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# Wavelet-based image compression using mathematical morphology and self organization feature map

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# WAVELET-BASED IMAGE COMPRESSION USING MATHEMATICAL MORPHOLOGY AND SELF ORGANIZING FEATURE MAP

By  
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A thesis  
submitted to Ryerson University  
in Partial fulfilment of the  
requirement for the degree of  
Master of Applied Science  
in the program of  
Electrical and Computer Engineering.

Toronto, Ontario, Canada, 2005

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# **ABSTRACT**

## **Wavelet-Based Image Compression using Mathematical Morphology and Self Organizing Feature Map**

**©Abdul Adeel Mohammed 2005**

**Master of Applied Science  
Department of Electrical and Computer Engineering  
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Image compression using transform coding technique has been widely used in practice. However, wavelet transform is the only method that provides both spatial and frequency domain information. These properties of wavelet transform greatly help in identification and selection of significant and non-significant coefficients from amongst the wavelet coefficients. Wavelet transform based image compression result in an improved compression ratio as well as image quality and thus both the significant coefficients and their positions within an image are encoded and transmitted. In this thesis a wavelet based image compression system is presented that uses mathematical morphology and self organizing feature map (MMSOFM). The significance map is preprocessed using mathematical morphology operators to identify and create clusters of significant coefficients. A self-organizing feature map (SOFM) is then used to encode the significance map. Experimental results are shown and comparisons with JPEG and JPEG 2000 are made to emphasize the results of this compression system.

# Acknowledgment

I would like to thank my supervisor Prof. Javad Alirezaie for his encouragement, guidance and support throughout my research and in writing of this manuscript. This work would have been impossible without his kindness, patience and feedback.

I would also like to thank School of Graduate Studies of Ryerson University for providing Graduate Student Scholarship and helping me in securing Ontario Graduate Scholarship (OGS).

Finally, I would like to thank my family especially my mother, my brother and my wife for their continuous support during this research.

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# Chapter 1

## Introduction

### 1.1 Need for image compression

THE advent of high speed computing devices and rapid development in the field of communication has created a tremendous opportunity for various computer based image applications. The amount of data required to store a digital image is continually increasing and overwhelming the storage devices. Another important issue in addition to high storage requirements is the transmission of image through high-bandwidth and low-bandwidth channels. A well designed data compression system is required to alleviate these problems. Data compression is a key to the rapid progress made in the field of information technology. It is highly impractical to put uncompressed images, audio and video on websites.

Image compression is the representation of an image in digital form with as few bits as possible while maintaining an acceptable level of image quality [2]. In digital form images are represented as binary numbers with very large bytes of data sets. For example, a small  $4'' \times 4''$  color picture scanned at 300 dpi with 24bits/pixel of true color will result in a file size of more than 4 mega bytes. This picture typically

requires a high storage space and a transmission time of more than a minute through a typical ISDN channel. There are two ways to solve this problem in a distributed environment i.e. to increase the channel bandwidth or to compress the image. High costs associated with high bandwidth channels makes it less attractive when compared to image compression.

## 1.2 Compression techniques

Over the last decade a variety of compression algorithms have been developed. A compression algorithm has a corresponding decompression algorithm in order to reconstruct the original image. A compression algorithm takes an input image  $\mathbf{X}$  to generate an output image  $\mathbf{X}_c$  with fewer bits, and a reconstruction algorithm operates on the compressed image  $\mathbf{X}_c$  to reconstruct  $\mathbf{Y}$ . Based on the reconstruction requirements of an image, the compression schemes can be divided into two broad categories, namely lossless compression and lossy compression.

### 1.2.1 Lossless compression

A lossless compression technique is one in which the reconstructed image  $\mathbf{Y}$  is identical to input image  $\mathbf{X}$ . When an image is compressed losslessly, the original image can be completely recovered from the compressed one. Such a compression techniques is employed in situations where any form of degradation in image quality is highly undesirable (example Medical Images). However a lossless compression system can achieve a limited amount of compression. Examples of lossless techniques are run-length coding, huffman coding, Lempel-Ziv-Welch (LZW) algorithm and arithmetic coding.

### 1.2.2 Lossy compression

A lossy compression technique as the name suggests is one in which there is a loss of information and the reconstructed image is different from the original. For example, there are several applications where it is acceptable for a reconstructed image to be different from the original as long as the differences do not result in annoying artifacts. Most current image compression algorithms fall into one of the three categories: Vector Quantization, Predictive Coding and Transform Coding [14], [15]. Vector quantization and predictive coding image compression techniques are not as competitive as transform coding techniques used in modern transform based lossy compressors, since they have inferior compression ratios and low peak signal to noise ratio (PSNR) [3].

#### Transform based compression

The simplest way of performing image compression is through the use of transform coding techniques [4], [5], [6],[7], which has been an active area of research for over a decade. Transform based coding techniques work by statistically decor-relating the information contained in the image so that the redundant data can be discarded [8]. Therefore a "dense" signal is converted to a "sparse" signal and most of the information is concentrated on a few significant coefficients. Transform based compression techniques allow efficient transmission, storage and display of images that would otherwise be impractical.

DCT (Discrete cosine transform)[10] based transform coding method was first applied for image compression by Ahmed, Natarajan and Rao [50]. It is a popular transform used by JPEG (joint photography expert group) image compression



standard for lossy compression of images. In JPEG compression image is divided into series of blocks, converted from spatial domain to frequency domain using a 2-D DCT, quantized and sent to a lossless entropy encoder . Due to the blocked nature of input correlation across the block boundaries cannot be eliminated. This results in noticeable and annoying "blocking artifacts", particularly at low bit rates.

More recently, the wavelet transform has emerged as a cutting edge technology, within the field of image compression. Wavelet-based coding [6] provide substantial improvement in picture quality at higher compression ratio. Over the past few years, a variety of powerful and sophisticated wavelet-based schemes for image compression have been developed and implemented. Wavelet based technique does not divide the image into blocks, but analyzes the whole image at a time. This prevents any blocking artifact within the reconstructed image and it's efficiency in terms of compression ratio/PSNR is much better than standard JPEG.

### 1.3 Summary of contributions

In order to improve the quality of compressed image, a hybrid method that uses wavelet transform, self organizing feature map and mathematical morphology is proposed. The proposed method eliminates all forms of redundancies: inter-pixel, psycho-visual and coding redundancy by using an improved decor-relating transform, quantizing both the significance map and significant coefficients and by using huffman coding respectively.

In the proposed compression algorithm wavelet transform was used to perform decor-relation of the input image and was preferred over the discrete cosine transform since it eliminates the inherent blocking effect produced by the discrete cosine

transform at high compression ratios. The wavelet filter used for performing the wavelet transform is a smooth biorthogonal wavelet filter that possesses symmetry so that the wavelet transform is implemented using mirror boundary conditions to reduce boundary artifacts. In addition to its symmetry the wavelet filter used is smooth so that the smoothness within an image is preserved.

Significance map is created by thresholding the wavelet coefficients using a hard threshold. These coefficients are obtained by filtering the image using a pair of scaling (low-pass filter) and wavelet (high-pass filter) functions. The significance map is processed using mathematical morphology operators to perform a clustering operation. Clustering is done to create clusters and to emphasize the significant coefficients from amongst the wavelet coefficients. The clustered significance map helps in preserving the fine details within an image and improves the peak signal to noise ratio of compressed image.

The clustered significance map is vector quantized using a self organizing feature map (SOFM). A SOFM was preferred over a classical LBG algorithm [41] to perform vector quantization due to its adaptability and its ability to preserve the input topology. In addition to this a SOFM is computationally less complex and less sensitive to initial codebook design than a LBG vector quantizer. The vector quantized significance map is used to extract the significant coefficients which are scalar quantized. The significant coefficient vector as well as the significance map are huffman encoded and the result is transmitted.

The proposed method is well suited for compressing medical images like CT images and MRI images due to their textual similarity. This similarity could be exploited since a self organizing feature map is used to perform vector quantization and thus improved image quality and reduced processing time are achieved.

- Proposing a new wavelet based image compression algorithm by implementing a Self Organizing feature map to perform vector quantization.
- Implementing a new biorthogonal spline wavelet filter to enrich the quality of compressed image.
- Achieving promising Peak Signal to Noise Ratio (PSNR) at a specified compression ratio.

## 1.4 Organization of thesis

The remainder of this thesis consists of 4 chapters which are organized as follows:

**Chapter 2:** *Literature Review*, discusses some of the methods used for image compression.

**Chapter 3:** *Wavelet and Neural Networks*, covers the mathematical properties of wavelets. Several types of wavelets are discussed, including Haar, Daubechies and biorthogonal spline wavelets. It also discusses how wavelets are applied to image compression and reviews how neural networks are used as tools for image compression and more specifically the use of self organizing feature map in this work as a vector quantizer.

**Chapter 4:** *Proposed compression algorithm*, details the proposed algorithm for image compression, summarizes the results and discusses the advantages of using this technique.

**Chapter 5:** *Conclusions and Future Work*, some considerations on how to enhance the work in the future are included.

# Chapter 2

## Literature Review

**I**MAGE compression has been a popular area of research for over two decades. During this period of time several image compression algorithms have been proposed and implemented. Some of the popular and recently proposed methods are reviewed in the following section. This chapter has been divided into three parts based on the approach used for image compression. The first section reviews image compression methods which are based on transform coding, second sections reviews image compression methods based on wavelet transform and finally image compression algorithms based on neural network are reviewed.

### 2.1 Image Compression based on Transform Coding

Transform based image compression is one of the most widely used image compression technique. The transformations operate on an image to produce a set of coefficients. A small subset of these coefficients is chosen and is sufficient to reconstruct an image with minimum distortion. In this section we will briefly review some of the important transform based algorithms proposed for image compression.

### 2.1.1 Compression using hybrid DPCM/DCT and TCQ

This method of image compression for hyperspectral images was proposed by Abousleman [16]. The compression system implemented in this work for hyperspectral imagery compression utilized trellis coded quantization (TCQ) [17]. In-order to decorrelate the data and achieve compression, differential pulse code modulation(DPCM) and discrete cosine transform (DCT) were used. Specifically, DPCM was used to decorrelate the hyperspectral data and a two-dimensional DCT was used for spatial decor-relation. Entropy constrained codebooks were designed using a modified version of the generalized Llyod algorithm.

The coder achieved a compression ratio of 70:1 with an average peak signal to noise ratio (PSNR) of 40.29 dB. The hybrid system proposed by Abousleman *et. al.* is of moderate complexity with the majority of the computations being done to evaluate the 2D DCT. In addition to improved average PSNR, the major advantage of using this system is the requirement of small amount of memory for both encoding and decoding process. The encoder requires only two bands to encode the entire hyperspectral sequence. Therefore the hybrid coder is well suited for sensor based applications.

### 2.1.2 Adaptive transform approach

Luc Vandendorpe, Benoit Maison and Fabrice Labeau [18] proposed an adaptive transform coding approach for image compression. Unlike conventional coding algorithms which require transmission of information regarding the nature of transform and the shape of each region, this method requires no data overheads. The proposed method continuously adapts the transform operator exclusively by means of data available at both the encoder and the decoder.

The adaptive transform was successfully tested on several images and the transform coefficients were encoded using uniform quantization, zig-zag scanning, run-length coding and entropy coding. Application of the proposed method for image compression resulted in a 20% improvement in bit-rate over conventional DCT. A greater coding efficiency than classical fixed transform method and its ability to encode motion compensated prediction error images produced by a video compression scheme are achieved.

### 2.1.3 Hybrid KLT-SVD coding

A hybrid image compression system based on Karhunen-Loeve transform (KLT) [19] and singular value decomposition (SVD) [20],[21], [22], [23] was proposed by Patrick Waldemar and Tor A. Ramstad [24]. This method proposes a transform adaptation technique for transform coding of images in order to exploit the variation in local statistics within an image. The method takes advantage of the relationship between KLT and SVD and their energy compaction properties. Experimental results indicated that this method outperformed regular KLT with better reconstruction of compressed image. However, the cost of using the hybrid approach in terms of bits was quite high when compared to KLT. Therefore a switching scheme between KLT and hybrid KLT-SVD transform is implemented in order to enhance the performance and reduce bit-rate.

### 2.1.4 Adaptive block-size transform coding

J.Bracamonte *et. al.* [25] proposed an adaptive block-size transform coding for image compression based on sequential JPEG (joint photographic expert group) algorithm with minimum information overhead. The proposed method is adaptive in the sense

that the image is divided into blocks of different sizes:  $N \times N$  and  $2N \times 2N$ . Input image is divided into different blocks and each block is categorized based on its image activity. Based on the block classification either an  $N$  point or a single  $2N$  point 2D DCT is applied on each block. The proposed algorithm takes advantage of the presence of large uniform regions within an image, which can be encoded as a single large unit instead of 4 smaller units as is done in a traditional fixed block-size transform.

The proposed adaptive block-size algorithm showed a significant improvement in compression ratio with respect to the non-adaptive transform. Blocks are classified as either a 0 block or a 1 block depending on the image activity. Higher the number of 0 blocks within an image, higher improvement in compression ratio is achieved compared to non-adaptive scheme. This method results in minimum information overhead, a significant reduction of the computational complexity and a coding efficiency that largely outperforms its non-adaptive counterpart.

### 2.1.5 Adaptive DCT coding with edge based classification

Discrete cosine transform (DCT) based coding is an efficient means of image compression coding. A number of research has been dedicated in order to improve efficiency of the DCT based coder. Itoh *et. al.* [26] proposed an adaptive DCT coding based on edge classification. The scheme is designed to correctly exploit the correlation between edge direction and distribution of DCT coefficients. The method works by first extracting the edges from an image and later an optimal block-size and scanning order are determined for each block based on the extracted edges. Therefor an adaptive DCT encoder which takes account of local variations within an image is achieved. Experimental results have indicated that the proposed method clearly outperforms other conventional methods [27] in terms of coding efficiency. Block diagram of the

proposed adaptive method is shown in Figure 2.1.

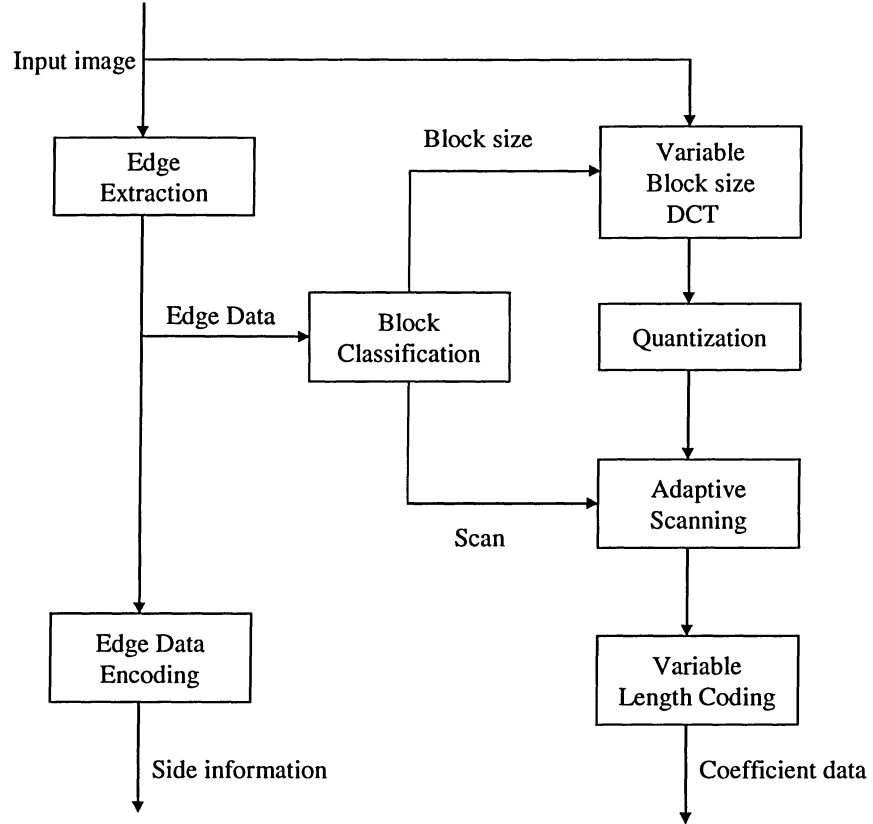


Figure 2.1: Block diagram of adaptive DCT coder

### 2.1.6 Dequantization of DCT based transform coding

S.Moon-Ho Song and Gunho Lee [28] proposed a new dequantization scheme for discrete cosine transform based encoding. The proposed algorithm is an improvement over the work done by Prost *et. al.* [29] and Philips *et. al.* [30]. Prost proposed



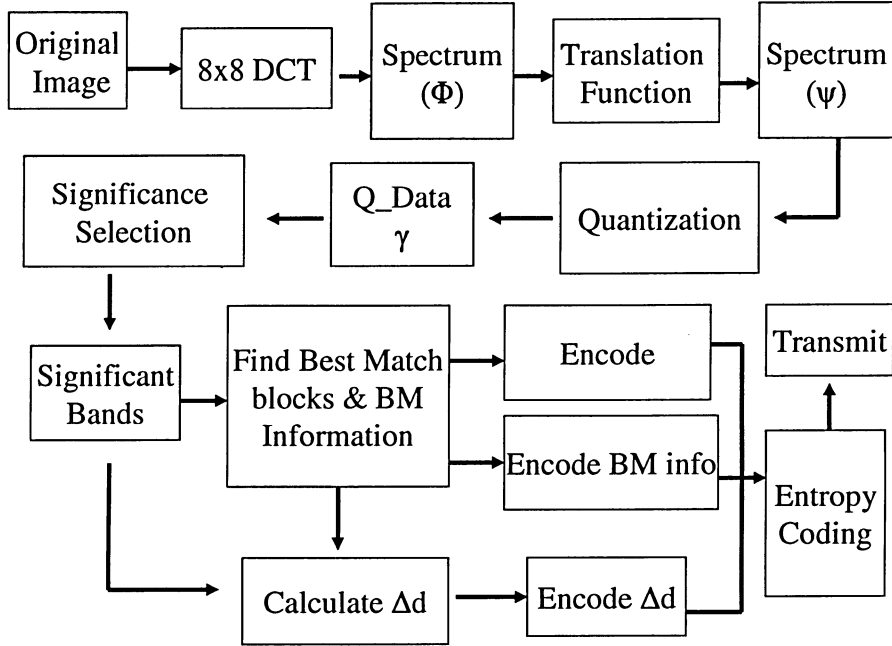
a dequantization scheme that modifies the quantization matrix used at the decoder i.e. the encoder uses one dequantization matrix and it modifies and sends another dequantization matrix at the decoder and Phillips later corrected the proposal. This approach for dequantization deviates from using just a different quantization matrix. The proposed dequantizer guarantees the mapping of quantized DCT coefficients to within  $k$  (quantizer spacing/2). This is achieved through a built-in non-linearity in the proposed iterative algorithm.

The performance of the proposed regularized dequantizer was evaluated and comparison to the standard JPEG approach. The regularized approach consistently provides higher PSNR level for all values of quality factor (QF). Higher the quantization step size, larger is the performance gain of the regularized dequantizer over the conventional dequantizer. Although the improvement in the actual PSNR values are typically in the range of 1dB but the improvement in visual quality is clearly evident.

### **2.1.7 Compression using spectral similarity in DCT**

Medical images are compressed before transmission and storage due to bandwidth and storage limitations. Compression reduces image fidelity especially at low bit rate (high compression). Reconstructed images suffer from blocking artifacts and the image quality will be severely degraded. Yung-Gi Wu and Shen-Chuan [31] proposed a simple strategy to increase compression ratio with small computational overhead and excellent decoded quality. Block diagram of the proposed algorithm is shown in Figure 2.2.

Application of this method to a wide range of medical images demonstrate that the proposed method achieves better performance when compared to other existing



**Figure 2.2:** Image compression system using spectral similarity in DCT

transform coding methods such as JPEG in terms of bit-rate and image quality. Although all medical images showed an improvement in PSNR in comparison with JPEG, angiogram image achieved a gain of about 13.5dB and PSNR gain for other medical images was around 4-8dB.

## 2.2 Image Compression using Neural Network

Image compression using neural networks have been an active area of research for over two decades. During this period of time numerous neural network based image compression algorithms have been proposed. Important image compression algorithms based on neural network will be reviewed in this section.

### 2.2.1 Predictive vector quantization

Robert Cierniak and Leszek Rutkowski [32] proposed a predictive vector quantization (PVQ) algorithm based on competitive neural networks and optimal linear predictors. The proposed algorithm is an improvement over their previous work [33]. In this method a semi-closed loop PVQ is implemented using a combination of vector quantization [14] and traditional differential pulse code modulation. This method achieves better compression ratio and lower mean square error(MSE) values when compared with an open-loop PVQ.

### 2.2.2 Compression by Self-Organized Kohonen Map

Compression scheme for digital still images using the Kohonen's neural network algorithm [35] was proposed by C.Amerijckx *et. al.* [34]. Kohonen's feature map is used for vector quantization and to realize a mapping between input and output space that preserves topology. After vectorization of the input image, discrete cosine transform is applied on the resulting vectors and the result is low pass filtered and vector quantized by kohonen's map. Block diagram of the proposed system is shown in Figure 2.3.

Images compressed with this compression scheme (Kohonen) achieve better PSNR when compared to JPEG for compression rates over 30. Although the difference in PSNR is not great but from a visual point of view images compressed with this method are much better than that of JPEG compressed images.

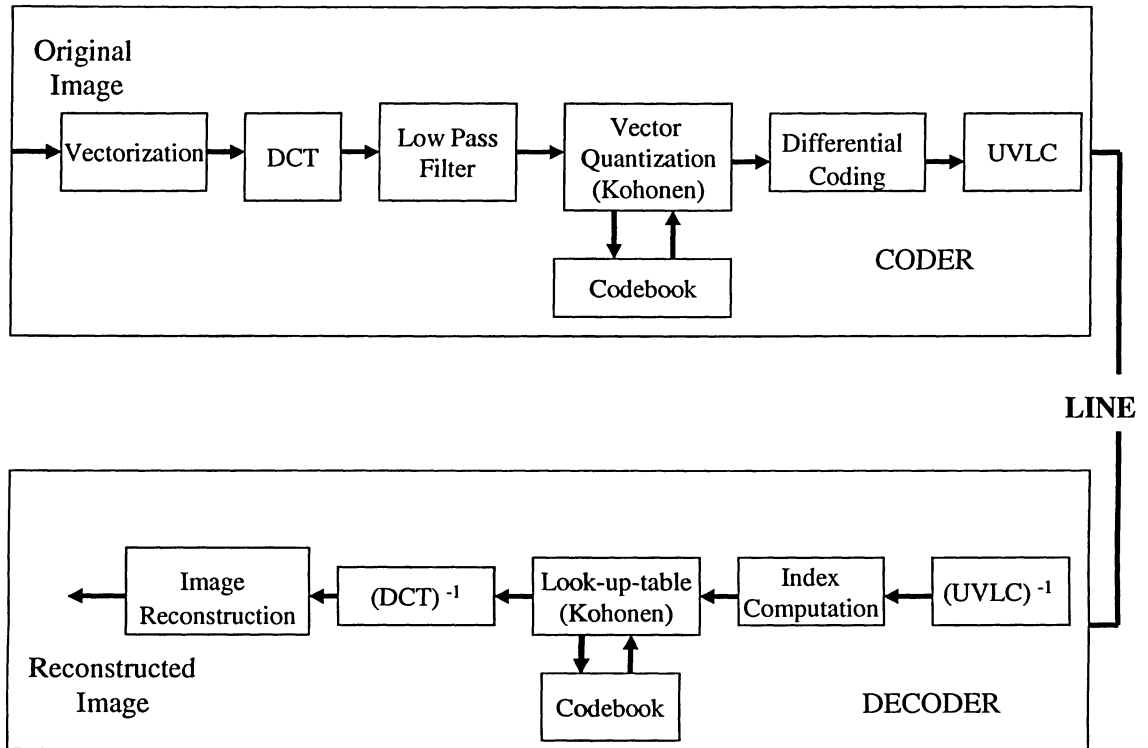


Figure 2.3: Block diagram of compression system using SOFM

### 2.2.3 Compression using modular differential pulse code modulation

S.A.Rizvi and N.M.Nasrabadi [37] proposed a new lossless image compression method called modular differential pulse code modulation (MDPCM). The proposed method consists of a vector quantizer classifier in conjunction with several neural network class predictors. The classifier predicts the class of the current pixel by using information regarding the class of four previously predicted pixels.

Performance of the compression algorithm varies with the number of predictors used. The modular predictors is implemented using one, two, three and four predictors

respectively. The performance of the MDPCM improves with an increase in the number of predictor from one to two. However adding more predictors has negligible effect on the performance of the modular predictor.

The proposed modular differential pulse code modulation outperforms conventional JPEG and achieves a bit-rate savings of almost 10% percent. Table 2.1 compares the results obtained with this method and JPEG.

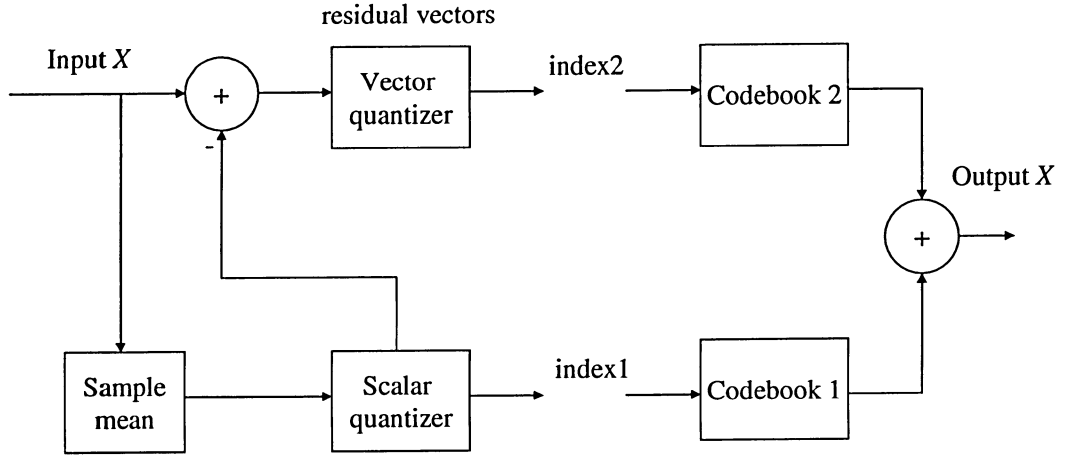
Image	Modular DPCM	Lossless JPEG
Lena	4.32 bpp	4.69 bpp
Boats	4.33 bpp	4.90 bpp
Goldhill	4.78 bpp	5.13 bpp
Average	4.48 bpp	4.98 bpp

**Table 2.1:** Performance comparison of MDPCM and JPEG

#### 2.2.4 Edge preserving image compression using neural networks

An edge preserving image compression technique based on unsupervised competitive neural network is proposed by Dong-Chul Park and Young-June Woo [37]. The proposed technique named weighted centroid neural network (WCNN) utilizes the characteristics of image blocks in and around the edges. Mean/residual vector quantization is used as the framework for the proposed technique. Block diagram of a mean/residual vector quantizer is as shown in the following Figure 2.4.

Edge strength of an image block is used as a tool to allocate code-vectors in the proposed WCNN. The proposed WCNN allocates more code-vectors to the image regions that contain edges and less code-vectors to those image regions that do not



**Figure 2.4:** Block diagram of mean/residual vector quantizer

contain edges. This adaptability in the allocation of code-vectors to different image regions results in a reconstructed image with improved edge characteristics than those obtained with a self organizing feature map and adaptive self organizing feature map.

### 2.2.5 Variable-rate residual vector quantizer for image compression

Venkatraman *et. al.* [38] proposed a variable rate residual vector quantizer to encode background information. The proposed algorithm is simple and elegant and is specifically designed to encode images with rich content i.e. synthetic aperture radar (SAR) image. Experimental results indicated that the variable-block-size vector quantizer preserves the texture and proves extremely useful for encoding background information that is necessary to establish context for target areas. The proposed encoding technique has numerical performance similar to Said and Pearlman wavelet-based encoder [39] and good qualitative performance. This technique also offers a logi-

cal ordering of the objects and very low decoder complexity. PSNR of the image compressed with this method is similar to that achieved using Said and Pearlman wavelet based coder but the variable vector quantizer based encoder represents the background texture with higher fidelity.

### 2.2.6 Image compression by hierarchical self organizing feature map

Dynamic hierarchical self organizing feature map (DHSOM) was proposed by D.Neto *et. al.* [40]. This algorithm is used for code-book design in vector quantization and used for image compression. The tree structured approach for code-book design is used for reducing high computational efforts in training and coding phase.

Application of the proposed DHSOM method for image compression have resulted in significant savings in training times when compared to other traditional algorithms based on Linde-Buzo-Gray [41] and self organizing feature map [35]. Although there is a small loss of quality in the compressed image when compared to other methods but the savings in processing time compensates for this loss. Table 2.2 compares the training time and PSNR values for different methods.

Method Used	Lena	PSNR: Lena	Zelda	PSNR: Zelda
LBG	207.57	27.5	208.49	29.76
Unidimensional SOM	255.68	27.53	260.40	29.69
DHSOM	57.49	26.7	58.82	29.06

**Table 2.2:** Comparison of training times and PSNR for different methods

## 2.3 Image compression using wavelets

Limitations in the application of Discrete cosine transform (DCT) for image compression have caused an inclination towards the use of Discrete Wavelet Transform (DWT). DWT can be efficiently used in image coding applications because of its data reduction capability. DWT have some important properties which makes it a better choice for image compression than DCT. In a DWT based compression system the entire image is transformed and compressed as a single data object rather than on a block by block (as in DCT based system) basis. DWT have higher decorrelation and energy compression efficiency and thus it provides better image quality at higher compression ratios. In this section we will briefly discuss some of the recently proposed wavelet based image compression algorithms.

### 2.3.1 Compression using shift-invariant wavelet

A new method of wavelet based image compression using shift-invariant dyadic wavelet filter was proposed by Y. Hui *et. al.* [42]. It has been shown in [43], [44] that shift-invariant(SI) DWT provides better energy compaction, a property extremely important for image coding applications. Several methods based on the best-basis-selection approach have been proposed to provide SI DWT. These methods are signal dependent and obtain shift-invariant wavelet transform for image compression by finding a decomposition path that minimizes shift-variance. The proposed shift-invariant wavelet transform method for image compression is independent of the input image and has better shift-invariant property compared with the conventional dyadic wavelet transform. Experimental results have indicated that the application of proposed method for image compression improved both the objective and subjective



quality of compressed image compared to the conventional wavelet transform based coding.

### **2.3.2 Line-based wavelet image compression**

A reduced memory line-based wavelet image compression algorithm was proposed by Christos Chrysafis and Antonio Ortega [45]. This approach is "line-based" in the sense that the images are read on a line by line basis and only the minimum required number of lines are kept in memory. This in effect reduces the memory requirements of the system with significant loss in performance. In this method a context-based encoder which does not need to be synchronized with the decoder and requires no global information is proposed and implemented. It stores only a local set of wavelet coefficients and uses it to encode the image.

Experimental results have indicated that this entropy coding algorithm works well with very low memory in combination with line-based transform. It also shows that its performance can be compared to the state of the art image coders at a fraction of their memory utilization. PSNR obtained with this method is similar to that achieved using Said and Pearlman's set partitioning in hierarchical tree (SPIHT) with a 10 fold memory savings. Line-based transforms implemented in this work have been incorporated into the JPEG 2000 verification model.

### **2.3.3 Image compression using quadtree approach**

Adrian Munteanu *et. al.* [46] proposed a new wavelet-based embedded compression algorithm. This algorithm supports lossy/lossless coding, quality scalability and region-of-interest coding. It also exploits the intra-band dependency and uses

a quadtree-based approach to encode the significance map. This technique is well suited for tele-medicine applications that require fast interactive handling of large image sets over networks with limited bandwidth.

Experimental results indicated that the proposed method outperformed SPIHT and lossless JPEG in terms of PSNR for practically all medical images and outperformed SPIHT and EZW (embedded zero wavelet coder) for natural images (Lena, Barbara). Although the PSNR gain is only 1dB but the proposed method offers region of interest (ROI) based encoding which is important for coding medical images.

### **2.3.4 Image compression using simplified stack-run coding**

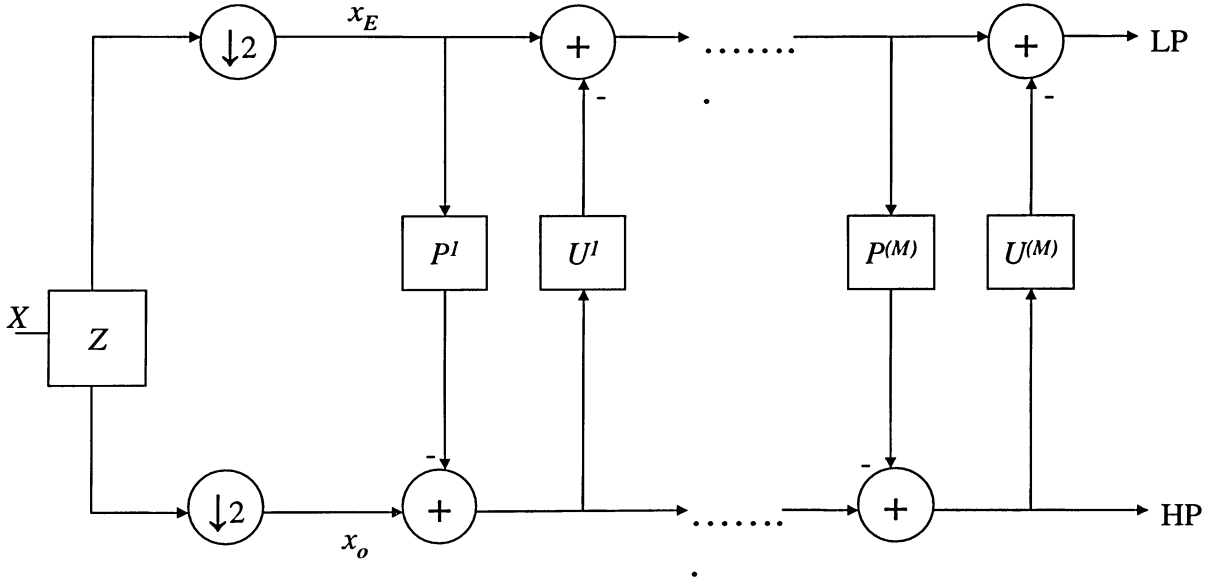
Wavelet based image compression scheme that possesses the simplicity of the stack-run coding (SR) scheme while achieving efficiency of the SPIHT algorithm was proposed by Yu Tian-Hu *et. al.* [47]. This method employs a multi-level dyadic wavelet decomposition, linear quantization with a proper dead zone, 1-D addressing complexity by raster scanning within sub-bands, variable length block coding, small alphabet representation of 1-D integer sequences and adaptive arithmetic entropy coding.

Simulation results have shown that the proposed wavelet based image compression scheme is computationally and as well as conceptually similar to SR. In addition to this, the coding efficiency of the proposed algorithm is competitive with that of SPIHT wavelet coder.

### **2.3.5 Image compression using projection based adaptive integer wavelet transform**

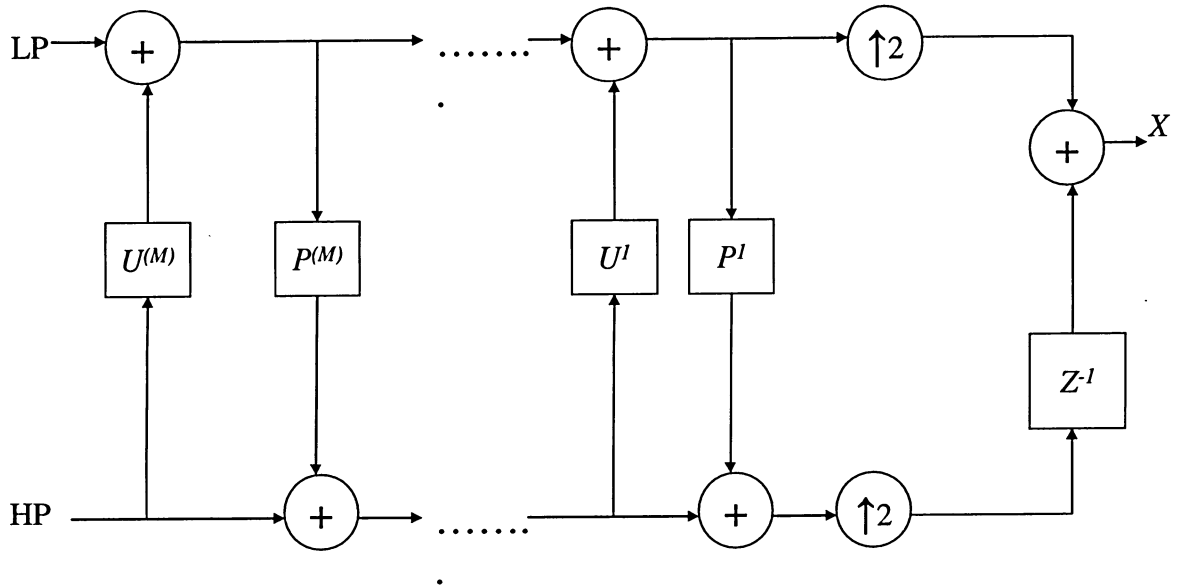
A projection-based reversible integer wavelet transform to reduce first-order entropy of transform coefficients and improve lossless compression is proposed by A. T. Deever

and S. S. Hemami [48]. JPEG 2000 lossless coding standard is based on reversible integer wavelet transform. This technique predicts the wavelet transform coefficients as a linear combination of other wavelet transform coefficients. It also yields optimal fixed prediction steps for lifting-based wavelet transforms and uses an adaptive prediction scheme that varies the final prediction step of the lifting-based transform. Block diagram of a lifting based integer forward and reverse wavelet transform is shown in Figure 2.5 and Figure 2.6 respectively.



**Figure 2.5:** Block diagram of lifting based integer forward wavelet transform

Simulation results show that the use of fixed projection prediction for the S transform yielded slightly improved compression performance compared to the S+P transform. Compression using S+Projection (S+Proj.) transform had a PSNR gain of 0.1 to 0.2 dB relative to compression with the S+P transform. Addition of projection step to (4,4) transform had minimal effect on lossy compression performance, typ-



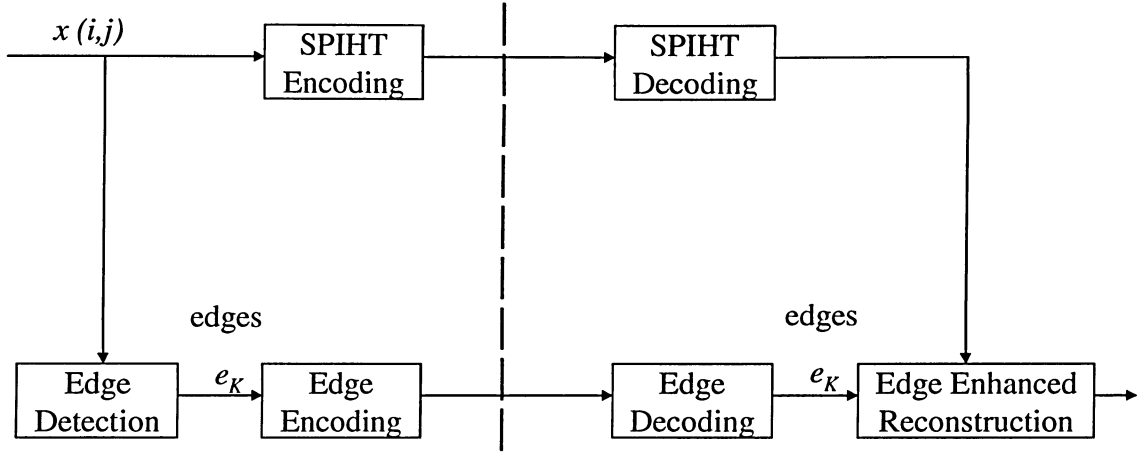
**Figure 2.6:** Block diagram of lifting based integer reverse wavelet transform

ically less than 0.05 dB for a variety of bit-rates. However application of adaptive projection to (2,2) transform under-performed when compared to the non-adaptive (2,2).

### 2.3.6 Low bitrate progressive image compression

A progressive image compression algorithm that focuses on preserving edge information was proposed by D. Schilling and P. C. Cosman [49]. In low-bandwidth applications images are sent or received at low bit rates and at these rates they suffer from significant distortion and artifacts making it difficult for viewers to understand. The proposed method aims to preserve important image features and edge information at high compression ration of 80 and above. The algorithm uses either a modified wavelet transform to "remove" edges and encodes the remaining texture information using SPIHT or transmits a standard SPIHT bit stream, and at the decoder applies

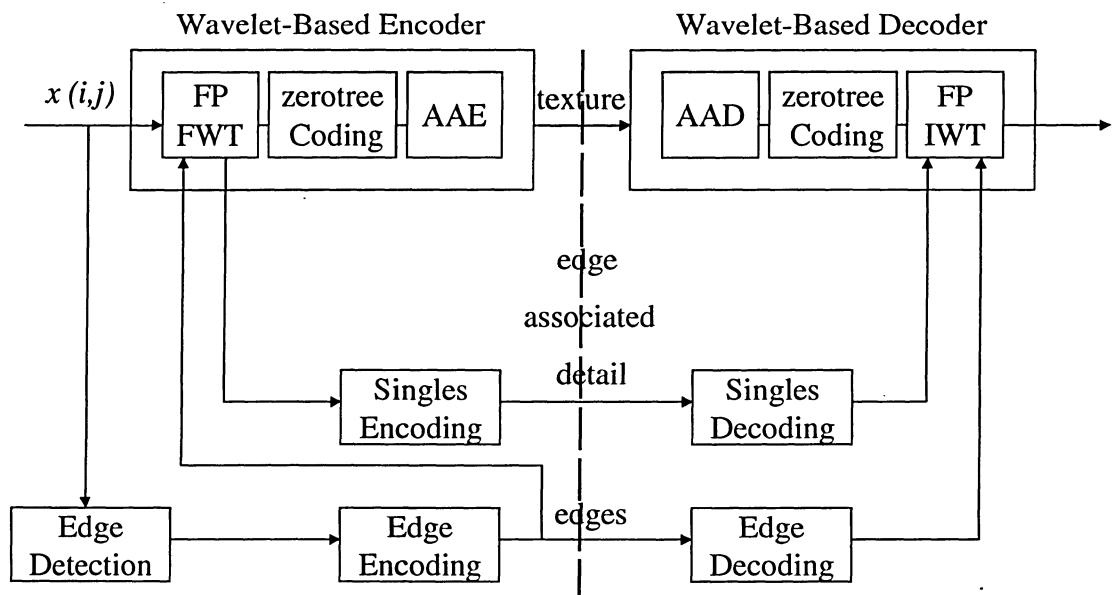
a nonlinear edge-enhancement procedure to improve the clarity of the encoded edges. Block diagrams of an edge enhancing image coder and feature preserving image coder are shown in Figure 2.7 and Figure 2.8 respectively.



**Figure 2.7:** Block diagram of edge enhancing image coder

Experimental results indicate that the images compressed using both edge enhancing image coder (EEIC) and feature preserving image coder (FPIC) at low bit rates have clearer edges than that with traditional wavelet-based image coders (SPIHT). Although for some images PSNR value of the compressed image using the proposed method is less than the PSNR achieved using SPIHT but the edge information is more pronounced.

As evident from the above review of previous works, the area of image compression is a very active research area. Nearly all aspects of image compression system i.e. complexity, memory requirements, PSNR gain, visual quality improvement etc. are



**Figure 2.8:** Block diagram of feature preserving image coder

being investigated and efforts are being made to improve upon them. In the subsequent chapter a hybrid image compression system using wavelets, neural network and mathematical morphology is proposed to improve both the PSNR and visual quality of compressed image.

## Chapter 3

# Wavelets and Neural Networks

**F**OURIER transform has been used as a principle tool for signal analysis since early 19th century. It was developed by French mathematician, J. Fourier, who showed that any periodic function can be expressed as a sum of periodic complex exponential function. Later this idea was generalized to non-periodic functions, periodic and non-periodic discrete time signals. In frequency domain Fourier transform constructs a sinusoidal basis to describe energy distribution of a signal. However Fourier transform is not well suited to describe local changes in frequency since the frequency component has infinite time support i.e. time (spatial) information is lost and it is impossible to specify when a particular phenomenon took place. Most of the practical signals and images contain non-stationary signal components and capturing them is a crucial step in classification.

To alleviate the limitations of Fourier transform, the windowed Fourier transform (Short Time Fourier Transform) was proposed. STFT works by dividing the signal into small segments where each segment is assumed to be stationary. STFT has several problems i.e. if we use an infinite length window; we get Fourier transform with perfect frequency resolution but no time information. On the other hand, to obtain a stationary sample we use a small enough window in which the signal is stationary. The narrower the window, the better is the time resolution and assumption

of stationarity, but poorer the frequency resolution [11]. Therefore to strike a balance between the time resolution and frequency resolution we turn our focus to wavelet transform, which is based on multiresolution analysis.

### 3.1 Wavelet Transform

Wavelet transform is the most recent solution to overcome the shortcomings of the Fourier transform and STFT. In wavelet analysis the fully scalable wavelet solves the problem of time and frequency resolution. The flexible window is moved along the signal and for every position the spectrum is calculated. The process is repeated several times with a variable window size and the collection of time-frequency representation of the signal is obtained. In this manner big wavelets give an approximate image of the signal, while the smaller wavelets zoom in on the details. Therefore, wavelets adapt automatically to both the high-frequency and low-frequency components of a signal by varying the window size. The wavelet transform is well suited for non-stationary signals, brief signals and signals with interesting components at different scales [1]. Wavelets are dilated and translated versions of a single function  $\Psi$ , which is called mother wavelet.

$$\Psi_{a,b}x = |a|^{-\frac{1}{2}} \Psi \frac{(x-b)}{a} \quad (3.1)$$

where  $\Psi$  satisfies the condition

$$\int_{-\infty}^{+\infty} \Psi(t) dt = 0 \quad (3.2)$$



The basic idea of the wavelet transform is to represent any arbitrary function  $f$  as a decomposition of wavelet basis or write  $f$  as an integral over  $a$  and  $b$ .

where  $a$  is the scale parameter and  $b$  is the position parameter.

When dealing with sampled data that is discrete in time we need to have a discrete representation of time and frequency, which is called discrete wavelet transform. We will briefly discuss the concept of multiresolution analysis before we discuss about the discrete wavelet transform.

## 3.2 Multiresolution Analysis

A signal/image can be viewed as combination of a smooth background and fluctuations(fine details). The distinction between the smooth part and the detail part of a signal is determined by the resolution. Image detail at one resolution will act as a smooth background at higher resolution. At a given resolution, a signal is approximated by ignoring all fluctuations below that scale. We can progressively increase the resolution; at each stage of the increase in resolution finer details are added to the coarser description, thus providing a successively better approximation of the signal.

A function  $f(t)$  at a resolution level  $j$  is denoted by  $f_j(t)$  and the details are denoted by  $d_j(t)$ . At the next higher resolution level  $j + 1$  the new approximation to  $f_j(t)$  is

$$f_{j+1}(t) = f_j(t) + d_j(t) \quad (3.3)$$

The original function is recovered as the resolution approaches to infinity.

$$f(t) = f_j(t) + \sum_{k=j}^{\infty} d_k(t) \quad (3.4)$$

Multiresolution analysis involves decomposition of the function space into a sequence of subspaces  $V_j$ . The subspace  $V_j$  is contained in all the higher subspaces. If the approximation of  $f(t)$  at a level  $j$  is denoted by  $f_j(t)$  then  $f_j(t) \in V_j$ . Since information at resolution level  $j$  is a part of information at a higher resolution level  $j + 1$ , mathematically  $V_j \in V_{j+1}(t)$  for all  $j$ .

We can therefore decompose our subspaces accordingly as

$$V_{j+1} = V_j \oplus W_j \quad (3.5)$$

where  $W_j$  is the detail space at a resolution level  $j$  and  $V_j$  is the approximation at resolution level  $j$ . The space  $V$  is decomposed in order to obtain

$$V_{j+1} = W_j \oplus V_j = W_j \oplus W_{j-1} \oplus V_{j-1} = \cdots = W_j \oplus W_{j-1} \oplus W_{j-2} \oplus \cdots \oplus W_0 \oplus V_0 \quad (3.6)$$

### 3.3 Discrete Wavelet Transform

Wavelet analysis is also based on a decomposition of a signal using an orthonormal family of basis functions. A wavelet has its energy concentrated in time and is well suited for the analysis of transient, time-varying signals.

A wavelet expansion is defined by a two-parameter family of functions

$$f(t) = \sum_j \sum_k a_{j,k} \psi_{j,k}(t) \quad (3.7)$$

where  $j$  and  $k$  are integers and the function  $\psi_{j,k}(t)$  is the wavelet expansion function which form an orthogonal basis. The two parameter coefficients  $a_{j,k}(t)$  are the discrete wavelet transform (DWT) coefficients. The DWT coefficients  $a_{j,k}(t)$  are obtained using the following formula

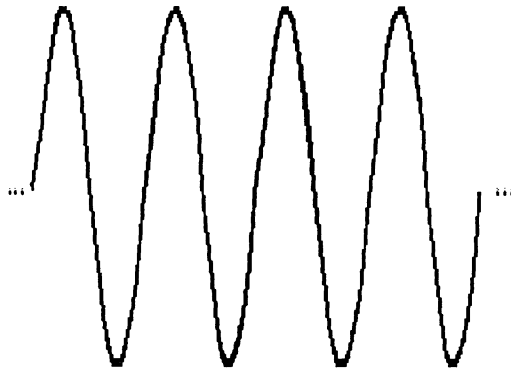
$$a_{j,k} = \int f(t) \psi_{j,k}(t) dt \quad (3.8)$$

The wavelet basis functions are a two-parameter family of functions that are related to the function  $\psi(t)$ , the mother wavelet by

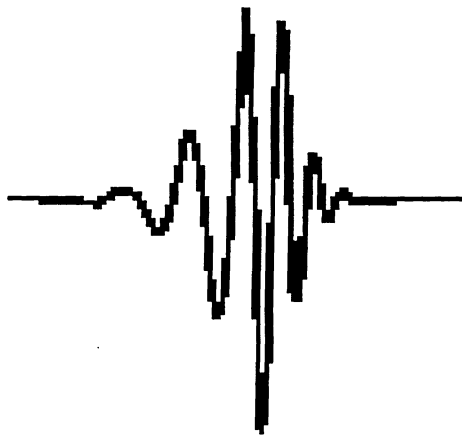
$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad (3.9)$$

where ' $k$ ' is the translation and ' $j$ ' is the dilation parameter. Therefore wavelet basis functions are obtained from a single wavelet by dilating and translating the single mother wavelet  $\psi(t)$ .

The concept of dilation and translation allows the wavelet transform to be localized in both time and frequency (scale) domain. Wavelet analysis is capable of revealing aspects of data that other transform techniques miss, like trends, breakdown points and discontinuities. By analyzing the sine wave and wavelets depicted in Figure 3.1 and Figure 3.2 respectively, we can clearly state that signals with sharp changes and peaks will be better analyzed with an irregularly shaped wavelet rather than with a smooth sinusoid.



**Figure 3.1:** Sinewave



**Figure 3.2:** Wavelet

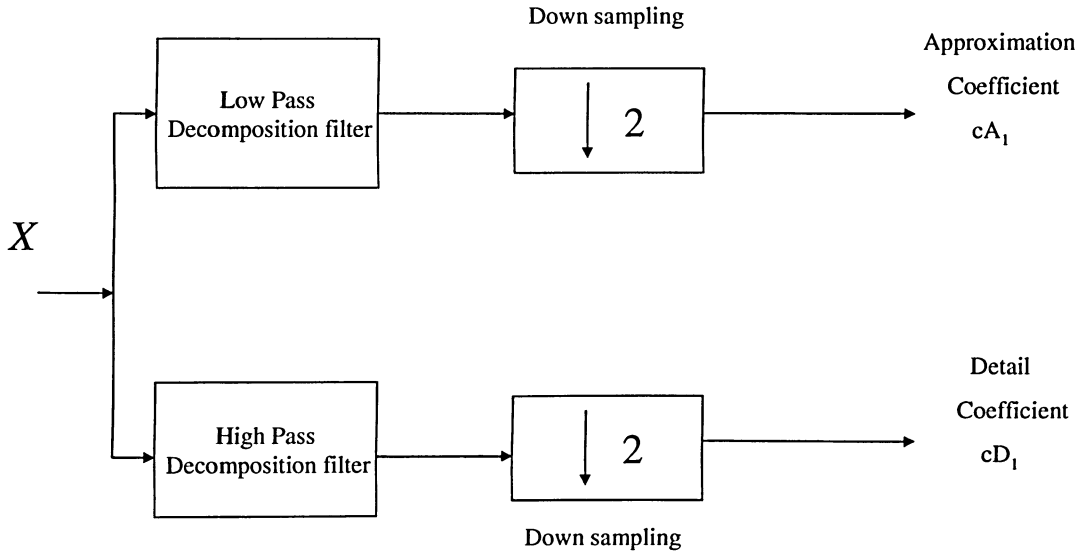
DWT is implemented using the Mallat algorithm [12] known as two-channel sub-band coder to obtain the discrete wavelet transform coefficients. A pair of FIR quadra-

ture mirror filters known as scaling filter and wavelet filter are used. The scaling filter is a low-pass filter " $\tilde{h}$ " and the wavelet filter is a high-pass filter " $\tilde{g}$ ".

Where " $\tilde{h}$ " is the low-pass reconstruction filter and " $\tilde{g}$ " is the high-pass reconstruction filter. Both " $\tilde{g}$ " and " $\tilde{h}$ " are related by the following equation.

$$g_n = (-1)^n h_{N-1-n}, \longrightarrow n = 0, 1, 2, \dots, N-1 \quad (3.10)$$

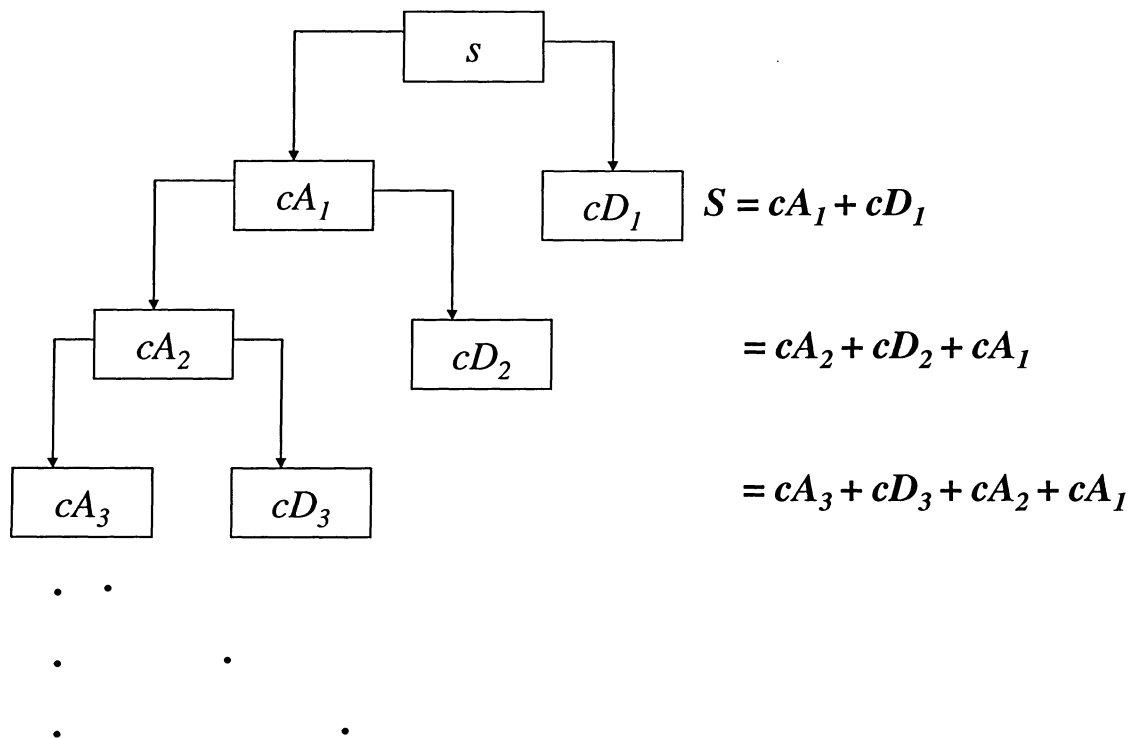
Filter implementation of DWT using a two channel subband coder is shown in Figure 3.3. For image processing applications the subband coder of Figure 3.3 is generalized as shown in Figure 3.5.



**Figure 3.3:** 1D DWT implementation using subband coding

In wavelet analysis, a 1-Dimensional signal is split into approximation and detail components. The approximation component is recursively decomposed into second

level approximation and detail coefficients and this process is repeated for  $n$  levels of decomposition (if required) as shown in Figure 3.4. Therefore for  $n$  level of decomposition there exists  $n + 1$  possible ways to decompose or encode a signal.



**Figure 3.4:** Multilevel Decomposition of 1D signal

Wavelet packet analysis is an extension of wavelet transform as both the approximation and detail coefficients are recursively decomposed at each level of decomposition. This results in an increased range of possibilities for signal analysis. Wavelet packet decomposition tree is as shown in Figure 3.6.

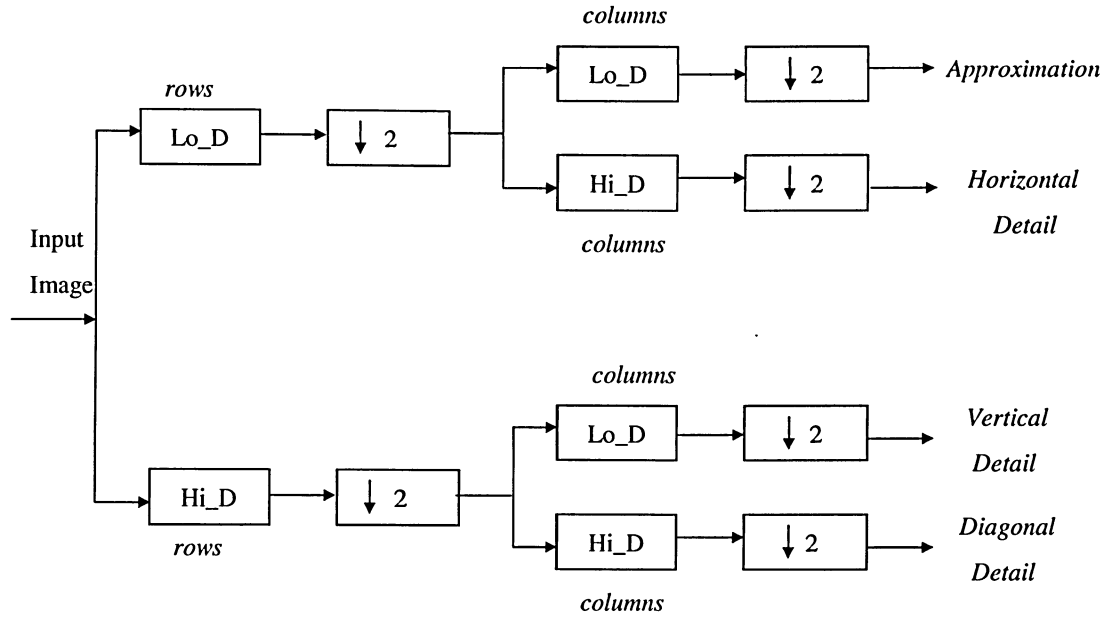
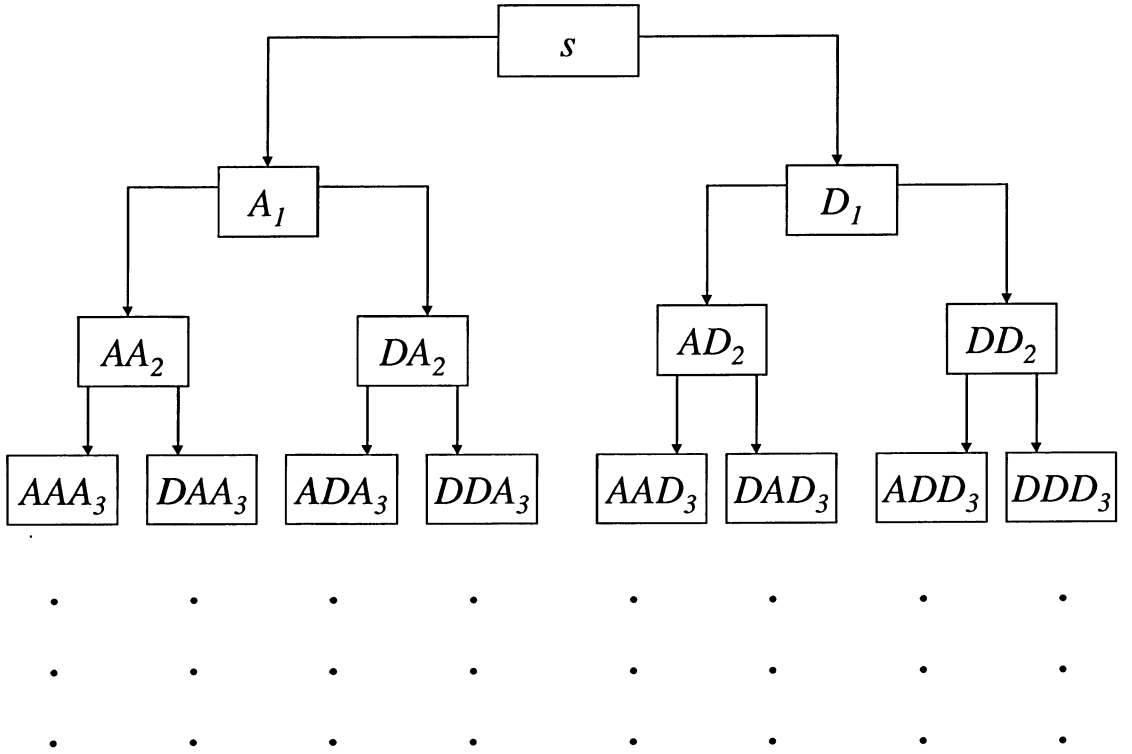


Figure 3.5: 2D DWT implementation using subband coding

### 3.4 Wavelet Filter

Wavelet bases are constructed with certain desired properties and quite a bit of freedom is exercised in choosing the wavelet function to generate a particular wavelet basis. Specific choice and method of construction of wavelet basis entirely depends on the requirements and motivation for its construction. There are two important classes of compactly supported wavelet bases, namely the compactly supported orthogonal and the biorthogonal wavelet bases. These wavelet bases give rise to FIR linear phase and FIR subband filtering schemes. Common examples of compactly supported orthogonal basis wavelets are the Haar wavelet basis and the Daubechies wavelet basis. In this section we will briefly discuss about the Biorthogonal spline wavelets and finally compare the wavelet properties of Haar, Daubechies and Biorthogonal spline



**Figure 3.6:** Wavelet Packet Transform: Generalization of Wavelet Transform

wavelets.

### 3.4.1 Biorthogonal Spline Filter

Most of the images are smooth and when dealing with images it is required that a wavelet filter should not deteriorate the smoothness of the image. Biorthogonal spline wavelets are a class of wavelet filters that use a smooth mother wavelet for image analysis. In addition to a smooth mother wavelet it is also required that mother wavelet is symmetric so that the corresponding wavelet transform could be implemented using mirror boundary conditions that reduce boundary artifacts. Except for the trivial case of Haar wavelets none of the wavelet filters are both symmetric and orthogonal.

Therefore to achieve symmetric property we relax the orthogonality constraint



and construct a biorthogonal basis. Decomposition of an image is obtained using the following equation.

$$c_{m,n}(f) = \sum_k g_{2n-k} a_{m-1,k}(f) \quad (3.11)$$

$$a_{m,n}(f) = \sum_k h_{2n-k} a_{m-1,k}(f) \quad (3.12)$$

where  $g_l = (-1)^l h_{-l+1}$  and  $h_n = 2^{1/2} \int \psi(x-n)\psi(2x)dx$ .

The image is reconstructed using the equation below

$$a_{m-1,f}(f) = \sum_n [\tilde{h}_{2n-l} a_{m,n}(f) + \tilde{g}_{2n-l} c_{m,n}(f)] \quad (3.13)$$

$\tilde{h}$ ,  $\tilde{g}$  are different from  $h$  and  $g$  and the relationship between them is given by the following equation

$$\tilde{g}_n = (-1)^n h_{-n+1}, g_n = (-1)^n \tilde{h}_{-n+1} \text{ and } \sum_n = h_n \tilde{h}_{n+2k} = \delta_{k,0} \quad (3.14)$$

Define

$$\phi(x) = \sum_n h_n \phi(x-2n) \quad (3.15)$$

$$\tilde{\phi}(x) = \sum_k \tilde{h}_k \tilde{\phi}(x-2n) \quad (3.16)$$

$$\psi(x) = \sum_n g_n \psi(x-2n) \quad (3.17)$$

$$\tilde{\psi}(x) = \sum_k \tilde{g}_n \tilde{\phi}(x - 2n) \quad (3.18)$$

Therefore we can rewrite  $a_{m,n}(f)$  and  $c_{m,n}(f)$  as:

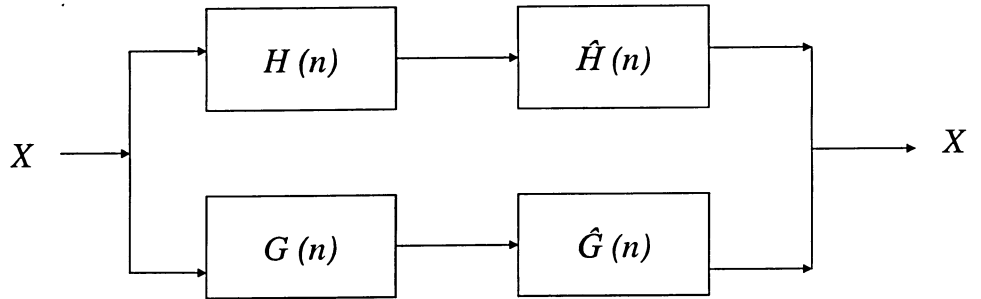
$$a_{m,n}(f) = 2^{-m/2} \int \phi_{m,n}(x) f(x) dx \quad (3.19)$$

$$c_{m,n}(f) = 2^{-m/2} \int \psi_{m,n}(x) f(x) dx \quad (3.20)$$

and the reconstruction equation thus becomes

$$f = \sum_m, n < \psi_{m,n}, f > \tilde{\psi}_{m,n} \quad (3.21)$$

Figure 3.7 gives a relationship between filter structure and wavelet functions:



**Figure 3.7:** Filter structure and associating wavelets

For symmetric filters, the condition of exact reconstruction on  $h$  and  $\tilde{h}$  can be written as

$$H(\xi) + \tilde{H}(\xi) + H(\xi + \pi) + \tilde{H}(\xi + \pi) = 1 \quad (3.22)$$

where

$$\tilde{H}(\xi) = 2^{-1/2} \sum_n \tilde{h}_n e^{-jn\xi} \quad (3.23)$$

and

$$H(\xi) = 2^{-1/2} \sum_n h_n e^{-jn\xi} \quad (3.24)$$

### 3.4.2 Comparison of filter properties

A comparative study of wavelet filter properties of Haar wavelet, Daubechies wavelet and Biorthogonal spline wavelet is as shown in Table 3.1.

Property	Haar	Daubechies	Biorthogonal Spline
Explicit Function	Yes	No	Yes
Orthogonal	Yes	Yes	No
Symmetric	Yes	No	Yes
Continuous	No	Yes	Yes
Compact support	Yes	Yes	Yes
Maximum regularity(order L)	No	No	Yes
Shortest scaling function(order L)	Yes	No	Yes

**Table 3.1:** Property comparison of different wavelet filters

Amongst the three wavelets discussed above only Haar wavelet and Daubechies wavelet possess orthogonality, which have some advantages:

1. Scaling and Wavelet functions are same for both forward and inverse transform.
2. Correlation in the signal between different subspaces is removed.

Haar wavelet is the simplest and the most fastest wavelet to implement but the major disadvantage of haar wavelet is its discontinuity, which makes it difficult to simulate a continuous signal. Daubechies invented the first continuous orthogonal compact support wavelet but this family of wavelet is non-symmetric. The advantage of the wavelet possessing symmetric property is that the wavelet transform can be implemented using mirror boundary conditions that reduce boundary artifacts. Therefore Biorthogonal spline wavelet filters are the best available wavelets for image compression. The B-spline wavelets are smooth and since splines are piecewise polynomial they are easy to manipulate.

### **3.5 Neural Network as a tool for Image Compression**

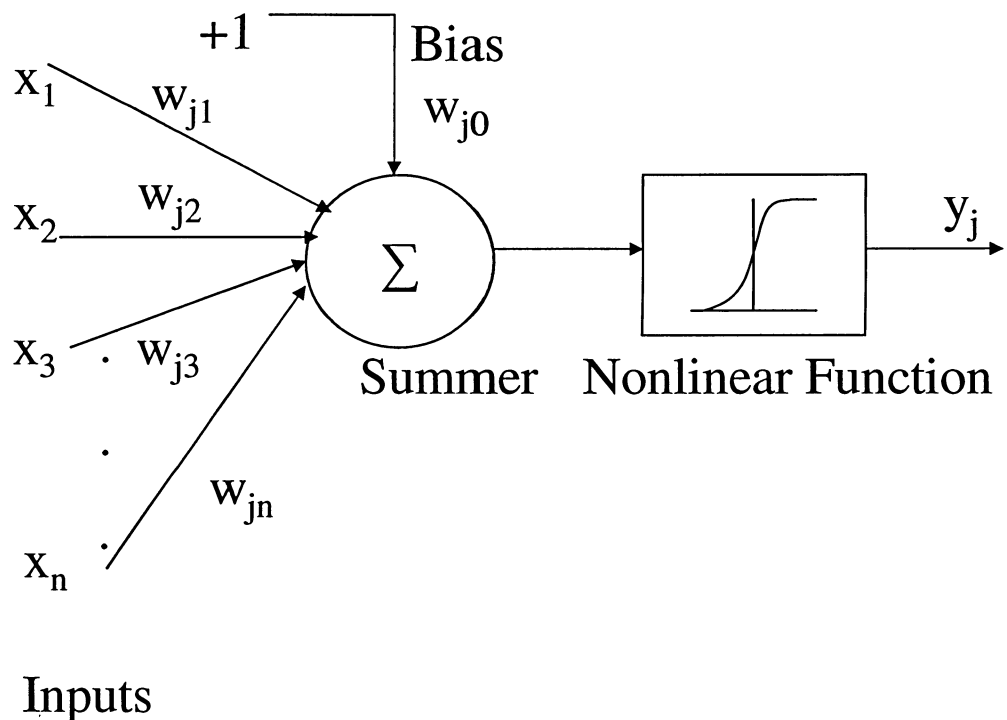
Neural networks are being used as signal processing tools for image compression for over a decade [13]. They are well suited for the task of image compression due to their massively parallel and distributed architecture. Physical characteristics of neural network are similar to that of a human visual system, which allows us to process visual information with ease [13]. For example, multilayer perceptions can be used as predictors in differential pulse code modulation system (DPCM). Such non-linear predictors outperform the linear predictors in terms of predictive gain. Another important area is the application of Hebbian learning algorithm to the extraction of principal components, which are the basis vectors for optimal Karhunen - Loeve Transform (KLT). The learning algorithms have computational advantages

over standard eigen decomposition techniques and are adaptive to changes in input. A clustering algorithm proposed by Kohonen is used to design code books for vector quantization of images. The neural network clustering algorithm, better known as Kohonen self organizing feature map (SOFM) is a two dimensional extensively interconnected unit of processors. The resulting code books are less sensitive to initial conditions than Linde-Buzo-Gray (LBG) algorithm since the topological ordering of the entries is exploited to further increase coding efficiency and reduce computational complexity.

In the Human Visual System (HVS) information is processed by massively parallel interconnected networks of processing units. This parallelism is evident right from the retina to the higher order structures in the visual cortex and the human brain [13]. The superiority of the parallel network over a serial structure is emphasized by the efficiency with which humans process images over speech. Characteristics of natural networks have been the source of inspiration for many artificial networks and many such natural systems have been successfully modeled by artificial systems. There are several application which the brain performs very well and one such application is image processing. Therefore for an artificial system to emulate this, the architectural properties of the artificial system should closely reflect the natural characteristics of the human system.

A neural network is defined as a "massively parallel distributed processor that has a natural propensity for storing experimental knowledge and making it available for use" [13]. An artificial neural network is an information processing paradigm, inspired by the biological nervous system such as the brain in order to process information. Physically it is a combination of a large number of simple computational units known as neurons which work in unison to solve specific problems. Due to its

parallel structure neural networks break down some of the computational bottlenecks which limit the performance of serial machines. The architecture of a simple neuron is as shown in the Figure 3.8.



**Figure 3.8:** Model of a Basic Neuron

In this chapter I will briefly discuss about some of the other major approaches to image compression in addition to vector quantization. These approaches include predictive coding: use neural network as non-linear predictors and transform coding: neural network based principal component analysis (PCA) using hebbian learning.

## 3.6 Predictive Coding

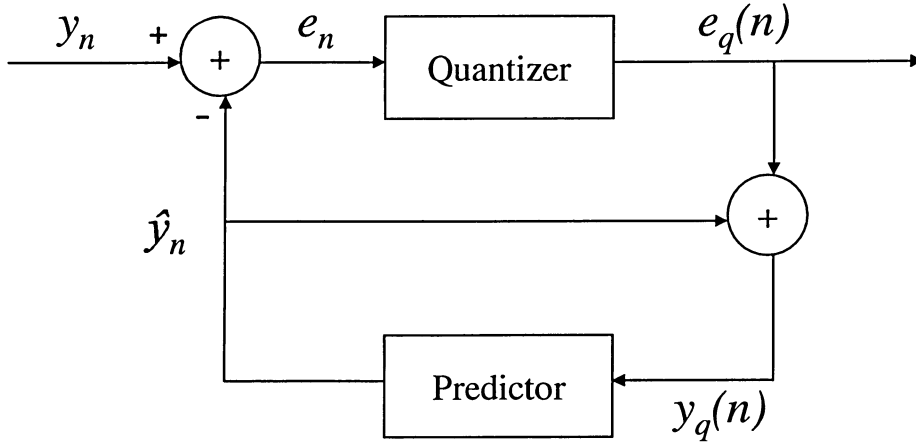
Virtually all images exhibit a high degree of correlation among neighboring pixels and this implies a high degree redundancy of data within the image. In an image compression system we take advantage of such redundancy and try to eliminate it by decorrelating the data. Thus by decorrelating the data a more efficient and hence compressed coding of image is possible. This decorrelation is accomplished through the use of linear/nonlinear predictive coders. I will briefly discuss about the linear coders and then focus on the nonlinear coders which are implemented using neural networks.

### 3.6.1 Linear Predictive Coding

A differential pulse code modulation(DPCM) system is the most common type of linear predictor used. The block diagram of such a predictor (DPCM) is as shown in Figure 3.9.

The DPCM predictor uses the neighboring pixels to calculate the estimate  $\hat{y}(n)$  of the current sample. The difference between the true value  $y(n)$  and the estimated value  $\hat{y}(n)$  namely the error  $e(n)$  is used for storage and transmission. As the accuracy of the predictor increases the variance of the difference decreases resulting in a better predictive gain and improved compression ratio.

To design a predictor a statistical model of the data is used to derive a function that relates the value of the current pixel to the neighboring ones in an optimal manner. One such statistical model that has been successfully applied to images is



**Figure 3.9:** Block Diagram of a DPCM System

the autoregressive model or the AR model. An AR model depends on the previous  $p$  outputs to determine the current output  $y(n)$ .

$$y(n) = \sum_{j=1}^p w_j y(n-j) + \epsilon_n \quad (3.25)$$

where  $w_j$  is the set of autoregressive coefficients and  $\epsilon_n$  is a set of zero mean independent and identically distributed random variables. The predicted value is a linear sum of neighboring samples and forms the basis of linear predictive coding as shown below.

$$\hat{y}_n = \sum_{j=1}^p w_j y(n-j) \quad (3.26)$$



In order to minimize the mean squared error  $E[(\hat{y} - y)^2]$ , the following relationship must hold true.

$$Rw = d \quad (3.27)$$

where  $R$  is the autocorrelation matrix and  $d$  is the cross covariance vector. Using  $R$  and  $d$  the unknown AR coefficient vector set  $W$  can be easily evaluated.

### 3.6.2 Non-Linear Predictive Coding Using Neural Networks

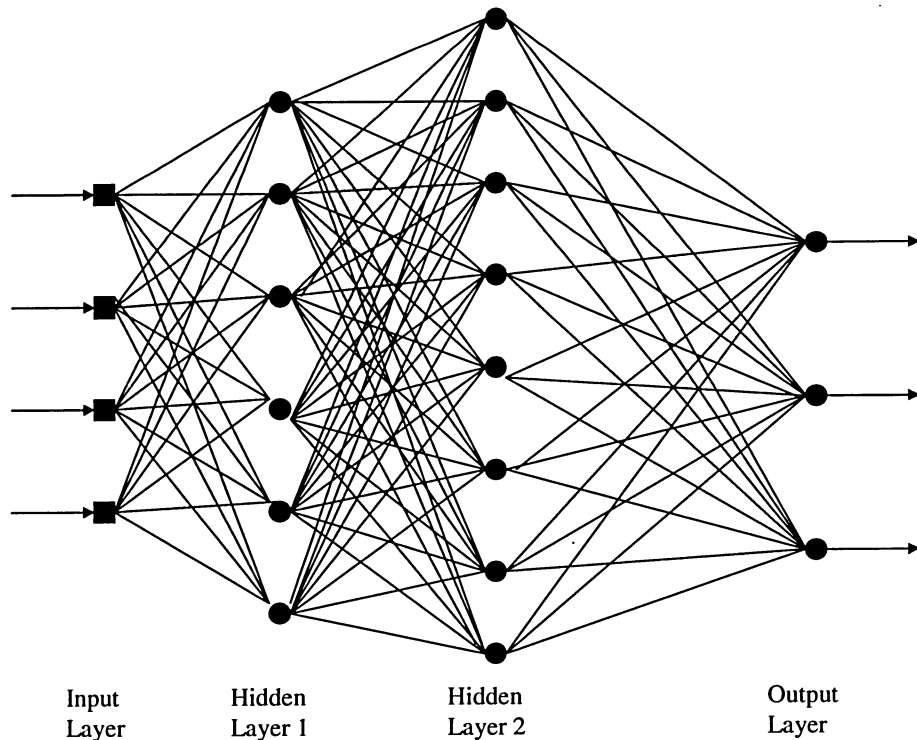
As discussed above predictors based on a linear weighted sum of neighboring pixels are relatively easy to design using the statistics of the image. However if the situation demands a nonlinear predictor the use of a linear predictor will result in suboptimal solution. The design of a nonlinear predictor is not as mathematically tractable as their counterpart. Therefore we take advantage of some of the useful properties of a neural network to optimally design a nonlinear predictor. There are several nonlinear predictors designed using neural networks however we will limit our discussion to the multilayer perceptron.

The main objective of designing a nonlinear predictor is to find an optimal parameter set  $W_0$  for a given nonlinear function based on the previous  $p$  inputs.

$$\hat{y}(n) = f(y(n-1), y(n-2), \dots, y(n-p), \mathbf{W}) \quad (3.28)$$

The above equation is used to evaluate the optimal parameter set such that the mean squared value of prediction error

$E[(\hat{y} - y)^2]$ , is minimized. Multilayer perceptron is one such predictor that is used to compute such class of nonlinear functions. Architecture of a multilayer perceptron is as shown in Figure 3.10.



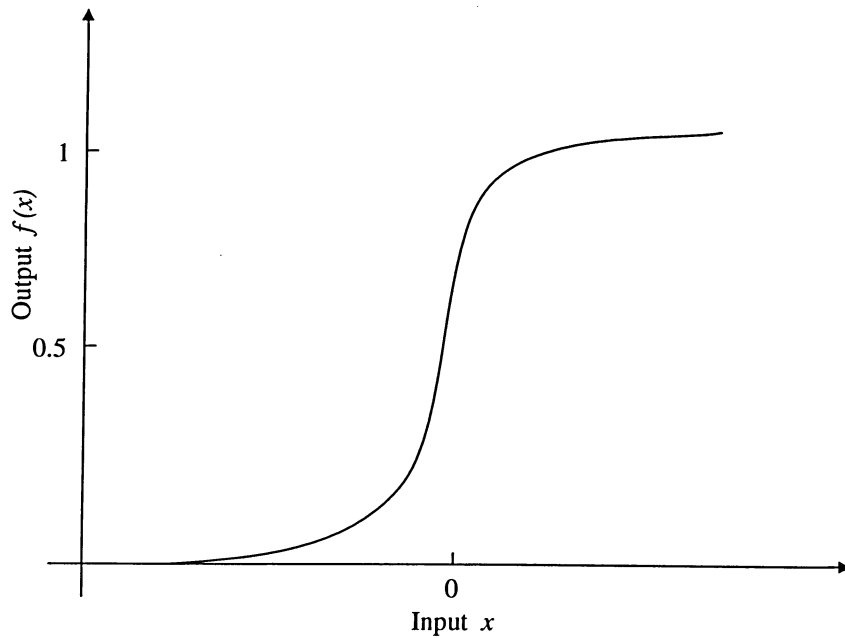
**Figure 3.10:** Architecture of Multilayer Perceptron

The basic computational unit in a multilayer perceptron is referred to as a "neuron". It consists of a set of "synaptic" weights, one for each input to the neuron, plus a bias weight and a nonlinear function as shown in Figure 3.8. The nonlinear function is also known as the activation function. Each neuron computes the weighted sum of inputs and the bias and passes it through the activation function to calculate the output.

$$y_j = f(\sum_i w_{ji}x_i + \theta_j) \quad (3.29)$$

The activation function  $f(\cdot)$  maps the infinite range weighted sum to a finite range output. The most common nonlinear activation function used is a sigmoid function, shown in Figure 3.11 and defined by the logistic function

$$f(v) = \frac{1}{1 + e^{-v}} \quad (3.30)$$



**Figure 3.11: Sigmoid Function**

In a multilayer configuration such as the multilayer perceptron the output of a previous layer forms an input to the next layer. Thus the inputs to the first layer

are considered as network inputs and the outputs of the final layer are considered as the network outputs. The weights of the network are randomly selected initially and are usually computed using a backpropagation algorithm. The backpropagation algorithm is a supervised learning algorithm which perform a gradient descent optimization on a squared error energy surface to reach a minimum. Due to the nonlinear nature of the network, the variance of the prediction error of a neural network is less than that of a linear predictor which results in an increased predictive gain.

Predictive coding algorithms are primarily used to exploit correlation between adjacent pixels. They predict the value of a given pixel based on the value of surrounding pixels. Since correlation exists among adjacent pixels in an image, the use of predictor significantly reduces the amount of information bits to represent an image. However such a lossy image compression technique is not as competitive as transform based technique since predictive techniques have inferior image compression ratios and worse image quality than that of transform based coding [3].

### 3.7 Transform Coding Using Neural Networks

Another important approach to image compression is the use of transformations that operate on an image to produce a set of coefficients. A subset of these coefficients that is adequate to reconstruct an image with a minimum of discernible distortion is chosen for storage and transmission.

Block transform coding is a simple and powerful transform coding technique wherein an image is divided into non-overlapping blocks of  $n \times n$  pixels. This can be considered as an  $N$ - dimensional vector  $x$  with  $N = n \times n$ . A linear transforma-

tion is applied on each block with  $M$  rows of  $W$ ,  $w_i$  being the basis vectors of the transformation. The resulting  $M$  dimensional coefficient vector is evaluated as

$$y = Wx \quad (3.31)$$

If the basis vectors are orthonormal, the reconstructed vector is calculated by the transpose of the forward transformation matrix as shown

$$\hat{x} = W^T y \quad (3.32)$$

The linear transformation with respect to minimizing the mean squared error is the Karhunen - Loeve Transform (KLT). The basis vectors of the transformation matrix  $W$  correspond to the  $M$  largest eigenvalues of the sample autocovariance matrix. Estimation of covariance of an image and the calculation of eigenvalues and eigenvectors is computationally intensive. A solution to the problems associated in the calculation of basis vectors through eigendecomposition of the covariance matrix is the use of iterative techniques based on neural network models.

### 3.8 Vector Quantization Using Neural Networks

Quantization is a process that maps a signal  $x(n)$  into a finite series of  $K$  discrete messages. For every  $K$ th message, there exists a pair of thresholds  $t_k$  and  $t_{k+1}$  and output value  $q_k$  such that  $t_k < q_k < t_{k+1}$ . Concept of scalar or one-dimensional quantization is extended to vector data of any arbitrary dimension. Instead of output

levels, vector quantization employs a set of representation vectors and matrices for one-dimensional and two-dimensional data respectively. The set of representation vector is often referred to as a codebook and the entries within the codebook are known as codewords. The thresholds are replaced by a decision surface defined by a distance metric such as euclidean distance. In vector quantization high degree of co-relation between neighboring pixels is exploited and the coding of vector can theoretically improve performance.

During coding the image is divided into blocks of fixed size  $n \times n$  pixels. For each block of input the codeword that results in a minimum euclidean distance is found and transmitted. On reconstruction, the same codebook is used and a simple look-up operation is performed and the image is reconstructed.

The classical method for codebook construction is by use of Linde, Buzo and Gray (LBG) algorithm [41]. According to this method  $K$  codebook entries are initially set to random values and on each iteration, each input space is classified based on euclidean distance. Each codebook is replaced by the mean of its resulting class and the iterations are continued until a minimum acceptable error is achieved.

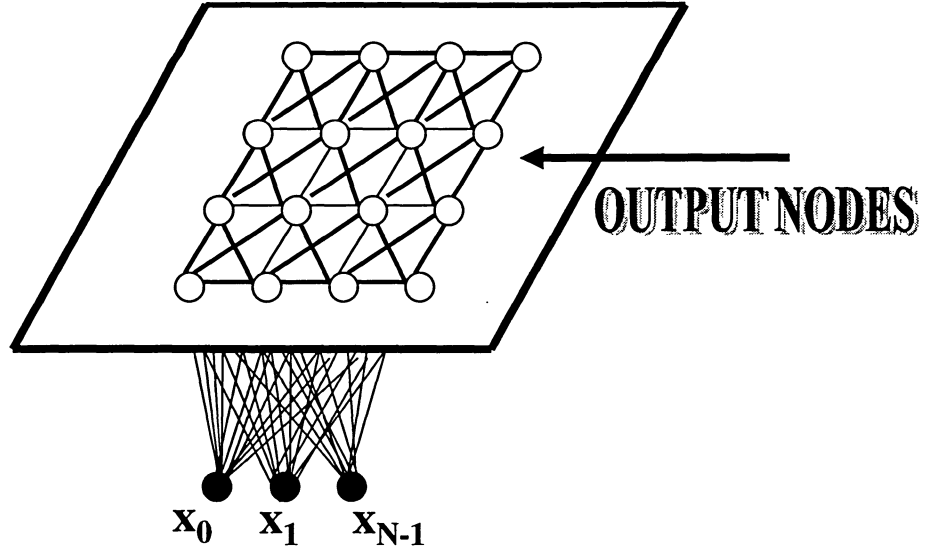
LBG algorithm results in a local minima but the global minima is not guaranteed. The algorithm is highly sensitive to initial codebook and is very slow since it requires an exhaustive search through the codebook for each iteration. These limitations of LBG has caused an inclination towards the use of neural networks for vector quantization. One such neural network based method used for vector quantization is the self organizing feature map (SOFM) which will be discussed below. In addition to reduced computational complexity, the self organizing feature map is an unsupervised learning which preserves the network topology.

### 3.8.1 Self Organizing Feature Map

Kohonen's self organizing feature map has formed the basis for a great deal of research into the application of neural network for codebook design in vector quantization. The SOFM network performs a mapping from a continuous input space to a discrete output space. In many clustering algorithms such as  $k$ -means clustering algorithm each input vector  $x$  is classified and only the winning class is updated during each iteration, whereas in a Kohonen's SOFM the winning class as well as its neighboring class is updated. This property of a SOFM helps in preserving the input topology i.e. points close to one another in input space are mapped to similar or neighboring processing elements in the output space. Therefore there exists a soft competition among processing elements (classes or neurons) in the output space.

Kohonen's self organizing feature map is a fully interconnected linear input layer with the output processing elements arranged in either one or two dimensional space. One dimensional neighborhood organizes the processing elements (PEs) in the form of a string so that each element has only two neighbors as shown in Figure 3.12. When the SOFM adapts to an input of higher dimensions, it stretches and curls itself to cover the entire input space. Whereas, a two dimensional neighborhood results in more neighbors and it creates more flexible mappings. Therefore when we choose a two-dimensional neighborhood in a two-dimensional space, we find that the PEs spread towards the data samples more rapidly.

The learning algorithm in a self organizing feature map is based on the idea of soft competition. Unlike in hard competition where there is only one winner i.e. one PE is active and all others are inactive, soft competition allows not only the winner PE but also its neighbors to be active. Soft competition creates a "bubble" of activity in the



**Figure 3.12:** Self Organizing Feature Map Network

neighborhood of the winning PE or the output neuron. Processing elements which are close to the winner are more active than those who are distant. The lateral weights vary with the distance from the PE, thus PEs that are close excite one another and those that are distant inhibit one another. Since more one PE is active for each input vector, therefore the winner as well as its neighbors have their weights updated for each input.

During learning the weights that connect the input to the output perform association between the weights and the inputs. The processing element whose weight is closest to the present input vector wins the competition. The winner as well as its neighbors have their weights updated according to the competitive rule as shown.

$$\mathbf{W}_i(n+1) = \mathbf{W}_i(n) + \alpha(\mathbf{X}(n) - \mathbf{W}_i(n)) \quad (3.33)$$



Where  $i$  is the winning PE,  $W(n)$  is the weight vector,  $W(n+1)$  is the updated weight vector and  $\alpha$  is the learning rate that varies between 0 and 1. During training the learning parameter  $\alpha$  shrinks to a small value for the algorithm to converge.

To simplify the computation, the lateral inhibition network is assumed to produce a Gaussian distribution centered at the winning processing element. Thus instead of computing the activity of the winner and the neighborhood for each input vector, we simply evaluate the activity of the winner and assume that the other PEs have an activity proportional to the Gaussian function at each PE's distance from the winner. Thus the competitive rule for a SOFM is updated to incorporate the neighborhood function as shown below.

$$\mathbf{W}_i(n+1) = \mathbf{W}_i(n) + \Lambda_{i,i^*}(n)\alpha(x(n) - \mathbf{W}_i(n)) \quad (3.34)$$

Where  $i^*$  is the winning element and  $\Lambda_{i,i^*}$  is a neighborhood function centered at the winning PE. Typically the neighborhood function  $\Lambda$  is Gaussian and it decreases with the iteration number (since variance decreases with iteration) and  $i^*$  is the winning element.

$$\Lambda_{i,i^*}(n) = \exp\left(\frac{-d_{i,i^*}^2}{2\sigma^2(n)}\right) \quad (3.35)$$

The Gaussian function starts by covering the full map and progressively gets reduced to a neighborhood of zero i.e. only the winning PE gets updated. As the size

of the neighborhood shrinks, the network shifts itself from a very soft competition to a hard competition.

SOFM creates a output space where topological relationships within the input-space neighborhoods are preserved i.e. distribution of data samples in the input space is approximately preserved. This property of a SOFM makes it suitable for density approximation. Some of the important properties of a SOFM which makes it suitable for use as a vector quantizer are:

- Feature vectors are good approximation to the original input space.
- Feature vectors are topologically ordered in a feature map such the correlation amongst them increases with reduced distance and vice-versa.
- Density of a feature map corresponds to the input density distribution so that regions with higher probability density have better resolution than regions with lower density.

## Chapter 4

# MMSOFM Compression Algorithm

**I**MAGE compression using the state of the art wavelet technology has been an active area of research for over a decade. During this period of time hundreds of wavelet based image compression algorithm have been proposed and implemented in order to improve the image characteristics. Some of the recently proposed image compression methods have already been discussed in Chapter 2. In this Chapter a hybrid image compression algorithm is proposed and discussed. The compression algorithm is hybrid in the sense that it is based on wavelets, neural networks and mathematical morphology (MMSOFM). It utilizes all the inherent properties of the mentioned methods and creates a reconstructed image with improved PSNR and subjective quality.

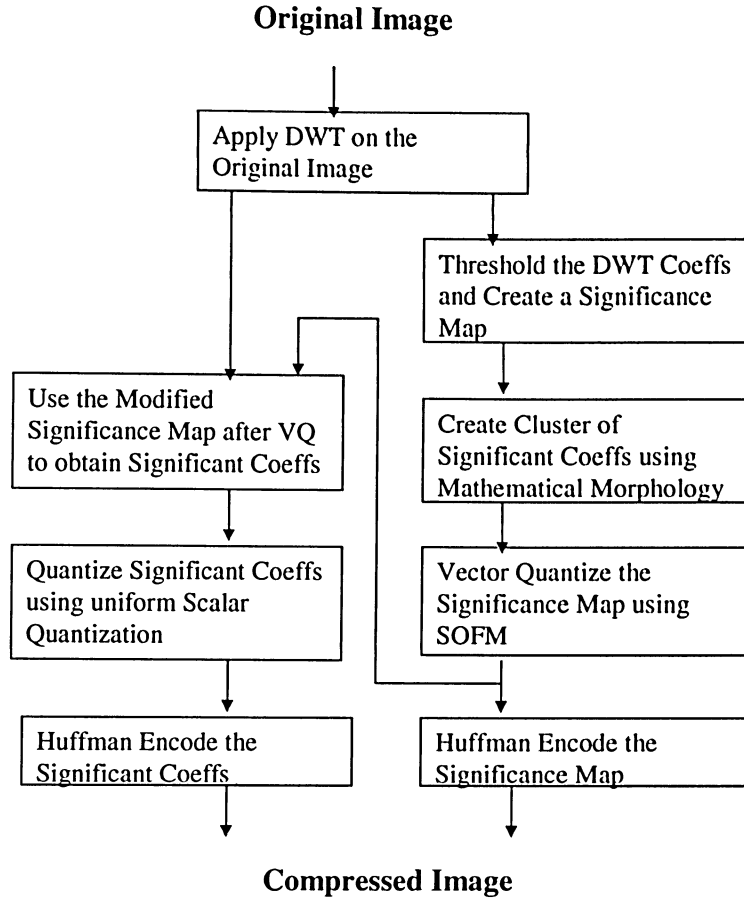
The proposed compression algorithm uses wavelets, mathematical morphology operators and neural network in order to enhance image characteristics. Wavelet transform was used to perform decorrelation of the input image and was preferred over the discrete cosine transform since it eliminates the inherent blocking effect produced by the discrete cosine transform at high compression ratios. The wavelet filter used for performing the wavelet transform is a smooth biorthogonal wavelet filter that possesses symmetry so that the wavelet transform is implemented using mirror boundary conditions to reduce boundary artifacts. In addition to its symmetry the

wavelet filter used is smooth so that smoothness within an image is preserved.

Significance map is created by thresholding the wavelet coefficients using a hard threshold. These coefficients are obtained by filtering the image using a pair of scaling (low-pass filter) and wavelet (high-pass filter) functions. The significance map is processed using mathematical morphology operators to perform a clustering operation. Clustering operation is done to create clusters and to emphasize the significant coefficients from amongst the wavelet coefficients. The clustered significance map helps in preserving the fine details within an image and improves the peak signal to noise ratio of compressed image.

The clustered significance map is vector quantized using a self organizing feature map (SOFM). A SOFM was preferred over a classical LBG algorithm to perform vector quantization due to its adaptability and its ability to preserve the input topology. In addition to this a SOFM is computationally less complex and less sensitive to initial codebook design than a LBG vector quantizer. The vector quantized significance map is used to extract the significant coefficients which are scalar quantized. The significant coefficient vector as well as the significance map are huffman encoded and the result is transmitted.

Discrete wavelet transform is applied on the original image to create wavelet coefficients. The coefficients are categorized as either significant or non-significant by analyzing the significance map. Significance map is created using a threshold operation and pre-processed using mathematical morphology in order to create clusters of significant coefficients and improve the quality of reconstructed image. A Self Organizing feature map then performs the vector quantization of the significance map. The resulting modified significance map is used to identify the significant and non-significant



**Figure 4.1:** Flow diagram of the MMSOFM

coefficients. The non-significant coefficients are eliminated and the resulting significant coefficient vector along with their positions within an image (significance map) is entropy encoded in order to ensure proper decoding of the significant vector bit stream. The proposed method includes the following three steps: (The system block diagram is shown in Figure 4.1).

- a) Extraction of wavelet coefficients.
- b) Identification and clustering of significant coefficients
- c) Vector quantization using self organizing feature map.

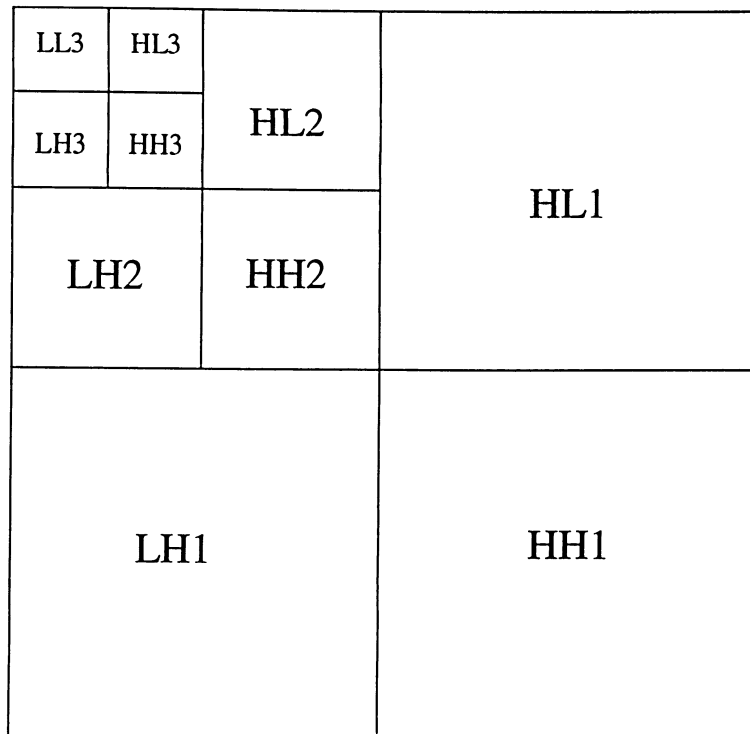
In the following subsections the aforementioned steps are discussed in more details.

## 4.1 Extraction of wavelet coefficients

Discrete wavelet transform (DWT) is implemented through a pair of high-pass and low-pass filters followed by down sampling (up sampling during reconstruction). The detail coefficients (high pass filtered image) are retained where as the approximation coefficients (low pass image) are further decomposed using the same pair of filter. Using a quadrature mirror filter the original  $n \times n$  Image is decomposed into wavelet coefficients. First the wavelet filters are applied along the rows of the image producing two sub-images of dimension  $n/2 \times n$  and then along the columns of the sub images to produce four sub-images of size  $n/2 \times n/2$  each i.e. low-low, high-low, low-high and high-high sub-bands. The coarser low-low (approximation coefficient) sub-band is again decomposed and the process continues.

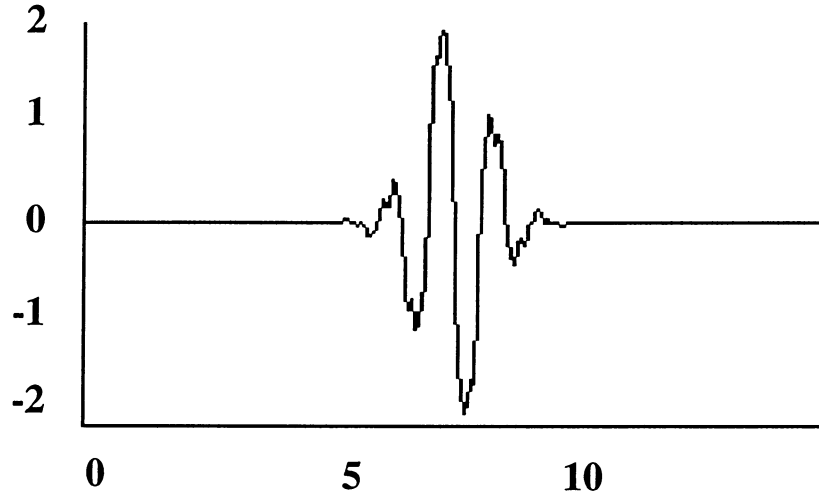
Figure 4.2 illustrates three levels of wavelet decomposition wherein HL1, LH1 and HH1 represent the finest/detail coefficient of the original image, HL2, LH2 and HH2 represent the finest/detail coefficient of sub-band LL1 and HL3, LH3 and HH3 represent the finest/detail coefficient of sub-band LL2. LL3 is the lowest frequency term which represents all the coarser levels.

A biorthogonal spline wavelet filter is used to perform the wavelet decomposition since it is a finite impulse response (FIR) filter with perfect reconstruction and regularity. In addition to this the biorthogonal spline wavelet filter is a smooth wavelet and possess linear phase, a property that helps in preserving edges in an image. Two wavelet filters, one for decomposition and one for reconstruction are used as shown in Figure 4.3 and Figure 4.4 respectively.



**Figure 4.2:** Three scale wavelet decomposition

Discrete wavelet transform is applied on the original image in order to reduce the amount of inter pixel redundancy. As a result of the decomposition many coefficients with in the high frequency (low scale) region are either zero or very close to zero, therefore these coefficients can be thresholded without appreciable loss of information (Image quality). A high percentage of wavelet coefficients are thresholded using a global threshold in order to create a significance map (location of significant coefficient in space) and eventually identify the significant coefficients.

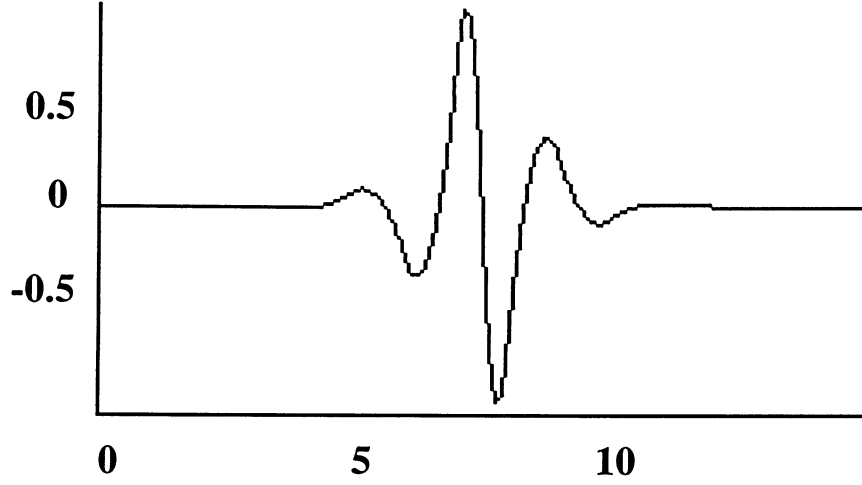


**Figure 4.3:** Biorthogonal wavelet decomposition filter

## 4.2 Identification and clustering of significant coefficients

After the application of wavelet transform to an image the most important factor is to correctly identify and cluster the significant coefficients. Prior to the identification of significant coefficients a significance map (identifies the positional information of each coefficient) is created since individual encoding of the coefficients along with a large number of zeros is highly inefficient. Thus only the significant coefficients and their positional information is encoded and transmitted to achieve compression. Wavelet coefficients are scanned from left to right and those that are lower than a pre-defined threshold signify the presence of a non-significant coefficient and a 0 is placed at the





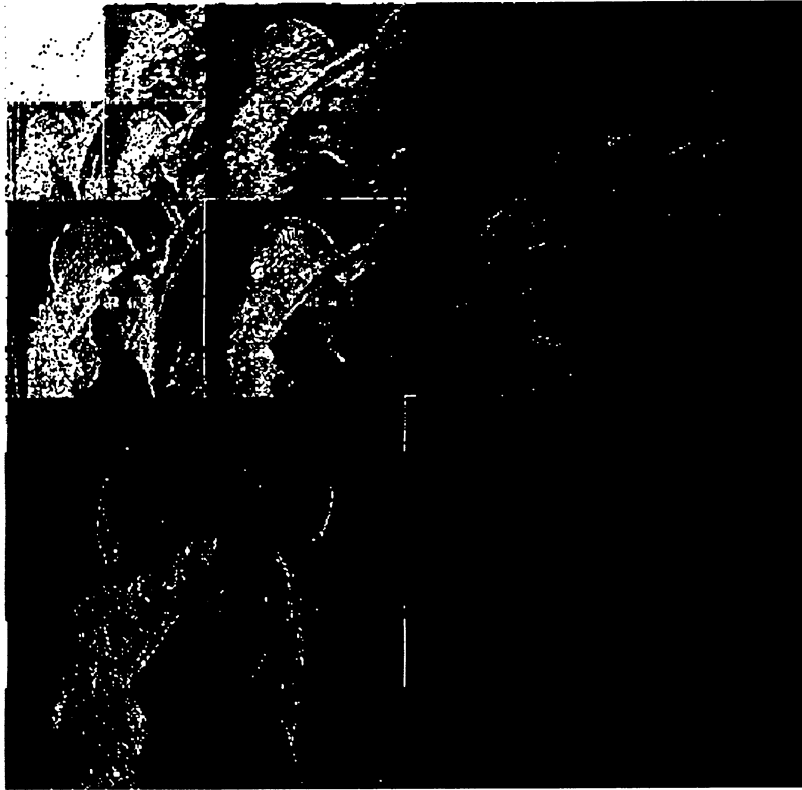
**Figure 4.4:** Biorthogonal wavelet reconstruction filter

corresponding spatial location in the significance map, whereas coefficients which are above threshold are considered as significant coefficients and a 1 is inserted in the significance map. Therefore a significance map is a binary map of ones and zeros, wherein a 0 indicates the presence of a non-significant coefficient and a 1 indicates the presence of significant coefficient. Hard threshold is the most common type of threshold function used and is defined by the following equations.

$$Y = 1 \rightarrow X \geq |T| \quad (4.1)$$

$$Y = 0 \rightarrow X < |T| \quad (4.2)$$

Where  $X$  is the coefficient value,  $Y$  is the output and  $T$  is the threshold value. Figure 4.5 shows the significance map obtained by thresholding the wavelet coefficients. The significance map obtained is pre-processed using mathematical morphology operators to create clusters of significant coefficients, emphasize the edge information and eventually improve the PSNR of compressed image.



**Figure 4.5:** Initial significance map

### 4.2.1 Mathematical Morphology Operators

Mathematical morphology is the analysis of signals/images in terms of their shape. It is used in image processing applications so as to preserve edge information and create clusters of significant coefficients [51]. The basic building blocks of mathematical morphology are dilations and erosions.

The basic effect of dilation on a binary image is to gradually enlarge the boundaries of regions of foreground pixels (i.e. white pixels, typically). Thus areas of foreground pixels grow in size while holes within those regions become smaller. Dilation of a binary input image is computed by superimposing the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position. If at least one pixel in the structuring element coincides with a foreground pixel (white pixel i.e. 1) in the image underneath, then the input pixel is set to the foreground value. If all the corresponding pixels in the image are background (black pixel i.e. 0), the input pixel is left at the background value. Erosion is the dual of dilation i.e. eroding foreground pixels is equivalent to dilating the background pixels. Effect of dilation and erosion on a binary image using a  $3 \times 3$  square structuring element (Figure 4.6) is shown in Figure 4.7 and Figure 4.8 respectively.

In order to create clusters of significant coefficients using mathematical morphology in this work the significance map was dilated twice and then eroded once using a  $3 \times 3$  circular structuring element. In this sequence of operation [9] the first dilation clusters the significant coefficient whereas the next dilation and erosion (also known as a closing operation) merely fills in small holes. The effect of mathematical morphology on initial significance map of Figure 4.5 is shown in Figure 4.9.

1	1	1
1	1	1
1	1	1

Set of coordinate points =  
 $\{ (-1, -1), (0, -1), (1, -1),$   
 $(-1, 0), (0, 0), (1, 0),$   
 $(-1, 1), (0, 1), (1, 1), \}$

Figure 4.6: Square structuring element

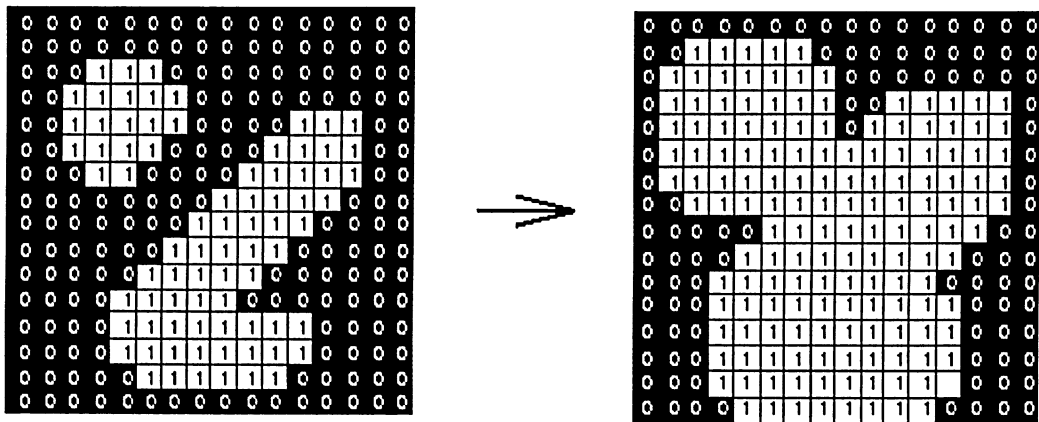


Figure 4.7: Dilation of binary map using a square structuring element

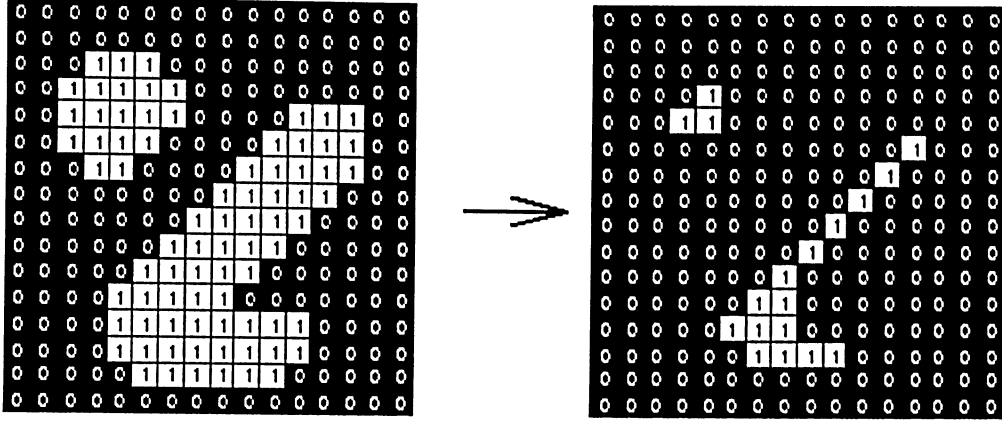


Figure 4.8: Erosion of binary map using a square structuring element

### 4.3 Vector quantization of significance map

The clustered significance map obtained after mathematical morphology provides spatial location of the significant coefficients within the wavelet coefficient matrix. Prior to the selection of these significant coefficients for quantization and encoding, the significance map is vector quantized using a self organizing feature map. Theoretical details about the self organizing feature map are explained in chapter 3. The modified significance map obtained after the vector quantization is used to extract the significant coefficients.

Steps involved in the quantization of the significance map are as follows:

- The significance map is divided into blocks of size  $m \times m$  and all the blocks except those which contain all zeros or all ones are part of the training process.

This is done to reduce the training time.



**Figure 4.9:** Significance map after mathematical morphology

- The synaptic weights are randomly chosen, the blocks are randomized and a neural network with only  $n - 2$  neurons is selected.
- Training of the network is carried out according to the learning rule described above and weights are updated according to Equation 3.34.
- After the training is done two neurons one with weights containing all zero and one with weights containing all ones are appended so as to represent blocks which contain all zeros and all ones respectively.

Result of the vector quantization of the significance map performed with blocks

of size  $4 \times 4$  and using 32 codewords is shown in Figure 4.10.



Figure 4.10: Significance map after vector quantization

The vector quantized significance map is used to extract the significant coefficients from the wavelet coefficients by scanning the significance map. The significance map is scanned from left to right and top to bottom and for each significant position found within the significance map (presence of 1), the wavelet coefficient at the corresponding row and column is selected as significant coefficient. At the end of scanning process a vector with all the significant coefficients is created which is quantized and encoded.

The significant coefficient vector and the significance map are coded using Huffman encoder [52]. It is a variable length encoding scheme which allocates bits depending on the occurrence frequency of each symbol: less frequent symbols are assigned longer bit strings and more frequent symbols are assigned smaller bit strings.

The described method reduces all forms of redundancies present in an image: inter-pixel redundancy (neighboring pixels have similar values) through a de-correlating transform, psycho visual redundancy (some color differences are imperceptible) through quantization of both the significance map and significant coefficients and finally coding redundancy (some pixel values are more common than others) by using Huffman encoding.

In order to reconstruct the compressed image first the significant coefficient vector is dequantized and then the significance map is scanned from left to right and top to bottom. For each significant position found within the significance map, a significant coefficient is selected from the coefficient vector in series starting from top. Eventually all the significant coefficients are placed in their original position and inverse wavelet transform is applied and the compressed image is obtained.

## 4.4 Experimental results

In this thesis variety of natural still images were compressed using the proposed method and comparable results with JPEG 2000 and superior results over standard JPEG are obtained. Subjective measure of image quality was made in terms of Peak Signal to Noise Ratio (PSNR). Mean squared error (MSE) and PSNR are calculated using the following equations:



$$MSE = \frac{1}{N^2} \sum_{i=1}^N \sum_{j=1}^N [X(i, j) - \hat{X}(i, j)]^2 \quad (4.3)$$

$$PSNR = 10 \log_{10} \frac{255}{N^2} \quad (4.4)$$

Where  $X$  is the original image and  $\hat{X}$  is the compressed image respectively.  $N$  represents the number of rows and columns in the original image (images which have similar number of rows and columns are selected for compression).

Large number of natural as well as CT-scan images were compressed using the proposed method and good peak signal to noise ratio and improved picture quality was achieved. A comparison of the PSNR values obtained with the proposed method, JPEG and JPEG 2000 for Lena image, Barbara image and CT image are shown in Table 4.1, Table 4.2 and Table 4.3 respectively. The image compressed using the proposed method is visually much better when compared to JPEG. In the JPEG compressed image blocking artifacts, ringing artifacts and other visible differences are much more pronounced when compared with the results obtained using proposed method. Though the PSNR achieved using MMSOFM is less when compared with JPEG 2000, but there is not much difference in the visual image quality of the compressed image obtained using JPEG 2000 and MMSOFM. Original Lena image, Lena image compressed using JPEG, Lena image compressed using JPEG 2000 and Lena image compressed using the proposed method at a compression ratio of 25 are shown in Figure 4.11. Whereas Barbara image, Barbara image compressed using JPEG, Barbara compressed using JPEG 2000 and Barbara image compressed using the proposed method at a compression ratio of 20 are shown in Figure 4.12 and finally CT-scan

image and its compressed version (compression ratio of 25) using JPEG, JPEG 2000 and proposed method is shown in Figure 4.13.

Method Used	Compression Ratio	PSNR
<b>JPEG</b>	10	20.1
	15	20.0
	20	19.8
	25	19.5
<b>JPEG 2000</b>	10	38.9
	15	37.1
	20	35.8
	25	34.7
<b>MMSOFM</b>	10	36.8
	15	33.7
	20	31.5
	25	29.8

**Table 4.1:** PSNR and compression ratio for lena image using JPEG, JPEG 2000 and MMSOFM

Method Used	Compression Ratio	PSNR
<b>JPEG</b>	10	19.9
	15	19.5
	20	19.0
	25	18.5
<b>JPEG 2000</b>	10	36.5
	15	33.1
	20	31.3
	25	29.5
<b>MMSOFM</b>	10	30.6
	15	26.5
	20	25.1
	25	24.6

**Table 4.2:** PSNR and compression ratio for Barbara image using JPEG, JPEG 2000 and MMSOFM

Method Used	Compression Ratio	PSNR
<b>JPEG</b>	15	41.8
	20	34.9
	25	32.0
	30	26.2
<b>JPEG 2000</b>	15	42.1
	20	39.8
	25	38.2
	30	37.1
<b>MMSOFM</b>	15	38.6
	20	36.5
	25	34.9
	30	32.5

**Table 4.3:** PSNR and compression ratio for CT-scan image using JPEG, JPEG 2000 and MMSOFM



(a)



(c)



(b)



(d)

**Figure 4.11:** Original Lena image (a) and Lena compressed with JPEG (b), JPEG 2000 (c) and MMSOFM (d)



(a)



(c)



(b)



(d)

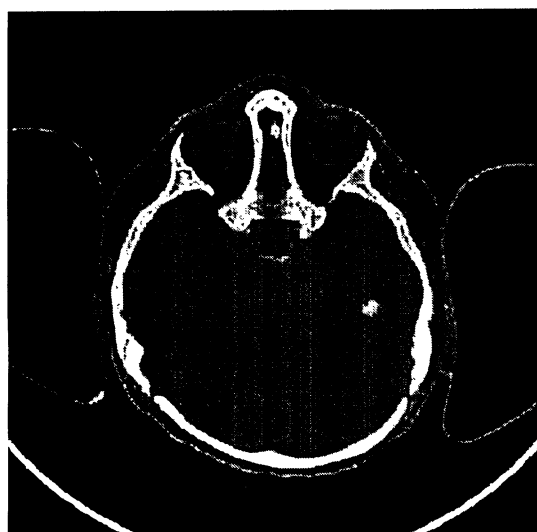
**Figure 4.12:** Original Barbara image (a) and Barbara compressed with JPEG (b), JPEG 2000 (c) and MMSOFM (d)



(a)



(c)



(b)



(d)

**Figure 4.13:** Original CT-scan image (a) and CT image compressed with JPEG (b), JPEG 2000 (c) and MMSOFM (d)

## Chapter 5

### Conclusion and future work

IN this thesis we have proposed an Image compression system utilizing wavelet and neural networks along with mathematical morphology operators. In Chapter 1, image compression was introduced, using the concept of transform coding at its root. Chapter 2 provided a literature review of the most recently proposed image compression algorithms based on transform coding, neural methods and wavelets. Then in Chapter 3, the theoretical details of wavelets and neural networks with their application to image compression were discussed. In Chapter 4, we proposed and implemented an algorithm to extract wavelet coefficients, cluster the significance map and finally to perform vector quantization of the significance map to identify significant coefficients. Experimental results with the proposed method and a comparison of different image compression algorithms is also done in Chapter 4. In Chapter 5 results of the overall system are summarized and some recommendation for future enhancement of the system are included.

## 5.1 Application of smooth wavelet filter to improve quality of compressed image

In this thesis we have used a smooth biorthogonal wavelet filter so as to preserve important edge regions within an image. In addition to its smoothness the filter used for obtaining wavelet coefficients is an FIR filter with linear phase property. The selected wavelet filter is also symmetric so that the corresponding wavelet transform could be implemented using mirror boundary conditions that reduce boundary artifacts. Therefore the use a smooth wavelet filter resulted in a compressed image with improved PSNR and visual quality.

## 5.2 Clustering using mathematical morphology

Mathematical morphology is an ideal tool for analyzing images based on their shape. It was used in the compression system to emphasize the edges within an image and to improve the overall image quality.

The biggest advantage in the application of morphology operators, i.e. dilation and erosion was the clustering of coefficients with in the significance map and eventually the identification of significant coefficients. The significant coefficients are slightly altered due to the vector quantization but the discrepancy in not huge.

In the proposed algorithm application of mathematical morphology achieved a PSNR gain in the compressed image. In addition to the PSNR gain, the subjective quality of compressed image i.e. both the texture information and edge information were well preserved.



## 5.3 Vector quantization

Self organizing feature map algorithm is an ideal learning machine for unsupervised learning approach. This technique was specifically used in the proposed algorithm since it preserves the input topology and performs better than a competitive network.

The test data i.e. blocks of significance map for training the feature map was divided into three categories namely: blocks that contain both 0 and 1, block that contain only 1 and finally blocks that contain only 0. Only blocks containing both 1 and 0 were selected for training whereas blocks that contain only 1 and only 0 were left out so as to improve the time performance of the vector quantizer.

## 5.4 Summary of Thesis contribution

In this work we have proposed an image compression system that consisted of three main components: First is the extraction of wavelet coefficients, which deal with the selection of a smooth biorthogonal wavelet filter. Second was the clustering of significant coefficients using mathematical morphology. The third component was the application of self organizing feature map to perform vector quantization of significance map and eventually identification of significant coefficients.

The proposed compression algorithm was successfully applied to broad classes of image and resulted in satisfactory PSNR. The proposed image coder is a general coding scheme and considerably improves the time performance. Image compressed using the proposed method was much better when compared to JPEG in terms of PSNR and visual quality. At lower compression ratios the performance of the proposed compression system is slightly lower than that of embedded zero wavelet (EZW), however

at higher compression ratio this method clearly under performs when compared with EZW. Unlike EZW the proposed method is independent of the parent-child relationship within the different subbands and will still be applicable even in the presence of a perfectly decorrelating wavelet transform. Performance of proposed MMSOFM is similar to JPEG 2000 in terms of visual image quality but the PSNR achieved using MMSOFM is lower than that obtained using JPEG 2000. However the proposed algorithm is simple and computationally less complex than JPEG 2000 which is based on lifting based integer wavelet transform and embedded block coding with optimal truncation.

## 5.5 Future work

More research is needed to improve the efficiency of the algorithm and to make it more competitive to the class of available wavelet coders.

Incorporating a lifting based integer wavelet transform with a smoother wavelet and the use of a hierarchical self organizing feature map could reduce the complexity and significantly improve the coding ability of the proposed algorithm.

The proposed method could be effectively employed for compressing medical images i.e. CT-scan and MRI images where there is a lot of textual similarity. The similarity amongst these images could be exploited for achieving better compression and image quality.

# Bibliography

- [1] B. B. Hubbard, "The World According to Wavelets", Wellesley, Massachusetts: A.K. Peters, 1998.
- [2] J. D. Gibson, T. Berger, T. Lookabaugh, D. Linghergh and R. L. Baker, "Digital Compression for Multimedia", San Francisco, California: Morgan Kaufmann Publishers, 1998.
- [3] K. Sayood, "Introduction to Data Compression ", 2nd Edition, San Diego, California: Academic Press, 2000.
- [4] J. D. Eggerton and M. D. Srinath, "A visually weighted quantization scheme for image bandwidth compression at low data rates", IEEE Transaction on Communication, Vol. 34, pp. 840-847, 1986.
- [5] R. A. DeVore, B. Jawerth and B. Lucier, "Image compression through wavelet transforms coding", IEEE Transaction on Information Theory, Vol. 38, pp. 719-746, 1992.
- [6] M. Vetterli and J. Kovavcevic, "Wavelets and Subband Coding", 1st Edition, Englewood Cliffs, New Jersey: Prentice Hall, 1995.
- [7] J. M. Shapiro, "Embedded image coding using zerotrees of wavelet coefficients", IEEE Transaction on Signal Processing, Vol. 41, pp. 3445-3462, 1993.

- [8] R. C. Gonzalez and R. E. Woods, "Digital Image Processing", 2nd Edition, Upper Saddle River, New Jersey: Prentice Hall, 2002.
- [9] E. Morales and F. Y. Shih, "Wavelet coefficients clustering using morphological operations and pruned quadtree", IEEE Transaction on Pattern Recognition, Vol. 33, 2000.
- [10] G. Strang, "The Discrete Cosine Transform", Society for industrial and applied mathematics, 1999.
- [11] S. G. Mallat, "A Wavelet Tour of Signal Processing", San Diego, California: Academic Press, 1999.
- [12] S. G. Mallat, "A theory of multiresolution signal decomposition: the wavelet representation", IEEE Transaction on Pattern Analysis and Machine Intelligence, Vol. 11, pp. 674-693, 1989.
- [13] Simon Haykin, "Neural Networks: A Comprehensive Foundation", Upper Saddle River, New Jersey: Prentice Hall, 1999.
- [14] A. Gersho and R. M. Gray, "Vector quantization and signal compression", Norwell, Massachusetts: Kluwer, 1992.
- [15] N. Jayant, J. Johnston and R. Safranek, "Signal compression based on models of human perception", IEEE Proceedings, Vol.81, pp. 1385-1421, 1993.
- [16] Glen. P. Abousleman, "Compression of hyperspectral imagery using hybrid DPCM/DCT and entropy-constrained trellis coded quantization", IEEE Transaction on Data Compression, pp. 322-331, 1995.
- [17] M. W. Marcellin and T. R. Fischer, "Trellis coded quantization of memoryless

and Gauss-Markov sources", IEEE Transaction on Communication, Vol. COM-38, pp. 82-93, 1990.

- [18] L. Vandendorpe, B. Maison and F. Labeau, "An adaptive transform approach for image compression", Proceedings of IEEE Digital Signal Processing workshop, Vol. 1, pp. 41-43, Loen, Norway, 1996.
- [19] C. W. Therrin, "Discrete random signals and statistical signal processing", Prentice Hall, 1992.
- [20] A. Albert, "Regression and the moore-penrose pseudoinverse", New York: Academic Press, 1972.
- [21] C. M. Goldrick, W. Dowling and A. Bury, "Image coding using the singular value decomposition and coding using the singular value decomposition and vector quantization" IEEE Transaction on Image Processing And Its Application, pp. 296-300, 1995.
- [22] T. Saito and T. Komatsu, "Improvement on singular value decomposition vector quantization", Electronics and Coimmunications, Part 1, Vol. 73, no.2, pp. 11-20, 1990.
- [23] J. F. Yang and C. L. Lu, "Combined techniques of singular value decomposition and vector quantization for image coding", IEEE Transaction on Image Processing, Vol. 4, no.8, pp. 1141-1145, 1995.
- [24] P. Waldemar and T. A. Ramstad, "Hybrid KLT-SVD image compression", IEEE International Conference on Acoustics, Speech and Signal Processing, Vol. 4, pp. 2713-2716, 1997.

- [25] J. Bracamonte, M. Ansorge and F. Pellandini, "Adaptive block-size transform coding for image compression", IEEE International Conference on Acoustics, Speech and Signal Processing, Vol. 4, pp. 2721-2724, 1997.
- [26] Y. Itoh, "An adaptive DCT coding with edge based classification", IEEE Transaction on Signal Processing, Vol. 1, pp. 83-86 1999.
- [27] E. Shusterman and M. Feder, "Image compression via improved quadtree decomposition algorithm", IEEE Transaction on Image Processing, Vol. 3, no. 2, pp. 207-215, 1994.
- [28] S. Moon-Ho Song and Gunho Lee, "Regulized Dequantization for DCT-based transform coding", IEEE International Conference on Acoustics, Speech and Signal Processing, Vol.4, pp. 2051-2054, 2000.
- [29] R. Prost, Y. Ding and A. Baskurt, "JPEG dequantization array for regularized decompression", IEEE Transaction on Image Processing, Vol. 6, no. 6, pp. 883-888, 1997.
- [30] W. Philips, "Correction to JPEG dequantization array for regularized decompression", IEEE Transaction on Image Processing, Vol. 6, no. 6, pp. 883-888, 1997.
- [31] Yung-Gi Wu and Shen-Chuan, "Medical Image Compression by Discrete Cosine Transform Spectral Similarity Strategy", IEEE Transactions on Information Technology in Biomedicine, Vol. 5, Issue 3, pp. 236-243, 2001.
- [32] R. Cierniak and L. Rutkowski, "Neural networks and semi-closed loop predictive vector quantization for image compression", IEEE Transactions on Image Processing, Vol. 1, pp. 245-248, 1996.

- [33] R. Cierniak and L. Rutkowski, "Image compression by competitive learning neural network and predictive vector quantization", *Applied Mathematics and Computer Science*, Vol.6, no. 3, 1996.
- [34] C. Amerijckx, M. Verleysen, P. Thissen and J. Legat, "Image Compression by Self-Organized Kohonen Map", *IEEE Transactions on Neural Networks*, Vol. 9, no. 3, pp. 503-507, 1998.
- [35] T. Kohonen, "Self-Organization and Associative Memory" 2nd Edition, Berlin: Springer-Verlag, 1997.
- [36] S. A. Rizvi and N. M. Nasrabadi, "Lossless image compression using modular differential pulse code modulation" *IEEE Transactions on Image Processing*, Vol. 1, pp. 440-443, 1999.
- [37] Dong-Chul Park and Young-June Woo, "Weighted centroid neural network for edge preserving image compression", *IEEE Transactions on Neural Networks*, Vol. 12, no. 5, pp. 1134-1146, 2001.
- [38] M. Venkatraman, H. Kwon and N. M. Nasrabadi, "Object-based SAR image compression using vector quantization", *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 36, no. 4, pp. 1036-1046, 2000.
- [39] A. Said and W. A. Pearlman, "A new fast and efficient image code using set partitioning in hierarchical tree", *IEEE Transactions on Circuit and Systems for video Technology*, Vol. 5, 1995.
- [40] J. M. Barbalho, A. Duarte, D. Neto, J. A. F. Costa and M. L. A. Netto, "Hierarchical SOM applied to image compression", *IEEE Transactions on Neural Networks*, Vol. 1, pp. 442-447, 2001.

- [41] Y. Linde, A. Buzo and R. M. Gray, "An algorithm for vector quantization design", IEEE Transactions on Communication, Vol. 28, pp. 84-95, 1980.
- [42] Y. Hui; C. W. Kok and T. Q. Nguyen, "Image compression using shift-invariant dyadic wavelets" IEEE Transactions on Image Processing. Vol. 1, pp. 61-64, 1997.
- [43] J. Liang and T. W. Parks, "A two-dimensional translation invariant wavelet representation and its applications", Proceedings of IEEE ICIP, Vol. 1, pp. 66-70, 1994.
- [44] S. D. Marco, P. N. Heller and J. Weiss, "An M-band 2-dimensional translation-invariant wavelet transform and applications", Proceedings of IEEE ICASSP, pp. 1077-1080, 1995.
- [45] C. Chrysafis and A. Ortega, "Line-based, reduced memory, wavelet image compression", IEEE Transactions on Image Processing, Vol. 9, Issue 3, pp. 378-389, 2000.
- [46] A. Munteanu, J. Cornelis, G. Van Der Auwera and P. Cristea, "Wavelet image compression - the quadtree coding approach", IEEE Transactions on Information Technology in Biomedicine, Vol. 3, no. 3, pp. 176-185, 1999.
- [47] Yu Tian-Hu, He Zhihai and S. K. Mitra, "Simple and efficient wavelet image compression", IEEE Transactions on Image Processing, Vol. 3, pp. 174-177, 2000.
- [48] A. T. Deever and S. S. Hemami, "Lossless image compression with projection-based and adaptive reversible integer wavelet transforms", IEEE Transactions on Image Processing, Vol. 12, no. 5, pp. 489-499, 2003.
- [49] D. Schilling and P. C. Cosman, "Preserving step edges in low bit rate progressive



image compression", IEEE Transactions on Image Processing, Vol. 12, no. 12, pp. 1473-1484, 2003.

- [50] N. Ahmed, T. Natarajan and K. R. Rao, "Discrete Cosine Transform", IEEE Transaction on Computer, Vol. C-23, no. 1, pp. 90-93, 1974.
- [51] J.Serra, "Image Analysis and Mathematical Morphology", New York: Academic Press, 1988.
- [52] D.A.Huffman, "A method for construction of minimum redundancy codes", Proceedings of the Institute of Electronics and Radio Engineers, Vol. 40, pp. 1098-1101, 1952.