beta x_{12}$. When δ varies, the profitability of the firm also varies. Assume that the carbon allowance prices, δ, vary, for the sake of simplicity, linearly from 2 to 5 CAD/tCo_2 and let the emission rate, β, be 0.2 CAD/tCO_2. Figure 1.1 also shows that the profit drops under carbon emission cost and the gap between the profit with and without carbon prices widens as the carbon price increases. This simple example exemplifies that the firm cannot let the profit variations unattended due to fluctuating exchange rate and carbon allowance prices. The situation even becomes grave when demand uncertainties are added to the uncertainties of exchange rate and carbon allowance prices. Firms can hedge these uncertainties utilizing real options and financial derivatives.

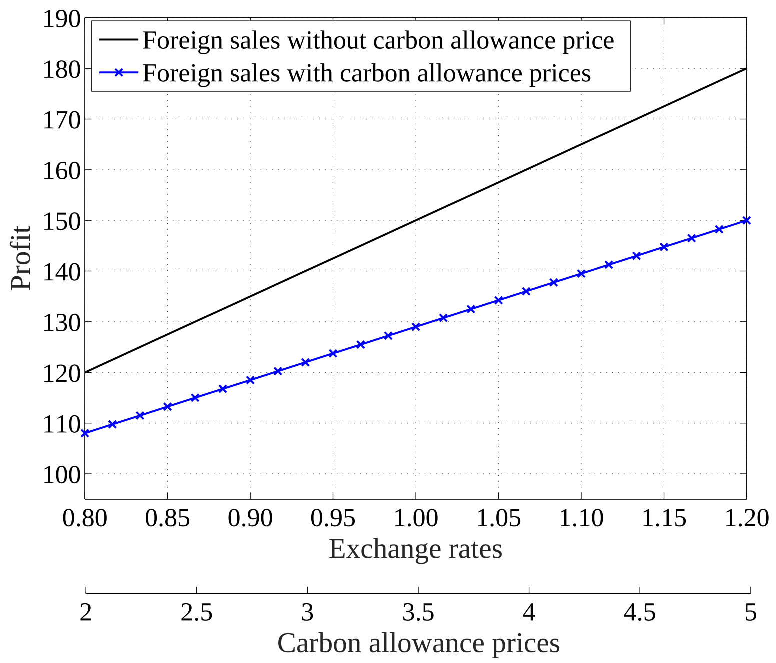

Figure 1.1: Variations in exchange rate and carbon allowance prices affect the profitability of a multinational firm.

1.3 Real options

Real options are "opportunities to delay and adjust investments and operating decisions over time in response to resolution of uncertainty" (Triantis 2000). Real options are viewed as operational risk management tools. Operational hedging are strategies to mitigate risk using operational instruments (Van Mieghem 2003). These include different kinds of processing and switching flexibilities among production locations or various production capacities. Four kinds of operational hedging are classified by Van Mieghem (2003): (a) Reserves and redundancy, which include safety capacity, safety inventory, and multi-sourcing; (b) diversification and pooling, for example, operating in diverse markets; (c) risk-sharing and transfer, which includes entering a financial contract with a third party; (d) reducing or eliminating root causes of risk.

A firm having capacity flexibility can safeguard demand variations by utilizing a set of discrete capacity levels. Automobile industries have this kind of capacity flexibility. The firm can adjust its capacity to meet a certain level of demand fluctuation. Consequently, there occurs a capacity adjustment cost in the form of capacity expansion, i.e., the adjustment from a lower to a higher capacity, and capacity contraction, i.e., the adjustment of capacity from a higher to a lower level capacity. Any demand above the level of maximum capacity is assumed to be unsatisfied and lost.

1.4 Financial derivatives

1.4.1 Forward and futures contracts

A forward contract ties two parties is an agreement in which one party having a long position agrees to buy an underlying asset from another party holding a short position at a specific price at a specific future time (Hull 2009). Forward contracts are traded over-the-counter and can be settled in cash at maturity, without delivering a physical asset. A future contracts is the same as a forward contracts except that it is traded on an exchange having standard features.

1.4.2 Call and put options

A call option is a right to buy an underlying asset, for example common stocks, index, and foreign currency, at a specified price on or before a specified future time. The buyer of an option has a long position and the seller has a short position. The specified price is known as strike price or exercise price. An option is a financial derivative, which derives its value from an underlying asset. A put option is a right to sell an underlying asset at an exercise or strike price. Both call and put options can be of two types, European and American. A European option is exercised at a specified time in future, while an American option can be exercised before the specified time (Hull 2009). Options differ from forward and future contracts such that forward and future contracts are commitment to fulfill an obligation and options are right to exercise by the option holder without any obligation. When the price of the underlying asset moves upward, call options get exercised. On the other hand, when the asset price drops unfavorably, put options protect the downside risk of the falling

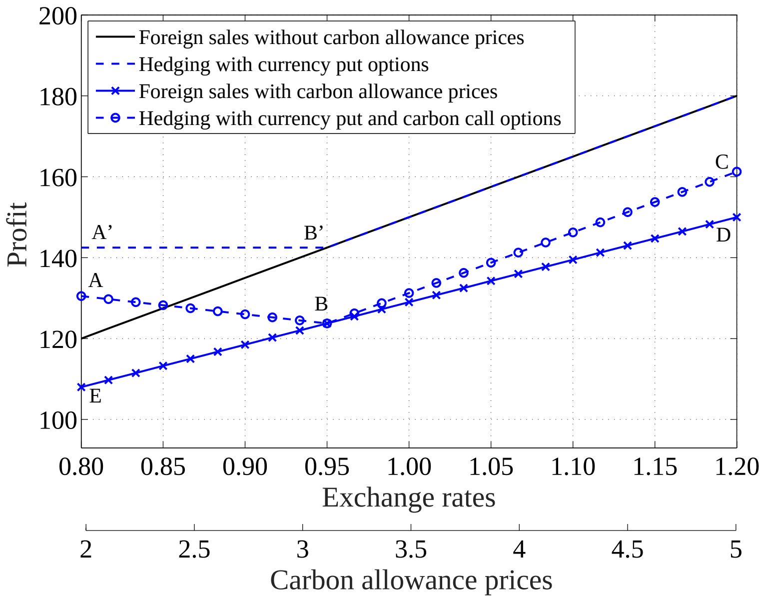

Figure 1.2: A simplified exhibit of hedging profit decline due to exchange rates through currency put and upside carbon emission cost through carbon call options.

asset prices.

Referring to the example in Section 1.2, the firm can hedge the downside risk of the exchange rates by taking a long position in currency put options. For the sake of simplicity, let the exercise or strike currency rate be 0.95 CAD/USD. This implies that whenever the exchange rate would fall below this exercise price of 0.95, the put options will be exercised to hedge the falling currency rate. The dotted line, A′B′, in Figure 1.2 shows that the firm can hedge the profit downfall by using currency put options. If the firm also faces a carbon price risk, the increasing carbon cost could be hedged by taking a long position in carbon call options. The example uses an exercise price of 3.125 for carbon options. That is, when the carbon price will increase above the exercise price, the call options would get exercised and hedge the upside risk of having higher carbon prices. The 'circled' dotted line, ABC, shows that the use of carbon call options can mitigate the loss due to carbon allowance prices. The line indicated by 'BE' has been shifted upward to the position of the line 'AB' due to the use of currency put options and the line denoted by 'BD' has been moved upward to the position of the line 'BC' due to the exercise of carbon call options. Figure 1.2 shows a simplified example how a firm could use both currency put and carbon call options. This thesis considers optimal production problems where exchange rates, carbon allowance prices, and demand are all stochastic, that is, their values move in an uncertain manner with respect to time.

1.5 Research motivation and objectives

Against the backdrop, this thesis is motivated to examine optimal production strategies for a multinational firm that experiences uncertainties in exchange rates, carbon allowance prices and demand. When exchange rates fall unfavorably, the firm may incur a huge financial loss. Moreover, the firm faces demand uncertainties from both markets. Stricter environmental regulations also force the firm to buy carbon allowances under a cap-and-trade mechanism. It would be an interesting research to examine how a firm tackles the uncertainties involving demand, exchange rates and carbon allowance prices. Consequently, the objectives of the thesis are to find answers to following research questions:

- How many forward contracts are needed to long, if the firm opts to long currency forward contracts for hedging exchange rate risk under different supply chain topologies?
- How does the firm decide capacity allocation over a multi-period time frame under the exchange rate fluctuation when the firm is considered to have capacity flexibility to meet the demand from a set of discrete capacity levels? If the firm opts to long currency put options, how many options to long to hedge the currency risk?
- How does the firm decide over the capacity allocation under a cap-and-trade emission scheme, if demand follows a stochastic two-state regime switching behavior? How can the correlated behavior of two-state regime-switching demand and geometric Brownian motion of carbon allowance prices be represented in a lattice? How many carbon call options to long?
- When demand, exchange rate, and carbon allowance prices all are considered to follow stochastic behavior along with overage and underage costs, how does the firm's decision alter? What if the carbon allowance prices follow a geometric Brownian motion with jump processes? How do overage and underage cost affect the allocation decisions? How many currency put options and carbon call options to long to hedge both currency and carbon risks?

1.6 Research contributions

The contribution of this thesis lies in, at least, four aspects:

- While the literature in Adkins (1993), Huchzermeier and Cohen (1996), Broll et al. (1999), and Ding et al. (2007) consider single period models, this thesis analyzes the production allocation problem for a multinational firm in a multi-period setting. No one considers a production allocation problem for a multinational firm under exchange rate uncertainties in a multi-period time-frame. The task is challenging in the sense that the decision taken at current time affects the future decision.

- The thesis considers the growing concern over environmental issues for multinational firms. To the best of the author's knowledge, no one studies the use of carbon options in hedging uncertainties associated with carbon allowance prices. In order to mitigate the upside risk of carbon emission costs, this thesis examines the use of carbon call options to mitigate the risk. Since in the literature, carbon allowance prices are modeled both as a geometric Brownian motion (e.g., Chesney and Taschini 2008) and as a geometric Brownian motion with jump process (e.g., Kou 2002), this thesis examines both processes for carbon allowance prices.

- The thesis also analyzes the stochastic product life cycles along with stochastic carbon emission prices. A novel lattice approach is utilized to model the correlated stochastic behaviour of regime-switching demand and carbon price uncertainties.

- Most of the literature are based on analyzing only one uncertainty, for example, Adkins (1993) examines the exchange rate risk, Bollen (1999) considers stochastic demand and Benjaafar et al. (2013) consider carbon emission issue. To the best of the author's knowledge, no one considers the uncertainties from demand, exchange rate and carbon emission prices simultaneously. This thesis examines the concurrent effect of demand, currency and carbon uncertainties. Moreover, the literature (e.g., Ding et al. 2007) did not examine the costs of overage (too much production) and underage (too less production). This thesis also addresses the overage and underage costs in case of stochastic demand.

1.7 Organization of the thesis

The thesis is arranged as follows. Chapter 2 reviews the existing literature, points out research gaps, and overall scheme of the thesis. Chapter 3 discusses the problem with a model that handles the exchange rate uncertainties with forward contracts. Chapter 4 examines a multi-period case. Chapter 5 considers regime–switching demand and geometric Brownian exchange rates. Chapter 6 considers all stochastic cases. Chapter 7 concludes the thesis.

Chapter 2

Literature Review

Flexibility is the ability to adapt to changes that may take place in a firm. Both manufacturers and customers require flexibility to deal with changes. Automobiles and electronic industries experience shorter product life cycles. Demand rises, in these industries, are followed by demand decays. Industries having capacity flexibility could capitalize the opportunities to get along such demand variations. Capacity flexibility involves managerial decisions in expanding capacity, contracting capacity, adjustment of capacity and/or postponing capacity allocation in order to minimize the negative effect on the profitability. Capacity flexibility can be deemed as one of the real options tool and also sometimes referred to as an operational hedging. In the literature, there are a number of methods that addressed capacity flexibility. Van Mieghem (2003) provides a good review on capacity management, investment and hedging.

2.1 Exchange rate uncertainties

A multinational firm having the supply chain network across different countries is vulnerable to risks from various sources such as exchange rate uncertainty, demand uncertainty, price uncertainty, political uncertainty, and lack of knowledge of customers and competitors in the foreign country (Kogut and Kulatilaka 1994). Among those risks, the one often addressed in the literature is exchange rate risk that can cause major loss. Especially, a firm having expenses in one currency and revenues in another currency may incur loss, when the change in the expense currency is greater than that in the revenue currency. For example, in 1982, Laker Airways filed bankruptcy when the US Dollar got stronger than the British Pound, because Laker Airways had expenses in US dollars and revenues in British Pounds. Cheung and Sengupta (2013) find that during 2000-2010 exchange rate appreciation negatively impacted exports in India. Therefore, it is very important for a firm to minimize risk from exchange rate uncertainty by using different risk hedging strategies that are explained later in this chapter.

2.2 Demand uncertainties

The product life cycle has also an impact on production and investment decisions. Products, like automobiles, semiconductors, and fashion items like cosmetic and toiletry products, exhibit specific patterns in their life cycles. Managers should understand the demand pattern and adjust their strategies accordingly. During the introduction period of the product, demand pressure is high and the production capacity should have adjustment capability to meet the growing demand (Golder and Tellis, 2004). The level of demand vary dramatically across the stages of life cycle and corresponding changes in the level of production should be made when demand starts to decay. Bass (1969) proposes a model for sales pattern during the growth stage of the product. According to the Bass model, the potential adopters of the product is influenced by both external factors, for example, through advertising, and internal factors, through communications with previous adopters. However, the assumption that the demand is likely to grow infinitely leads to an erroneous valuation of a firm.

Product life cycle models, on the other hand, has the notion that demand decays at some point because of saturated market and incoming new products. The changing underlying stochastic processes over the course of product life cycle affect the future profitability and hence the value of the firm. Chi and Liu (2001) evaluate a stochastic model for market entry and exit decisions during the uncertain product life cycle. They assume that the product life cycle follows a stochastic process characterized by a standard Brownian motion. Bollen (1999) frames a valuation that considers the product life cycle. The product life cycle is assumed to follow a regime switching process with two regimes, growth and decay. He has developed a lattice with five branches, in which one regime is represented by a binomial and the the other by a trinomial branches. The step size of the branches is adjusted to match the five branches with even space. He demonstrates that ignoring the product life cycle undervalues the option to contract and overvalues the option to expand a project. In fashion industry, cosmetic products like fragrance and perfumes are sensitive to changing consumer preferences and the cosmetic products possess evolving and shorter product life cycles. Cucchiella et al. (2010) propose a real option valuation model for cosmetic products assuming a Poisson distribution for switching between the stages of the product life cycle. They utilize a binomial lattice-base approach to find the net present value of the firm.

Demand uncertainties also cause underage and overage costs. From the perspective of a newsvendor model, producing too many products than the actual demand will induce overage cost, the firm will remain with many leftover. On the other hand, producing too little will induce an underage cost, the firm cannot meet the demand and lose the goodwill. Chen and Parlar (2007) consider a single-period newsvendor model with stochastic demand. The newsvendor opts for a put option to mitigate financial losses due to low demand. The newsvendor transfers his risk to the option writer for an amount equal to a risk premium plus the expected option payoff. In return, he/she receives the exercise price from the option writer for each unit of demand that drops below the strike quantity. The model determines the order quantity, exercise price and strike quantity of the option.

2.3 Carbon allowance prices uncertainties

Like a greenhouse glass, the gasses in the atmosphere, for instance, carbon dioxide, nitrous oxide, methane, perfluorocarbons, hydrofluorocarbons, and sulphur hexafluoride, allow the sun rays to come into the earth, keep the earth warm, and shield the warmth trapped in the atmosphere. That is why these gases are called greenhouse gases. The greenhouse gases hold heat and radiate back some heat to the earth's surface. Without these greenhouse gases, the earth would be very cold having an average temperature of -18^{0}C instead of having the current average earth temperature of around 15^{0}C. Nature herself maintains the balance of amount greenhouse gases in the atmosphere. But the problem is that the emission of huge amount of carbon dioxide by human activities like deforestation, industrialization, and burning of fossil fuels - coal, natural gas, and gasoline - has caused an increase in the earth temperature. The concentration of carbon dioxide in the atmosphere today is 42% more than the amount in the beginning of the industrial era in 1760. The earth's surface temperature is, therefore, increasing. According to a report by the Goddard Institute for Space Studies of NASA, the average temperature of the earth has increased 0.8^{0}C since 1880 and much of this increase has occurred recently in the 20th century.

The consequences of the global warming are very alarming and grave. The arctic region is suffering the most as mean-temperatures in the western Canada and Alaska have increased twice the earth average. As a result, the arctic ice is reducing rapidly. Scientists apprehend that the region might be ice-free by the 2040. The ice-loss has already affected the existence of polar bears and other indigenous creatures. The Koyoto Protocol, an international treaty signed in Koyoto, Japan in 1995 has agreed to reduce the amount of carbon dioxide based on the premises that the global warming exists and the carbon dioxide produced by human activities is the main cause of the global warming. To reduce the detrimental effect of greenhouse gases, mainly carbon dioxide emissions, many countries in Europe and recently Canada have devised policies to harness carbon emissions.

There two carbon reduction policies, carbon tax and cap-and-trade. Carbon tax is the fee or monetary price placed on the emission of carbon. Under the carbon tax policy, the government imposes a fixed amount of tax on emissions. The tax is set by a government with an aim to reducing pollution by carbon emissions. Government authorities can implement this policy by putting a surcharge on carbon-based fuels and industrial processes. The provinces of British Columbia and Quebec in Canada use carbon taxes as a part of their policies to reduce carbon emissions and to encourage investments in renewable energy. In a cap-and-trade system, government puts a cap or limit on the overall carbon emissions level from industry and tightens the cap yearly to achieve a set emissions target. As the cap or the limit reduces and tightens each year, the tightening of the emission limit causes industries to cross their emission limits or caps and purchase spare quota from another company. The government monitors and distributes emission quotas through auctions. This encourages firms to decrease their emissions and inspires to sell emission quotas instead of purchase of quotas. Under the cap-and-trade system, the market dynamics evaluate the emission price.

The U.S. successfully used the cap-and-trade policy to reduce emissions of sulphur dioxide and nitrous oxide, which are responsible for acid rain. Tokyo launched its own cap-and-trade system in 2010. The European Union (EU) Emissions Trading System (ETS) covers approximately more than $11,000$ power and manufacturing industries including production of metals, papers, and cardboards. According to InterContinentalExchange (ICE) Inc., during the period of from 2007 to 2010, the emission index varied from 8.50 to over 30 Euro per tonne of CO_2. The Western Climate Initiative (WCI) is formed by four Canadian provinces, Ontario, Quebec, Manitoba, and British Columbia, together with seven U.S. states in order to implement a cap and trade scheme from January, 2012. The aim of WCI is to reduce emission by 15% by 2020 from the emission level of 2005 (Mnif and Davison 2011). EU carbon allowances are traded through exchanges, such as European Climate Exchange (ECX) based in the UK, European Energy Exchange (EEX) based in Germany, BlueNext based in France, and NASDAQ based in the US. The carbon products are traded in the form of spots, futures, forwards, options and swaps. For example, the carbon product coded as 'NECSEP1-13.50' stands for a carbon call option traded at NASDAQ for the strike price of 13.50 (Lapierre, 2015). During the period from 2005 to 2008, trading volumes in EU emission allowances has increased from 2.327 millions tons to 7.903 millions tons of carbon.

As stated earlier, the market based mechanism to control carbon emissions is known as carbon cap-and-trade method. Companies can buy or sell such certificates and decide on their amount of CO_2 released into the atmosphere. The right to produce a particular amount of CO_2 has now become a tradable commodity. A company with lower carbon emissions can benefit from selling its allowances to higher carbon emitting companies (Benz and Trück 2009). Benjaafar et al. (2013) propose mixed-integer optimization models that discuss the integration of carbon emission issues into operational activities of procurement, production, and inventory management. In modeling carbon allowance prices, Seifert et al. (2009) assume that the uncertainty in emission price dynamics is driven by a standard Brownian motion. Benz and Trück (2009) analyze the log return of carbon spot prices from January, 2005 to December 2006 as a Markov regime-switching model with two regimes (base and spike regimes). Chesney and Taschini (2008) propose the carbon price dynamics as a geometric Brownian motion. However, Huang (2010) assumes that the process could follow either an arithmetic Brownian or a mean reverting motion. Daskalakis et al. (2009) study carbon allowance prices from the Dutch European Climate Exchange (ECX), Norway based Nord pool, and France based Powernext during the period from December, 2005 to December, 2007. They examine six different stochastic processes to model carbon prices and observe that the carbon allowance prices has a better fit with the geometric Brownian motion with jump processes than that of other stochastic models.

2.4 Operational hedging

Operational flexibility is the ability to foresee changes in dynamic market conditions and ability to respond to these changes by utilizing the firm's operations capabilities (Huchzermeier and Cohen 1996). Firms utilize the volatility in the environment by exercising these options. Operational hedging involves managerial decisions, such as selecting the production facility or market, postponing capacity allocation, expanding capacity, contracting capacity, and switching capacity to diminish detrimental impacts of exchange rate and demand uncertainties (Boyabatli and Toktay 2004). Strategies of operational hedging are deemed as real options. This is the option to exercise operational capabilities, for instance, expansion or contraction of capacity levels, in face of demand, prices, and/or exchange rates uncertainties experienced by a multinational firm. This is also an application of real options in the context of global supply chain. A firm having sales and productions in two different countries experiences both exchange rate and demand risks. Financial tools like forward contracts and options are used to mitigate exchange rate risks. However, demand risk cannot be effectively mitigated by using financial tools. Astute adjustment of resources, such as postponing production decision until product demand is revealed can reduce the mismatch between supply and demand (Boyabatli and Toktay 2004). In the literature, operational hedging is analyzed along with financial hedging. An empirical study by Pantzalis et al. (2007) show that firms can utilize operational hedges in managing their risks.

Huchzermeier and Cohen (1996) show that a multinational firm may select the production location, i.e, network structure. The firm may delay the production decision on how much to produce until the demand and exchange rate are observed. When a firm has the right to postpone the production quantity decision, the firm possesses the flexibility to choose a different network structure that has an excess capacity. Their results demonstrate that the firm value increases through the operational hedging. It reflects that real options has the capability to enhance the value of the firm under uncertainty (Huchzermeier and Cohen 1996). In a risk-neutral setting, where profit volatility is not of concern, Ding et al. (2007) show that the use of real options have value-enhancing capabilities. Kogut and Kulatilaka (1994) utilize operational flexibilities by having manufacturing plants located in two different countries with multi-period sales. They assume that the exchange rate follows a mean-reverting process while the demand is assumed to be known. Hodder and Jucker (1985) employ operational hedging by using geographically dispersed plant locations under price and exchange rate fluctuations. Dasu and Li (1997) alter the production quantities to minimize exchange rate risk for plants operating in more than one location. For a single-period problem with probabilistic demand, Kazaz et al. (2005) used a two–stage stochastic programming approach. Capacity allocation takes place in the first stage and the production decision is made in the second stage after the realization of exchange rate. Van Mieghem (2003) defines capacity as "a measure of processing abilities and limitations that stem from the scarcity of various processing resources and is represented as a vector of stocks of various processing resources."

Examples of operational hedging also include Ding et al. (2007) and Huchzermeier and Cohen (1996). While Ding et al. (2007) propose postponement of logistics decision, Huchzermeier and Cohen (1996) suggest holding of extra capacity and delaying the commitment of capacity investments, and Liu and Nagurney (1996) develop a model that switches among supply chain network structures. These operational strategies are known as real options and used to reduce the risk exposure by diminishing the downside risk in the long run. All of these are real options in form of operational flexibility. The flexibility is gained by utilizing of excess capacity and/or stochastic recourse.

2.5 Financial hedging

Financial hedging, on the other hand, refers to minimizing risks by having positions in financial instruments, such as call options, put options, and forward contracts (e.g., Broll and Wong 1999; Broll et al. 1999). Financial hedging is considered as a preferred method for hedging exchange rate uncertainty. Servaes et al. (2009) reported that 82% of international firms uses foreign exchange rate derivatives. Cohen and Huchzermeier (1996) conclude that global firms utilizing financial hedging have higher expectancy for profit than those firms that are not using it. Financial hedging is advantageous in numerous ways. For example, risk from taxes, cost of financial distress, and managerial risk aversion can be managed using financial hedging (Pindyck 1988). In Broll and Eckwert (2000), forward contract is utilized to mitigate the exchange rate risk in a two-period hedging strategy. One of the drawbacks of using the forward contract is that it limits the profit due to an agreed upon fixed exchange rate even when the exchange rate changes favorably (Hull 2009). However, using forward contract protects a firm against the downside risk of exchange rate (Hull 2009; Liu and Nagurney 1996). For a single period problem, Adkins (1993) hedges the exchange rate risk by using put options, where the exchange rate follows a geometric Brownian motion and the number of put options is calculated by minimizing the variance of the profit. Wong (2003b) minimizes the utility function of the profit to determine the number of put options, where the exchange rate is a random variable described by a probability distribution.

2.6 Integrated operational and financial hedging

Firms cannot manage their risks only by using financial instruments, but risk can be reduced by an integrated approach (Miller 1992). Currently many firms use integrated method encompassing both operational and financial hedging (e.g., Mello et al. 1995; Chowdhry and Howe 1999; Ding et al. 2007; Chod et al. 2010). Mello et al. (1995) consider shifting the production to a low cost country with an integration of financial markets in a continuous time to minimize exchange rate risks. Chowdhry and Howe (1999) consider the fixed total capacities, which are allocated between the local and foreign plants against the exchange rate movement in single period sales. The optimal hedging policy is obtained through foreign currency contracts with considering uncertainty of demand. Chod et al. (2010) examine the operational flexibility and financial hedging against the demand uncertainty. The work showed that if demands are positively correlated, the product flexibility and financial hedging are complements. One approach to dealing with capacity allocation is to delay the capacity allocation until the currency and demand uncertainties are settled. Ding et al. (2007) discuss the allocation delay along with financial option contracts on the currency exchange rate. Exchange rate usually follows a geometric Brownian motion and option pricing is determined by the Black and Scholes (1973) model. The optimal number of options is obtained by maximizing the marginal utility function for a single period problem. A brief summary of the literature review is provided in Table 2.1.

While the existing literature highlights single period exchange rate risk hedging problems, this thesis presents a multi–period risk hedging problem, in which decisions are taken in each period depending on the level of capacity and the movement of the exchange rate. Path dependent profits are calculated at each period integrating real options, in the form of capacity flexibilities, and financial options, in the form of currency put options.

Table 2.1: A brief literature review on the exchange rate risk hedging strategies

Real options approach: operational flexibility	
Author(s)	Objective/Description
Kogut and Kulatilaka (1994)	Propose a model that coordinates shifting of production between plants in different countries as the exchange rate moves stochastically.
Huchzermeier and Cohen (1996)	A global supply chain model that switches among different manufacturing flexibility by a combination of global supply flexibility and supply chain linkage choices.
Kazaz et al. (2005)	A two–stage capacity–and–production planning model that utilizes the allocation decisions until the exchange rate is realized.
Aytekin and Birge (2006)	An analytical model to investigate the operating policies in response to the exchange rate uncertainty.
Financial derivatives approach: by using forward/options	
Adkins (1993)	Hedging exchange rate risk by using put options.
Broll and Eckwert (2000)	A two–period hedging model using currency forward contracts.
Wong (2003a)	A foreign currency risk hedging model by using cross hedging forward contracts.
Approaches integrating operational flexibility and financial derivatives	
Chowdhry and Howe (1999)	A mean-variance model that combines operational hedging and financial contracts under demand and exchange rate uncertainties.
Ding et al. (2007)	A two–stage capacity–and–production planning model that invetigates the effect of integrating capacity allocation and currency options on capacity investment decisions for a risk averse firm.

2.7 Research gaps

While the literature on exchange rate risk hedging is rife with single period models, for example, Huchzermeier and Cohen (1996), Adkins (1993), Broll et al. (1999), and Ding et al. (2007), there is, to the best of the author's knowledge, no model that deals with the integrated operational and financial risk hedging problem in a multi-period time frame. Firms in the real world do not take decision over a single period time frame, rather they take decisions over a multi-period time frame. Multi-period models involve dynamic decisions, since the decision for the next period is contingent on the current decision. This thesis examines production allocation decisions over a multi-period time frame for a firm that has capacity flexibility and that is also exposed to exchange rate uncertainties.

Growing environmental concern has an impact on the production allocation decisions. While Benjaafar et al. (2013) consider the carbon emission cost for inventory management, no one studies the use of carbon options in hedging the carbon allowance price uncertainties. This thesis examines the use of carbon call options to hedge the upward cost of carbon emissions. In order to examine possible stochastic behavior of carbon allowance prices on the production allocation problem, carbon prices are modeled in the form of geometric Brownian motion and geometric Brownian motion with jump processes.

The thesis addresses another gap in the literature. The thesis considers uncertainties from demand, exchange rate and carbon allowance prices simultaneously. For a two-stage supply chain consisting of a domestic production plant having sales in both domestic and foreign markets, demand and carbon prices uncertainties arise from the domestic market, while exchange rate uncertainties appear from selling products to a foreign market. Therefore, the firm having domestic production with sales in both domestic and foreign markets faces three uncertainties simultaneously stemming from demand, carbon prices, and exchange rates. Most of the literature, consider only one uncertainty at a time, for example, Adkins (1993) considers only exchange rate uncertainty, Bollen (1999) considers stochastic demand, and Benjaafar et al. (2013) consider uncertainty of carbon prices. This thesis, however, investigates concurrent uncertainties of demand, exchange rate, and carbon allowance prices. Furthermore, current literature, for example, Ding et al. (2007) did not consider the cost of overage (too much production) and underage (too little production). This thesis takes into account the underage and overage costs, in case of stochastic demand.

2.8 The overall scheme of the thesis

The overall scheme of the thesis is given in the following Table 2.2. Chapter 3 discusses a single period production allocation problem with stochastic exchange rate and known demand. Chapter 4 discusses a multi–period production allocation problem with stochastic exchange rate and known demand. Chapter 5 addresses a multi–period problem with regime–switching demand product cycles and carbon prices uncertainties. Chapter 6 discusses a single period problem in which demand, exchange rate and carbon prices are considered uncertain.

Table 2.2: The overall scheme of the thesis

Chapter	Demand	Exchange rate	Carbon price	Models	Objective
3	known	stochastic (Firm longs currency forward contracts	–	• domestic production • domestic production with foreign subsidiary • foreign production with domestic subsidiary • fully flexible model	production allocation & number of forward currency contracts so as to maximize the mean-variance utility
4	known (Firm has capacity flexibility)	stochastic (Firm longs currency put options)	–	• fixed capacity (no options) • fixed capacity with options • flexible capacity (no options) • flexible capacity with currency options	production allocation & number of currency put options so as to maximize the mean-variance utility
5	stochastic (Firm has capacity flexibility)	–	stochastic (Firm longs carbon call options)	• fixed capacity (no options) • fixed capacity with options • flexible capacity (no options) • flexible capacity with carbon options	production allocation & number of carbon call options so as to maximize the expected value
6	stochastic (with overage & underage cost)	stochastic (Firm longs currency put options)	stochastic (Firm longs carbon call options)	• carbon prices modeled as a jump process • stochastic demand, exchange rates, and carbon prices	production allocation & number of currency and carbon options so as to maximize the mean-variance utility

2.8.1 The mean-variance approach

Firm managers usually show risk averse attitude. An utility approach requires assessing the utility function of decision makers and it is essentially a difficult task. On the other hand, the mean-variance (MV) approach requires only the mean and the variance of the profit to determine the objective function and thus is easily implementable and applicable (Tekin and Özekici 2015). The MV approach trades off between the high return in term of mean and the low risk in term of variance.

The mean-variance approach has two important advantages: (a) they are implementable. That is, only two moments, the mean and variance, are required and these two moments can be easily evaluated; and (b) the approach provides reasonable insights and explanations even when the decision maker might not know his/her utility function (Van Mieghem 2003). The MV function value, $\mathrm{U}(\pi)$, can be expressed as:

$$\mathrm{U}(\pi) = \mathbb{E}(\pi) - \gamma \mathbb{V}(\pi), \tag{2.1}$$

where $\mathbb{E}(\pi)$ is the expected profit, $\mathbb{V}(\pi)$ is the variance of the profit, and $\gamma \geq 0$ is the mean-variance ratio. The MV ratio, γ, measures the pace at which the firm compensates the variance for expected profit value (Ding et al. 2007). In the case where the utility function is modeled as $\mathrm{U}(\pi) = -\exp(-\gamma\pi)$, the value of γ can be determined as, $\gamma = -\mathrm{U}''(\pi)/\mathrm{U}'(\pi)$. Chen and Parlar (2007), for the quadratic utility function $\mathrm{U}(\pi) = a_0 + a_1\pi - a_2\pi^2$, shows that minimizing the variance of the profit maximizes the expected utility. In their study, Chen and Parlar (2007) assume the values of $(a_0, a_1, a_2) = (0, 1, 0.000005)$. In the study of Ding et al. (2007), the value of γ is assumed to be 0.0002.

2.8.2 Dynamic programming approach

It is a recursive approach to solving multi-period optimal decision problems, when the current decision influences the future payoff. The problem is formulated in a Hamilton-Jacobi-Bellman equation, which is solved for the asset value by backward induction using a discount rate. The solution moves in a backward recursive direction, discounting the future values and folding them into the current decision. This discount rate should reflect the opportunity cost of capital of similar risk. In a dynamic programming approach, intermediate values can be visible and the decision process is transparent that enables to handle real options features efficiently (Bollen 1999). Section 4.2.1 in Chapter 4 illustrates how the dynamic programming approach is utilized to determine the expected profit of the firm.

2.9 Summary

This chapter discusses the literature review, overall scheme of the thesis, mean-variance function, and dynamic programming approach. The following chapters illustrate different models, respective methodologies, solution approaches, and results.

Chapter 3

Optimal production decision under exchange rate risk using forward contracts

This chapter presents an optimal production decision under exchange rate risk. The uncertainties in the currency risk is hedged by using forward contracts. In this chapter, we develop models for a global firm producing and selling in a domestic and a foreign country. The firm can produce its product either in one of the countries or in both countries. Depending on where the firm produces its product, we consider four different models, namely, domestic production model, domestic production with foreign subsidiary model, foreign production with domestic subsidiary model, and two–plant fully flexible model. For each model, risk associated with exchange rate is hedged by using forward contracts. While maximizing the profit, the optimal hedging strategies (or production quantities) for all models are analyzed from Section 3.1.1 to Section 3.2.3.

3.1 Models

The domestic country and the foreign country are indexed as 1 and 2, respectively. Demand in each country is assumed to be deterministic and denoted by D_1 and D_2, while unit sale prices are given as s_1 and s_2 in respective currencies. Capacity in each country is denoted by k_1 and k_2, respectively. The product cost function is assumed to be convex (e.g., see Pindyck (1988)) and given by $g_i(.)$ $(i = 1, 2)$ as follows:

$$g_i(x_{ij}) = a_i + b_i x_{ij} + c_i x_{ij}^2. \tag{3.1}$$

where, a_i, b_i, and c_i are constant and $c_i > 0$. Variable x_{ij} $(i, j = 1, 2)$ denotes the sale quantity from country i to country j. Production capacity in each facility is allocated at time 0 and the product is sold at time T. The number of forward contracts exercised at time T is Δ, which was purchased at time 0. The random spot exchange rate at time t is denoted by e_t.

3.1.1 Two-plant fully flexible system (General case)

In this model, the firm produces products in both foreign and domestic facilities and meet the demand in both domestic and foreign markets (see Figure 3.1). This supply chain network provides complete flexibility for the global firm. The sales quantity from domestic country 1 to domestic market 1 is denoted by x_{11}. In the same manner, x_{12} is the sale quantity from domestic facility 1 to foreign market 2. Similarly, x_{21} denotes the sales quantity from foreign facility 2 to domestic country 1 and x_{22} is the sale quantity from foreign facility 2 to foreign country 2. At time t, the profit, π, expressed in terms of the domestic currency is the sales revenue minus the production cost as shown in Equation (3.2).

$$\pi = s_1[x_{11} + x_{21}] + s_2 e_t[x_{12} + x_{22}] - g_1(x_{11} + x_{12}) - e_t g_2(x_{21} + x_{22}), \tag{3.2}$$

where, $[x_{11}+x_{21}]$ is the amount of products sold in the domestic market, s_1 is the selling price, and $s_1[x_{11}+x_{21}]$ is the revenue from selling the products in domestic country 1. Similarly, $s_2 e_t[x_{12}+x_{22}]$ is the revenue from selling in foreign country 2. The cost of production in domestic country is $g_1(x_{11} + x_{12})$ and the cost of production in foreign country 2 is $e_t g_2(x_{21} + x_{22})$. The exchange rate risk associated with the profit, π, can be hedged by exercising Δ number of forward contracts at time, t. These forward contracts were bought at time $t = 0$. The hedged profit, π_h, therefore, can be expressed, as in Equation (3.3), in terms of the profit, π, plus the payoff from the forward contract, $\Delta(e_t^f - e_t)$, where, Δ is the number of forward contracts purchased at time, $t = 0$, e_t^f is the forward exchange rate, and e_t is the exchange rate at time, t.

$$\pi_h = \pi + \Delta(e_t^f - e_t), \tag{3.3}$$

where $e_t^f = \mathbb{E}[e_t]$. The optimal number of forward contracts is calculated by maximizing the utility of the profit, π_h. The mean-variance utility function provides clear insights to decision makers and is given by Equation (3.4).

$$U(\pi_h) = \mathbb{E}(\pi_h) - \gamma \mathbb{V}(\pi_h), \tag{3.4}$$

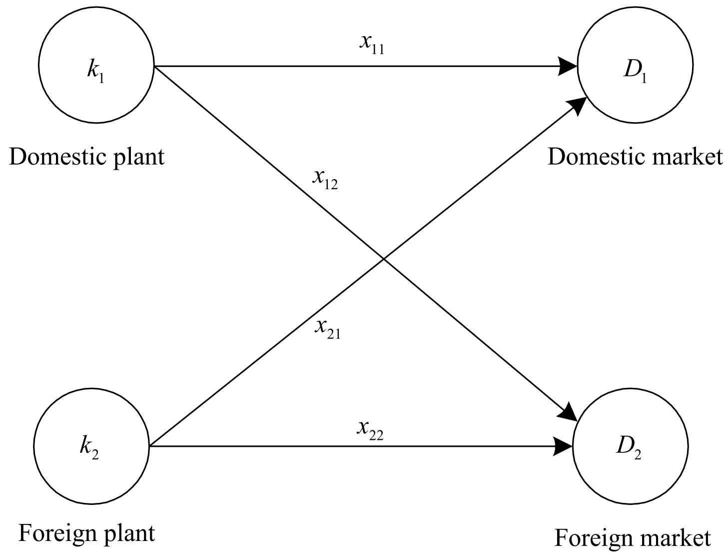

Figure 3.1: Domestic and foreign plants with domestic and foreign markets

where $\mathbb{E}(\pi_h)$ is the expected profit, γ is the mean-variance ratio, and $\mathbb{V}(\pi_h)$ is the variance of the profit. In order to find the number of forward contracts, Δ, the utility of the hedged profit, π_h, is maximized by differentiating Equation (3.4) with respect to Δ.

$$\frac{\partial U(\pi_h)}{\partial \Delta} = -\gamma \frac{\partial \mathbb{V}(\pi_h)}{\partial \Delta} + \frac{\partial \mathbb{E}(\pi_h)}{\partial \Delta} \tag{3.5}$$

Now, it can be proved that $\mathbb{E}(\pi_h) = \mathbb{E}[\pi]$ as follows:

$$\begin{aligned} \mathbb{E}(\pi_h) &= \mathbb{E}[\pi] + \Delta\ \mathbb{E}[e_t^f - e_t] \\ &= \mathbb{E}[\pi] + \Delta\{\mathbb{E}[e_t^f] - \mathbb{E}[e_t]\} \\ &= \mathbb{E}[\pi] + \Delta[e_t^f - e_t^f] \\ &= \mathbb{E}[\pi]. \end{aligned} \tag{3.6}$$

Since $\mathbb{E}[\pi]$ does not depend on Δ, differentiating $\mathbb{E}[\pi]$ with respect to Δ, we have $\frac{\partial \mathbb{E}(\pi)}{\partial \Delta} = 0$. Hence the optimal number of forward contracts, Δ, can be obtained as follows.

$$\begin{aligned} \frac{\partial U(\pi_h)}{\partial \Delta} &= -\gamma \frac{\partial \mathbb{V}(\pi_h)}{\partial \Delta} \\ \frac{\partial U(\pi_h)}{\partial \Delta} &= -\gamma \frac{\partial \mathbb{E}(\pi_h^2)}{\partial \Delta} + \gamma \frac{\partial \mathbb{E}(\pi_h)^2}{\partial \Delta} \\ \frac{\partial U(\pi_h)}{\partial \Delta} &= -2\gamma\Delta\mathbb{E}[(e_t^f - e_t)^2] - 2\gamma\mathbb{E}[(e_t^f - e_t)e_t(s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}))] \end{aligned} \tag{3.7}$$

Equating $\frac{\partial U(\pi_h)}{\partial \Delta} = 0$ in Equation (3.7), we have the following Equation (3.8):

$$-2\gamma\Delta\ \mathbb{E}[(e_t^f - e_t)^2] - 2\gamma\mathbb{E}[(e_T^f - e_t)e_t(s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}))] \quad = \quad 0 \tag{3.8}$$

Therefore, the number of futures contracts, Δ, are as follows:

$$\Delta \quad = \quad -\frac{\mathbb{E}[(e_t^f - e_t)e_t(s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}))]}{\mathbb{E}[(e_t^f - e_t)^2]} \tag{3.9}$$

$$\Delta \quad = \quad \frac{s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}))]\ \mathbb{E}[e_t^2 - e_t^f e_t]}{\mathbb{E}[{e_t^f}^2 + e_t^2 - 2e_t^f e_t]} \tag{3.10}$$

$$\Delta \quad = \quad \frac{s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}))]\ (\mathbb{E}[e_t^2] - {e_t^f}^2)}{\mathbb{E}[{e_t^f}^2 + e_t^2 - 2e_t^f e_t]} \tag{3.11}$$

The denominator, $\mathbb{E}[{e_t^f}^2 + e_t^2 - 2e_t^f e_t]$, in Equation (3.11), can be expressed as follows:

$$\begin{aligned}\mathbb{E}[{e_t^f}^2 + e_t^2 - 2e_t^f e_t] &= \mathbb{E}[{e_t^f}^2] + \mathbb{E}[e_t^2] - \mathbb{E}[2e_t^f e_t] \\ &= {e_t^f}^2 + \mathbb{E}[e_t^2] - 2e_t^f \mathbb{E}[e_t] \\ &= {e_t^f}^2 + \mathbb{E}[e_t^2] - 2{e_t^f}^2 \\ &= \mathbb{E}[e_t^2] - {e_t^f}^2 \end{aligned} \tag{3.12}$$

The numerator, $(\mathbb{E}[e_t^2] - {e_t^f}^2)$, in Equation (3.11), is cancelled out by Equation (3.12). Therefore, the value of Δ is as follows:

$$\Delta \quad = \quad s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22}) \tag{3.13}$$

Based on this optimal number forward contract, the hedge profit can be given by following Equation (3.14).

$$\pi_h = s_1[x_{11} + x_{21}] + s_2 e_t^f [x_{12} + x_{22}] - g_1(x_{11} + x_{12}) - e_t^f g_2(x_{21} + x_{22}) \tag{3.14}$$

It is observed from Equation (3.14) that the uncertainty involving the exchange rate is removed by using forward currency rate, e_t^f. The hedged profit now can be maximized subject to local and foreign demands and capacities. The problem is a non-linear optimization problem and the number of sale quantities, x_{11}, x_{12}, x_{21}, and x_{22} can be obtained optimally by solving the following model.

$$\begin{aligned}
\max_{x_{11},x_{12},x_{21},x_{22}} \pi_h &= s_1[x_{11}+x_{21}] + s_2 e_t^f [x_{12}+x_{22}] \\
&\quad -g_1(x_{11}+x_{12}) - e_t^f g_2(x_{21}+x_{22}) && (3.15)\\
s.t. \quad x_{11}+x_{12} &\leq k_1 && (3.16)\\
x_{21}+x_{22} &\leq k_2 && (3.17)\\
x_{11}+x_{21} &\leq D_1 && (3.18)\\
x_{12}+x_{22} &\leq D_2 && (3.19)\\
x_{11}, x_{12}, x_{21}, x_{22} &\geq 0 && (3.20)
\end{aligned}$$

Constraint (3.16) states that the total local sale quantities are limited by the domestic production capacity of k_1. Equation (3.17) refers to the total foreign sale quantities are constrained by the capacity, k_2, of the foreign country. Constraints (3.18) and (3.19) set the limitations to the total distribution quantities by the local market demand, D_1, and the foreign market demand, D_2, respectively. Non–negativity constraints are imposed by Equation (3.20). The non–liner optimization problem can be solved by using the Kuhn–Tucker method. Therefore, the Khun–Tucker conditions are:

$$\frac{\partial \pi_h}{\partial\, x_{11}} - \alpha_1 - \alpha_3 = 0 \tag{3.21}$$

$$\frac{\partial \pi_h}{\partial\, x_{21}} - \alpha_2 - \alpha_3 = 0 \tag{3.22}$$

$$\frac{\partial \pi_h}{\partial\, x_{12}} - \alpha_1 - \alpha_4 = 0 \tag{3.23}$$

$$\frac{\partial \pi_h}{\partial\, x_{22}} - \alpha_2 - \alpha_4 = 0 \tag{3.24}$$

$$\alpha_1(k_1 - x_{11} - x_{12}) = 0 \tag{3.25}$$

$$\alpha_2(k_2 - x_{21} - x_{22}) = 0 \tag{3.26}$$

$$\alpha_3(D_1 - x_{11} - x_{21}) = 0 \tag{3.27}$$

$$\alpha_4(D_2 - x_{12} - x_{22}) = 0 \tag{3.28}$$

where, $\alpha_1, \alpha_2, \alpha_3,$ and $\alpha_4 \geq 0$. Differentiating Equation (3.14) with respective to x_{11}, x_{21}, x_{12} and x_{22}, respectively, we get:

$$
\begin{aligned}
\frac{\partial \pi_h}{\partial\ x_{11}} &= s_1 - [b_1 + 2c_1(x_{11} + x_{12})] && (3.29)\\
\frac{\partial \pi_h}{\partial\ x_{21}} &= s_1 - [b_2 + 2c_2(x_{21} + x_{22})]e_t^f && (3.30)\\
\frac{\partial \pi_h}{\partial\ x_{12}} &= s_2 e_t^f - [b_1 + 2c_1(x_{11} + x_{12})] && (3.31)\\
\frac{\partial \pi_h}{\partial\ x_{22}} &= s_2 e_t^f - [b_2 + 2c_2(x_{21} + x_{22})]e_t^f && (3.32)
\end{aligned}
$$

Substituting Equations (3.29) to (3.32) into Equations (3.21) to (3.24) we have:

$$
\begin{aligned}
\alpha_1 &= -b_1 + b_2 e_t^f - 2c_1 k_1 + 2c_2 e_t^f k_2 && (3.33)\\
\alpha_2 &= 0 && (3.34)\\
\alpha_3 &= -b_2 e_t^f - 2c_2 D_1 e_t^f - 2c_2 D_2 e_t^f + 2c_2 e_t^f k_1 + s_1 && (3.35)\\
\alpha_4 &= -b_2 e_t^f - 2c_2 D_1 e_t^f - 2c_2 D_2 e_t^f + 2c_2 e_t^f k_1 + e_t^f s_2 && (3.36)
\end{aligned}
$$

The problem has multiple solutions depending on the relationship among the values of sales prices, s_1 and s_2, and cost functions, $g_1(\cdot)$ and $g_2(\cdot)$. Assuming that the sales in the local market is more profitable than the foreign market, i.e., $s_1 > s_2 e_t^f$, and the cost of production in the local market is cheaper, $g_1(\cdot) < g_2(\cdot)$, and $D_1 + D_2 = k_1 + k_2$, the optimal number of sale quantities can be obtained as follows:

$$
\begin{aligned}
x_{11} &= D_1 - x_{21} && (3.37)\\
x_{12} &= -D_1 + k_1 + x_{21} && (3.38)\\
x_{22} &= D_1 + D_2 - k_1 - x_{21} && (3.39)
\end{aligned}
$$

Figure 3.2 shows the optimal sale quantities sold in domestic and foreign markets for a varying level of capacity, k_2, keeping the local capacity, k_1, fixed. It is observed that when there is no foreign subsidiary with $k_2 = 0$, then the local plant tries to meet the local demand, as $x_{11} = \min(k_1, D_1)$. Since the local sales are more profitable, when the foreign capacity, k_2, is increased from zero, the sales quantity, x_{21}, from foreign subsidiary to the local market also increases in order to fill the local market demand. When k_2 reaches to a value equal to the difference of D_1 and k_1, the value of x_{11} tends to decrease, while the value of x_{12}, tends to rise as the local production plant starts fulfilling the foreign demands. One can observe that the behavior of x_{11} is a mirror of that of x_{12},

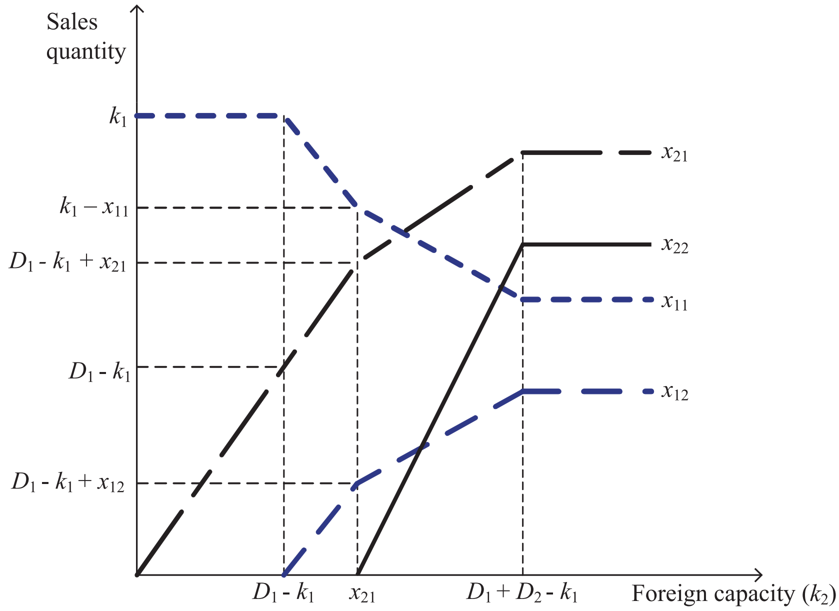

Figure 3.2: The optimal production quantities for a fully flexible model.

as x_{11} deceases, x_{12} increases. The production quantity, x_{22} remains zero until k_2 reaches a value equal to x_{21}. After fulfilling the local market completely, the foreign production plant starts to fill the foreign market demand and x_{22} commences to increases from the point where the value of k_2 equals to x_{21}. Note that as $s_1 > s_2 e_t^f$, the demand of the local market has more priority because of its having higher revenue from local sales.

The profit curve for a fully flexible model is shown in Figure 3.3. When foreign capacity, $k_2 = 0$ at time $t = T$, the profit is $s_1 k_1 - g_1(k_1)$. The reason is that sales in the local market is considered to be more profitable than the sales in the foreign market. That is why, when $k_2 = 0$, the firm seeks to maximize its profit by selling products to the local market. As k_2 increases, the profit increases and reaches at its maximum, when $k_2 = D_1 + D_2 - k_1$. Note that the model is generic and fits well for other assumptions as well.

Figure 3.4 shows the effect of using forward contracts on the hedged and non-hedged profits. When the exchange rate at time $t = 0$ is e_0, the non-hedged profit is $s_1 D_1 + s_2 e_0 D_2 - g_1(k_1) - e_0 g_2(k_2)$ and the hedged profit is $s_1 D_1 + s_2 e_T^f D_2 - g_1(k_1) - e_T^f g_2(k_2)$. From the figure, it is observed that when the exchange rate is below e_T^f, the hedged profit is more than the non-hedged profit. When the exchange rate is above e_T^f, the hedged profit remains the same, although the non-hedged profit is high. Therefore, it can be concluded that forward contracts hedge the downside risk of the firm.

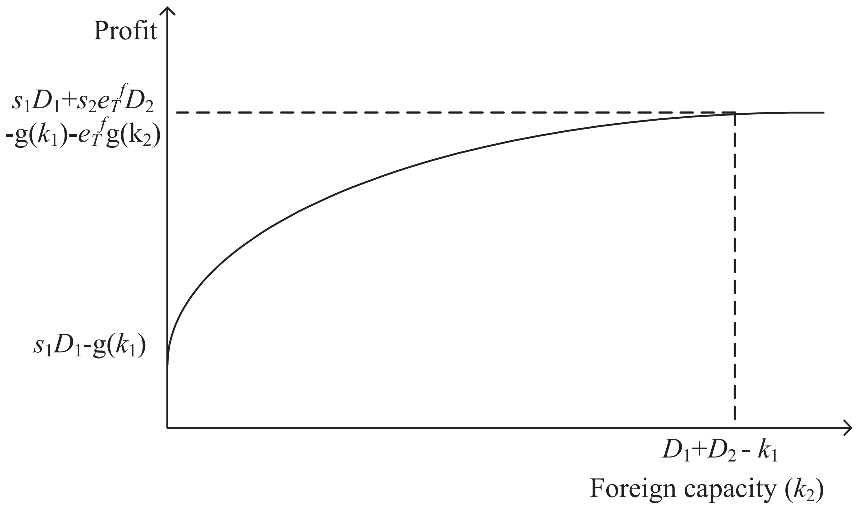

Figure 3.3: The hedged profit for different levels of capacity for a fully flexible model.

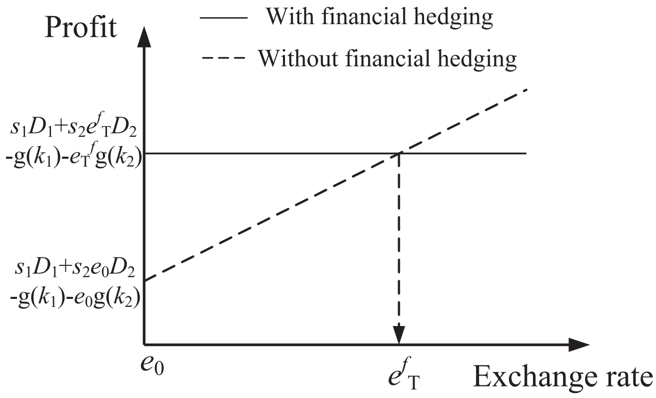

Figure 3.4: The hedged and non-hedged profits for a fully flexible model.

3.2 Numerical studies

In this section, we provide numerical examples discussed in Section 3.1. The following parameters are considered in these examples: $s_1 = 110$ CAD, $s_2 = 100$ USD, $D_1 = 150$ units, $D_2 = 50$ units, $a_1 = 1.0$, $b_1 = 1.0$, $c_1 = 0.3$, $a_2 = 0.8$, $b_2 = 0.5$, $c_2 = 0.15$. The forward exchange rate at time $t = T$ is $e_T^f = 1.2$ CAD/USD.

3.2.1 Domestic production

From the generalized fully flexible model described above, special cases can be derived. This section illustrates, as shown in Figure 3.5, that the firm has a production facility in the domestic country. However, the firm sells its production in both domestic and foreign markets. Substituting $x_{21} = 0$ and $x_{22} = 0$ in Equation (3.13) and Equation (3.15), we find $\Delta = s_2x_{12}$ and the hedged profit at time $t = T$ is $\pi_h = [s_1 x_{11} + e_T^f s_2 x_{12}] - g_1(x_{11} + x_{12})$.

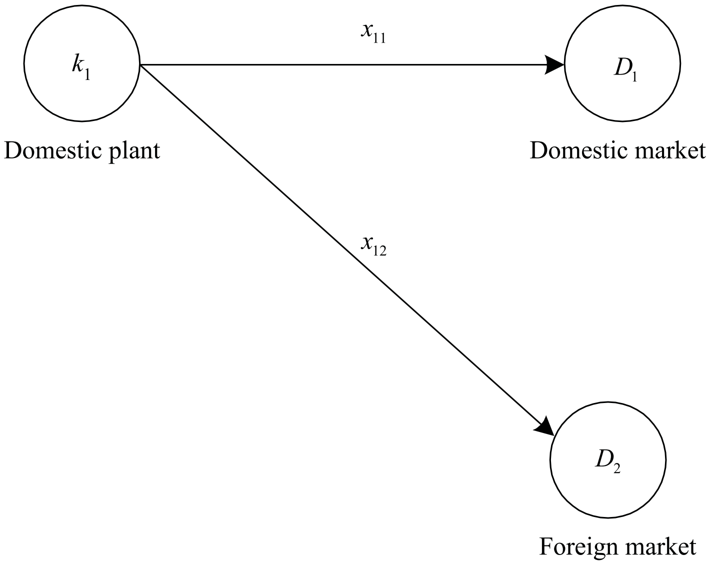

Figure 3.5: Domestic plant with domestic and foreign markets.

The optimization problem now can be stated as:

$$\max_{x_{11}, x_{12}} \quad \pi_h = [s_1 x_{11} + s_2 e_T^f x_{12}] - g_1(x_{11} + x_{12}) \tag{3.40}$$

$$x_{11} + x_{12} \leq k_1 \tag{3.41}$$

$$x_{11} \leq D_1 \tag{3.42}$$

$$x_{12} \leq D_2 \tag{3.43}$$

$$x_{11}, x_{12} \geq 0 \tag{3.44}$$

The Kuhn–Tucker conditions can be written as:

$$\frac{\partial \pi_h}{\partial \ x_{11}} - \alpha_1 - \alpha_3 = 0 \tag{3.45}$$

$$\frac{\partial \pi_h}{\partial \ x_{12}} - \alpha_1 - \alpha_4 = 0 \tag{3.46}$$

$$\alpha_1(k_1 - x_{11} - x_{12}) = 0 \tag{3.47}$$

$$\alpha_3(D_1 - x_{11}) = 0 \tag{3.48}$$

$$\alpha_4(D_2 - x_{12}) = 0 \tag{3.49}$$

Solving Equations (3.45)–(3.49) for the given parameters, we obtain, $\alpha_1 = 0$, $\alpha_3 = 0$, $\alpha_4 = 10$, $x_{11} = 132$, and $x_{12} = 50$. Figure 3.6 depicts the optimal quantities sold in domestic and foreign markets for different levels of capacity in the domestic plant. The exchange rate risk is hedged by using forward contracts. It is observed that when the capacity in the domestic plant is less than 50 units, which is same as the foreign market demand, the product is sold only in the foreign market. It is due to the fact that the forward contract guarantees a revenue of $1.2 \times \$100 = \$120$ per unit, which is more than the revenue of \$100 per unit from the domestic market. When the capacity of the domestic plant is more than 50 units, the foreign demand is fully satisfied and the rest is sold in the domestic market up to 132 units. This implies that the optimal capacity at the domestic plant is $50 + 132 = 182$ units meaning that the domestic market demand, which is 150, will not be fully satisfied. As a result, the total profit, as shown in Figure 3.7, is at the peak when the optimal capacity is 182 units.

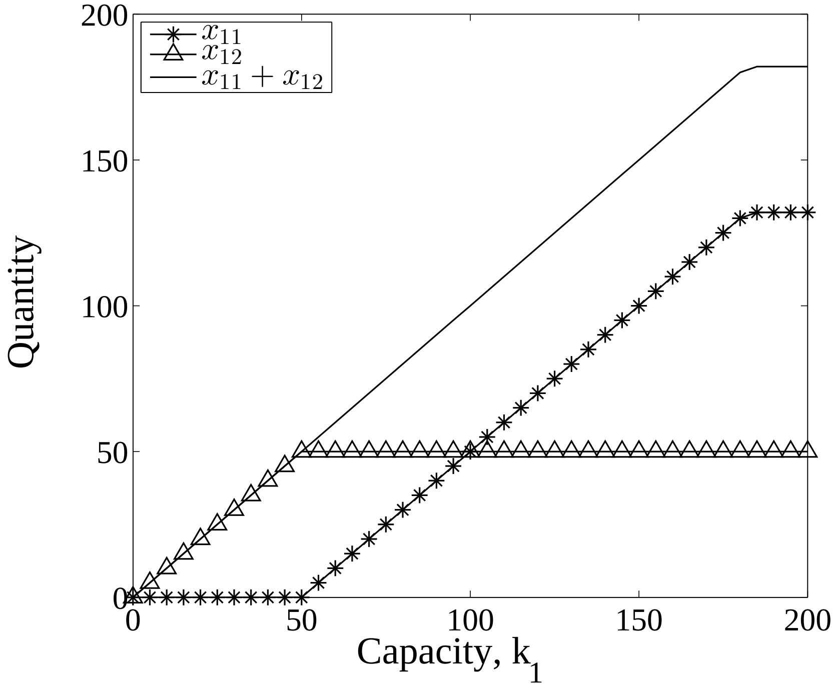

Figure 3.6: Quantities sold in domestic and foreign markets for different levels of capacity in the domestic plant

3.2.2 Domestic production with foreign subsidiary

In this model, in addition to domestic production, the firm also has a subsidiary production facility in the foreign country to meet excess demand from the foreign market only. The supply chain is shown in Figure 3.8. This model is preferred when producing in the domestic facility is cheaper and the domestic production capacity is not sufficient to meet the demand from both foreign and domestic markets. Setting $x_{21} = 0$ in Equation (3.2), we have at $t = T$:

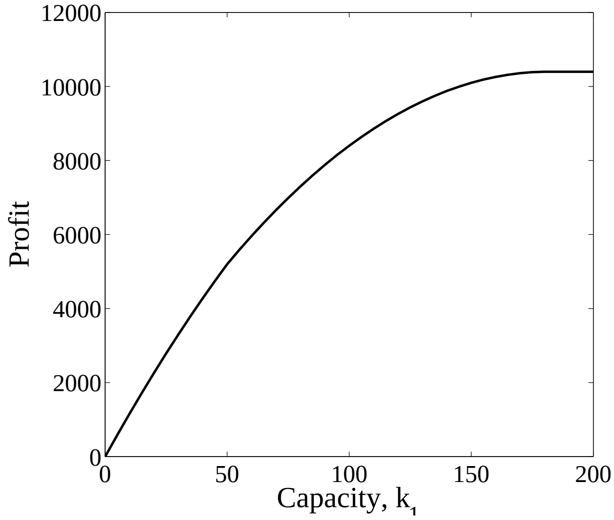

Figure 3.7: The total hedged profit for different levels of capacity in the domestic plant

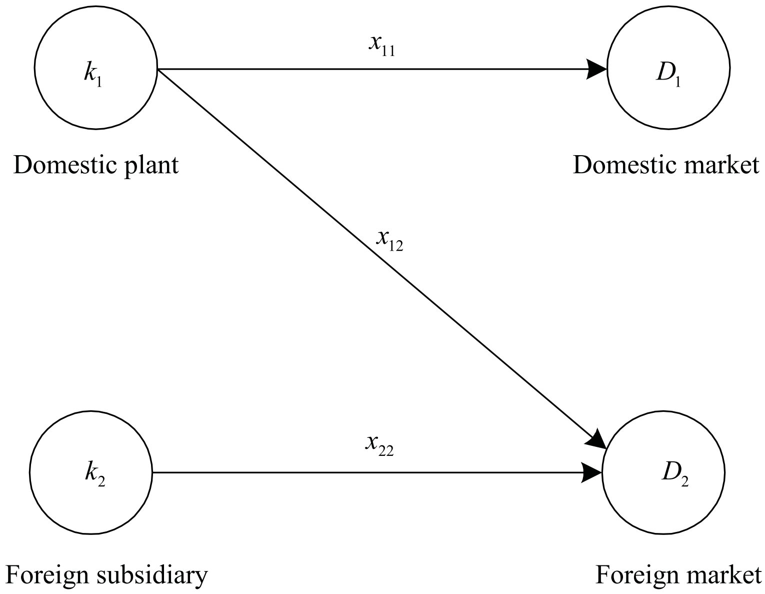

Figure 3.8: Domestic plant and foreign subsidiary with domestic and foreign markets

$$\pi = s_1 x_{11} + s_2 e_T [x_{12} + x_{22}] - g_1(x_{11} + x_{12}) - e_T g_2(x_{22}), \tag{3.50}$$

Again substituting $x_{21} = 0$ in the general case, the optimal number of forward contracts, $\Delta = s_2 x_{12} + g_2(x_{21})$ and the hedged profit, π_h, can be expressed as:

$$\pi_h = s_1 x_{11} + s_2 e_T^f [x_{12} + x_{22}] - g_1(x_{11} + x_{12}) - e_T^f g_2(x_{22}), \tag{3.51}$$

The optimization problem can be written as:

$$\begin{aligned}
\max_{x_{11},x_{12},x_{22}} \quad \pi_h &= s_1 x_{11} + s_2 e_T^f [x_{12} + x_{22}] \\
&\quad - g_1(x_{11} + x_{12}) - e_T^f g_2(x_{22}) && (3.52)\\
s.t. \quad x_{11} + x_{12} &\leq k_1 && (3.53)\\
x_{22} &\leq k_2 && (3.54)\\
x_{11} &\leq D_1 && (3.55)\\
x_{12} + x_{22} &\leq D_2 && (3.56)\\
x_{11}, x_{12}, x_{22} &\geq 0 && (3.57)
\end{aligned}$$

By using the Kuhn–Tucker conditions, following equations can be obtained as:

$$\begin{aligned}
\frac{\partial \pi_h}{\partial\ x_{11}} - \alpha_1 - \alpha_3 = 0 && (3.58)\\
\frac{\partial \pi_h}{\partial\ x_{12}} - \alpha_1 - \alpha_4 = 0 && (3.59)\\
\frac{\partial \pi_h}{\partial\ x_{22}} - \alpha_2 - \alpha_4 = 0 && (3.60)\\
\alpha_1(k_1 - x_{11} - x_{12}) = 0 && (3.61)\\
\alpha_2(k_2 - x_{22}) = 0 && (3.62)\\
\alpha_3(D_1 - x_{11}) = 0 && (3.63)\\
\alpha_4(D_2 - x_{12} - x_{22}) = 0 && (3.64)
\end{aligned}$$

Solving Equations (3.58)–(3.64) for the values stated earlier in Section 3.2, it is found that $\alpha_1 = 0$, $\alpha_2 = 75.4$, $\alpha_3 = 19$, $\alpha_4 = 29$, $x_{11} = 150$, $x_{12} = 0$, and $x_{22} = 50$. Figure 3.9 shows the optimal quantities sold in domestic and foreign markets for different levels of capacity in the foreign subsidiary. It can be seen that the capacity in the foreign subsidiary is completely utilized regardless of its capacity level. The demand from the foreign market is always met with use of domestic and foreign subsidiary capacities because of higher revenue per unit as discussed in the previous example. Domestic market demand is not fully satisfied when the capacity of the foreign subsidiary is 0. However, as capacity in the foreign subsidiary is more than 18 units, demand from the domestic market is fully satisfied. Figure 3.10 shows the total hedged profit. Since the production cost in the foreign subsidiary is lower than that of the domestic plant, the profit keeps increasing as the capacity at the foreign subsidiary increases and then the total profit levels out once the foreign capacity is reached to 50 units. Hence, the optimal capacity at the domestic plant is 150 units and the optimal capacity at the foreign plant is 50 units. In this example, having low-cost foreign subsidiary helps to increase the quantity sold and consequently it increases the total profit.

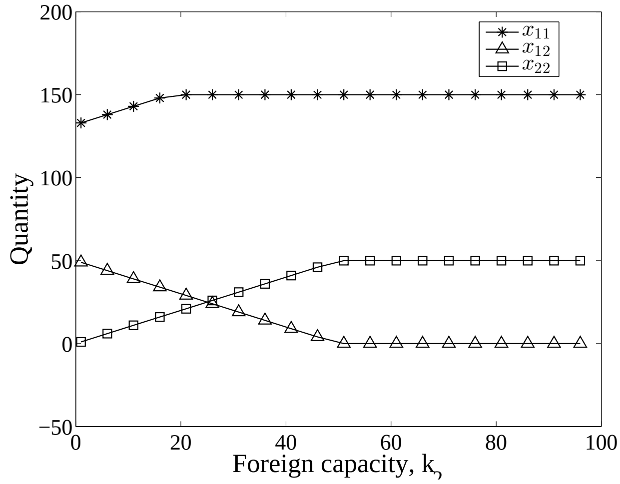

Figure 3.9: Quantities sold in domestic and foreign markets for different levels of capacity in the foreign subsidiary

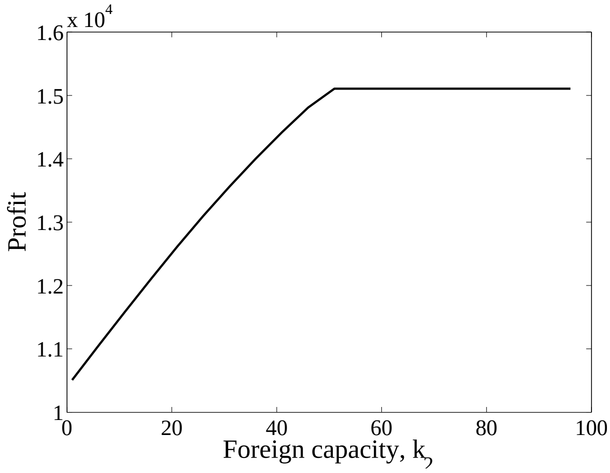

Figure 3.10: The total hedged profit for different levels of capacity in the foreign subsidiary

3.2.3 Foreign production with domestic subsidiary

In this model, in addition to foreign production, the global firm has a domestic subsidiary production facility to meet excess demand from the local market as shown in Figure 3.11 This model is preferred when producing in the foreign facility is cheaper.

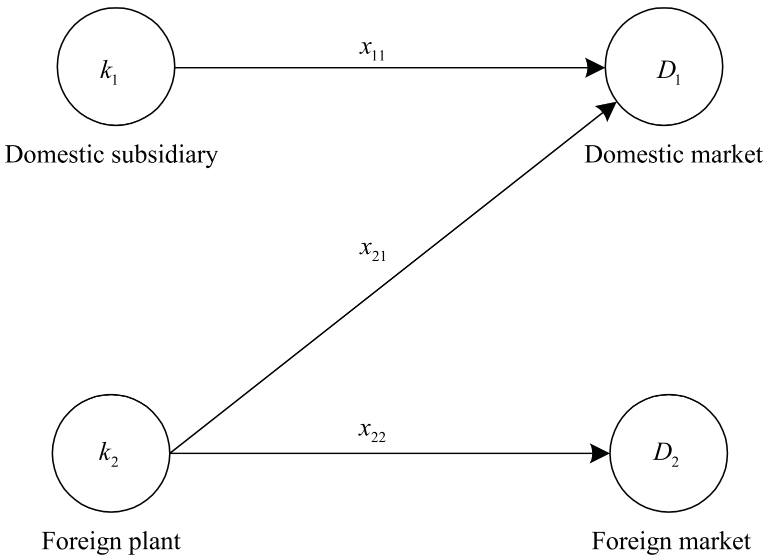

Figure 3.11: Foreign plant and domestic subsidiary with domestic and foreign markets

Similar to previous sections, by substituting $x_{12} = 0$ in Equation (3.14), we obtain the hedged profit, π_h, at $t = T$ as follows:

$$\pi_h = s_1[x_{11} + x_{21}] + s_2 e_T^f [x_{22}] - g_1(x_{11}) - e_T^f g_2(x_{21} + x_{22}), \tag{3.65}$$

The optimal number of forward contracts, $\Delta = s_2[x_{12} + x_{22}] - g_2(x_{21} + x_{22})$ and The optimization problem can be written as:

$$\begin{aligned}
\max_{x_{11},x_{21},x_{22}} \quad \pi_h &= s_1[x_{11} + x_{21}] + s_2 e_T^f [x_{12} + x_{22}] \\
&\quad -g_1(x_{11}) - e_T^f g_2(x_{21} + x_{22}) && (3.66) \\
s.t. \quad x_{11} &\leq k_1 && (3.67) \\
x_{21} + x_{22} &\leq k_2 && (3.68) \\
x_{11} + x_{21} &\leq D_1 && (3.69) \\
x_{22} &\leq D_2 && (3.70) \\
x_{11}, x_{21}, x_{22} &\geq 0 && (3.71)
\end{aligned}$$

The Kuhn-Tucker conditions can be written as:

$$\frac{\partial \pi_h}{\partial\, x_{11}} - \alpha_1 - \alpha_3 = 0 \tag{3.72}$$

$$\frac{\partial \pi_h}{\partial\, x_{21}} - \alpha_2 - \alpha_3 = 0 \tag{3.73}$$

$$\frac{\partial \pi_h}{\partial\, x_{22}} - \alpha_2 - \alpha_4 = 0 \tag{3.74}$$

$$\alpha_1(k_1 - x_{11}) = 0 \tag{3.75}$$

$$\alpha_2(D_1 - x_{11} - x_{21}) = 0 \tag{3.76}$$

$$\alpha_3(k_2 - x_{21} - x_{22}) = 0 \tag{3.77}$$

$$\alpha_4(D_2 - x_{22}) = 0 \tag{3.78}$$

Solving Equations (3.72)–(3.78) for the given values in Section 3.2, we obtain $\alpha_1 = 0$, $\alpha_2 = 0.4$, $\alpha_3 = 64$, $\alpha_4 = 74$, $x_{11} = 75$, $x_{21} = 75$, and $x_{22} = 50$. Figure 3.12 shows the optimal quantities sold in domestic and foreign markets for different levels of capacity in the foreign plant. When the capacity at the foreign plant is 0, the domestic subsidiary completely meet the demand from the domestic market. As the capacity at the foreign plant increases from 0 to 50, it only meets the foreign market demand. Once the foreign capacity exceeds the foreign market demand, the excess quantities are sold in the domestic market until each foreign plant and domestic subsidiary equally shares the demand from the domestic market. As a result, the optimal capacity at the foreign plant is 125 units and it is 75 units at the domestic subsidiary. The total hedged profit in Figure 3.13 plateaus once the optimal capacity is reached. In this example, demands from domestic and foreign markets are completely met. However, the total hedged profit is higher, since a large amount of the total demand is met by the low-cost foreign plant.

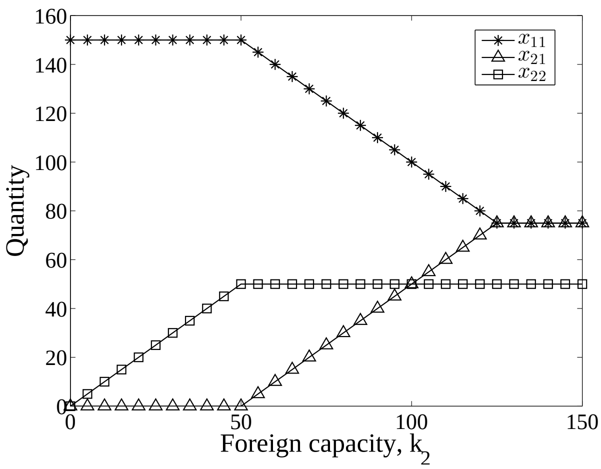

Figure 3.12: Quantities sold in domestic and foreign markets for different levels of capacity in the foreign plant

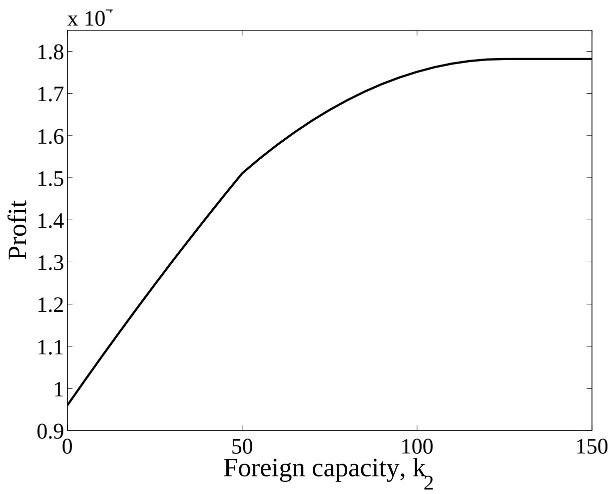

Figure 3.13: The total hedged profit for different levels of capacity in the foreign plant

3.2.4 A fully flexible system

Figure 3.14 shows the optimal quantities sold in domestic and foreign markets for different levels of capacity in the foreign plant. When the capacity at the foreign plant is 0, the domestic plant completely meets the demand from the foreign market due to higher revenue per unit as discussed in example 1; and also domestic plants supplies 182 units to domestic market. As the capacity at the foreign plant increases from 0 to 50 units, it meets the demand from the domestic market. When the capacity at the foreign plant is 50 units, the capacity at the domestic plant is reduced to 150 units. Moreover, when the foreign plant capacity increases beyond 50 units, both plants share the demand from both markets and the maximum profit is archived at the optimal domestic plant capacity of 74 units and foreign plant capacity of 126 units. Beyond these capacity levels, the total hedged profit is levels out as seen in Figure 3.15. Even though this model is fully flexible, demands from domestic and foreign markets are completely met at the same optimal capacity allocation and the same total hedged profit is obtained.

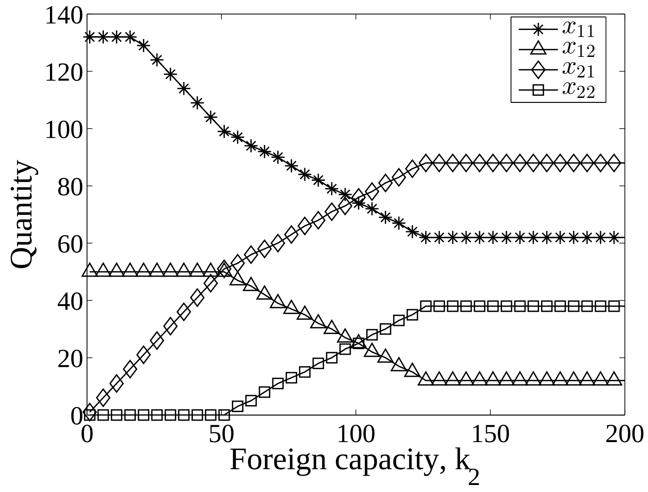

Figure 3.14: Quantities sold in domestic and foreign markets for different levels of capacity for a fully flexible system.

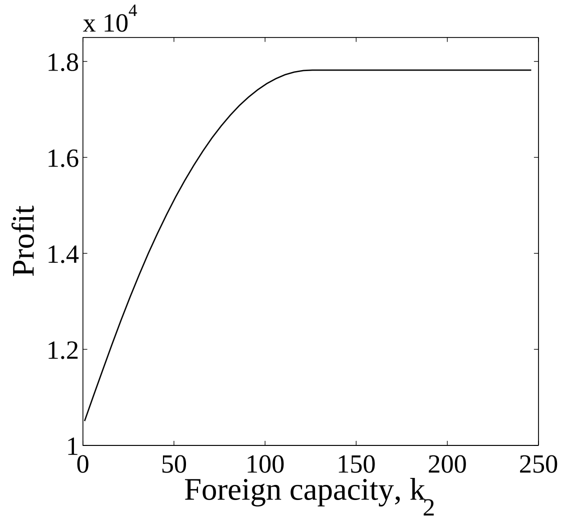

Figure 3.15: The total hedged profit for different levels of capacity for a fully flexible system.

3.3 Sensitivity tests with foreign exchange rates

This section provides sensitivity tests with respect to changing foreign exchange rates for the proposed models.

3.3.1 Domestic production

This is an example for the domestic production model presented in Section 3.2.1. Figure 3.16 shows the profit versus exchange rate at time $t = 0$, e_t, for different levels of domestic capacity, when forward exchange rate at time $t = T$, e_T^f, is 1.2. Solid lines are for hedged profit and dashed lines are for non-hedged profit. This figure clearly demonstrates that downside risk is completely hedged by forward contracts.

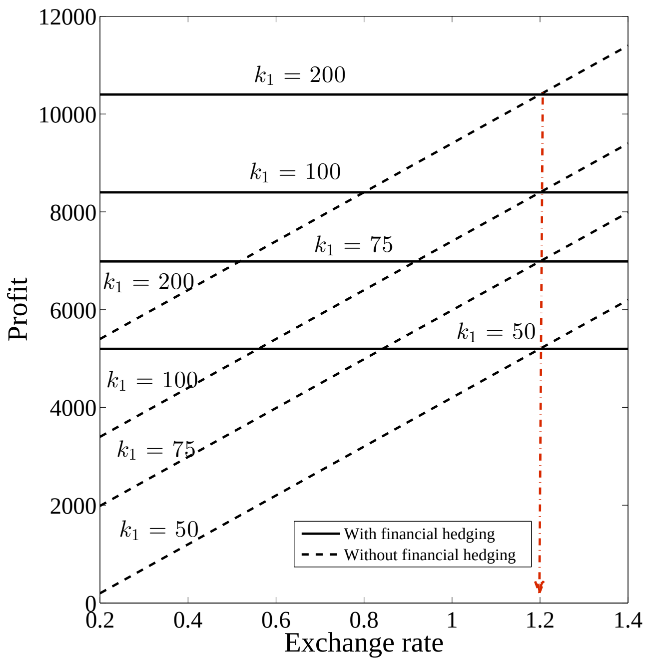

Figure 3.16: Hedged and non-hedged profits for domestic production model

3.3.2 Domestic production with foreign subsidiary

This is an example for the domestic production with foreign subsidiary model. In this example, we consider a fixed domestic production capacity, $k_1 = 200$, while foreign capacity, k_2, is varied from 0 to 50. The reason to choose $k_1 = 200$ is to ensure that we have enough capacity to meet domestic demand of 150 units and foreign demand of 50 units, when the foreign subsidiary capacity is 0. Similarly, the maximum level of k_2 is chosen as 50 units to guarantee that the maximum capacity in the foreign subsidiary alone is enough to meet the foreign market demand. Figure 3.17 presents the profit versus exchange rate at time T (i.e., e_T), for different levels of foreign capacity, when forward exchange rate at time $t = T$, e_T^f, is 1.2. Solid lines are for hedged profit and dashed lines are for non-hedged profit. This figure shows that downside risk is mitigated by forward contracts.

3.3.3 Foreign production with domestic subsidiary

This is an example for the foreign production with local subsidiary model. In this example, we consider fixed domestic production capacity $k_1 = 150$, while foreign capacity, k_2, is varied from 0 to

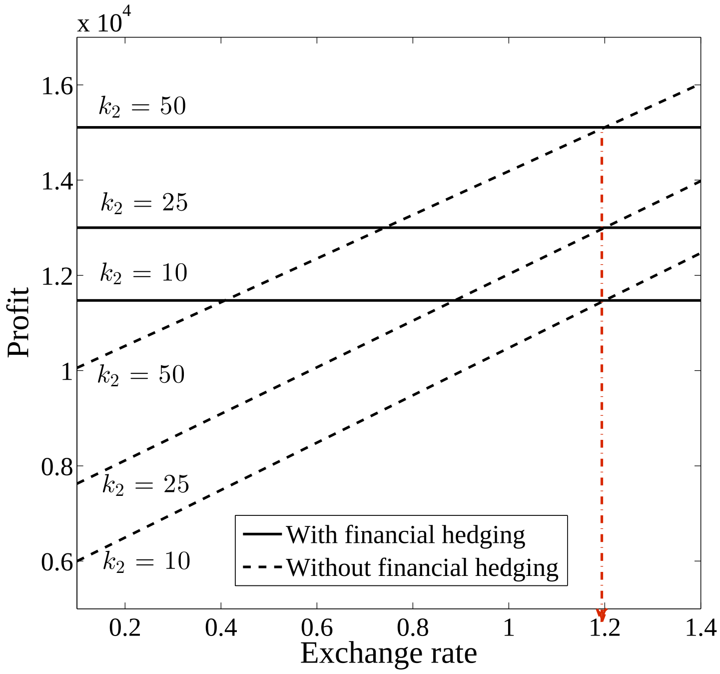

Figure 3.17: Hedged and non-hedged profits for domestic production with foreign subsidiary

140. The reason to choose $k_1 = 150$ is to ensure that the domestic subsidiary has enough capacity to meet the demand from the domestic market, when the foreign plant capacity is 0. Similarly, the maximum level of k_2 is chosen as 200 units to guarantee that the maximum capacity in the foreign plant alone is enough to meet the demand from foreign and domestic markets. Figure 3.18 presents the profit versus exchange rate, i.e., e_t, for different levels of foreign capacity, when forward exchange rate at time $t = T$, e_T^f, is 1.2. Solid lines and dashed lines are for hedged and non-hedged profits, respectively. This figure shows that downside risk is mitigated by forward contracts. It is also observed that when capacity increases, the expected profit also increases.

3.3.4 A fully flexible system

This example is for a two-plant fully flexible system. The domestic production capacity, k_1, is fixed at 200, while foreign capacity, k_2, is varied from 0 to 200. The reason to choose $k_1 = 200$ and the maximum value of $k_2 = 200$ are to ensure that any of the plants has capacity to satisfy the demand from both domestic and foreign markets. Figure 3.19 presents the profit versus exchange rate for different levels of foreign capacity. Solid lines and dashed lines are for hedged and non-hedged profits, respectively. This figure shows that downside risk is hedged by forward contracts and the hedged profit is greater than non-hedged profit for an exchange rate less than 1.2.

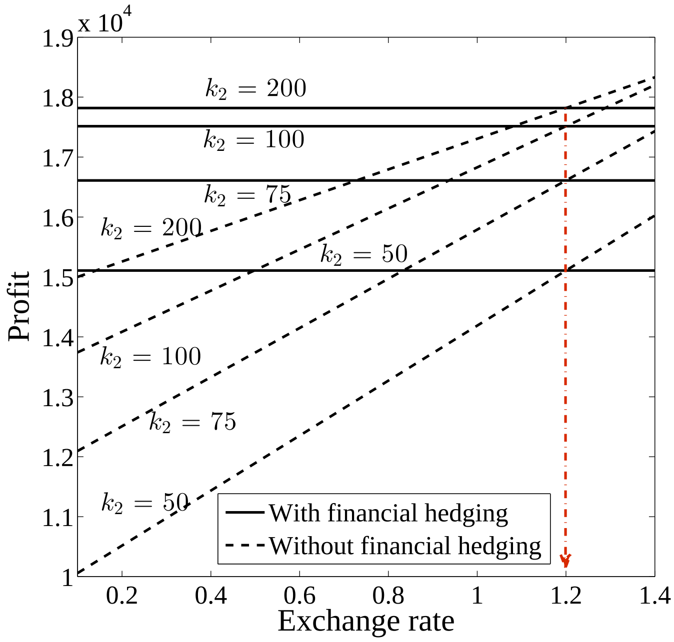

Figure 3.18: Hedged and non-hedged profits for domestic subsidiary with foreign production

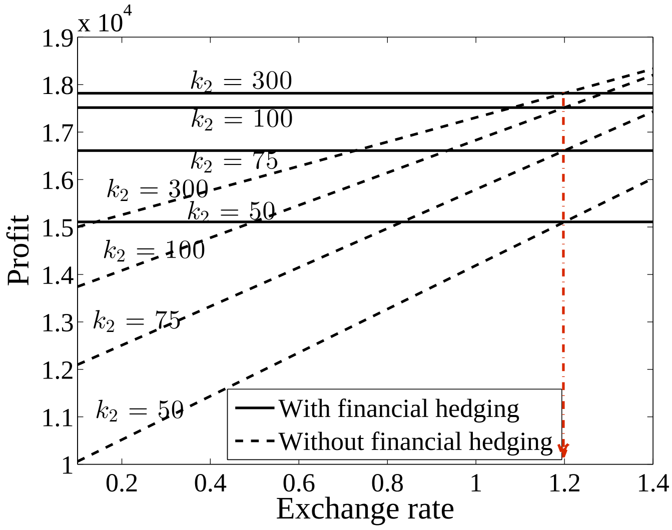

Figure 3.19: Hedged and non-hedged profits for a fully flexible system.

3.4 Summary

In this chapter, we study two-level supply chain models having productions and markets in both domestic and foreign countries. Consequently, the uncertainty in exchange rate and the non–linear production costs play a crucial role in production and sales decisions. The exchange rate risk is mitigated by using forward contracts. We consider four types of supply-chain models between two countries, which are domestic production model, domestic production model with foreign subsidiary, foreign production model with domestic subsidiary and fully flexible model.

The domestic production model has no flexibility and is considered as a base model for the sake of comparison with other models. The domestic production model with foreign subsidiary is preferable at times when the local production cost is low and local capacity does not fulfill all demands. In contrary, if the foreign production has the ability to meet the foreign demand and foreign production cost is substantially lower than domestic cost, the optimized model consists of local production for the local market and foreign production for foreign market. The foreign production model with local subsidiary is preferable when the foreign production cost is low, thus avoiding the production from local market. The fully flexible model consists of all the above models. We utilize one stage hedging to optimize the firm's profit by allocating the capacity between the two countries as required based on firm's demand and capacity constraints. Production capacity is allocated at initial stage for a fixed forward exchange rate. Through this, the firm can make business decision upfront. The models hedge downside risk and guarantee a given amount of profit in the event of exchange rate uncertainties.

Chapter 4

Multi-period optimal production decision under currency exchange risk using options

In this chapter, a multi-period optimal production decision problem for a multinational firm is analyzed. The firm possesses capacity adjustment ability, i.e., the ability or flexibility to choose a capacity level from a set of discrete capacity levels. Such flexibility, specifically mentioned as capacity flexibility, enables the firm to meet demand variations. The exchange rate is considered uncertain and is hedged by using both real and financial options. In a multi-period setting, i.e., $t = 1, 2, \ldots, T$, the exchange rate uncertainty is hedged by using both real options (in the form of capacity expansion and contraction flexibilities) and financial options (in the form of currency put options). At period $(t-1)$, two decisions are made: (a) how much capacity to expand or contract and (b) the number of put options to long. Both the operational and financial decisions are exercised at the next period t and the capacity allocations are implemented and the currency put options are exercised accordingly. For a flexible system, there occurs a certain cost for capacity expansion or contraction in period $(t-1)$. This is discussed in detail in Section 4.1.3. The option premium is determined by using the Black and Scholes (1973) model. For example, the put option premium paid at period $(t-1)$ is P_{t-1}, which is given by $\exp(-r_f h)\{K_t\Phi(-d_2) - e_t\Phi(-d_1)\}$, where e_t is the exchange rate in period t, and K_t is the strick price of the put option in period t. The option can be exercised in period t. Since an option on an underlying asset can be traded at different exercise prices, K_t can be a vector. The value of $d_1 = \frac{\ln(e_t/K_t)+(r-r_f+\sigma^2/2)h}{\sigma\sqrt{h}}$ and $d_2 = d_1 - \sigma\sqrt{h}$. The $\Phi(.)$ is the cumulative distribution function for the standard normal distribution (Hull 2009). In period t, put options are excised when $K_t > e_t$, otherwise put options remain unexercised and the premium paid at time $(t-1)$ is lost.

4.1 Models

A firm having a domestic manufacturing plant with a domestic market and an international market is considered. The firm produces a single type of product and satisfies the multi-period known demands from both markets. The manufacturing plant has a flexible system that has the ability to change its capacity levels. In particular, the manufacturing system has the ability to increase (expansion flexibility) or decrease (contraction flexibility) its capacity in response to the exchange rate fluctuations so as to maximize the MV utility of the firm at each period of time. The price for the product in both domestic and foreign markets is fixed in their respective currencies. As the exchange rate between the two countries is stochastic, the revenue from the foreign market is uncertain. Therefore, the manufacturer faces the risk from the exchange rate uncertainty. The stochastic exchange rate between the two countries is modeled by using a geometric Brownian motion (e.g., Dixit 1989; Adkins 1993). In a multi-period setting, the geometric Brownian motion is discretized by using a binomial tree (e.g., Cox et al. 1979; Hull 2009). The probability that the exchange rate moves along the upward branch is denoted by p, which is $\frac{(\exp((r-r_f)h)-d)}{(u-d)}$, where $u = \exp(\sigma\sqrt{h})$, $d = 1/u$, r and r_f are the risk free interest rates in the local and foreign countries, respectively, σ is the volatility of the exchange rate, and h is the time step between periods $(t-1)$ and t. The probability that the exchange rate moves along the downward branch is given by $(1-p)$.

Production cost is calculated in the domestic currency. The production cost function, $G(.)$, is assumed to be non–linear and depends on the current capacity level, and is given by Equation (4.1) (e.g., Pindyck 1988; Bollen 1999).

$$G(Q_t) = a_1 Q_t + a_2 Q_t^2/2M_t + a_3 M_t, \tag{4.1}$$

where a_1, a_2, a_3 are coefficients, Q_t is the production quantity at time t, and M_t is the capacity at time t, where $Q_t \leq M_t$, meaning production quantity at time t does not exceed the capacity at t. Moreover, $Q_t = x_{11_t} + x_{12_t}$, where x_{11_t} and x_{12_t} are sales quantities in local and foreign markets at time t, respectively.

In order to investigate the effects of operational flexibility, financial options, and the integration of real options (operational flexibility) and financial options in a multi–period time–frame, four models are considered. Model I is a supply chain model with a fixed capacity system having neither operational flexibility nor utilization of financial options. This model is assumed as a reference model. In order to understand the effect of using financial hedging via financial options, this reference model is examined with put options in mitigating the exchange risk. We refer to this model as Model II. Again, in order to understand the benefit of the operational hedging, the reference model incorporates capacity flexibilities, expansion and contraction, at each period of time. This model is called a flexible capacity system, Model III. Finally, in order to investigate the effect of the integrated approach of using both operational hedging and financial hedging, Model III, the flexible capacity system, is investigated with the addition of currency put options. We refer it to as Model IV.

4.1.1 Model I: Fixed capacity system

In the fixed capacity system, the production capacity is to be determined at the beginning of the planning period. This production capacity will then remain same throughout the planning period. For example, in producing a specialty synthetic fiber, a significant amount of cost is associated with setting up the line. Once a line is fixed, it will produce the fiber at a fixed rate. If a higher rate is required, a new line with a higher capacity has to be installed. If this specialty fiber is now to export in a foreign market, there involves the uncertainty in the profit due to the exchange rate fluctuations. However, the model presented here is a generic one and is not coupled with any specific example. The objective is to maximize the mean–variance profit function so that the demands from a local and a foreign markets are met.

At time 0, the fixed capacity system has an option of choosing a capacity level among $\{M_1, M_2, \ldots, M_N\}$. This is denoted by M_i, where $i = \{1, 2, \ldots, N\}$. For a given planning period of T, the objective is to maximize the expected utility of the profit as:

$$\mathrm{U}(\pi_t) = \{\mathbb{E}(\pi_t(e_t, M_i)) - \gamma \mathbb{V}(\pi_t(e_t, M_i))\}, \tag{4.2}$$

where γ is the mean–variance (MV) ratio, $\mathbb{E}[\pi_t(e_t, M_i)]$ is the expected maximum profit, and $\mathbb{V}(\pi_t(e_t, M_i))$ is the variance of the profit at time t, given the exchange rate, e_t, and the capacity level, M_i, acquired at time $t = 0$.

The profit, π_t, can be obtained by subtracting the cost of production, $g(x_{11_t} + x_{12_t})$, from the sales revenue from the domestic market, $s_1 q_t^l$, and the sales revenue from the foreign market, $s_2 e_t x_{12_t}$.

$$\pi_t = s_1 x_{11_t} + s_2 e_t x_{12_t} - g(x_{11_t} + x_{12_t}), \tag{4.3}$$

where, s_1 is the local or domestic unit price, x_{11_t} is the production or sales quantity in the domestic market, $s_2 e_t$, is the foreign unit price converted to the local currency, e_t is the exchange rate, and $g(x_{11_t} + x_{12_t})$ is the total cost of the production in the domestic plant as defined in Equation (4.1).

The optimal production quantities to produce at each time period for the local and foreign markets, x_{11_t} and q_{f_t}, are determined by the following optimization problem.

$$\begin{aligned} \max_{M_i} \ \mathrm{U}(\pi_t) & & & \quad (4.4) \\ \text{s.t.} \quad x_{11_t} + x_{12_t} &\leq \min(M_i, D_t) & & \quad (4.5) \\ x_{11_t} &\leq D_{1_t} & & \quad (4.6) \\ x_{12_t} &\leq D_{2_t} & & \quad (4.7) \\ x_{11_t}, x_{12_t} &\geq 0 & & \quad (4.8) \end{aligned}$$

where the constraint (4.5) states that the total production is limited by the capacity of the plant and the total demand $D_t = D_{1_t} + D_{2_t}$. The constraint (4.6) states that the total domestic sales must be less than or equal to the total domestic demand. The constraint (4.7) states that the total foreign sales must be less than or equal to the total foreign demand. The constraint (4.8) states that all sales quantities must be positive. The numerical result for this model is provided in Section 4.2.

4.1.2 Model II: Fixed capacity system along with using currency put options

This section investigates what if the firm having a fixed capacity system, as described in Model I, adopts currency put options in order to mitigate the exchange rate risk. Consequently, Model I is modified with an addition of using currency put options. Put option contracts purchased in each period are exercised in the next period depending on the exchange rate in that period. The profit, π_t, now takes the form as below:

$$\pi_{h_t} = s_1 x_{11_t} + s_2 e_t x_{12_t} - g(x_{11_t} + x_{12_t}) + Y_t\{\max((K_{e_t} - e_t), 0) - P_{t-1}\exp(rh)\}, \quad (4.9)$$

where, Y_t is the number of put options to be exercised at time t and that were bought at time $(t-1)$, K_{e_t} is the strike or exercise price, and P_{t-1} is the option premium paid at the earlier time period, and h is the time interval between time $(t-1)$ and t. If we take the expectation of Equation (4.9), we have the following expression.

$$\begin{aligned} \mathbf{E}(\pi_{h_t}) &= \mathbf{E}(s_1 x_{11_t} + s_2 e_t x_{12_t} - g(x_{11_t} + x_{12_t})) + \mathbf{E}(Y_t\{\max((K_{e_t} - e_t), 0) - P_{t-1}\exp(rh)\}) \\ &= \mathbf{E}(s_1 x_{11_t} + s_2 e_t x_{12_t} - g(x_{11_t} + x_{12_t})) \quad (4.10) \end{aligned}$$

The optimization problem at each period can be described as follows:

$$\begin{aligned}
\max_{M_i} \ \mathrm{U}(\pi_{h_t}) & & & (4.11)\\
\text{s.t. } \ x_{11_t} + x_{12_t} & \leq & \min(M_i, D_t) & \quad (4.12)\\
x_{11_t} & \leq & D_{1_t} & \quad (4.13)\\
x_{12_t} & \leq & D_{2_t} & \quad (4.14)\\
x_{11_t}, x_{12_t} & \geq & 0 & \quad (4.15)
\end{aligned}$$

The optimal number of options

It is assumed that the individual firm is more concerned to minimize its own income variance relative to the variance of market prices (McKinnon 1967). Therefore, the optimal number of put options can be obtained by minimizing the variance of the profit of the firm. The variance of the profit, π_t, as shown in Equation (4.11), is dependent on the following terms of the profit,

$$\begin{aligned}
\mathrm{U}(\pi_{h_t}) &= \mathbb{E}(\pi_{h_t}) - \gamma \mathbb{V}(\pi_{h_t})\\
&= \mathbb{E}(\pi_t) + Y_t \mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)]\\
&\quad -\gamma[\mathbb{V}(\pi_t) + \mathbb{V}[Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)]]\\
&\quad +2\{\mathrm{cov}(\pi_t, Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\}] \qquad (4.16)
\end{aligned}$$

Differentiating Equation (4.16) with respect to Y_t and equating to zero as $\partial \mathrm{U}(\pi_{h_t})/\partial Y_t = 0$, we obtain the optimal number options to buy.

Therefore,

$$\begin{aligned}
\frac{\partial \mathrm{U}(\pi_{h_t})}{\partial Y_t} &= \frac{\partial \mathbb{E}(\pi_t)}{\partial Y_t} + \mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)]\\
&\quad -\gamma\left[\frac{\partial}{\partial Y_t}\mathbb{V}(Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\right.\\
&\quad \left. +2\frac{\partial}{\partial Y_t}\{\mathrm{cov}(\pi_t,\ Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\}\right] \qquad (4.17)
\end{aligned}$$

Since, $\partial\mathbb{E}(\pi_t)/\partial Y_t = 0$, equating Equation (4.17) to zero, we obtain

$$\mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}e^{rh}] = \gamma\Bigg[\frac{\partial}{\partial Y_t}\mathbb{V}(Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)]) + 2\frac{\partial}{\partial Y_t}\{\mathrm{cov}(\pi_t,\ Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\}\Bigg] \tag{4.18}$$

We now take out the term, $\frac{\partial}{\partial Y_t}\mathbb{V}[Y_t\{\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)\}]$, and simplify as follows:

$$\begin{aligned}\frac{\partial}{\partial Y_t}\mathbb{V}[Y_t\{\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)\}] &= \frac{\partial}{\partial Y_t}\{Y_t^2\mathbb{V}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)]\} \\ &= 2\mathbb{V}([\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\end{aligned} \tag{4.19}$$

We obtain,

$$Y_t^* = \frac{\mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)] - 2\frac{\partial}{\partial Y_t}\{\mathrm{cov}(\pi_t,\ Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\}}{2\gamma\mathbb{V}([\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])} \tag{4.20}$$

Similar to the justifications and approaches used in Tekin and Özekici (2015) and Chen and Parlar (2007), a risk-neutral probability measure is also imposed here under a complete arbitrage-free market. Chen and Parlar (2007) has justified this condition under a quadratic utility function. They also prove that the expected utility maximization is equivalent to the profit variance minimization. Therefore, $\mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp rh] = 0$ and we obtain,

$$Y_t^* = \frac{-\frac{\partial}{\partial Y_t}\{\mathrm{cov}(\pi_t,\ Y_t[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])\}}{\gamma\mathbb{V}([\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)])} \tag{4.21}$$

Let these terms to be mentioned as:

$$\begin{aligned}A &= \pi_t = s_1 q_t^l + s_2 e_t x_{12_t} - g(x_{11_t} + x_{12_t}), \\ B &= Y_t\{\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)\}.\end{aligned}$$

The variance of A is expressed as follows:

$$\mathbb{V}(A) = \mathbb{E}[(A - \mathbb{E}(A))^2] = s_2 x_{12_t}\mathbb{E}[(e_t - \mathbb{E}(e_t))^2].$$

The variance of B is expressed as follows:

$$\begin{aligned}\mathbb{V}(B) &= \mathbb{E}[(B-\mathbb{E}(B))^2] &(4.22)\\ &= Y_t^2\mathbb{E}\left[\{\max(K_{e_t}-e_t,0)-P_{t-1}\exp(rh)\}^2\right], &(4.23)\end{aligned}$$

$\mathbb{E}(B)=0$, because of the fair price of the option. Now, the covariance, $\text{cov}(A,B)$, is:

$$\begin{aligned}\text{cov}(A,B) &= \mathbb{E}[\{A-\mathbb{E}(A)\}\{B-\mathbb{E}(B)\}] &(4.24)\\ &= Y_t s_2 x_{12_t}\mathbb{E}\left[\{e_t-\mathbb{E}(e_t)\}\{\max((K_{e_t}-e_t),0)-P_{t-1}\exp(rh)\}\right] &(4.25)\end{aligned}$$

To find the optimal number of put options, the variance of profit is minimized with respect to Y_t and solved for Y_t. These are given by Equations (6.35)–(4.27):

$$\begin{aligned}\frac{\partial\mathbb{V}(A+B)}{\partial Y_t} &= 2s_2x_{12_t}\mathbb{E}\left[\{e_t-\mathbb{E}[e_t]\}\{\max(K_{e_t}-e_t,0)-P_{t-1}\exp(rh)\}\right]\\ &+ 2\,Y_t\mathbb{E}\left[\{\max(K_{e_t}-e_t,0)-P_{t-1}\exp(rh)\}^2\right]. &(4.26)\end{aligned}$$

$$Y_t=\frac{s_2x_{12_t}\mathbb{E}\left[e_t\left\{P_{t-1}\exp(rh)-\max(K_{e_t}-e_t,0)\right\}\right]}{\mathbb{E}\left[\{\max(K_{e_t}-e_t,0)-P_{t-1}\exp(rh)\}^2\right]}. \tag{4.27}$$

Substituting the numerator of Equation (4.27) as $C=s_2x_{12_t}\mathbb{E}[e_t\{P_{t-1}\exp(rh)-\max(K_{e_t}-e_t,0)\}]$ and the denominator as $D=\mathbb{E}[\{\max(K_{e_t}-e_t,0)-P_{t-1}\exp(rh)\}^2]$, we can express, $Y_t=C/D$. The numerator, C, can be simplified as:

$$C=s_2x_{12_t}\mathbb{E}[e_tP_{t-1}\exp(rh)-e_tK_{e_t}\mathbf{1}_{\{e_t<K_{e_t}\}}+e_t^2\mathbf{1}_{\{e_t<K_{e_t}\}}], \tag{4.28}$$

where, $\mathbf{1}_{\{e_t<K_{e_t}\}}$ is an indicator function such that $\mathbf{1}_{\{e_t<K\}}=1$, when $e_t<K_{e_t}$ and $\mathbf{1}_{\{e_t\geq K_{e_t}\}}=0$, otherwise. Therefore,

$$C=s_2x_{12_t}\left[\mathbb{E}[e_t]P_{t-1}\exp(rh)-K_{e_t}\int_0^K e_tf(e_t)\,de_t+\int_0^K e_t^2f(e_t)\,de_t\right] \tag{4.29}$$

The denominator, D, from Equation (4.27) can be expressed as:

$$\begin{aligned}D &= \mathbb{E}[\max(0,K_{e_t}-e_t)^2+P_{t-1}^2\exp(2rh)-2P_{t-1}\exp(rh)\max(0,K_{e_t}-e_t)] &(4.30)\\ &= \mathbb{E}[K_{e_t}^2]+\int_0^K e_t^2f(e_t)\,de_t-2K_{e_t}\int_0^K e_tf(e_t)\,de_t-P_{t-1}\exp(rh) &(4.31)\\ &= K^2(\Pr(e_t<K_{e_t}))+\int_0^K e_t^2f(e_t)\,de_t-2K_{e_t}\int^K {}_{t0}e_tf(e_t)\,de_t-P_{t-1}^2\exp(2rh)\end{aligned}$$

Therefore, the optimal number of put options, Y_t, is given by Equation (4.32).

$$Y_t = \frac{s_2 e_t \{\mathbb{E}[e_t] P_{t-1} \exp(rh) - K \int_0^K e_t f(e_t)\, de_t + \int_0^K e_t^2 f(e_t)\, de_t\}}{K^2(\Pr(e_t < K)) + \int_0^K e_t^2 f(e_t)\, de_t - 2K \int_0^K e_t f(e_t)\, de_t - P_{t-1}^2 \exp(2rh)}. \tag{4.32}$$

In Equation (4.32), $\mathbb{E}[e_t] = \exp(\mu_t + \sigma^2/2)$ and

$$\mu_t = \ln(e_t) + \frac{(r - r_f)T}{n} - \frac{\sigma^2 T}{n}, \tag{4.33}$$

$$\int_0^K e_t f(e_t) de_t = \exp\left(\mu_t + \sigma^2/2\right) \Phi\left(\frac{\ln(K_{e_t}) - \mu_t - \sigma^2/2}{\sigma}\right), \tag{4.34}$$

$$\int_0^K e_t^2 f(e_t)\, de_t = \exp\left(2\mu_t + 2\sigma^2\right) \Phi\left(\frac{\ln(K_{e_t}) - \mu_t - 2\sigma^2}{\sigma}\right). \tag{4.35}$$

Plugging theses values in Equation (4.32), the optimal number of options to long at each period can be determined.

4.1.3 Model III: Flexible capacity system

In each period, the flexible capacity system has an option to choose a capacity level among $\{M_1, M_2, \ldots, M_N\}$. This is denoted by M_i, where $i = \{1, 2, \ldots, N\}$. Hence, from one period to another period, capacity can be expanded, contracted, or unchanged. In a flexible system, the capacity level can be adjusted at each period of time. That is, the capacity at any time t, M_t, is such that $M_t \in M_i$. The firm's expected profit can now be calculated by Equation (4.36).

$$\text{Expected profit} = -c_0 M_0 + \sum_{t=1}^{T} \exp(-rt) \left[E(\pi_t^*(e_t, M_{t-1})) + S(M_{t-1}, M_t)\right], \tag{4.36}$$

where, $M_0 (\in M_i)$ is the initial capacity level acquired at time 0, M_{t-1} and M_t are the capacity levels in time $t-1$ and t, respectively, $S(M_{t-1}, M_t)$ is the cost of switching the capacity from M_{t-1} to M_t. The cost of switching, $S(M_{t-1}, M_t)$, can be either the cost of capacity expansion or the cost of capacity contraction. The expansion flexibility is the ability to increase the capacity of the system and the contraction flexibility is the ability to decrease the capacity of the system (e.g., Bollen 1999; Wahab et al. 2008). In this model, the optimal capacity level for each period is calculated. The risk of exchange rate fluctuation is mitigated by expansion or contraction of the capacity. Therefore, the cost of expansion and contraction has to be considered.

Expansion capacity: The capacity increases between two consecutive sales period and that results in an increased cost. The cost to expand the capacity level from M_{t-1} to M_t (where $M_{t-1} < M_t$)

is given by the following Equation (4.37).

$$S(M_{t-1}, M_t) = c_3 c_0 (M_{t-1} - M_t) - c_5. \tag{4.37}$$

c_3 represents a percentage of the initial installment cost of unit capacity, c_5 is the fixed cost and c_0 is the unit capacity cost at time 0. In Equation (4.37), both $c_3 > 0$ and $c_5 > 0$.

Contraction capacity: When the capacity is reduced from level M_{t-1} to M_t (where $M_{t-1} > M_t$), it results in cost reduction for the firm. This is given as follows:

$$S(M_{t-1}, M_t) = c_4 c_0 (M_t - M_{t-1}) - c_5. \tag{4.38}$$

where c_4 is the percentage of unit cost, which can be negative or positive. In this model, it is assumed to be positive (e.g., Bollen 1999).

The optimization model is the same as the fixed capacity model except the capacity level, M_t, is chosen at each time period. The objective function is given by Equation (4.39).

$$\begin{aligned} & \max_{x_{11_t}, q_{f_t}} \mathrm{U}(\pi_t) && (4.39) \\ s.t. \quad x_{11_t} + x_{12_t} & \leq \min\ (M_t, D_t) && (4.40) \\ x_{11_t} & \leq D_{1_t} && (4.41) \\ x_{12_t} & \leq D_{2_t} && (4.42) \\ x_{11_t}, x_{12_t} & \geq 0 && (4.43) \end{aligned}$$

4.1.4 Model IV: Flexible capacity system with put options

This model uses the currency put options along with the flexible capacity system described in Section 4.1.3. That is, both real options and put options are used to hedge the exchange rate risk. The number of put options to long is calculated by using Equation (6.38). The optimization model is given as follows:

$$\begin{aligned} & \max_{x_{11_t}, x_{12_t}, Y_t} \mathrm{U}(\pi_t) && (4.44) \\ s.t.\ x_{11_t} + x_{12_t} & \leq \min\ (M_t, D_t) && (4.45) \\ x_{11_t} & \leq D_{1_t} && (4.46) \\ x_{12_t} & \leq D_{2_t} && (4.47) \\ x_{11_t}, x_{12_t} & \geq 0 && (4.48) \end{aligned}$$

4.2 Numerical studies

This section includes the numerical analysis of each model defined in Section 4.1.1 through Section 4.1.4. The first part of these numerical studies illustrates a simple example of a flexible system with put options model considering three periods of sales. In Section 4.2.2 to Section 4.2.2, numerical results show how the expected profit varies for each model as demand, the volatility of exchange rate, unit capacity, and overhead cost are varied for both linearly changing and constant demands.

4.2.1 An illustrative example

This section illustrates the model via a numerical example for a planning period of 3 months. The cost coefficients are: $a_1 = -1$, $a_2 = 1.0$, and $a_3 = 1.5$. The volatility of the exchange rates is $\sigma = 0.11$. The coefficients of capacity expansion and contraction are: $s_1 = 1$, $s_2 = 0.9$, $s_3 = 150$, and $c_0 = 15$. The sale prices at the foreign and local markets are $s_2 = 30$ and $s_1 = 45$, respectively. Demands in the foreign markets for months 1, 2, and 3 are 40, 55, and 70, respectively and the local demands for months 1, 2, and 3 are 40, 50, and 60, respectively. The risk free rates in the local and foreign markets are respectively, $r_l = 0.03$ and $r_f = 0.06$. The initial exchange rate is assumed to be 2.0 with an exercise price of $K = 2.10$. The MV ratio, γ is assumed to be 0.002. Let the capacity levels be: $M_i = \{M_1 = 70, M_2 = 100, M_3 = 130\}$.

Figure 4.1 shows the movement of the exchange rate in a binomial lattice. At node 'A', the initial exchange rate is assumed to be 2.00. In the next period, the exchange rate may go up to 2.0645 at node 'B' or go down to 1.9375 at node 'C'. The value below the the exchange rate is the number of put options to buy at each period. The number of options is determined as if one contract represents one unit of foreign currency. However, it can be easily converted for a specific foreign currency contact, for example, one contract represents 31,250 British Pounds (e.g., Hull 2009). Figure 4.2 shows the values of the profit calculated by using the non-linear optimization model presented in Equations (5.4)–(41). The first value at each node corresponds to the profit at the capacity level of 70, the second value refers to the profit at the capacity level of 100, and the third one corresponds to the capacity level of 130. The path dependent profits are shown in two separate columns. For example, at node 'H' the column at the left shows profits following the path 'A–B–D–H' and the right column shows the profits values corresponding to the path 'A–B–E–H', because at nodes 'D' and 'E', the number of put options bought is different. In order to calculate the expected profit at node 'D', for example, at the capacity level of 130 and without switching the capacity, we get: $12651 = 5634 + e^{(-0.03\times 1/12)}[0.45275(7190) + (1 - 0.45275)6906]$, by taking into account path dependent profit at node 'H', and profit values corresponding to capacity level of 130 at nodes 'G' and 'H'. Similarly, for the capacity pair of $(130, 100)$ (i.e., capacity level of 130 at node 'D' and capacity level of 100 at both nodes 'G' and 'H'), the expected profit at node 'D' is: $11589 = 5634 + e^{(-0.03\times 1/12)}[0.45275(5870) + (1 - 0.45275)5586] + \mathrm{S}(130, 100)$. For the capacity pair of (130,70), the expected profit at node 'D' is: $10273 = 5634 + e^{(-0.03\times 1/12)}[0.45275(4550) + (1 - 0.45275)4266] + \mathrm{S}(130, 70)$. Therefore, the expected profit at node 'D' for a given capacity level of 130 is $12651 = \max(10273, 11589, 12651)$. Following the similar steps, we can obtain expected profit values for the capacity levels of 70, and 100 at node 'D'. These steps are repeated at each node and the expected profit values at each node are given in Figure 4.3. The maximum expected profit at the initial node 'A' is $14213 = \max(15113 - 15 \times 70, 15602 - 15 \times 100, 16163 - 15 \times 130)$.

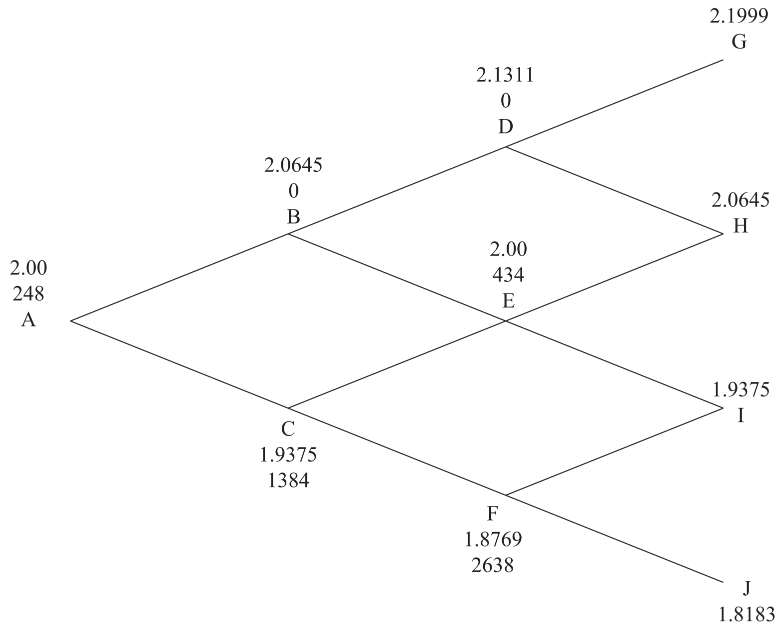

Figure 4.1: A binomial lattice showing the movement of exchange rate (above) and the number of options (below) at each node.

4.2.2 Numerical results

The following sections demonstrate numerical results for a planning period of 24 months. The models are implemented for three different cases: (a) an increasing demand, (b) a constant demand, and (c) a decreasing demand trend over the whole planning period. The reason is that the life cycle of a typical product usually has a growth regime (an increasing demand trend), maturity regime (a constant demand trend), and decay regime (a decreasing demand trend), and Guide et al. (2006) mention that an analysis can be focused on a particular regime of the product life cycle in managing and designing a supply chain.

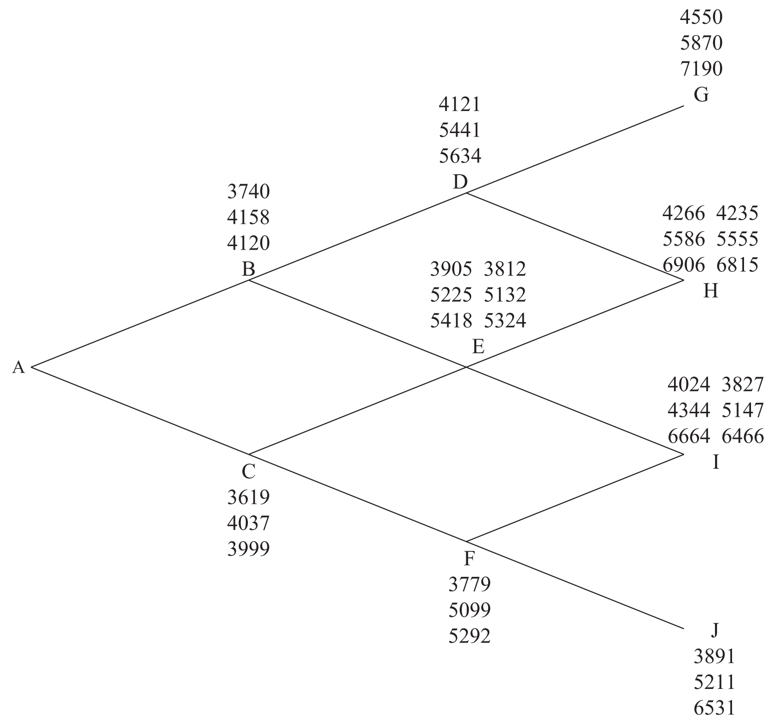

Figure 4.2: A binomial lattice showing profits of the firm at each node.

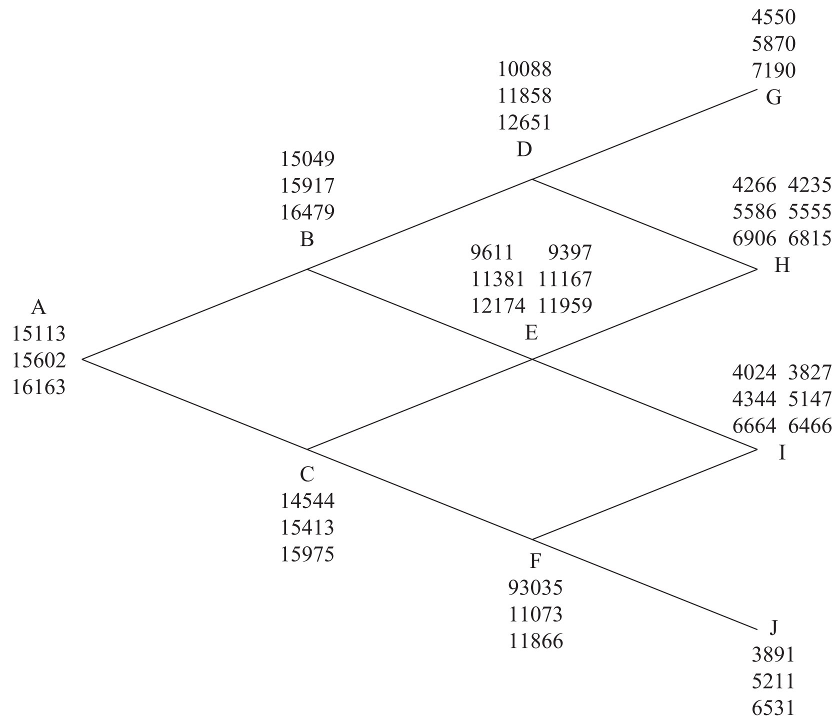

Figure 4.3: Expected profit of the firm corresponding to Figure 4.2.

Linearly changing demand

In this section, it is assumed that the demand increases linearly both in the domestic country and in the foreign country in each sale period as $D_{1_t} = D_0^l + 10t$ and $D_{2_t} = D_0^f + 15t$, where the initial local and foreign demands are respectively, $D_0^l = 40$ and $D_0^f = 40$, and $0 \leq t \leq 24$. Figure 4.4 shows the effect of the exchange rate volatility on the expected profit for a fixed system (Model I), a flexible system (Model II), a fixed system with put options (Model III), and a flexible system with put options (Model IV).

It is observed from Figure 4.4 that when the volatility of exchange rate increases, the expected value of the firm does not vary significantly. The fixed system along with using the currency put options yields a little higher expected value than that of the fixed system without using put options. The capacity expansion and contraction options are also important for the firm. The expected value of the flexible system is little higher than that of the fixed system. The difference in the expected values between fixed system and fixed system with currency put options is not very significant.

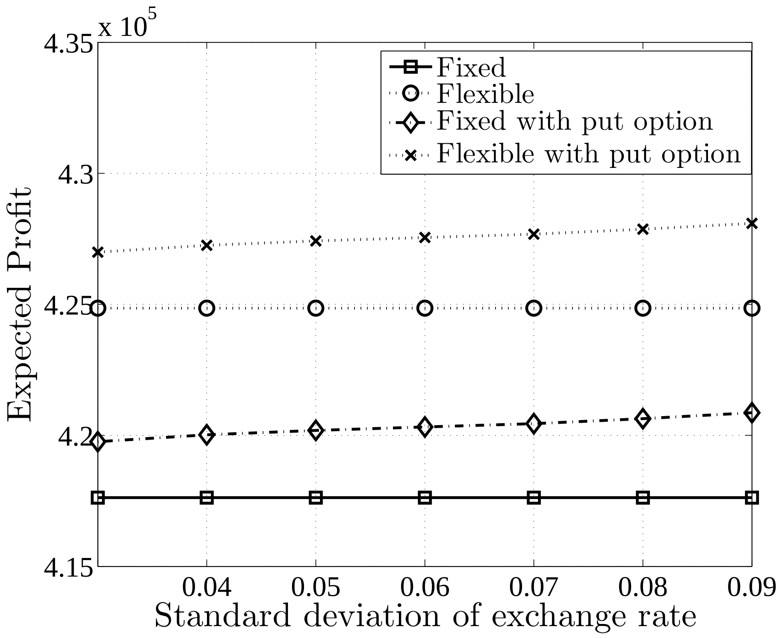

Figure 4.4: The variation of the expected profit under the currency exchange rate volatilities

The profit variances for each model with respect to the changes in the exchanger rate volatility is presented in Figure 4.5. It is observed that the variance of the profit increases as the the exchange rate volatility increases. It is also observed from Figure 4.5 that the use of currency options causes to reduce the variance of the profit.

Figure 4.6 shows the values of the expected utility with respect to the the changes in exchange rate volatilities. This figure reflects the effect of the combination of the expected profit in Figure 4.4 and the variance of the profit in Figure 4.5. As the variance of the profit decreases with an increase in the currency rate volatilities, overall the expected utility decreases as the volatility of the exchange rate increases as observed from Figure 4.6.

Figure 4.7 shows the variation the expected profit with respect to the unit capacity cost. When

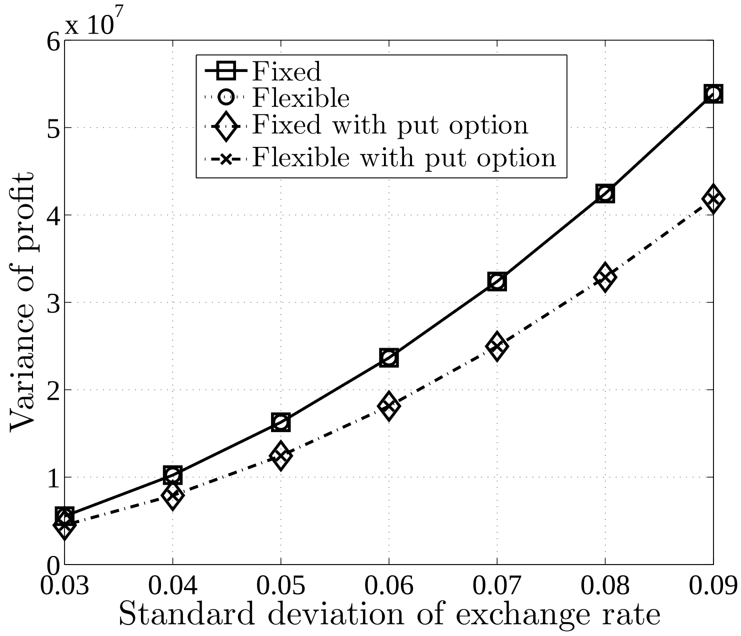

Figure 4.5: The variance of the profit under the currency exchange rate volatilities

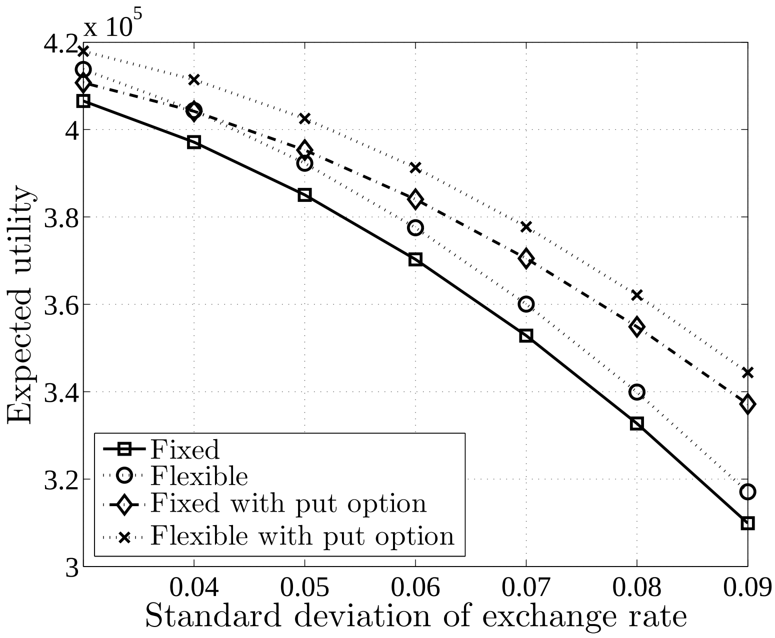

Figure 4.6: The variation of the expected utility against the changes in exchange rate volatilities

the unit capacity cost increases, the expected value for the fixed system and fixed system with put options decreases. Similarly, the expected value for the flexible system and flexible system with currency put options also decreases. Figure 4.8 shows the effect of the overhead cost on the expected profit for each model. In this case, when the overhead cost increases, the expected profit of each model decreases. However, flexibility of the system compensates for increasing overhead cost and hence the downward move of the expected profit of the flexible system decreases. The highest profit

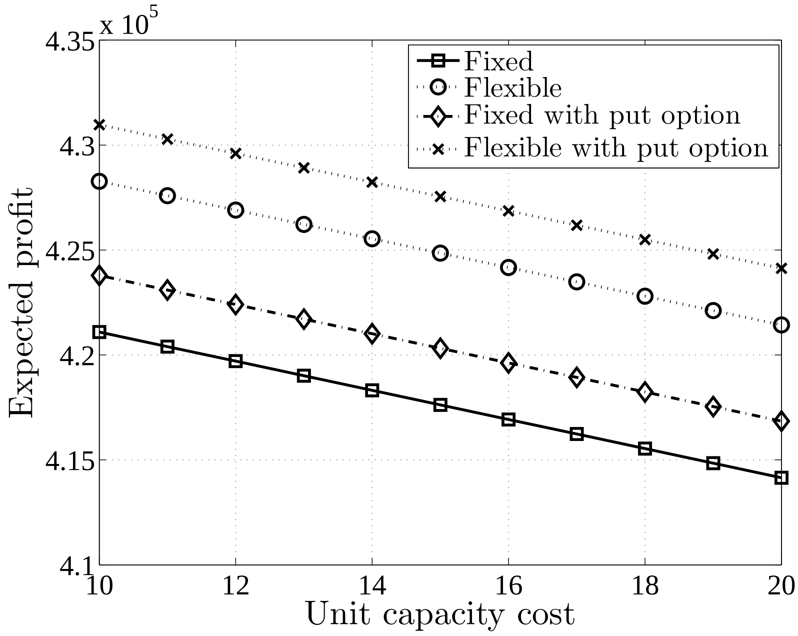

Figure 4.7: The variation of the expected value against the unit capacity cost

is obtained by integrating real options with currency put options. Figure 4.9 shows the variation of

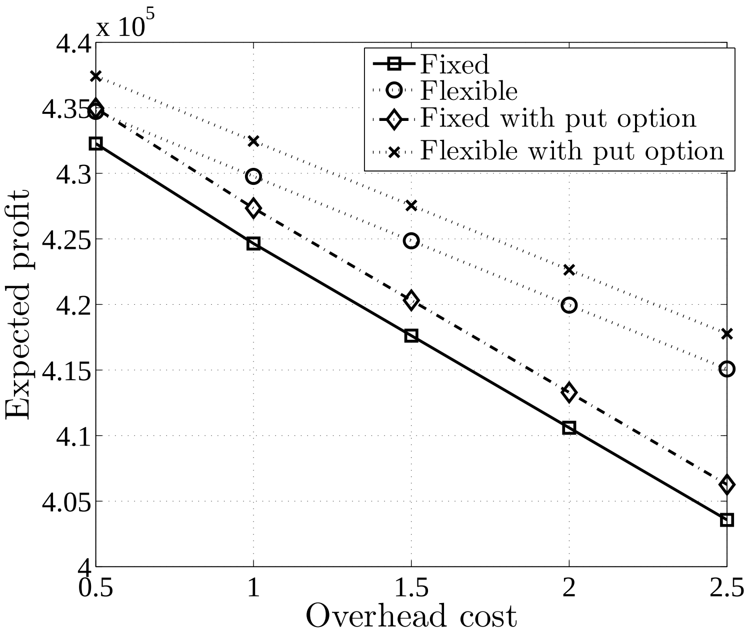

Figure 4.8: The variation of the expected value against the overhead cost

the expected utility against different mean-variance ratios. It is observed that the expected utility moves downward as the mean-variance ratio, γ, increases. The reason is that a higher value of γ puts more weightage on the variance and as a result the expected utility decreases.

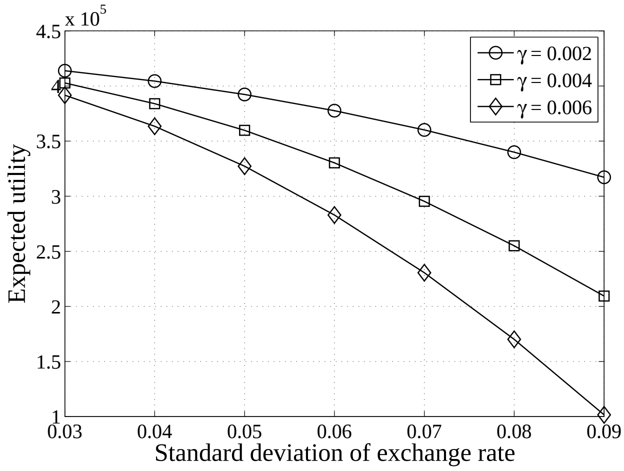

Figure 4.9: The variation of the expected utility against various mean-variance ratios

4.2.3 Constant demand

In this section, the demand is considered to be constant in each sale period as $D_0^l = 400$ and $D_0^f = 300$ in the domestic country and foreign country, respectively. In this case, with respect to the volatility of exchange rate, Figure 4.10 shows the expected profit of fixed capacity system, flexible system, fixed system with currency put options, and flexible system with currency put options.

It is observed from Figure 4.10 that due to the constant demand there is no room for the contraction or expansion capacity flexibility. Consequently, both fixed and flexible systems have the same expected values. However, exercising of the currency put options makes a difference in the expected values. Hence, the upper lines in Figure 4.10 refer to the fixed and flexible systems with currency put options, while the lower lines denote the fixed and flexible systems without using currency put options. Figures 4.11 shows the variances of the profit with respect to the changes in currency volatilities. It is observed that the variance of the profit increases as the currency volatility increases. It is also observed that the use of currency options reduces the variance of the profit.

The variation of the expected utility with respect to the currency volatilities is shown in Figure 4.12. As the variance of the profit increases with an increase in the currency volatilities, as observed from Figure 4.11, the expected utility decrease with respect to an increase in currency volatilities.

Figure 4.13 and Figure 4.14 show that the expected profit decreases as the overhead cost and the unit capacity cost increase.

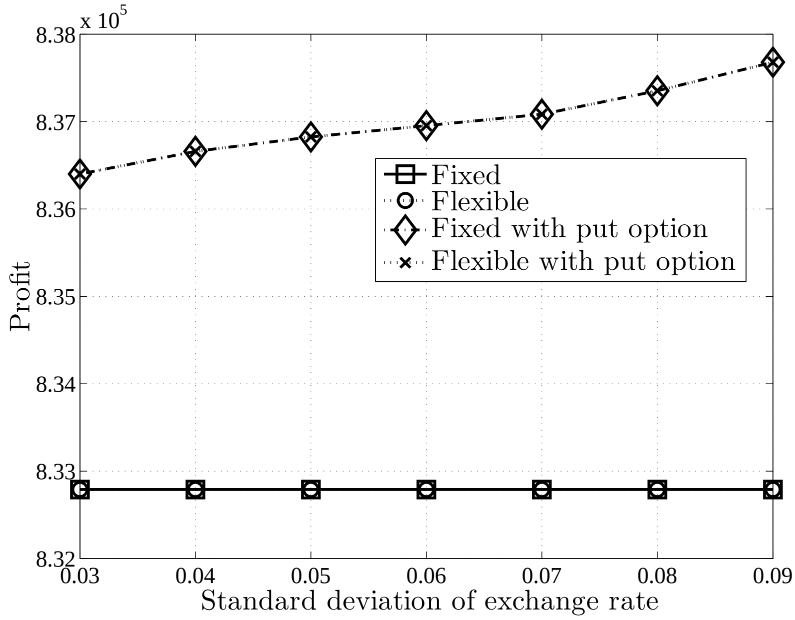

Figure 4.10: The variation of the expected profit under the currency exchange rate volatilities

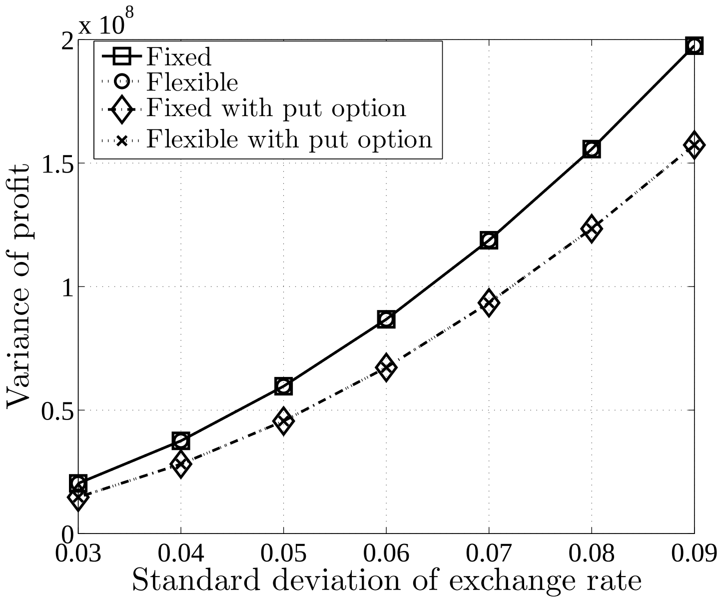

Figure 4.11: The variance of the profit under the currency exchange rate volatilities

4.3 Summary

In this chapter, the effect of exchange rate on the expected profit, variance and the expected utility of the firm having flexible capacity levels is examined in a multi-period setting. The firm has markets in both domestic and foreign countries. The exchange rate is considered to follow a geometric Brownian motion. Four models are introduced. The first model is the fixed capacity system that neither offers any flexibility in capacity nor uses any currency put options, and this is

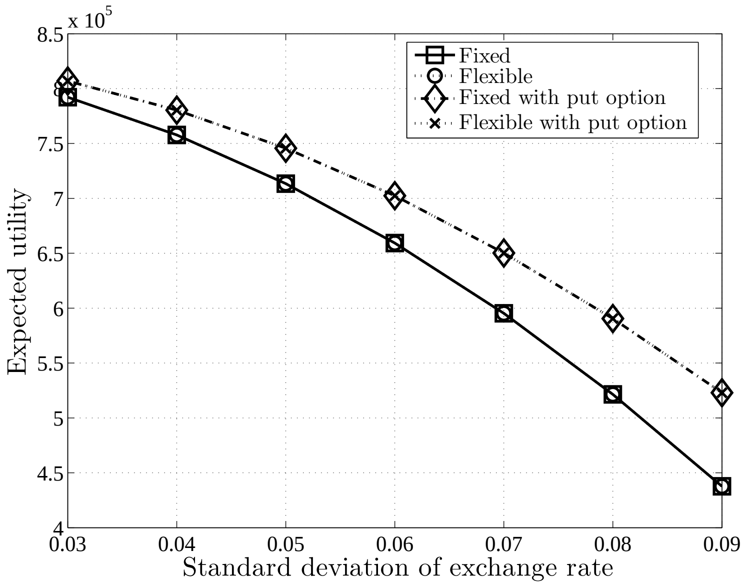

Figure 4.12: The expected utility under the currency exchange rate volatilities

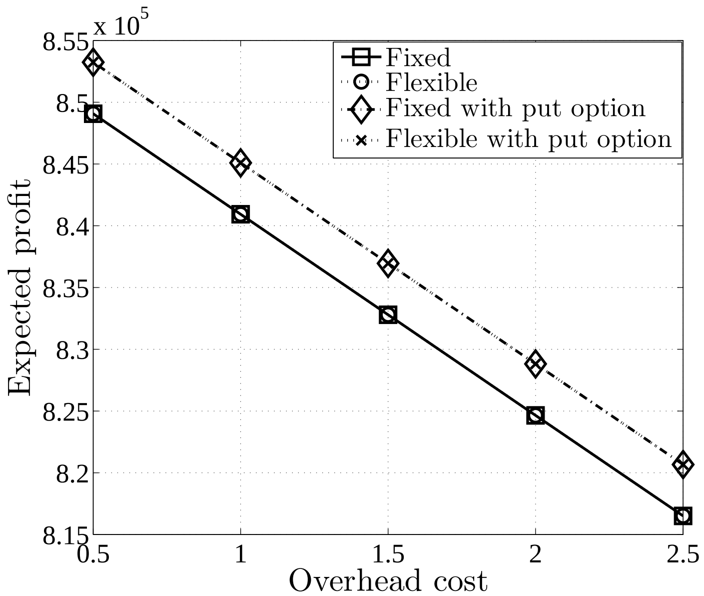

Figure 4.13: The variation of the expected profit against the overhead cost

referred as a base model. The second model is developed with a fixed capacity system along with currency put options. As a result of the fixed capacity, this system does not have any real options, but currency options are used to hedge the exchange rate risk. The third model is the flexible system, which has the flexibility in the capacity in each sale period. In this model, real options in the form of capacity flexibilities are used to hedge the exchange rate risk. The fourth model is the flexible system with currency put options. This model represents the integration of both real and finance options. Results demonstrate that the firm has the highest expected profit when real options in the form of capacity flexibilities along with currency put options are utilized.

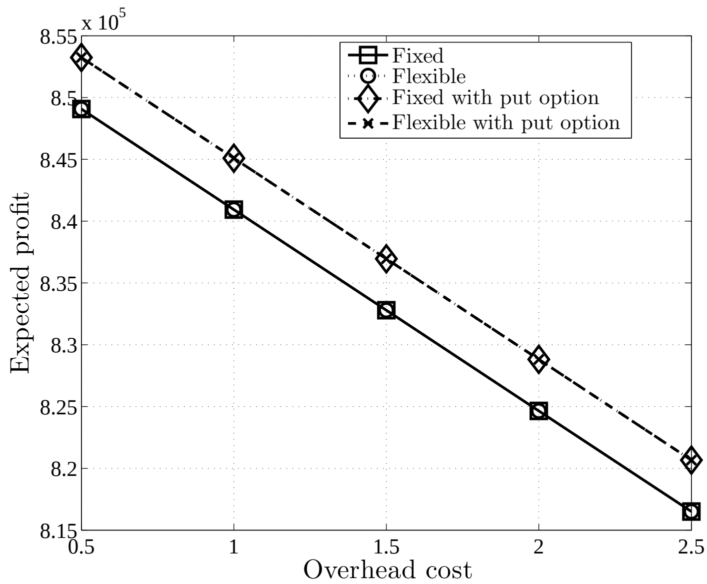

Figure 4.14: The variation of the expected profit against the unit capacity cost

It is also observed that when the exchange rate has a higher volatility, the firm achieves a higher expected profit; and when the unit capacity cost increases or the overhead cost increases, the expected profit of the firm decreases. Either when demand increases or when demand decreases, real options can offset the adverse effect of both exchange rate variations and cost variations. When demand remains constant, real options can not offset such adverse effect but currency put options can offset. In a nutshell, real options are useful when there is a demand variation, but financial options can be useful even when there is no demand variation. Integration of both real and financial options benefits the most.

Chapter 5

Multi-period optimal production decision with product life cycles using carbon call options

This chapter considers that the product has a stochastic life cycle and the firm operates under carbon dioxide emission regulations. The aim is to maximize the value of the firm through the selection of an optimum production level along with the purchase of an optimal number of carbon call options at each interval of time. This is consequently a multiperiod production decision problem. It is assumed that the product passes through a life cycle of growth and decay phases or regimes. The stochastic demand, therefore, is either in growth or decay regime and is represented by, $\theta_t \in (\theta_t^g, \theta_t^d)$. The firm produces $\min(\theta_t, M_t)$ number of units, where M_t is the optimal capacity level at time t. The firm emits carbon dioxide by an amount of $\beta \min(\theta_t, M_t)$, where β is the amount of carbon dioxide per unit of production. If δ_t is the price of carbon at time t, the cost of carbon is $\delta_t \beta \min(\theta_t, M_t)$. If s is the sales price and c is cost of production per unit, the firm earns a revenue of $(s-c)\min(\theta_t, M_t)$. Therefore, the profit, π_t, can be expressed as $\pi_t = (s-c)\min(\theta_t, M_t) - \delta_t \beta \min(\theta_t, M_t)$. Since, at each period, both the demand, θ_t, and the carbon prices, δ_t are uncertain and stochastic, the profit π_t is also uncertain and stochastic. An increasing carbon price causes to drop the profit, π_t. The downward profit can be counterbalanced by exercising X_t number of carbon call options with a strike price, K_{c_t}. Therefore, the profit, π_t along with carbon options can be expressed as $\pi_t = (s-c)\min(\theta_t, M_t) - \delta_t \beta \min(\theta_t, M_t) + X_t(\max(\delta_t - K_{c_t}, 0) - P_{t-1})$, where P_{t-1} is the price of the call option purchased at time $(t-1)$. When δ_t goes high, the emission cost also goes high. However, if the firm opts to long carbon call options, a payoff from exercising the options when the carbon prices are higher than the strike price, K_{c_t}, is incurred and the firm can offset the downward profit caused by higher carbon prices. The correlated movement of θ_t and δ_t is represented by a lattice, which is a discretized representation of continuous processes in the form of nodes and branches. At each decision point of the lattice, the profit, π_t, is maximized by obtaining the optimum capacity level, M_t, and the optimum number of carbon options, X_t.

In order to analyze the effect of capacity flexibility, a fixed capacity system is introduced as a reference system, in which the firm chooses a fixed capacity level, M_0, from a set of capacity levels, $\mathbf{M} = \{M_1, M_2, \ldots, M_i\}$, at the beginning of planning period, $t = 0$, and maintains this capacity throughout the planning period. On the other hand, a flexible capacity system is also analyzed, in which the firm is able to switch its capacity, M_t, such that $M_t \in \mathbf{M}$, from one time period to the next time period. In switching the capacity from one period to another period, the firm experiences a switching cost of either expansion or contraction. Both fixed and flexible capacity systems are studied and compared with and without utilizing carbon options. The unit capacity cost is c_0 and the unit production cost is given by c_1. The unit sale price is denoted by s, the risk free interest rate is r, and T is the planning horizon.

5.1 The model

In order to examine the effect of various combination of carbon options and capacity flexibility, four alternative valuation models are considered. The first model considers a fixed capacity system without using any carbon options. The second model investigates a fixed capacity system with carbon options. The third model analyzes a flexible capacity systems without using carbon options. The fourth model examines a flexible capacity system along with using carbon options.

As stated earlier, the firm is considered to possess a capacity vector, $\mathbf{M} = \{M_1, M_2, \ldots, M_i\}$. In a fixed capacity system, an optimal capacity level, $M_0 \in \mathbf{M}$, is chosen in the beginning of the planning period at time $t = 0$. The capacity, M_0, remains the same at each time interval, $t \in \{0, 1, 2, \ldots, T\}$. The objective is to select an optimal capacity, at $t = 0$, that will provide the maximum net present value. The fixed capacity system could be of two types: one is without using carbon options and the other is with carbon options.

5.1.1 A fixed capacity system without using carbon options

The profit, $\pi_t(\theta_t, M_0)$, can be expressed as:

$$\pi_t(\theta_t, M_0, \delta_t) = (s - c_1)[\min(\theta_t, M_0)] - \delta_t \beta[\min(\theta_t, M_0)] \tag{5.1}$$

where, $(s - c_1)$ is the profit per unit of sales, and $\min(\theta_t, M_i)$ reflects that if the demand, θ_t, is less than the capacity, M_0, the firm produces at the level of θ_t and if $\theta_t \geq M_0$, then the firm produces up to the capacity level of M_0.

5.1.2 A fixed capacity system with carbon options

If the firm chooses to use carbon call options, the firm could take a long position in call options and exercise a number of call options, X_t, purchased in the previous period, if δ_t exceeds the strike price, K_{c_t}. The price to buy an option at time $(t-1)$ is P_{t-1}. The profit, $\pi_{c_t}(\theta_t, M_0, \delta_t)$, then can be expressed as:

$$\begin{aligned} \pi_t(\theta_t, M_0, \delta_t) &= (s - c_1)[\min(\theta_t, M_0)] - \delta_t \beta[\min(\theta_t, M_0)] \\ &+ X_t\{\max[(\delta_t - K_{c_t}), 0] - P_{t-1}\exp(rh)\}, \end{aligned} \tag{5.2}$$

where h is the time interval between two consecutive periods.

5.1.3 A flexible capacity system without using carbon options

For a flexible capacity system, there exists an option to choose a capacity level, M_t, from the capacity vector, $\mathbf{M}$, at each time interval, t. This system could be of two types: a flexible system with carbon options and a flexible system without using carbon options. The profit, π_t, for a flexible capacity system can be expressed as follows:

$$\pi_t(\theta_t, M_t, \delta_t) = (s - c_1)[\min(\theta_t, M_t)] - \delta_t \beta[\min(\theta_t, M_t)] - c_0 M_t, \tag{5.3}$$

where $(s - c_1)[\min(\theta_t, M_t)]$ is the revenue from sales, $\delta_t \beta[\min(\theta_t, M_t)]$ is the cost of carbon emission, and $c_0 M_t$ is the cost installing the capacity level, M_t, at each time, t.

5.1.4 A flexible capacity system with carbon options

For a flexible capacity system with carbon call options, the profit, π_t, can be expressed as:

$$\begin{aligned} \pi_t(\theta_t, M_t, \delta_t) &= (s - c_1)[\min(\theta_t, M_t)] - \delta_t \beta[\min(\theta_t, M_t)] - c_0 M_t \\ &+ X_t\{\max[(\delta_t - K_{c_t}), 0] - P_{t-1}\exp(rh)\} \end{aligned} \tag{5.4}$$

where, X_t is the optimal number of carbon options to be exercised at time t, P_{t-1} is the option price, K_{c_t} is the strike or exercise price for carbon options. The optimal number of carbon options, X_t, is obtained by minimizing the variance of the profit, the details of which is discussed in Section

5.2.3.

Expansion capacity: The cost to expand the capacity level from M_t to M_{t+1} (if $M_{t+1} > M_t$) is given by Equation (5.5).

$$S(M_t, M_{t+1}) = c_3 c_0 (M_{t+1} - M_t) + c_5, \tag{5.5}$$

where, c_3 represents a percentage of the initial installment cost of unit capacity, c_5 is the fixed cost and c_0 is the unit capacity cost. In Equation (5.5), $c_3, c_5 > 0$.

Contraction capacity: When the capacity is reduced from level M_t to M_{t+1} (if $M_{t+1} < M_t$) results in cost reduction for the firm. This is given as follows

$$S(M_t, M_{t+1}) = c_4 c_0 (M_t - M_{t+1}) + c_5, \tag{5.6}$$

where, c_4 is the percentage of unit cost where it can be negative or positive. Here, it is assumed to be positive.

5.2 Modeling carbon prices and demand dynamics

The carbon futures price δ_t is assumed to follow a geometric Brownian motion (Seifert et al., 2009) as expressed by Equation (5.7).

$$d\delta_t = \mu_c \delta_t dt + \sigma_c \delta_t dz_c, \tag{5.7}$$

where δ_t is the carbon futures prices at time t, μ_c and σ_c are the drift and volatility for the process of carbon spot price, respectively, and dz_c is the increment of the Wiener process with a mean 0 and standard deviation $\sqrt{dt}$.

A product life cycle includes the phases of growth, maturity, and decaying. Introduction and growth phases or regimes are characterized by an increasing demand, the maturity phase represents a stable demand and the decaying phase refers to a declining demand. Bass (1969), however, proposes a two-phase product life cycle with exponential growth and decay. Bollen (1999) also presents a two-regime product life cycle. In this study, the growth regime is assumed to follow a geometric Brownian motion with the drift, μ_g and the volatility, σ_g, and the decaying regime is assumed to follow another geometric Brownian motion with the drift of μ_d and the volatility of σ_d. The drift of growth regime is assumed to be positive and the drift of the decaying regime is assumed to be negative. The growth regime, θ_g, is defined by the following geometric Brownian motion:

$$d\theta_t^g = \mu_g \theta_t^g dt + \sigma_g \theta_t^g dz_g, \tag{5.8}$$

where μ_g is the drift of the growing demand, σ_g is the volatility of the process, and dz_d is the increment of the Wiener process. The decaying regime, θ_t^d, is defined by the following geometric Brownian motion:

$$d\theta_t^d = \mu_d \theta_t^d dt + \sigma_d \theta_t^d dz_d, \tag{5.9}$$

where μ_d is the drift of decaying demand, σ_d is the volatility of the process, and dz_d is the increment of the Wiener process. The instantaneous correlation between the Wiener increments dz_c and dz_g is ρ_{cg} and between dz_c and dz_d is ρ_{cd}.

5.2.1 Lattice representation of correlated movements of demand and corresponding carbon prices

In order to determine the firm value, a numerical solution approach is required that generates the correlated movements of carbon allowance prices and regime-switching demand processes defined by Equations (5.7)–(5.9). A lattice approach is a discretization of a continuous process and offers advantages over a Monte Carlo simulation approach in terms of simplicity, computational time and flexibility. In valuing the price an option, Cox et al. (1979) develop a lattice approach that discretizes the stock price movements following a geometric Brownian motion. Boyle (1988) presents a lattice model representing multiple underlying assets following a geometric Brownian motion. For a two-state regime-switching underlying variable, Bollen (1999) develops a pentanomial lattice approach for valuing European and American options. Wahab et al. (2010) value swing options on the electricity prices using a three-state regime-switching model. In this thesis, a lattice that represents the correlated two-state regime-switching processes and a geometric Brownian motion is utilized in valuing a firm having a flexible capacity system.

In order to build a correlated lattice, a four step procedure is employed: (i) transforming two correlated processes into uncorrelated ones, (ii) making two groups of uncorrelated processes by selecting one uncorrelated process from each pair, (iii) building a pentanomial lattice for each group, and (iv) forming a combined lattice by taking Cartesian product of both pentanomial lattices. Following sections provide the details of each of these steps.

Let ψ_1 and ψ_2 are the uncorrelated processes, (θ_t^g, δ_t), between the demand growth regime θ_t^g and the carbon prices δ_t (e.g., Hull and White 1990). Therefore,

$$\psi_1 = \sigma_c \ln\theta_t^g + \sigma_g \ln\delta_t \tag{5.10}$$

$$\psi_2 = \sigma_c \ln\theta_t^g - \sigma_g \ln\delta_t \tag{5.11}$$

Using the Ito's lemma, $d(\ln\delta_t)$ and $d(\ln\delta_t)$ can be expressed as:

$$d(\ln\delta_t) = [\mu_c - \frac{\sigma_c^2}{2}]dt + \sigma_c dz_c \tag{5.12}$$

$$d(\ln\theta_t^g) = [\mu_g - \frac{\sigma_g^2}{2}]dt + \sigma_g dz_g \tag{5.13}$$

Now the uncorrelated processes in Equations (5.10)–(5.11) can be expressed as: (Hull and White, 1990):

$$d\psi_1 = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_g + (\mu_g - \frac{\sigma_g^2}{2})\sigma_c]dt + \sigma_g\sigma_c\sqrt{2(1+\rho_{cg})}dz_{\psi_1} \tag{5.14}$$

$$d\psi_2 = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_g - (\mu_g - \frac{\sigma_g^2}{2})\sigma_c]dt + \sigma_g\sigma_c\sqrt{2(1-\rho_{cg})}dz_{\psi_2} \tag{5.15}$$

where dz_{ψ_1} and dz_{ψ_2} are the uncorrelated Wiener processes. Similarly considering the regime, (θ_t^d, δ_t), the uncorrelated processes can expressed as follows:

$$d\psi_3 = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_d + (\mu_d - \frac{\sigma_d^2}{2})\sigma_c]dt + \sigma_d\sigma_c\sqrt{2(1+\rho_{cd})}dz_{\psi_3}, \tag{5.16}$$

$$d\psi_4 = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_d - (\mu_d - \frac{\sigma_d^2}{2})\sigma_c]dt + \sigma_d\sigma_c\sqrt{2(1-\rho_{cd})}dz_{\psi_4}, \tag{5.17}$$

where, dz_{ψ_3} and dz_{ψ_4} are the uncorrelated Wiener processes. From these four uncorrelated processes expressed by Equations (5.14)–(5.17), two possible combination of growth and decay regimes along with carbon prices processes are formed and they are represented by two separate pentanomial lattices respectively. A pentanominal lattice can efficiently describes a two state regime switching behavior (Bollen 1999). Accordingly, a pentanomial lattice is formed from the combination of processes, $d\psi_1$ and $d\psi_3$ represented by $(\theta_t^g, \delta_t)_1$ and $(\theta_t^d, \delta_t)_1$, respectively. The other pentanomial lattice is formed from the combination of the processes, $d\psi_2$ and $d\psi_4$ represented by respectively by $(\theta_t^g, \delta_t)_2$ and $(\theta_t^d, \delta_t)_2$. The means and volatilities are represented by μ_{1j} and σ_{1j}, and μ_{2j} and σ_{2j}, where $j = 1, 2$ represent the growth and decay regimes, respectively. Table 5.1 summarizes the mean and volatilities of these processes.

The step sizes of each lattice are given by the following equations (see Bollen 1999):

$$\phi_{\omega j} = \sqrt{\sigma_{\omega j}^2 + \mu_{\omega j}^2 (dt)^2}, \quad \omega = 1, 2 \tag{5.18}$$

where, $\omega = 1$ represents the first group of processes formed from the processes, $(\theta_t^g, \delta_t)_1$ and $(\theta_t^d, \delta_t)_1$ and discretized into one pentanomial lattice and $\omega = 2$ represents the second group of

Table 5.1: Instantaneous means and volatilities of four uncorrelated processes

Processes	Instantaneous means	Instantaneous volatilities
$(\theta_t^g, \delta_t)_1$	$\mu_{11} = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_g + (\mu_g - \frac{\sigma_g^2}{2})\sigma_c]$	$\sigma_{11} = \sigma_g \sigma_c \sqrt{2(1+\rho_{cg})}$
$(\theta_t^d, \delta_t)_1$	$\mu_{12} = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_d + (\mu_d - \frac{\sigma_d^2}{2})\sigma_c]$	$\sigma_{12} = \sigma_d \sigma_c \sqrt{2(1+\rho_{cd})}$
$(\theta_t^g, \delta_t)_2$	$\mu_{21} = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_g - (\mu_g - \frac{\sigma_g^2}{2})\sigma_c]$	$\sigma_{21} = \sigma_g \sigma_c \sqrt{2(1-\rho_{cg})}$
$(\theta_t^d, \delta_t)_2$	$\mu_{22} = [(\mu_c - \frac{\sigma_c^2}{2})\sigma_d - (\mu_d - \frac{\sigma_d^2}{2})\sigma_c]$	$\sigma_{22} = \sigma_d \sigma_c \sqrt{2(1-\rho_{cd})}$

processes formed from the processes, $(\theta_t^g, \delta_t)_2$ and $(\theta_t^d, \delta_t)_2$ and discretized into another pentanomial lattice. As stated earlier, $j = 1, 2$ represent the growth and decay regimes in each group or lattice. Adjustment of the step size is given by the following expression:

$$\phi_{\omega j} = \begin{cases} \phi_{\omega k}, & \text{if } j = \text{k}, \\ R_j \phi_\omega, & \text{if } j \neq \text{k}, \end{cases}$$

where, R is the ranking process of j.

In a pentanominal lattice, each regime is represented by a trinomial lattice. The middle branch is shared by both regimes. To reduce the number of nodes in the lattice, the nodes are merged by adjusting the step size of one regime so that the step sizes of two regimes have a 1:2 ratio. The binomial branch probabilities (up and down) are given by following expressions:

$$P_{\omega k u} = \frac{1}{2}\left[1 + \frac{\mu_{\omega k} dt}{\phi_{\omega k}}\right] \tag{5.19}$$

$$P_{\omega k d} = 1 - P_{\omega k u} \tag{5.20}$$

After adjusting the step size, the conditional branch probabilities for the trinomial lattice are given by following equations.

$$P_{\omega j u} = \frac{1}{2}\left[\frac{\phi_{\omega j}^2}{(R_j \phi_\omega)^2} + \frac{\mu_{\omega j} dt}{R_j \phi_\omega}\right] \tag{5.21}$$

$$P_{\omega j d} = \frac{1}{2}\left[\frac{\phi_{\omega j}^2}{(R_j \phi_\omega)^2} - \frac{\mu_{\omega j} dt}{R_j \phi_\omega}\right] \tag{5.22}$$

$$P_{\omega k m} = 1 - P_{\omega j u} - P_{\omega j d} \tag{5.23}$$

In order to restore the values of correlated demand and carbon prices, the nodes of two individual pentanomial lattices are merged together to form a two-dimensional lattice. As nodes are merged, a node can represent any of two combined regimes. Figure 5.2 shows a combined lattice constructed by merging two individual pentanomial lattices at $t = 1$. At each node, the lattice has the values of demand and carbon prices.

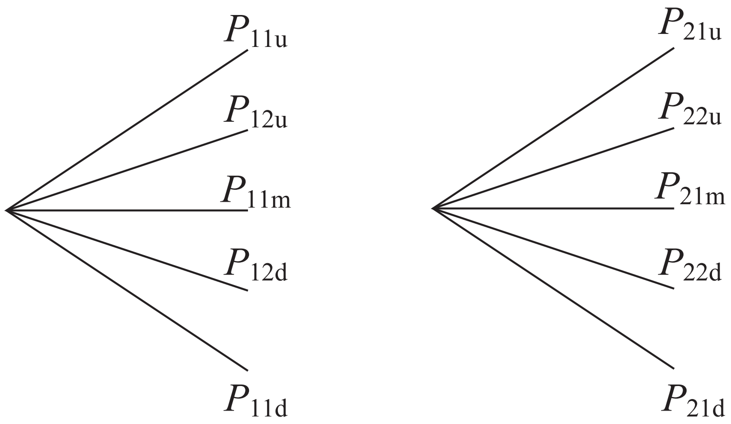

Figure 5.1: Construction of two pentanomial lattices: group 1 (left) and group 2 (right).

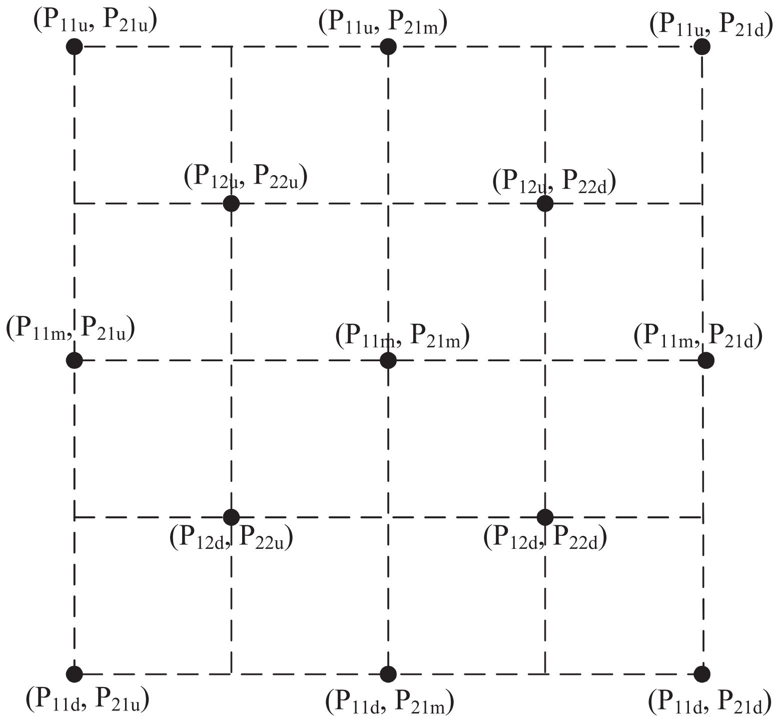

Figure 5.2: Construction of a three-dimensional lattice built from two individual pentanomial lattices at $t = 1$.

At time, t, the values of correlated demand and carbon prices can be retrieved by using an inverse transformation. At a combined regime (θ_t^g, δ_t), the value of correlated demand and carbon prices are given as follows:

$$\theta_t^g = \exp(\frac{\psi_1 + \psi_2}{2\sigma_c}) \quad \text{and} \quad \delta_t = \exp(\frac{\psi_1 - \psi_2}{2\sigma_g}) \tag{5.24}$$

Similarly at combined regime (δ_t, θ_d) the value of correlated demand and exchange rate are given by follows:

$$\theta_t^d = \exp(\frac{\psi_3 + \psi_4}{2\sigma_c}) \quad \text{and} \quad \delta_t = \exp(\frac{\psi_3 - \psi_4}{2\sigma_g}) \tag{5.25}$$

5.2.2 The expected value of the firm

After constructing the lattice, the value of the firm can be determined by commencing the calculation from the end of the lattice. Using a backward recursion approach, the calculations move towards the present node. At each node, the value is calculated as a function of carbon price, the demand regime, and the capacity level of the previous period. Since the previous period's optimum capacity level is unknown, all possible prior capacity levels are taken into consideration. Let $\pi(\Re, M_t, t)$ represent the value of the firm conditional on the capacity level, M_t, and the state of the combined regime denoted by $\Re$. The notion of $\Re$ is to indicate the combined regime with the demand status and the carbon prices. Therefore, $\Re$ can be any of the combined regimes as $(\theta_t^g, \delta_t)_1$, $(\theta_t^d, \delta_t)_1$, $(\theta_t^g, \delta_t)_2$, or $(\theta_t^d, \delta_t)_2$, the details of which are discussed in Section 5.2. The expected value at time t is given by the following equation.

$$\pi(\Re, M_t, t) = \max_{M_t \in \mathbf{M}} \{\pi(\theta_t, M_t, \delta_t) + S(M_t, M_{t+1}) + \mathbb{E}[\pi(\Re, M_{t+1}, t+1]\}, \tag{5.26}$$

where, the $\mathbb{E}[., ., ., .]$ is the discounted expected value across the different combined regimes.

If the current combined regime, $\Re$, belongs to the growth regime, (θ_t^g, δ_t), it could remain in the same combined regime, (θ_t^g, δ_t), or could switch to the combined regime, (θ_t^d, δ_t). Therefore, the expected profit is:

$$\begin{aligned} \mathbb{E}[\pi(\Re, M_{t+1}, t+1]\} =& \exp(-rh)[(1-p(t))\mathbb{E}[\pi((\theta_t^g, \delta_t), M_{t+1}, t+1)] \\ &+ p(t)\mathbb{E}[\pi((\theta_t^d, \delta_t), M_{t+1}, t+1)]], \end{aligned} \tag{5.27}$$

where $p(t)$ is the probability of switching from regime (θ_t^g, δ_t) to regime (θ_t^d, δ_t).
If the current combined regime is (θ_t^d, δ_t), the combined regime in the next period can be again (θ_t^d, δ_t). Therefore, the expected profit would be:

$$\mathbb{E}[\pi(\Re, M_{t+1}, t+1] = \exp(-rh)[\mathbb{E}[\pi((\theta_t^d, \delta_t), M_{t+1}, t+1)] \tag{5.28}$$

An important aspect is to use an appropriate discount rate. One approach is use the risk–free interest rate as a discount rate, when returns are the risk–adjusted expected return. The risk adjusted returns are obtained by deducting the risk premium, i.e., the excess return over the risk–

free return, from the process drift. Since the proposed model uses carbon futures prices and futures prices are the expected value of the future spot prices in a risk–neutral world, the risk premium for carbon prices is assumed to be zero. In case of stochastic demand, the demand risk is not also priced assuming that the uncertainty of switch from growth to decay is diversifiable (Bollen 1999). It is assumed that the firm constantly replaces the existing product with improved ones. However, there are various ways to evaluate the market price of risk, for example, employing an equilibrium pricing model using the Capital Asset pricing Modeling (CAPM). Calculating the risk adjusted growth rate from the security derivative prices is another method of evaluating the market price of risk (Hull 2009).

5.2.3 The optimal number of options

From Equation (5.4), we have the following expression:

$$\begin{aligned} \pi_t(\theta_t, M_t, \delta_t) &= (s-c_1)[\min(\theta_t, M_t)] - \delta_t\beta[\min(\theta_t, M_t)] - c_0 M_t \\ &+ X_t\{\max[(\delta_t - K_{c_t}), 0] - P_{t-1}\exp(rh)\} \end{aligned} \tag{5.29}$$

The expected utility is expressed as $\mathrm{U}(\pi) = \mathbb{E}(\pi) - \gamma\mathbb{V}(\pi)$. Therefore,

$$\mathbb{E}(\pi) = (s-c_1)\mathbb{E}[Q_1] - \beta\mathbb{E}[\delta_t Q_1] - c_0 M_t + X_t\mathbb{E}[\max(0, \delta_t - K_t) - \exp(rh)P_{t-1}] \tag{5.30}$$

where $Q_1 = \min(\theta_t, M_t)$. Assuming $Q_1 = \min(\theta_t, M_t)$, the components of Equation (5.4) can be expressed as: $A = (s-c_1)Q_1$, $B = \beta\delta_t Q_1$, and $C = X\{\max(\delta_t - K_{c_t}, 0) - P_{t-1}\exp(rh)\}$, where h is the time interval between $t-1$ and t. The variance of the profit, pi_t, given in Equation (5.4) can be expressed by the following expression.

$$\mathbb{V}[\pi_t] = \mathbb{V}[A] + \mathbb{V}[B] + \mathbb{V}[C] + 2\mathrm{cov}[A, B] + 2\mathrm{cov}[B, C] + 2\mathrm{cov}[A, C], \tag{5.31}$$

Maximizing the expected utility with respect to X_t and equating it to zero, we obtain:

$$\frac{\partial \mathrm{U}}{\partial X_t} = \frac{\partial \mathbb{E}[\pi]}{\partial X_t} - \gamma\frac{\partial \mathbb{V}[\pi]}{\partial X_t} \tag{5.32}$$

Since,

$$\frac{\partial \mathbb{E}[\pi]}{\partial X_t} = \mathbb{E}[\max(0, \delta_t - K_{c_t}) - P_{t-1}\exp(rh)] \tag{5.33}$$

Therefore setting $\partial \mathrm{U}/\partial X_t = 0$ in Equation (5.32), we obtain,

$$
\begin{aligned}
0 &= \mathbb{E}[\max(0, \delta_t - K_{c_t}) - P_{t-1}\exp(rh)] - \gamma \frac{\partial \mathbb{V}[\pi]}{\partial X_t} && (5.34)\\
\gamma \frac{\partial \mathbb{V}[\pi]}{\partial X_t} &= \mathbb{E}[\max(0, \delta_t - K_{c_t}) - P_{t-1}\exp(rh)] && (5.35)\\
\frac{\partial \mathbb{V}[\pi]}{\partial X_t} &= \frac{1}{\lambda}\mathbb{E}[\max(0, \delta_t - K_{c_t}) - P_{t-1}\exp(rh)] && (5.36)
\end{aligned}
$$

A complete arbitrage-free market under some risk-neutral probability measure is also imposed according to the justifications and approaches used in Tekin and Özekici (2015) and Chen and Parlar (2007). Therefore, $\mathbb{E}[\max(K_{e_t} - e_t, 0) - P_{t-1}\exp(rh)] = 0$ and we obtain,

$$
\frac{\partial \mathbb{V}[\pi]}{\partial X_t} = 0 \qquad (5.37)
$$

Chen and Parlar (2007) has justified the condition that maximizing the expected utility is equivalent to minimizing the variance of the profit under a quadratic utility function. Consequently, the firm purchases a number of carbon option so that it minimizes the variance of the profit given by the expression in Equation (5.29). where $A = (s - c_1)[\min(\theta_t, M_t)]$, $B = \delta_t\beta[\min(\theta_t, M_t)]$, and $C = X_t\{\max[(\delta_t - K_{c_t}), 0] - P_{t-1}\exp(eh)\}$. Equation (5.31) can be simplified as follows:

$$
\begin{aligned}
\mathbb{V}[\pi_t] =&(s - c_1)^2\mathbb{V}[Q_1] + \beta^2\mathbb{V}[\delta_t Q_1] + X^2\mathbb{V}[\{\max(\delta_t - K_{c_t}, 0) - P_{t-1}\exp(rh)\}]\\
&+ 2\mathrm{cov}[Q_1, \delta_t Q_1] - 2\mathrm{cov}[\delta_t\beta Q_1, X\{(\delta_t - K)^+ - P_{t-1}\exp(rh)\}]\\
&+ 2\mathrm{cov}[(s - c_1)Q_1, X\{(\delta_t - K)^+ - P_{t-1}\exp(rh)\}], \qquad (5.38)
\end{aligned}
$$

where $Q_1 = [\min(\theta_t, M_t)]$, $\mathbb{V}[A] = (s - c_1)^2\mathbb{V}[Q_1]$, $\mathbb{V}[B] = \beta^2\mathbb{V}[\delta_t Q_1]$, and $\mathrm{cov}[Q_1, \delta_t Q_1]$. Since, $\mathbb{V}[A]$, $\mathbb{V}[B]$, and $\mathrm{cov}[Q_1, \delta_t Q_1]$ do not dependent on X_t, the differentiation of these terms with respect to X_t turns out to be zero. Since $\mathbb{E}[(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)] = 0$, we can express $\mathbb{V}[\{\max(\delta_t - K_{c_t}, 0) - P_{t-1}\exp(rh)\}] = \mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}^2]$. Therefore,

$$
\begin{aligned}
\frac{\partial \mathbb{V}[\pi]}{\partial X} =& 2X\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}^2] - 2\mathrm{cov}[\delta_t\beta Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}]\\
&+ 2\mathrm{cov}[(s - c_1)Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] \qquad (5.39)
\end{aligned}
$$

The number of options, X, then can be obtained by setting $\frac{\partial \mathbb{V}[\pi_t]}{\partial X_t} = 0$.

$$
X_t = \frac{\beta\mathrm{cov}[\delta_t Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] - (s - c_1)\mathrm{cov}[Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}]}{\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}^2]} \qquad (5.40)
$$

The covariance terms in Equation (5.40) can be simplified as:

$$\operatorname{cov}[\delta_t Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh\}] = \mathbb{E}[\delta_t Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] - \mathbb{E}[\delta_t Q_1]\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] \tag{5.41}$$

$$\operatorname{cov}[Q_1, \{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] = \mathbb{E}[Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] - \mathbb{E}[Q_1]\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] \tag{5.42}$$

Since $\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] = 0$, replacing the expressions for the covariances in Equation (5.40), we obtain the following expression for X_t:

$$X_t = \frac{\beta\mathbb{E}[\delta_t Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] - (s - c_1)\mathbb{E}[Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}]}{\mathbb{E}[\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}^2]} \tag{5.43}$$

The numerator, $\mathbb{E}[\delta_t Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}]$, in Equation (5.43) can be expressed as:

$$\mathbb{E}[\delta_t Q_1\{(\delta_t - K_{c_t})^+ - P_{t-1}\exp(rh)\}] = \mathbb{E}[\delta_t Q_1\{(\delta_t - K_{c_t})\mathbf{1}_{\{\delta_t > K_{c_t}\}}] - \mathbb{E}[\delta_t Q_1 P_{t-1}\exp(rh)\}] \tag{5.44}$$

$$= \mathbb{E}[\delta_t^2 Q_1 \mathbf{1}_{\{\delta_t > K_{c_t}\}} - \delta_t Q_1 K_{c_t}\mathbf{1}_{\{\delta_t > K_{c_t}\}}] - \mathbb{E}[\delta_t Q_1 P_{t-1}\exp(rh)], \tag{5.45}$$

where $\mathbf{1}_{\{\delta_t > K_{c_t}\}}$ is an indicator function, which is equal to 1, if $\delta_t > K_{c_t}$, otherwise 0. The expression, $\mathbb{E}[\delta_t^2 Q_1 \mathbf{1}_{\delta > K_c}]$, in Equation (5.45) can further be simplified as follows:

$$\begin{aligned}
\mathbb{E}[\delta_t^2 Q_1 \mathbf{1}_{\delta > K_c}] &= \mathbb{E}[\mathbb{E}[(\delta_t^2 Q_1 \mathbf{1}_{\{\delta_t > K_c\}}) \mid \theta]] && (5.46)\\
&= \mathbb{E}[Q_1 \int_{K_c}^{\infty} \delta_t^2 f(\delta_t \mid \theta_t) d\delta_t] && (5.47)\\
&= \int_0^{\infty} Q_1 f(\theta_t) \left(\int_{K_c}^{\infty} \delta_t^2 f(\delta_t \mid \theta_t) d\delta_t \right) d\theta_t && (5.48)
\end{aligned}$$

5.3 A numerical example

A numerical example illustrates the implementation of the model. Table 5.2 shows the coefficients of the processes defined by Equations (5.7)–(5.9). These coefficients are the means and variances of and carbon allowance prices and the demand processes. Based on these coefficients shown in Table 5.2, the mean and variance of the uncorrelated processes defined by Equations (5.14)–(5.17) are calculated as shown in Table 5.3. For example, $\mu_{11} = 0.0241 = \{0.06 - (0.04^2/2)\} \times 0.22 + \{0.3 - (0.22^2/2))\} \times 0.04$.

Table 5.2: Parameters of product demand and carbon

Product demand	Mean	Volatility
Growth	$\mu_g = 0.30$	$\sigma_g = 0.22$
Decay	$\mu_d = -0.25$	$\sigma_d = 0.14$
Carbon price	$\mu_c = 0.06$	$\sigma_c = 0.04$
Correlation coefficients		
Growth	$\rho_{cg} = 0.10$	
Decay	$\rho_{cd} = 0.20$	
Regime switching parameters		
Mean	33.6 months	
Variance	6 months	

Table 5.3: A numerical example of mean variances of the four uncorrelated processes

Process	Mean	Volatility
$\psi_1 \rightarrow (\theta_t^g, \delta_t)_1$	$\mu_{11} = 0.0241$	$\sigma_{11} = 0.0131$
$\psi_3 \rightarrow (\theta_t^d, \delta_t)_1$	$\mu_{12} = 0.0020$	$\sigma_{12} = 0.0118$
$\psi_2 \rightarrow (\theta_t^g, \delta_t)_2$	$\mu_{21} = 0.0021$	$\sigma_{21} = 0.0087$
$\psi_4 \rightarrow (\theta_t^d, \delta_t)_2$	$\mu_{22} = 0.0187$	$\sigma_{22} = 0.0071$

The step sizes are determined by using Equation (5.18). For example,

$$\phi_{11} = 0.0043 = \sqrt{0.0241^2 \times (1/12)^2 + 0.0131^2 \times (1/12)}$$

Similarly, $\phi_{12} = 0.0025$, $\phi_{21} = 0.0034$, and $\phi_{22} = 0.0026$. For the adjustment of the step size for group 1, since $\phi_{12} < \phi_{11}$, the adjusted step size for group 1 will be, $\phi_1 = \max(\frac{\phi_{11}}{2}, \frac{\phi_{12}}{1}) = 0.0025$. Similarly, $\phi_2 = 0.0051$. Figure 5.3 (left) shows the branch probabilities for group 1 from the combination of the uncorrelated processes, $(\theta_t^g, \delta_t)_1$ and $(\theta_t^d, \delta_t)_1$ and Figure 5.3 (right) shows the branch probabilities for group 2 from the combination of the uncorrelated processes, $(\theta_t^g, \delta_t)_2$ and $(\theta_t^d, \delta_t)_2$. The branch probabilities are calculated by using Equations (5.21)–(5.23). For example, $P_{12u} = 0.4650 = 0.50 \times [1 + \{-0.0021 \times (1/12)/0.0025\}]$.

Figure 5.4 shows the lattice representation of the combined regimes 1 and 2 at $t = 2$. The 'circle' in the figure refer to nodes generated from the combined regime 1, which represents the combination of the uncorrelated processes of $(\theta_t^g, c)_1$ and $(\theta_t^d, c)_1$. There are 9 nodes denoted by

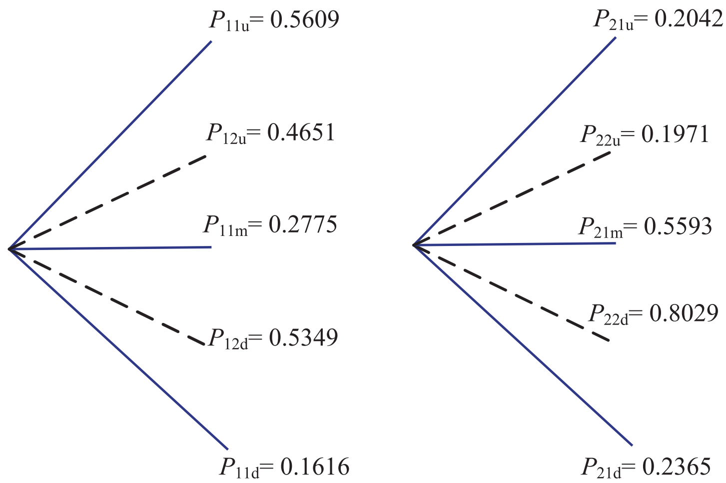

Figure 5.3: Branch probabilities of the respective pentanomial lattices: (left) group 1; (right) group 2.

circles at $t = 2$. These 9 nodes are generated from the combination of three branches from the process, $(\theta_t^g, \delta_t)_1$ and three branches from the process, $(\theta_t^d, \delta_t)_1$. Similarly, there are 4 nodes denoted by the 'crosses' representing the combined regime 2 generated from the combination of two branches from the process, $(\theta_t^g, \delta_t)_2$ and two branches from the process, $(\theta_t^d, \delta_t)_2$. Each node in Figure 5.4 has three values – the profit of the combined regime 1, the profit of the combined regime 2 and the branch probability to reach this node at $t = 2$ from the node at $t = 1$.

Figure 5.5 shows the the lattice representation of the combined regimes 1 and 2 at $t = 3$. Each node from the lattice at $t = 2$ generates 13 nodes at $t = 3$. Since some nodes will merge together at $t = 3$ as indicated by overlapping 'circles' and 'crosses' in Figure 5.5, there will be 41 nodes in the lattice at $t = 3$. The expected value at regime 1 can be calculated by multiplying the values at regime 1 to their respective branch probabilities as shown in Equation (5.49).

$$\begin{aligned}
27860.3159 \quad = \quad & 0.132663 \times 27080.31824 + 0.313687 \times 28906.61 + 0.114548 \times 30872.86 + \\
& 0.065624 \times 25455.5727 + 0.15517 \times 27200 + 0.056663 \times 29029.86 + \\
& 0.038232 \times 23935.3346 + 0.090402 \times 25570.63 + 0.033012 \times 27291.18 \qquad (5.49)
\end{aligned}$$

Similarly, the expected value at regime 2 is calculated in Equation (5.50).

$$\begin{aligned}
26613.6678 \quad = \quad & 0.373395 \times 27121.58 + 0.091685 \times 28968.63 + \\
& 0.429466 \times 25513.04 + 0.105453 \times 27250.35 \qquad (5.50)
\end{aligned}$$

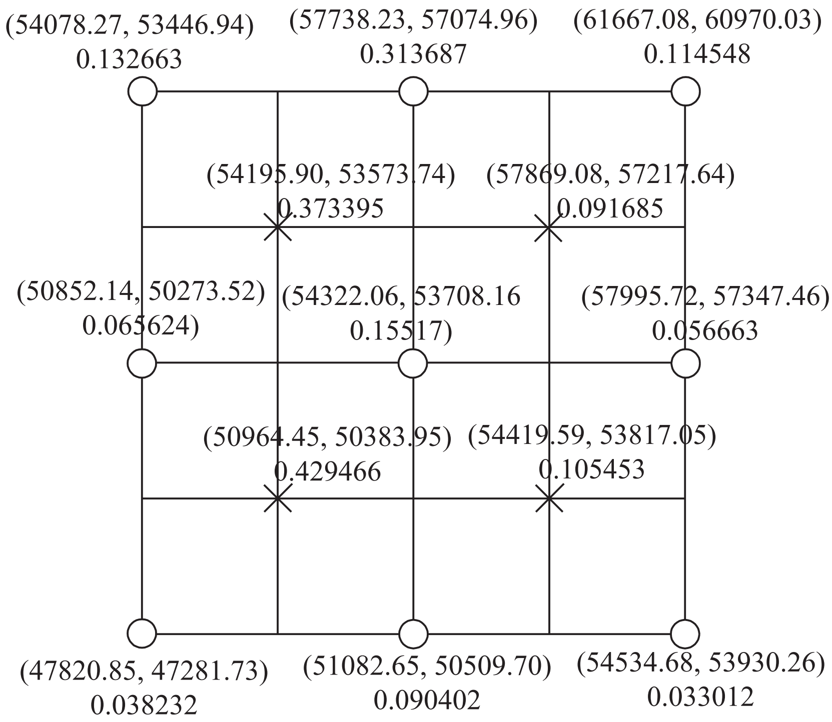

Figure 5.4: Nodes of the lattice at t=2.

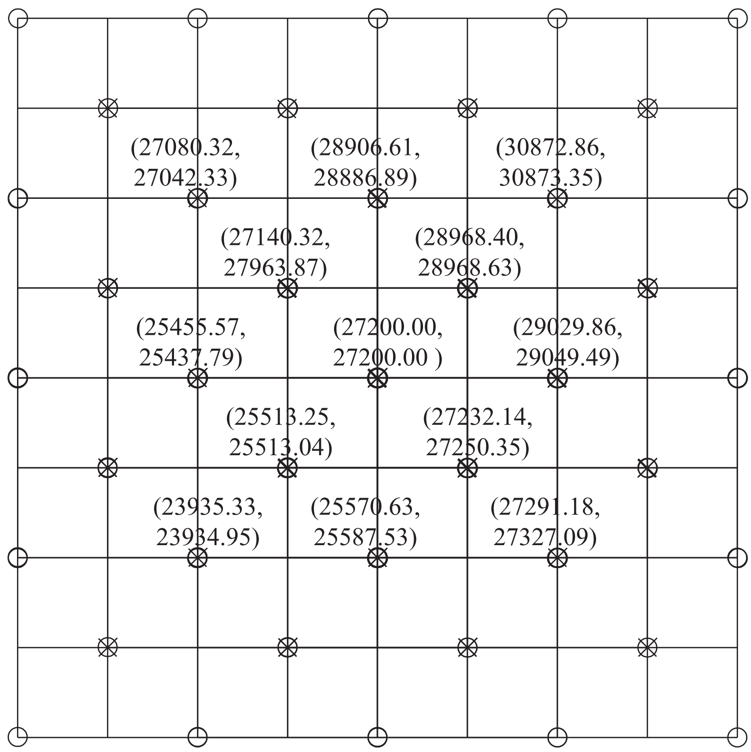

Figure 5.5: Nodes of the lattice at t=3.

The expected profit in the middle node at $t = 2$, is determined by using Equations (5.27)–(5.28).

$$\begin{aligned} 54323.7409 &= 27200 + \exp(-0.05 \times (1/12)) \\ &\quad \times (0.50 \times 27860.3159 + 0.50 \times 26613.6678) \end{aligned} \tag{5.51}$$

$$53703.0087 = 27200 + \exp(-0.05 \times (1/12)) \times (1.0 \times 26613.6678) \tag{5.52}$$

5.4 Numerical results

This section presents a numerical analysis of a firm that produces a single product with a regime–switching life cycle and has an associated carbon emission cost per unit of production. The expected values of the firm is analyzed with respect to the changes in the mean and volatility of the demand pattern and carbon prices. The sale price, s, is assumed to be \$40 per unit. The initial demand of the product is 1000 units. The cost of the initial capacity installment, c_0, is \$10 per unit. Whenever the flexible capacity system expands or contracts its maximum production by an unit, there occurs a fixed cost of $c_5 = 300$ and a variable cost of $c_3 = 1.0$ for capacity expansion and $c_4 = 0.9$ for capacity contraction, as a fraction of the unit capacity installation cost, c_0. The carbon emission rate, β, is assumed to be 0.40tCO_2 per unit of production. As stated in Section 5.2, the carbon prices are assumed to follow the geometric Brownian motion with an initial price of \$7 per tCO_2 emission. The carbon call option strike price, K_c, is assumed to be \$10 per unit of option. The risk free discount rate, r, is 5%. Regime–switching demand and the stochastic carbon prices parameters are given in Table 5.2.

Figure 5.6 shows the expected value of the firm with respect to the changes in the mean of the growth regime of the demand. It is observed that as the mean demand increases, the expected value of the firm also increases. The reson is that the increasing demand naturally tends to bring more revenue and consequently more expected profit for the firm. It is also observed that when the firm employs flexible capacity and adjusts its capacity from one time interval to the next time interval, the firm experiences a higher expected value than the value when the firm decides to utilize a fixed capacity system throughout the planning period. The options of capacity flexibility evidently add value to the firm. Another observation is that the integration of real option, in the form of capacity flexibility, and the financial options, in the form of carbon call options, causes a higher expected value in comparison to when only real options is used. The reason is when the impact of carbon price uncertainties are hedged through the carbon call options, the firm can increase its capacity levels, that contributes to an increasing expected profit. Similar observations are found in Figure

5.7 when the average decay demand increases.

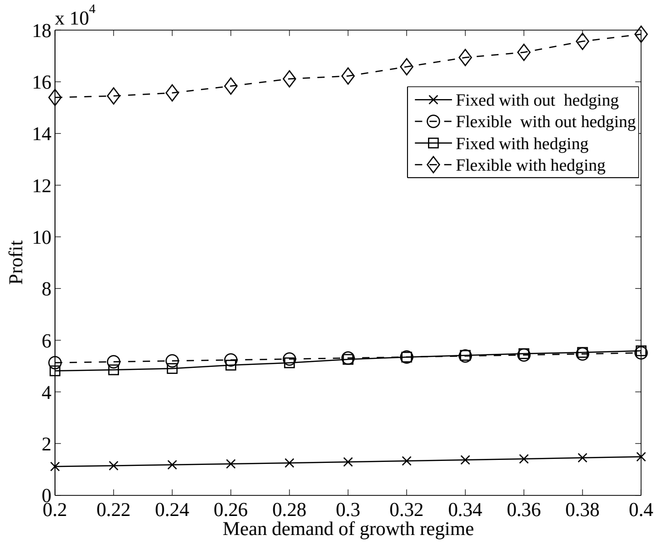

Figure 5.6: Expected value of the firm with respect to the mean demand of the growth regime.

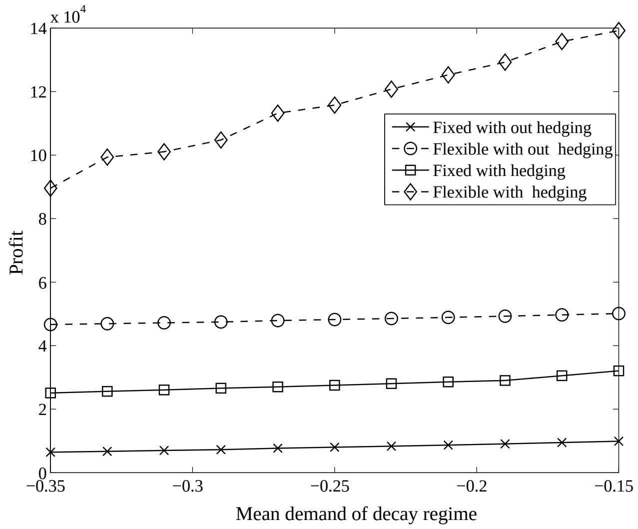

Figure 5.7: Expected value of the firm with respect to the mean demand of the decaying regime.

Figure 5.8 shows the expected profit of the firm with respect to the varying mean of carbon prices. It is observed that the expected profit decreases, as the mean carbon prices increase. However, the usage of the carbon call options along with flexible capacity options offers the highest expected profit for the firm.

The effect of the changes in the volatilities of the growth and decaying regimes of the demand pattern can be observed from Figure 5.9 and Figure 5.10. As the volatility of the demand growth regime increases, the expected profit for both fixed and flexible systems using carbon options increase, while the expected profit for both fixed and flexible systems without using carbon options remain almost unchanged. However, an increase in the volatility of the demand decay regime

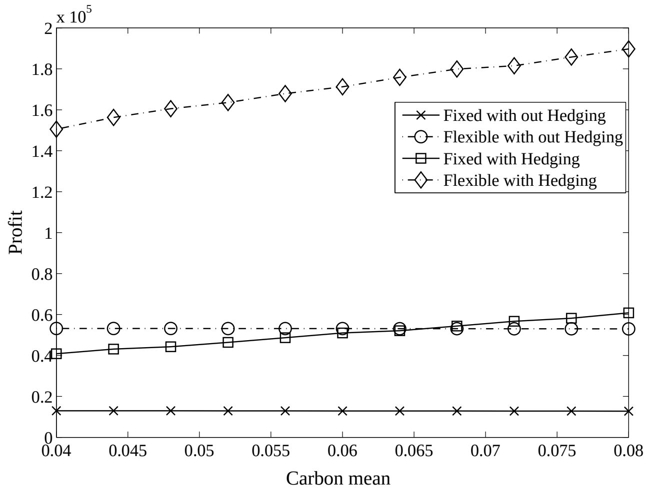

Figure 5.8: Expected value of the firm with respect to the mean of carbon allowance prices.

decreases the expected profit for both fixed and flexible systems with carbon call options. The reason is that the firm can reap more profit, when the uncertainty in the growth regime of the demand is higher. On the other hand, when the volatility in the decaying regime of the demand increases, the firm becomes reserved in utilizing the carbon options and the expected value decreases.

Figure 5.11 shows the expected profit with respect to the changes in the volatility of carbon allowance prices. It is observed that as the volatility of the carbon allowance prices increase, the expected value for both fixed and flexible systems using the carbon options increases. carbon options add more values to the firm both in the cases of fixed and flexible capacity systems. Uncertainty in the carbon prices encourages the firm to use more options that contributes to the higher expected profit of the firm.

The effect of carbon strike price, K_c, on the expected value of the firm is presented in Figure 5.12. It is observed that as K_c increases, the expected values of the firm for both fixed and flexible systems with carbon options decrease. The reason is that an increasing carbon strike price causes a drop in option payoff, which eventually affects the expected profit of the firm. However, the expected value for the flexible system with options drops more rapidly than that of the fixed system. As stated earlier, when carbon price uncertainties are hedged through carbon call options, the firm increases its ability to utilize the capacity flexibility, and it contributes to an increasing expected profit. The reverse phenomenon is also true that when the payoff from carbon options tends to decrease, the system retards to use its capacity flexibility and tends to move toward a fixed system.

Figure 5.13 and Figure 5.14 show the sensitivity of the correlation coefficients between carbon prices and demand in both growth and decay regimes, respectively, on the expected value of the

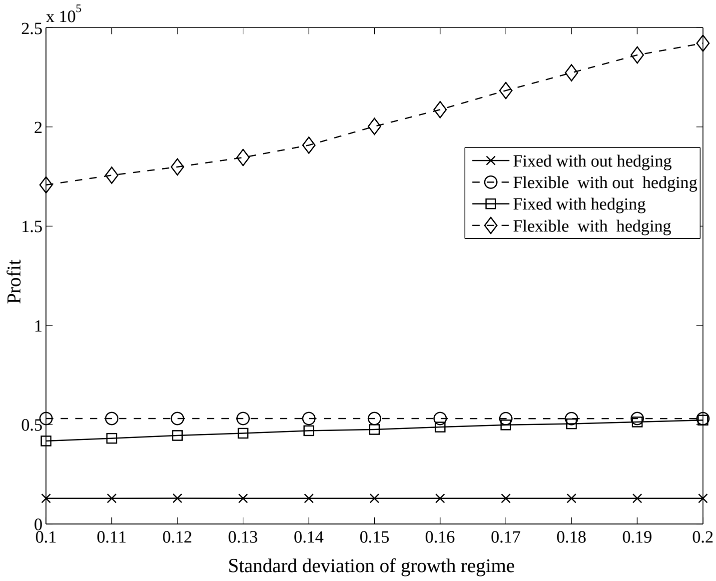

Figure 5.9: Expected value of the firm with respect to the volatility of the growth regime.

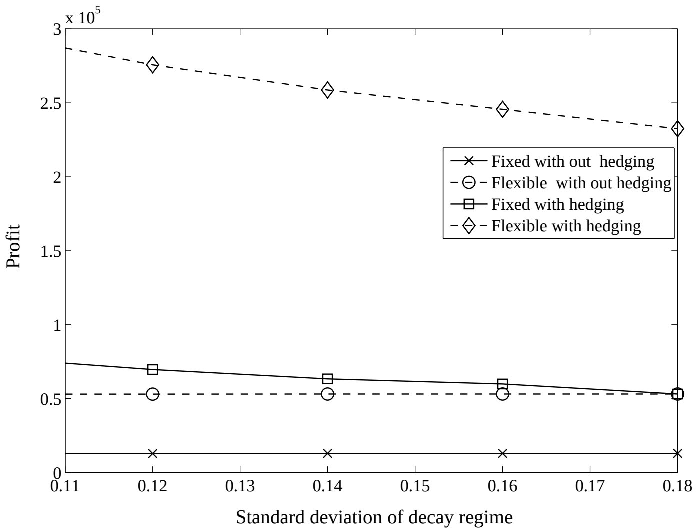

Figure 5.10: Expected value of the firm with respect to the volatility of the decaying regime.

firm.

5.5 Summary

This chapter considers two risks, namely, demand and carbon price uncertainties for a firm that possesses a flexible capacity system. The proposed model considers an integration of operational hedging in the form of capacity allocation and financial option with carbon call option. The model assumes a multi-period time frame in which demand follows a regime–switching behavior. The carbon prices are assumed to follow a geometric Brownian motion. Four possible scenarios are considered, namely, a fixed capacity system without using carbon options, a fixed capacity system

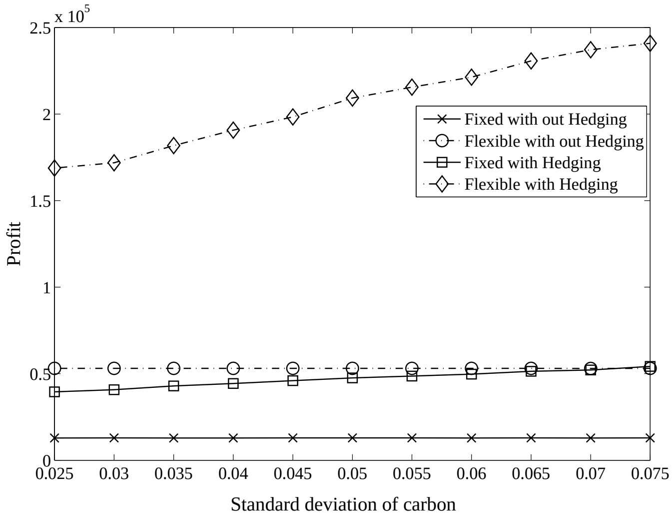

Figure 5.11: Expected value of the firm with respect to the carbon allowance prices volatilities.

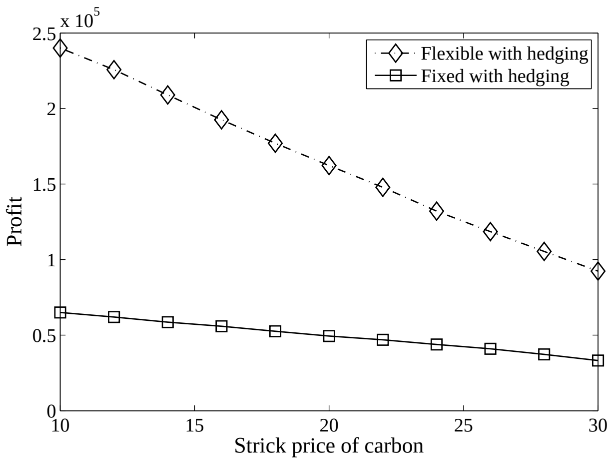

Figure 5.12: Expected value of the firm with respect to the series of carbon strike prices, K_c.

with carbon options, a flexible capacity system with carbon options and a flexible capacity system with carbon options. The number of option is calculated by minimizing the variance of the profit and the optimal capacity allocation by maximizing the expected profit. The proposed approach helps the firm make business decision upfront by avoiding downside risk. It also guarantees a favorable profit regardless of the uncertainties in demand and carbon prices.

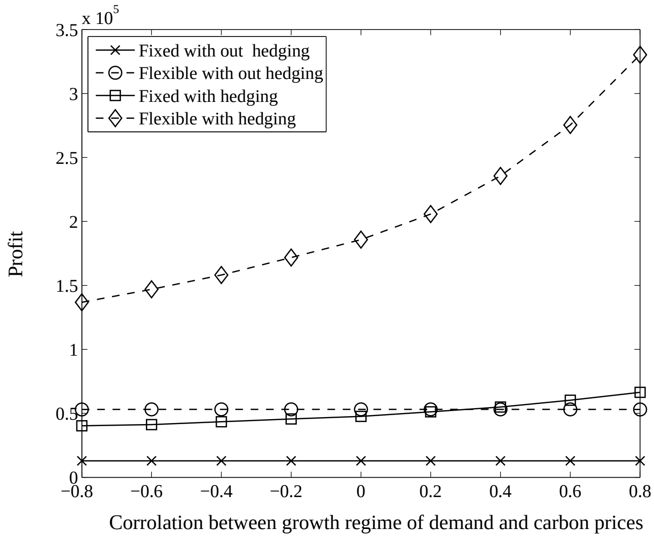

Figure 5.13: Expected value of the firm with respect to the correlation coefficients between the carbon prices and demands in the growth regime.

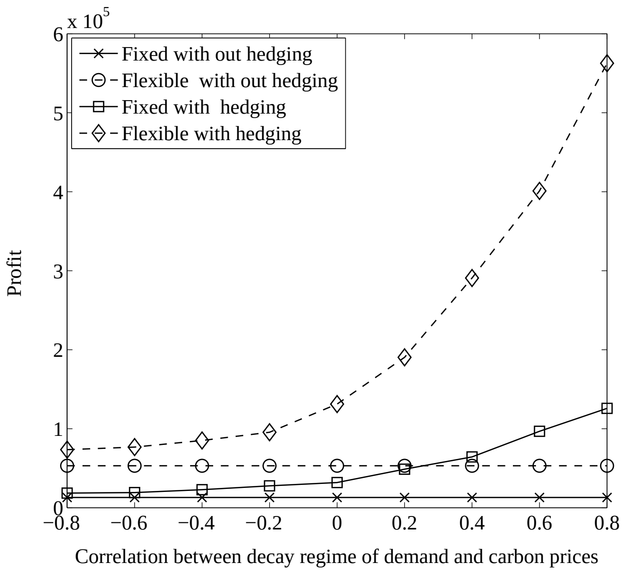

Figure 5.14: Expected value of the firm with respect to the correlation coefficients between the carbon prices and demands in the decay regime.

Chapter 6

Optimal production decision under exchange rate, demand, and carbon price uncertainties

This chapter considers uncertainties of exchange rates, demand, and carbon allowance prices simultaneously. An interesting feature of this chapter is that the demand uncertainties are considered with underage and overage costs. The underage cost occurs when the production does not meet the demand. The overage cost arises when the production exceeds the demand. Furthermore, the carbon allowance prices are considered to follow a geometric Brownian motion with a jump process. In order to compare the effect of underage and overage costs along with the usages of currency and carbon options, two models are developed – one in Section 6.2 with known demand, known exchange rates, and with stochastic jump process for carbon; and the other in Section 6.3 with stochastic demand, stochastic exchange rates using options, and stochastic carbon prices with jumps. Consequently, the option pricing for stochastic jump process is also discussed.

6.1 The models

The quantity the firm sells to domestic market is denoted by variable x_{11}, whereas the quantity the firm sells its locally produced goods to foreign markets is represented by x_{12}. The optimal capacity allocation for the firm is given by χ and c_0 is the unit capacity cost. Unit production cost is given by c_1. D_1 and D_2 represent domestic and foreign market's demand respectively. Variable s_1 denotes the unit sale price in the local market in its local currency, whereas s_2 denotes unit sale price in foreign market. The spot price of the exchange rate is represented by e. The carbon spot price at time t is denoted by δ_t. The strick price of exchange rate is denoted by K_e and the strick price of carbon is denoted by K_c. P_e and P_c are prices of currency put option and carbon call option, respectively. The risk free interest rate is r and T is the time between the production and sale.

6.2 Stochastic carbon prices with given exchange rate and demand

In this section, it is assumed that the firm experiences carbon allowance prices uncertainty, while exchange rate and demand are assumed to be known. The stochastic behavior of the carbon allowance prices is assumed to follow a geometric Brownian motion with jump process, the detail of which is discussed in the following section. Accordingly, the profit is given by Equation (6.1):

$$\pi = (s_1 - c_1)x_{11} + (s_2 e - c_1)x_{12} - \beta\delta_t(x_{11} + x_{12}) - c_0\chi \exp(rT) + X\left\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\right\}, \tag{6.1}$$

where $(s_1 - c_1)$ is the unit profit from the local sale, $(s_2e - c_1)$ is the unit profit from the foreign sale, $c_0\chi$ is the total capacity cost, $\beta\delta_t(x_{11}+x_{12})$ is total emission cost, $\max(\delta_t - K_c, 0)$ is the payoff from the carbon call options, and $P_c \exp(rT)$ is the premium value at time, T. Since exchange rate and demand are known and carbon prices are stochastic, we consider only carbon call options in this case. The expected utility is the function of the expected profit and the variance of profit and it is expressed as:

$$U(\pi) = \mathbb{E}(\pi) - \gamma\mathbb{V}(\pi), \tag{6.2}$$

where $\mathbb{E}(\pi)$ is the expected profit, $\mathbb{V}(\pi)$ is the variance of the profit, and γ is the mean-variance ratio.

6.2.1 Modeling carbon allowance prices as a stochastic jump process

Carbon cap-and-trade method is a market based mechanism to control carbon emissions. A monitoring authority imposes a cap or limit on the emission of carbon dioxide and allows firms to trade emission permits among them. In Europe, one emission certificate, also known as European Union Allowance (EUA), allows to emit one tonne of CO_2. Companies can buy or sell such certificates and decide on their amount of CO_2 released into the atmosphere. The right to produce a particular amount of CO_2 has now become a tradable commodity. A company with lower carbon emissions can benefit from selling its allowances to higher carbon emitting companies (Benz and Trück 2009). In modeling carbon allowance prices, Seifert et al. (2009) assume that the uncertainty in emission price dynamics is driven by a standard Brownian process. Benz and Trück (2009) analyze the log return of carbon spot prices from January, 2005 to December 2006 as a Markov regime-switching model with two regimes (i.e., base and spike regimes). Daskalakis et al. (2009) analyze and compare six empirical and theoretical configurations of carbon prices behavior and find that the geometric Brownian motion with jump process is a better model in terms of parameter significance.

In this chapter, the carbon spot price is assumed to be driven by a Brownian motion along with a Poisson jump process. The jump process is assumed to be double exponentially distributed, which consists of two exponential functions joined together on a threshold (Daskalakis et al. 2009). Accordingly, the carbon price δ_t is assumed to follow the price dynamics as follows:

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t}(V_i - 1)\right), \tag{6.3}$$

where W_t is a standard Brownian motion, N_t is a Poisson process with rate λ and V_i is a sequence of independent identically distributed nonnegative random variables. The logarithm of the variable, V, $Y = \log(V)$ has an asymmetric double exponential distribution with the density:

$$f_Y(y) = p \cdot \eta_1 \exp(-\eta_1 y)\mathbf{1}_{y\geq 0} + q \cdot \eta_2 \exp(-\eta_2 y)\mathbf{1}_{y<0}, \tag{6.4}$$

where $\eta_1 > 1$, $\eta_2 > 0$. The probabilities of upward and downward jumps are given by $p, q \geq 0$, and $p + q = 1$. That is, Y_i is given by the following equation.

$$\ln\ (V_i) = Y_i = \begin{cases} \xi^+ & \text{with probability } p, \\ -\xi^- & \text{with probability } q, \end{cases}$$

where ξ^+ and $-\xi^-$ are exponential random variables with means $1/\eta_1$ and $1/\eta_2$. Solving Equation (6.3), we obtain the following dynamics for carbon price:

$$\delta_t = \delta_0 e^{\sigma W_t + (\mu - \frac{1}{2}\sigma^2)t} \prod_{i=1}^{N_t} V_i, \tag{6.5}$$

where δ_0 is carbon price at time $t = 0$.

6.2.2 Carbon Option pricing for a jump process

The price of an European call option on the underlying carbon price that follows a geometric Brownian motion with jump process can be obtained as (Kou 2002):

$$
\begin{aligned}
P_c = {} & \delta_0 \gamma \left(r + \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \tilde{\lambda}, \tilde{p}, \tilde{\eta_1}, \tilde{\eta_2}; \ln(\frac{K_c}{\delta_t}), T \right) \\
& - K_c \exp(-rT) \gamma \left(r - \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \lambda, p, \eta_1, \eta_2; \ln(\frac{K_c}{\delta_t}), T \right),
\end{aligned}
\tag{6.6}
$$

where $\tilde{p} = \frac{p}{1+\zeta} \times \frac{\eta_1}{\eta_1 - 1}$, $\tilde{\eta_1} = \eta_1 - 1$, $\tilde{\eta_2} = \eta_2 + 1$, $\tilde{\lambda} = \lambda(\zeta + 1)$, and $\zeta = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$, and

$$
\gamma(\mu, \sigma, \lambda, p, \eta_1, \eta_2; \ln(\frac{K_c}{\delta_0}), T) := \int_{K_c}^{\infty} f(\delta_t) d(\delta_t) = P(\delta_T > K_c) \tag{6.7}
$$

6.2.3 Number of carbon call options

Differentiating Equation (6.2) with respect to X, we obtain

$$
\frac{\partial \mathrm{U}(\pi)}{\partial X} = \frac{\partial \mathbb{E}(\pi)}{\partial X} - \gamma \frac{\partial \mathrm{V}(\pi)}{\partial X} \tag{6.8}
$$

The expected profit is given by the following equation:

$$
\mathbb{E}(\pi) = (s_1 - c_1)x_{11} + (s_2 e - c_1)x_{12} - \beta(x_{11} + x_{12})\mathbb{E}(\delta_t) - c_0\chi \exp(rT) + X\mathbb{E}\left[\max(\delta_t - K_c, 0) - P_c \exp(rT)\right] \tag{6.9}
$$

Differentiating Equation (6.9) with respect to X, we obtain

$$
\frac{\partial \mathbb{E}(\pi)}{\partial X} = \mathbb{E}\left[\max(\delta_t - K_c, 0) - P_c \exp(rT)\right] \tag{6.10}
$$

Under the no-arbitrage pricing method, the expected payoff from the carbon call option is zero. Therefore,

$$
\mathbb{E}\left[\max(\delta_t - K_c, 0) - P_c \exp(rT)\right] = 0 \tag{6.11}
$$

Therefore, maximizing the utility is equivalent to minimizing the variance of the profit.

Since exchange rate, e, and demand, x_{11} and x_{12} are given, the first four terms in Equation (6.1), $(s_1 - c_1)x_{11}$, $(s_2 e - c_1)x_{12}$, $\beta\delta_t(x_{11} + x_{12})$, and $c_0\chi \exp(rT)$ are constant. Assume that, $A_C = -\beta(x_{11} + x_{12})\delta_t$ and $B_C = X\left\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\right\}$. The variance of profit can be given by following expression.

$$
\mathbb{V}[\pi] = \mathbb{V}[A_C] + \mathbb{V}[B_C] + 2\mathrm{cov}[A_C, B_C] \tag{6.12}
$$

Variance of the profit, $\mathbb{V}[\pi]$, is the summation of variance of $A_C = -\beta(x_{11}+x_{12})\delta_t$ and the variance of $B_C = X\left\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\right\}$. Because of known demand and exchange rate, the terms $(s_1 - c_1)x_{11}$, $(s_2 e - c_1)x_{12}$, $\beta\delta_t(x_{11}+x_{12})$, and $c_0\chi\exp(rT)$ are also known and these terms do not affect the variance of profit. The variances and covariance of A_C and B_C can be expresses as in Equations (6.13)–(6.15):

$$\mathbb{V}[A_C] = \beta^2(x_{11}+x_{12})^2\mathbb{V}[\delta_t] \tag{6.13}$$

$$\mathbb{V}[B_C] = X^2\mathbb{E}\left[\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\}^2\right] \tag{6.14}$$

$$\mathrm{cov}[A_C, B_C] = -X\beta(x_{11}+x_{12})\mathbb{E}\left[\{(\delta_t - \mathbb{E}(\delta_t))\}\left\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\right\}\right] \tag{6.15}$$

The optimal number of carbon call option, X, is obtained by minimizing the variance of profit as follows.

$$\begin{aligned}\frac{\partial\mathbb{V}[\pi]}{\partial X} =& 2\beta(x_{11}+x_{12})\mathbb{E}\left[\{-\delta_t + \mathbb{E}[\delta_t]\}\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\}\right] \\ &+ 2X\mathbb{E}\left[\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\}^2\right]\end{aligned} \tag{6.16}$$

Setting Equation (6.16) to zero, Y is given by Equation (6.17).

$$X = \frac{2\beta(x_{11}+x_{12})\mathbb{E}\left[\{\delta_t - \mathbb{E}[\delta_t]\}\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\}\right]}{\mathbb{E}\left[\{\max(\delta_t - K_c, 0) - P_c\exp(rT)\}^2\right]} \tag{6.17}$$

Equation (6.17) can be simplified to

$$X = \beta(x_{11}+x_{12})\left\{\frac{\int_{K_c}^{\infty}\delta_t^2 f(\delta_t)d(\delta_t) - K_c\int_{K_c}^{\infty}\delta_t f(\delta_t)d(\delta_t) - \mathbb{E}(\delta_t)P_c\exp(rT)}{\int_{K_c}^{\infty}\delta_t^2 f(\delta_t)d(\delta_t) - 2K_c\int_{K_c}^{\infty}\delta_t f(\delta_t)d(\delta_t) - P_c^2\exp(2rT) + K_c^2\int_{K_c}^{\infty}f(\delta_t)d(\delta_t)}\right\}. \tag{6.18}$$

Further simplification of the integral in Equation (6.18) can be obtained by using the gamma function given in Equation (6.7). Therefore, from Equation (6.7) we obtain:

$$\int_{K_c}^{\infty}\delta_t f(\delta_t)d(\delta_t) = \delta_0\gamma(r + \frac{1}{2}\sigma^2 - \lambda\zeta, \sigma, \tilde{\lambda}, \tilde{p}, \tilde{\eta_1}, \tilde{\eta_2}; \ln(\frac{K_c}{\delta_0}), T). \tag{6.19}$$

where $\tilde{p} = \frac{p}{1+\zeta} \times \frac{\eta_1}{\eta_1 - 1}$, $\tilde{\eta_1} = \eta_1 - 1$, $\tilde{\eta_2} = \eta_2 + 1$, $\tilde{\lambda} = \lambda(\zeta+1)$, and $\zeta = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2+1} - 1$. To calculate the value of $\int_{K_c}^{\infty}\delta(t)^2 f(\delta_t)d(\delta_t)$, we have to find the process of δ_t^2. Assuming $Z_t = \delta_t^2 =$

$[\delta_0 e^{\sigma W_t + (\mu - \frac{1}{2}\sigma^2)t} \prod_{i=1}^{N_t} V_i,]^2$ and by using Itô's lemma, we obtain:

$$\begin{aligned} Z_t &= \delta_0^2 e^{2\sigma W_t + (2\mu - \sigma^2)t} \prod_{i=1}^{N_t} V_i^2 \\ &= Z_0 e^{2\sigma W_t + (2\mu - \sigma^2)t} \prod_{i=1}^{N_t} V_i^2 \\ &= Z_0 e^{2\sigma W_t + (2\mu - \sigma^2)t} \prod_{i=1}^{N_t} e^{2Y_i}, \end{aligned}$$

where

$$Y_i(Z) = 2Y_i = \begin{cases} 2\xi^+ & \text{with probability } p, \\ -2\xi^- & \text{with probability } q, \end{cases}$$

where ξ^+ and ξ^- are exponential random variables with means $1/\eta_1$ and $1/\eta_2$. Hence $\sigma_{Z_t} = 2\sigma$, $\mu_{Z_t} = 2\mu - \sigma^2$, $\lambda_{Z_t} = \lambda$, $\eta_{1,Z_t} = \eta_1/2$, and $\eta_{2,Z_t} = \eta_2/2$.

$$\int_{K_c}^{\infty} \delta_t^2 f(\delta_t) d(\delta_t) = \delta_0 \gamma(r + \frac{1}{2}\sigma_{\delta_t^2}^2 - \lambda\zeta, \sigma_{\delta_t^2}, \tilde{\lambda}, \tilde{p}, \tilde{\eta_1}, \tilde{\eta_2}; \ln(\frac{K_c}{\delta_0}), t). \tag{6.20}$$

Substituting Equation (6.19) and Equation (6.20) in Equation (6.18), we obtain the number of carbon call options, Y_0.

6.2.4 Numerical results

This section illustrates numerical results. In this regard following parameters are considered: x_{11} = 500, x_{12} = 350, s_1 =50, c_1 = 20, s_2 = 40, e_0 = 2, c_0 = 5, r = 0.05, T = 0.5, δ_0 = 100, β = 0.02, η_1 = 10, η_2 = 5, p = 0.5, σ = 0.16, λ = 20, and K_c = 98. Figure 6.1 shows the number of call options to long against the strike price, K_c. It is observed that as the strike price increases, the number of call options required to long also increases. A higher value of strike price reflects more risk. In order to hedge this higher risk, the firm needs to buy more options. Figure 6.2 shows the number of call options to long with respect to the number of jumps, λ. As the jump rate increases the number of call options required to long decreases. Increasing jump tends to occur due to a sudden increase in carbon prices, that in turn results in to reduce the number of call options to long. As the carbon price volatility increases, the number of options to buy also increases. Figure 6.3 shows the number of call options to long with respect to the volatility of the carbon prices. Since increasing volatility increases the carbon price uncertainties, the number of options required to hedge the risk also increase. Figure 6.4 and Figure 6.5 show the expected profit and variance of the profit, respectively, with regard to the changing volatility of carbon allowance prices. It

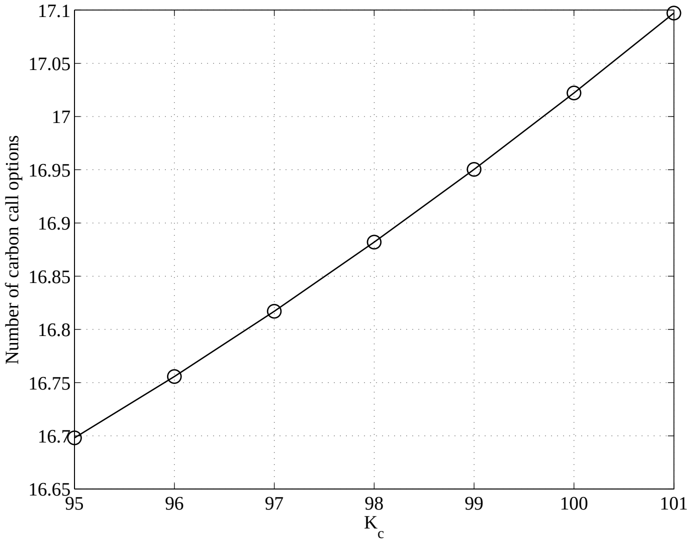

Figure 6.1: Number of carbon call options with respect to the strike carbon prices, K_c

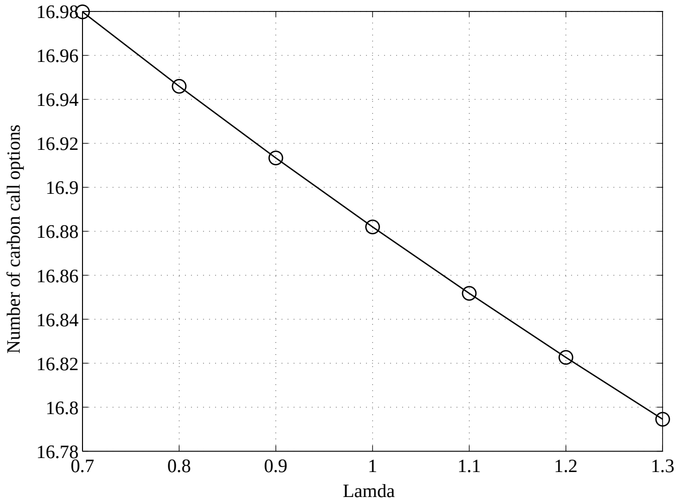

Figure 6.2: Number of carbon call options to long with respect to jump rate, λ

is observed that while the expected profit is more when the volatility of carbon prices increases, the variance is reduced due to the utilization of carbon call options. The firm achieves a higher expected value with a reduced volatility in profit.

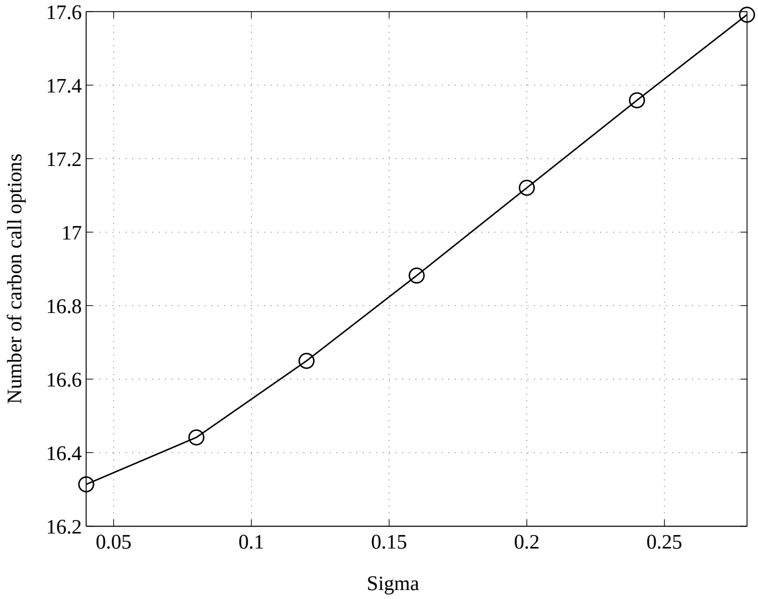

Figure 6.3: Number of carbon call options to long with respect to the volatility of carbon prices, σ

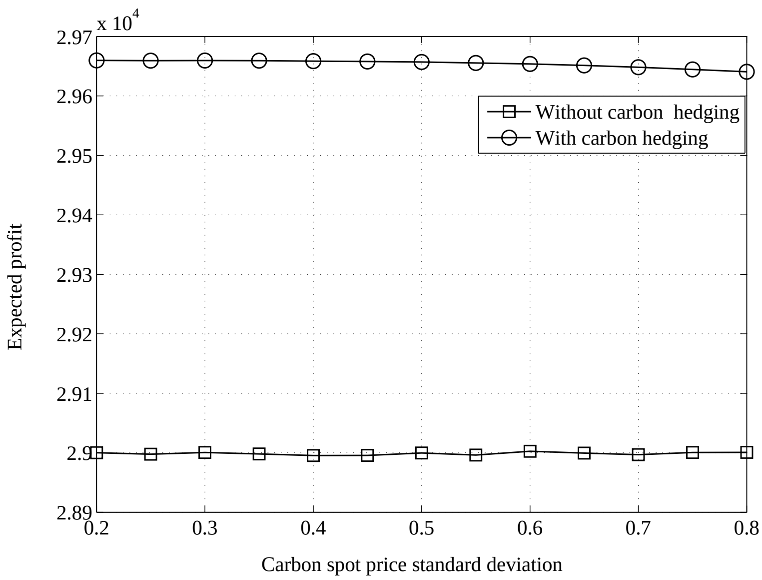

Figure 6.4: Expected profit of the firm with respect to the volatility of carbon prices, σ

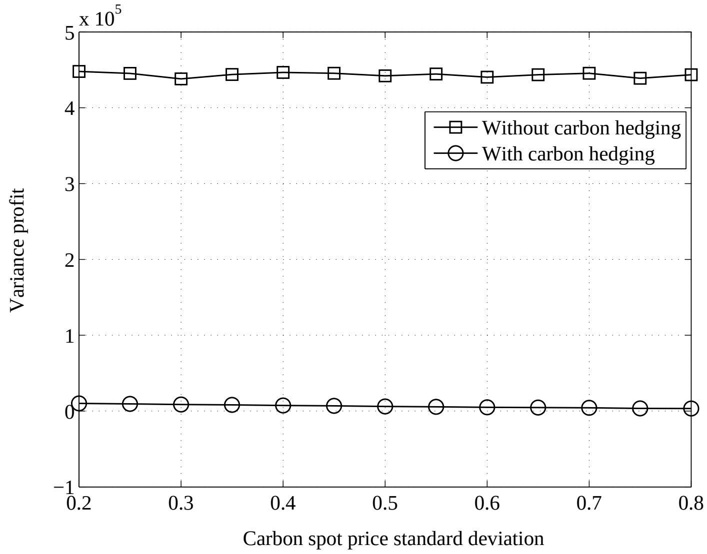

Figure 6.5: Variance of the profit with respect to the volatility of carbon prices, σ

6.3 Stochastic exchange rate, demand, and carbon prices

When both carbon call and currency put options are considered, the profit is expressed as follows:

$$\begin{aligned}\pi_{hc} =&(s_1 - c_1)Q_1 + (s_2 e - c_1)Q_2 - c_0 X \exp(rT) - \beta\delta_t(Q_1 + Q_2) - U_c - O_c \\ &+ Y \{\max(K_e - e, 0) - P_e \exp(rT)\} + X \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}\end{aligned} \tag{6.21}$$

where $(s_1 - c_1)$ is the unit profit from the local sale, $\min(\chi, D_1)$ is the number of quantity sold in the local market that is obtained from the minimum of the capacity, χ, and the local demand, D_1. The unit profit from the foreign sale is $(s_2 e - c_1)$, where the exchange rate, e, is stochastic, $c_0\chi$ is the total capacity cost, $\min(\max(\chi - D_1, 0), D_2)$ is the leftover quantity sold to foreign country, $\max(K_e - e, 0)$ is the currency put option payoff, $\max(\delta_t - K_c, 0)$ is the carbon call option payoff, U_c is the underage cost and O_c is the overage cost. The overage cost is calculated by multiplying the unit production cost and number of quantity left over in the storage. The left over quantity is different between the total production quantity, χ and total sales quantity in both market, $\min(\chi, D_1) + \min(\max(\chi - D_1, 0), D_2)$.

$$O_c = c_1(\chi - \min\,(\chi, D_1) + \min\,(\max(\chi - D_1, 0), D_2)). \tag{6.22}$$

The overage cost is calculated by sum of profit loss by not satisfying the local and foreign markets. Profit loss by local market is given by not satisfying the local demand. Which is given by unit profit gain from the local sale, which is $s_1 - c_1$. different between the local demand and sold quantity in local market is $D_1 - \min(\chi, D_1)$.

$$U_c = (s_1 - c_1)\{D_1 - \min(\chi, D_1)\} + (s_2 e - c_1)\{D_2 - \min(\max(\chi - D_1, 0), D_2)\} \tag{6.23}$$

The utility of the profit can be expressed as:

$$\mathrm{U}(\pi_{hc}) = \mathbb{E}(\pi_{hc}) - \gamma\mathbb{V}(\pi_{hc}) \tag{6.24}$$

In order to find the number of carbon call option, Equation (6.3) is differentiated with respect to Y as in Equation (6.3):

$$\frac{\partial U(\pi_{hc})}{\partial X} = \frac{\partial \mathbb{E}(\pi_{hc})}{\partial X} - \gamma\frac{\partial \mathbb{V}(\pi_{hc})}{\partial X} \tag{6.25}$$

Because of no-arbitrage risk-neutral assumption, $\frac{\partial \mathbb{E}(\pi_{hc})}{\partial X} = 0$. Therefore, maximizing the expected utility is equivalent to minimizing the variance.

6.3.1 Number of carbon call options

To calculate the optimal number of carbon call option X for the firm, we minimize the variation of profit.

$$\begin{aligned}
\mathbb{V}[\pi_{hc}] =& \mathbb{V}[(s_1 - c_1)Q_1] + \mathbb{V}[(s_2 e - c_1)Q_2] + \mathbb{V}[\beta(Q_1 + Q_2)\delta_t] + \mathbb{V}[U_c + O_c] \\
& + \mathbb{V}[Y\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] + 2\mathbf{cov}[(s_1 - c_1)Q_1, (s_2 e - c_1)Q_2] \\
& - 2\mathbf{cov}[(s_2 e - c_1)Q_2, \beta(Q_1 + Q_2)\delta_t] - 2\mathbf{cov}[\beta(Q_1 + Q_2)\delta_t, X\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\
& + 2\mathbf{cov}[X\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}, (s_1 - c_1)Q_1] \\
& + 2\mathbf{cov}[X\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}, (s_2 e - c_1)Q_2] - 2\mathbf{cov}[(s_1 - c_1)Q_1, (U_c + O_c)] \\
& - 2\mathbf{cov}[(s_2 e - c_1)Q_2, (U_c + O_c)] + 2\mathbf{cov}[(U_c + O_c), \beta(Q_1 + Q_2)\delta_t] \\
& - 2\mathbf{cov}[X\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}, U_c + O_c)]
\end{aligned} \tag{6.26}$$

Differentiating the variance of profit with respect to the number of call options,Y, is given by Equation (6.27).

$$\begin{aligned}
\frac{\partial \mathbb{V}[\pi_{hc}]}{\partial X} =& 2X\mathbb{E}[\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}^2] - 2\mathbb{E}[(Q_1 + Q_2)\delta_t \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\
& + 2\mathbb{E}[(s_2 e - c_1)Q_2 \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\
& + 2\mathbb{E}[(s_1 - c_1)Q_1 \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\
& - 2\mathbb{E}[(U_c + O_c)\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}]
\end{aligned} \tag{6.27}$$

Therefore optimal number of call options is given by following equation:

$$X = \frac{G}{H}, \tag{6.28}$$

where

$$\begin{aligned} G = \mathbb{E}[(Q_1 + Q_2)\delta_t \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\ - \mathbb{E}[(s_2 e - c_1) Q_2 \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\ - \mathbb{E}[(s_1 - c_1) Q_1 \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \\ - \mathbb{E}[(U_c + O_c) \{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}] \end{aligned} \tag{6.29}$$

$$H = \mathbb{E}[\{\max(\delta_t - K_c, 0) - P_c \exp(rT)\}^2] \tag{6.30}$$

In this case, local and foreign demands influence the jump size of the carbon spot price. We assume that the relation between the spot price and demand is given by following relationship:

$$p = b_1 \Psi(D_1) + (1 - b_1)\Theta(D_2), \tag{6.31}$$

where $0 \leq b_1 \leq 1$ and $\Psi(.), \Theta(.)$ are function of local and foreign demands. We also assume that, $\Psi(D_1) = (\frac{D_1 - a}{b - a})$ and $\Theta(D_2) = (\frac{D_2 - c}{d - c})$, where $a, b, c,$ and d are positive constant and satisfy the following conditions: $d < c, b < a$. Therefore the jump probability p is function of local and foreign demand. The expected profit for the global firm is given by Equation (6.32).

$$\begin{aligned} \mathbb{E}[\pi_{hc}] = & (s_1 - c_1) \; \mathbb{E}[Q_1] + \mathbb{E}[(s_2 e - c_1) Q_2] - c_0 \chi \exp(rT) \\ & - \mathbb{E}[U_c] - \mathbb{E}[O_c] - \beta \mathbb{E}[(Q_1 + Q_2)\delta_t] \end{aligned} \tag{6.32}$$

6.3.2 Number of currency put options

The optimal number of options, Y, is obtained by maximizing the utility of the profit, $U(\pi) = E(\pi) - \gamma V(\pi)$. Here, $E(\pi)$ is not dependent on X. Therefore, the optimal number of currency put option is obtained by minimizing the variance of profit. Assume that A_e and B_e are defined as follows:

$$\begin{aligned} A_e = & (s_2 - c_1) D_1 + (s_2 e - c_1) D_2 - c_0 \chi \exp(rT) \\ B_e = & Y \{\max(K - e, 0) - P_e \exp(rT)\} \end{aligned}$$

Then the variance of π_h is expressed as:

$$\begin{aligned}
\mathbb{V}_{[\pi_h]} =& \mathbb{V}\left[(s_2 - c_1)D_1 + (s_2 e - c_1)D_2 - c_0\chi \exp(rT) + Y\{\max(K - e, 0) - P_e \exp(rT)\}\right] \\
=& \mathbb{V}[A_e + B_e] \\
=& \mathbb{V}_{[A_e]} + V_{[B_e]} + 2\mathrm{cov}[A_e, B_e]
\end{aligned} \tag{6.33}$$

Now, $\mathbb{V}_{[A_e]}$, $V_{[B_e]}$, and $\mathrm{cov}[A_e, B_e]$ can be expanded as follows:

$$\begin{aligned}
\mathbb{V}_{[A_e]} =& \mathbb{E}\left[(s_1 - c_1)D_1 + (s_2 e - c_1)D_2 - c_0\chi \exp(rT)\right. \\
& \left. - \mathbb{E}\left[(s_1 - c_1)D_1 + (s_2 e - c_1)D_2 - c_0\chi \exp(rT)\right]\right] \\
\mathbb{V}_{[B_e]} =& Y^2 \mathbb{E}\left[\{\max(K - e, 0) - P_e\}^2\right] \\
\mathrm{cov}[A_e, B_e] =& \mathbb{E}\left[\{(s_2 - c_1)D_1 + (s_2 e - c_1)D_2 - c_0\chi \exp(rT)\right. \\
& \left. - \mathbb{E}[(s_2 - c_1)D_1 + (s_2 e - c_1)D_2 - c_0\chi \exp(rT)]\} Y\{\max(K - e, 0) - P_e \exp(rT)\}\right]
\end{aligned} \tag{6.34}$$

To find the optimal number of put options, the utility of profit is differentiated with respect to X. Since exchange rate and carbon prices are independent of each other, we individually deduce the number of carbon call options and the number of exchange rate put options. Hence, $\frac{\partial \mathbb{E}_{[\pi]}}{\partial Y} = 0$ and the variance of profit differentiated with respect to Y is given by Equation (6.35).

$$\frac{\partial \mathbb{V}_{[\pi]}}{\partial Y} = 2\mathbb{E}\left[\{A_e - \mathbb{E}[A_e]\}\{\max(K - e, 0) - P_e \exp(rT)\}\right] + 2Y\mathbb{E}\left[\{\max(K - e, 0) - P_e \exp(rT)\}^2\right] \tag{6.35}$$

From Equation (6.35), the optimal number of put option is obtained as Equation (6.36).

$$Y = \frac{\mathbb{E}\left[(\mathbb{E}[A_e] - A_e)\{\max(K - e, 0) - P_e \exp(rT)\}\right]}{\mathbb{E}\left[\{\max(K - e, 0) - P_e \exp(rT)\}^2\right]} \tag{6.36}$$

The second order derivation of the variance of the profit with respect to X is given by the Equation(6.37), which is always positive. This shows that the optimal number of option, X, reduces the variance of profit.

$$\frac{\partial^2 V_{[\pi]}}{\partial Y^2} = 2\mathbb{E}\left[\{\max(K - e, 0) - P_e \exp(rT)\}^2\right] \tag{6.37}$$

Equation(6.36) can be simplified as follows.

$$
\begin{aligned}
Y =& P/Q \\
P =& -\mathbb{E}\left[(A_e - \mathbb{E}[A_e])\{\max(K-e,0) - P_e \exp(rT)\}\right] \\
=& s_2 X_2 \mathbb{E}\left[(\mathbb{E}\left[e\right] - e)\max(K-e,0)\right] \\
=& s_2 X_2 \{\mathbb{E}\left[e\right] P_e \exp(rT) - K\int_0^K e f(e)\, de + \int_0^K e^2 f(e)\, de\} \\
Q =& \mathbb{E}\left[\{\max(K-e,0) - P_e \exp(rT)\}^2\right] \\
=& \mathbb{E}\left[\max(0,K-e)^2 + P_e^2 e^{2rT} - 2P_e \exp(rT)\max(0,K-e)\right] \\
=& K^2 P(e<K) + \int_0^K e^2 f(e)\, de - 2K\int_0^K e f(e)\, de - P_e \exp(rT) \\
=& K^2 P(e<K) + \int_0^K e^2 f(e)\, de - 2K\int_0^K e f(e)\, de - P_e^2 e^{2rT}
\end{aligned}
$$

From Equation(6.35), the optimal number of options X can be given by Equation (6.38)

$$
Y = \frac{s_2 X_2 [\mathbb{E}\left[e\right] P_e \exp(rT) - K\int_0^K e f(e)\, de + \int_0^K e^2 f(e)\, de]}{K^2 P(e<K) + \int_0^K e^2 f(e)\, de - 2K\int_0^K e f(e)\, de - P_e^2 e^{2rT}} \tag{6.38}
$$

6.3.3 Numerical results: Stochastic exchange rate, demands, and carbon prices

From Figure 6.6 four significant observations can be deduced: (1) It is observed that the expected profits without overage and underage costs (see the lines with square and circle markers in Figure 6.6) are higher than the profits with underage and overage costs. The firm with no obligation to comply with the overage and underage costs enjoys more freedom on its production allocation, that causes to obtain a higher expected profit; (2) options does not play a significant role in gaining the expected profit. That is why, the curves for the expected profits with and without options merge together; (3) Below the optimal production allocation of 850, the expected profit drops. The reason is that at a lower capacity allocation, the firm can not meet the demand fluctuation effectively, which results in higher overage and underage costs and consequently the profit goes downward; (4) if overage and underage costs are not considered, the firm tends to overestimate the expected profit.

The variance of the profit with respect to production allocation is shown in Figure 6.7. Following observations can be drawn from this figure: (1) at a lower capacity allocation, for example, less than 550, when overage and underage costs are not considered, the variance of the profit turns out to be lower. However, with overage and underage costs, the variance of the profit is higher; (2) at a higher capacity allocation, for instance, more than 900, the overage and underage costs do not

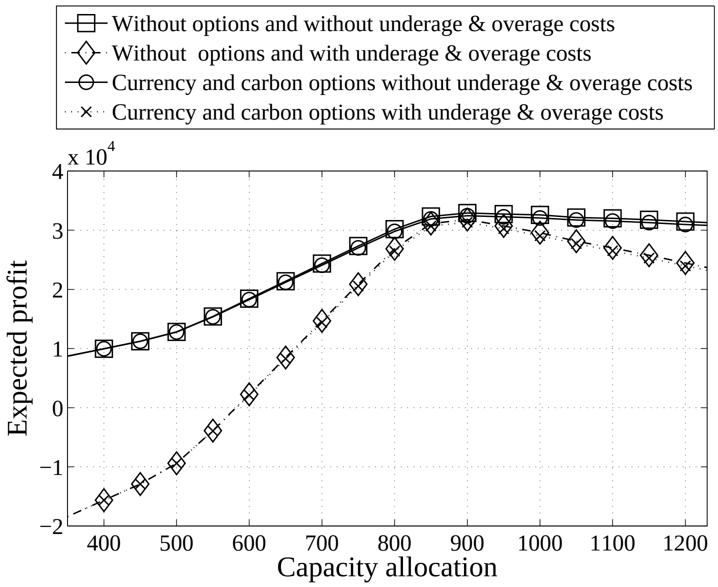

Figure 6.6: Expected profit of the firm with respect to production allocation, χ

affect the variance significantly; (3) at a higher capacity allocation, the variance of the profit is influenced by the utilization of options. It is observed that options reduce the profit variance at a higher capacity allocation; (4) At a lower capacity allocation, the options do not play a significant role in reducing the variance of the profit; and (5) below the capacity allocation of 600, overage and underage costs affect the variance of the profit, and above the capacity allocation of 600, options influence the variance.

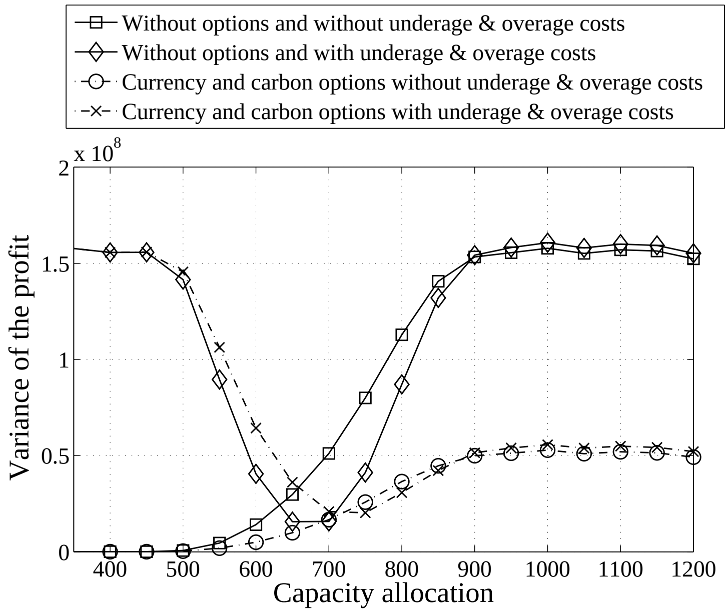

Figure 6.7: Profit variance of the firm with respect to production allocation, χ

Chapter 7

Conclusion

This thesis focuses on the analysis of optimal production allocation problems for a multi-national firm under exchange rate, demand, and carbon prices uncertainties. The firm possesses the ability to adjust its production capacity by having different levels of capacity. The uncertain behavior of these variables – exchange rates, demand, and/or carbon allowance prices, alone or combined, can significantly impact the profitability of the firm, and can cause the firm plunge into deep financial crisis. In order to encounter these uncertainties, the firm can adopt an real options approach in the form of capacity adjustment flexibility, or can opt to utilize financial instruments like forward contracts and options, or can adopt an approach combining both real and financial options. This thesis analyzes a relative comparison these approaches under different scenarios and finds the best approach that maximizes a mean-variance function for a risk-averse manager.

When the firm undergoes exchange rates uncertainties with known demands, the firm can utilize forward contracts. The stochastic behavior of exchange rates is modeled as a random process. The aim is to decide how many contracts to buy and how many production quantity to allocate optimally to both local and foreign markets that maximizes the expected utility of the firm. Four different scenarios are considered: fully flexible, domestic production, domestic production with foreign subsidiary, and foreign production with domestic subsidiary systems. Results show that the firm can hedge the risk from the exchange rate uncertainties by utilizing forward contracts.

In a multi-period setting, the firm can also choose to buy currency put options to hedge exchange rate uncertainties and exercise the options when exchange rates drop below a strike exchange rate. The objective is to decide how many options to buy at each period of time that maximizes the mean-variance utility function subject to the allocation of optimal production capacities to both local and foreign markets. The firm utilizes both capacity flexibility and currency put options. Investigation of different scenarios, for instance, fixed capacity system, fixed capacity system with currency put options, flexible capacity system, and flexible capacity system with currency put options, show that the integration of real and financial options benefits the firm most.

A product can have a stochastic life cycle, in which the demand for the product may rise in a growth regime, and then can fall after some time in a decay regime. A firm may function under stricter emission regulations and also produces a product that has a regime-switching behavior. This thesis also examines this case in a mutil-period setting. The correlated behavior of regime-switching product life cycle along with a stochastic geometric Brownian motion behavior of carbon allowance prices is modeled in a lattice approach.

A firm experiences uncertainties from demand, carbon prices, and exchange rates simultaneously. Moreover, the firm may face overage (too many production) and underage (too low production) costs. The firm opts for both carbon call options and currency put options. The aim is to decide how many call and put options to buy considering overage and underage costs. Results show that there exists an optimal capacity allocation below which the expected profit drops sharply, if overage and underage costs are considered. If the firm does not consider the overage and underage costs, the firm overvalues its expected value.

7.1 Summary of contributions

The contributions of this thesis include four aspects. Firstly, this thesis examines production allocation problems in a multi-period setting for a multi-national firm under exchange rate uncertainties. To the best of the author's knowledge, this multi-period production allocation approach for a multinational firm under exchange rate uncertainties is the first of its kind.

Secondly, to the best of the author's knowledge, the literature did not study the use of carbon options in hedging uncertainties associated with carbon allowance prices. This thesis analyzes the use of carbon call options to hedge the carbon emission risk. Carbon prices are modeled in the form of both geometric Brownian motion and geometric Brownian motion with jump processes.

Thirdly, this thesis investigates the stochastic product life cycle under environment emission regulations in which the stochastic carbon emission prices is modeled as a geometric Brownian motion. The correlated stochastic behavior of regime-switching demand along with carbon price uncertainties is modeled in a novel lattice approach. This approach is also the first of its kind.

Fourthly, this thesis analyzes the integrated effect of demand, currency and carbon uncertainties for a multi-national firm. Moreover, the costs of overage when production exceeds demand and underage when production falls short of demand. This thesis addresses the overage and underage costs in the case of stochastic demand.

7.2 Challenges, limitations, and future research

One challenge is to modeling exchange rates appropriately. The right modeling of a stochastic process is essential to deduce the correct expected profit and the variance of the profit. This thesis assumes that the exchange rates follow a geometric Brownian motion. However, Bollen et al. (2000) model exchange rates as a regime-switching model. There is probably no unique stochastic model that fits all time frames. The similar statement is also true for the carbon allowance prices uncertainties. Extracting model parameters is another challenge. In this thesis, we use jump parameters from Kou (2002). Daskalakis et al. (2009) analyze a total of 273 data from European Energy Exchange (EEX) carbon prices from March, 2005 to April, 2006 and find that prices vary from 10 to 29 Euros per EUA with an average of 21.66 Euros. They observe two extremes of about 15% in magnitude in the daily movements. They find a leptokurtic distribution with positively skewed returns. The jump parameters are derived from the maximum likelihood (ML) approach.

Model reproducibility and model validation are two concerns for any research work on modeling and simulation. For model reproducibly, this thesis includes detailed numerical examples that show step by step procedure to implement the proposed models. Detailed calculations are provided for three period problems with adequate illustrations and figures. For instance, jump/step sizes, branch probabilities, profits, and corresponding expected values at each time period are clearly explained. Any reader can easily verify these models using real life data. with a different set of data anyone can calibrate, verify, and reproduce these models. The programming languages used in this thesis are MATLAB and C. For model validation, real world exchange rates data are utilized to extract geometric Brownian motion parameters. Moreover, parameters for demand and carbon prices uncertainties used in this thesis are in accordance with similar research publications in the literature. The developed models are generic ones; anyone can extract parameter values from real-world data, plug these parameters into the proposed models, and can assess the expected profit of a manufacturing firm.

The current research can be extended in many ways. One could be to consider a supply chain model in which production and sales occur in more than two countries. The network would be an intricate one and the hurdle would be to model correlated exchange rates from two different countries and to find an appropriate hedging strategy. Another way to extend the research is to consider options along with forward contracts to hedge exchange rate risks. The objective is to compare results among the scenarios when only forward contracts are used, only options are used, and when both of these financial tools are used. Considering stochastic product life cycles along with stochastic exchange rate in a multi-period time frame could be another interesting extension.

Since environmental regulations on manufacturing companies are becoming stricter day by day, incorporating the cost of the carbon dioxide emission into this model could be another challenge. Another extension of the research could be to consider three uncertainties, demand, carbon prices, and exchange rate, simultaneously over a multi-period planning horizon. The challenge is to model the correlated behavior of these variables and to determine the expected value of the firm.

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