A Downtown On-Street Parking Model with Urban Truck Delivery Effects

by

Ahmed Amer

Master of Business Administration, Strathclyde Business School, UK, 2011

Bachelor of Engineering, Cairo University, Cairo, Egypt, 2000

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Ahmed Amer
Department of Civil Engineering
Ryerson University

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Abstract

This study presents an on-street parking model for downtowns in urban centers that incorporates the often-neglected parking demand of commercial vehicles. The behavior of truck deliveries is distinctly different from commuter parking: trucks do not cruise for parking spaces when parking is saturated, instead they are more likely to double-park near their destinations and occupy a travelling street lane.

The study generalizes the downtown on-street parking model from Arnott and Inci (2006) to investigate the relationship between commercial and passenger vehicles' parking behaviors, and provide tools for policy makers to optimize the trade-offs in parking space allocation, pricing, and network congestion. The social optimum can be obtained by solving a nonlinear optimization problem.

The model is applied to a case study of downtown Toronto. It is shown that developing an inclusive policy, one that captures the effect of all road users including commercial vehicles, leads to considerable efficiency gains.

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1 INTRODUCTION

As the rate of urbanization increases, societies struggle to develop policies to make the most efficient use of space to cope with congestion. Parking management is one such policy. Poorly implemented parking policies can lead to “cruising” for parking spaces, which can account for more than 30% of downtown traffic in some cases (Shoup, 2005). On the other hand, parking pricing strategies can be more effective than road pricing strategies because of a greater public acceptance. The effectiveness of parking policies can also be enhanced by such engineered technologies as real time information systems (e.g. Cao and Menendez, 2015) like *SFpark.org* or data-driven parking pricing (Qian and Rajagopal, 2013; Mackowski et al., 2015).

Researchers have developed analytical means of evaluating trade-offs in pricing, capacity, information technologies, and spatial-temporal allocation of parking spaces with respect to their welfare effects on cruising, traffic congestion, transit use, and activity patterns, among others. However, urban freight is largely neglected in these studies, despite the significant differences in freight parking use patterns from commuter patterns, the high demand for freight parking or loading/unloading, and the exacerbated effects that truck delivery inefficiencies have on multiple aspects of urban sustainability—congestion, safety, air quality, etc. (e.g. Chow et al., 2010; You et al., 2015). In a recent study of freight parking demand in New York City, Jaller et al. (2013) confirmed that parking policies often overlook urban freight.

Urban freight parking needs are inherently different from commuter parking. Unlike commuters, delivery trucks typically need spaces to temporarily park to load or unload goods at destinations in the central business district. Trucks take up more space, require close proximity to destinations (Tipagornwong and Figliozzi, 2015), and require access routes to parking locations with greater turning radii. For example, parcel delivery services like FedEx, UPS, and Purolator accounted for more than $1.5M in parking fines in Toronto in 2006 (Haider et al., 2009). Jaller et al. (2013) highlight a list of example parking policies available to policy-makers: parking management systems, car-share provision, in-lieu fee, maximum parking standard, parking freeze, residential parking permits, transferable parking rights, variably priced parking, among others. These policies typically overlook freight or commercial vehicle parking demand.

In a focus group survey of thirteen industry sectors, Morris et al. (1998) identified parking as one of the key transportation barriers for freight mobility. Focus groups indicated congestion, inadequate docking space, inadequate curb space for commercial vehicles, and oppressive parking regulations as examples. Recommendations included off-peak deliveries, reducing passenger vehicle traffic, improving mass transit to reduce private passenger vehicles, creating “truck only” areas like the garment district in New York City, using integrated information systems, or introducing consolidation centres outside the city. While some strategies like off-peak deliveries have been studied further (e.g. Holguín-Veras et al., 2011), there are generally no analytical downtown parking models that consider freight delivery activities. The few efforts that do exist are either traffic simulation-based (Nourinejad et al., 2014) or do not consider equilibrium interactions of truck deliveries and passenger parking (Tipagornwong and Figliozzi, 2015). As such, many of the recommendations or issues in urban freight and city logistics related to parking cannot be analytically addressed.

I propose a downtown on-street parking equilibrium model that incorporates the effects of urban freight. The model generalizes a state-of-the-art on-street parking model (Arnott and Inci, 2006) to include effects of space allocation for truck deliveries, truck double parking, and consequences in traffic flow capacities. To the best of my knowledge, this is the first parking equilibrium model that considers all these trade-offs. I then apply the theoretical model to a case study of downtown Toronto to support first-best and second-best space allocation policies for truck deliveries. The model can be easily customized to other downtown regions around the world to support similar policy recommendations.

2 LITERATURE REVIEW

Analytical commuter parking models are relatively new compared to other transportation models. Some of the earliest models of note examined the dual nature of parking as a private and public good. Glazer and Niskanen (1992) noted that economists (e.g. Vickrey, 1954) generally assumed curbside parking to be a private good to justify marginal cost pricing. On the contrary, the authors demonstrated that insufficient parking spaces lead to cruising behavior, which results in increased costs for both travelers looking for parking as well as in-transit travelers. When the roadway is sub-optimally priced or free, there should be a positive lump sum parking fee that covers that cost.

Another feature of the dual nature of parking observed by Arnott et al. (1991) and Anderson and de Palma (2004) is that the pricing by a market of private operators is both monopolistic and competitive. Each operator sets the price as profit-maximizing due to the all-or-nothing demand for a single space (this behavior has been empirically confirmed by Kobus et al., 2013), but is competitive with other parking spaces for a user. Because operators may ignore the costs they impose on cruising, it is possible that the competitive pricing may result in welfare reduction relative to no pricing at all.

Arnott et al. (1991) used Vickrey's (1969) bottleneck congestion model to derive insights on the spatial and temporal nature of parking pricing. When parking is free, the authors showed how driver behavior to naturally park "outwards"—occupy spots in order of decreasing accessibility—leads to increased inefficiencies. Time-varying road pricing may eliminate queueing and reduce schedule delay costs, but distance-based parking pricing is needed to induce a more efficient "inward" parking behavior. They concluded that it is easier to implement an efficient parking fee policy than efficient road tolling policy. Their bottleneck model of parking has been extended by Zhang et al. (2008) to consider both morning and evening commutes, by Zhang et al. (2011) to investigate the efficiency of parking permits, by Qian et al. (2012) to examine parking clusters, and by Yang et al. (2013) to add capacity constraints and parking reservations. Fosgerau and de Palma (2013) studied the effects of early bird specials with time-varying parking pricing. While Lam et al.'s (2006) work is not directly a bottleneck parking model, they considered departure time choice at a network level using variational inequalities. The model requires route enumeration, which makes it difficult to apply to large scale study areas.

Arnott and Rowse (1999) used a circular city structure to analyze the randomness of parking availability and cruising to examine dynamic parking pricing and justify parking information systems. The model structure resulted in non-unique equilibria, however, and required a number of assumptions including ignoring traffic congestion. Anderson and de Palma (2004) incorporated cruising in a simpler model to arrive at several major conclusions. First, the socially optimal parking configuration is independent of the cost of cruising. However, the equilibria of both unpriced parking and privately operated parking have smaller optimal parking spans as cruising costs increase, though the price of parking is always better off than the unpriced parking.

Arnott and Inci (2006) first introduced a parking equilibrium model with traffic flow behavior to explicitly measure cruising effects. They found that regardless of the curbside parking capacity, it is efficient to price the spots to the point where cruising can be eliminated without parking becoming unsaturated. On the other hand, if pricing is fixed, then it is second-best optimal to increase the number of curbside spaces until cruising is eliminated without parking becoming unsaturated.

In more recent years, research on parking has shifted to interactions between multiple decision-makers. Calthrop and Proost (2006) studied curbside parking in the presence of off-street parking using a Stackelberg game with a single garage operator as a follower. Arnott (2006) extended his earlier traffic flow explicit parking model to include spatial competition between parking garages and curbside parking under a public authority. The study includes a variant model that accounts for mass transit, allowing policy-makers to evaluate trade-offs between system-wide transit designs and parking policies. Several conclusions were made: competition between parking operators determines the full price of parking; cruising costs adjust the curbside parking pricing to match the garage parking; increasing saturated curbside parking prices reduces cruising and traffic congestion; mass transit can significantly affect second-best parking policy, which can be exploited by considering maximum garage parking standards (done so in Boston, New York, and San Francisco). Arnott and Rowse (2009) illustrated the model with a detailed numerical example.

The two leading analytical parking model structures in the literature appear to be Arnott and Inci (2006) model and the Vickrey bottleneck parking model, each with their own benefits and limitations. Neither class of models currently deals with truck delivery behavior. As a consequence, we cannot evaluate the effects of congestion impacts between trucks, personal in-transit vehicles, cruising vehicles, and double-parking vehicles; curbside space capacity for trucks; time windows for deliveries; integrated information systems or advanced connected truck technologies; or consolidation centers. In this study, Arnott and Inci (2006) model is generalized to include truck traffic and delivery behavior. It turns out this generalization is not a trivial matter (i.e. adding a second class) as the behavior and measurement of consequences are quite different for trucks.

3 THE PROPOSED GENERALIZED MODEL

The model is developed based on Arnott and Inci (2006) downtown parking and traffic congestion model with a key extension made to consider commercial vehicles parking. The model is chosen to allow policy makers to control the double-parking behavior of CVs along with the cruising behavior of passenger cars. In the next four sections I describe the different parts of the model.

3.1 Assumptions and Downtown Setting Description

Before describing the proposed generalized model, it is important to mention a few notes to help distinguish between passenger cars and commercial vehicles as they are intended in this study.

First, the size of commercial vehicles and their maneuvering capabilities are quite different from passenger cars, and it is sensible that we distinguish between the size of the typical curbside parking spaces available for passenger cars and those required for commercial vehicles. In this study I consider only light commercial vehicles for which curbside parking is more applicable than the loading/unloading docks. The latter are predominantly meant for larger vehicles with different types of cargo and with much longer parking periods at the destination.

With this perspective in mind, light commercial vehicles are still different from passenger cars and would require special parking spaces; therefore I assume that for a specific curbside parking space it would be necessary to assign it to either type of these vehicles. To distinguish between passenger and commercial vehicle parking, I denote the number of passenger cars parking per unit area as P_p and the number of commercial vehicles parking as P_c.

Another important note is the distinction between passenger car and commercial vehicle behavior when curbside parking spaces are fully occupied. In such case, passenger car drivers typically cruise around the area until an available space is found. This particularly occurs when curbside parking is underpriced, as it makes economic sense to search for cheap parking spaces.

Commercial vehicles, on the other hand, do not cruise for parking. Due to the need to load or unload goods, if no parking spaces were immediately available in close vicinity to their destination, commercial vehicles will resort to double-parking as the cheapest choice available. This major difference is incorporated in the model.

(Daily news, 2014)

(CNBC, 2015)

Figure 1 Trucks double-park near its destination

The term double-parking typically indicates a parking situation in which the truck stops to park in a lane designated for travelling vehicles and next to an occupied parking lane as indicated in the image above. However, the term could also be extended to cover similar situations in which trucks illegally stop to park in a travelling lane that is not necessarily being next to an occupied curbside parking as demonstrated in the image below. In both cases the truck occupies a travelling lane, which in this study is considered as a lane drop that creates a bottleneck in the traffic flow as will be discussed in later sections. In should be noted in this respect that a lane drop from four lanes to three lanes would have different effect than lane drop from three lanes to two lanes. This is further addressed in the sensitivity analysis in section 5.3.1.3.

(Toronto, 2015)

(Chicago, USA)

Lastly, whereas passenger car demand is elastic to parking and traffic costs, truck deliveries are not so elastic (Tipagornwong and Figliozzi, 2015). The trucking company transfers that additional cost to the receiver, who in turn may transfer it to the customer, so as a result the freight demand is fairly inelastic.

As I make clear these notes, I proceed with describing the model's assumptions and variables. The model assumes a downtown area that features a Manhattan style street network with city blocks of side length b and street width equal to w, where parking is provided uniformly on-street. Table 1 below shows an initial set of variables used to describe the different types of vehicles travelling on downtown streets.

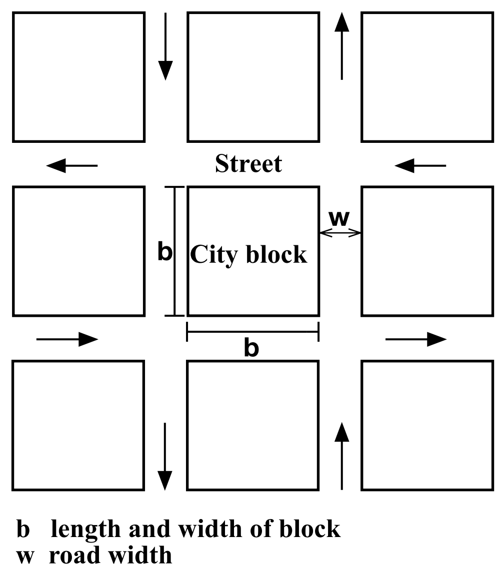


Figure 2 Downtown features a Manhattan street network

Table 1 Set of variables describing travelling vehicles

Notation	Description
D_p	Passenger car trip demand per unit time-area (veh/hr-mi^2)
D_c	Commercial vehicles trip demand per unit time-area (veh/hr-mi^2)
T_p	Stock of in-transit passenger cars per unit area (veh/mi^2)
C	Stock of cruising passenger cars per unit area (veh/mi^2)
T_c	Stock of in-transit commercial vehicles per unit area-time (veh/mi^2)
H	Stock of double-parking commercial vehicles per unit area-time (veh/mi^2)
P_p	Parking spaces allocated to passenger cars per unit area (space/mi^2)
P_c	Parking spaces allocated to commercial vehicles per unit area (space/mi^2)
θ	Ratio of a commercial vehicle parking space to that of a passenger car parking space.
m_p	Distance travelled by passenger cars in downtown before arriving to destination (mi)
m_c	Distance travelled by commercial vehicles in downtown before arriving to destination (mi)
l_p	Parking duration of passenger cars (hr)
l_c	Parking duration of commercial vehicles (hr)
ρ_p	Value of time of passenger cars ($/hr)
ρ_c	Value of time of commercial vehicles ($/hr)

For passenger cars, the demand for travel per unit area is D_p, and drivers are assumed to have a homogeneous value of time ρ_p. They must travel a distance m_p through the downtown before arriving to their destinations. T_p is the stock of passenger vehicles in-transit per unit area until they arrive to their destinations. Once there, they park for a period of time l_p if parking is available. Otherwise, they cruise until a space is available. C is the stock of cruising vehicles per unit area.

Commercial vehicles have different travel behavior and therefore another set of variables $(D_c, \rho_c, l_c, m_c, T_c)$ is used to identify the above characteristics. Cruising is only recognized for passenger cars and the stock of double-parking vehicles per unit area H is only considered for commercial vehicles.

With the introduction of these variables, we are able to distinguish between four types of travelers that make up the traffic composition in the streets of the downtown. First, for passenger cars we have vehicles in-transit to their destinations T_p and vehicles cruising for parking C. Second, for commercial vehicles we have vehicles in-transit to destination T_c and other vehicles H that occupy part of the street space by double-parking. Finally, there are two other types of vehicles P_p and P_c that occupy a non-travelling part of the street space.

3.2 Travel congestion

Travel is subject to flow congestion and in this section I aim to distinguish between the congestion effects that every type of travelling vehicles (T_c, T_p, C, H) contribute to the traffic. However, it is suitable to first review key concepts related to traffic flow models.

3.2.1 Traffic Flow Models

Traffic flow models are used in planning, designing, and in monitoring traffic operations. The main objective of these models is to provide a generalized mathematical representation of the relationship between key traffic stream characteristics. (May, 1990) described three key traffic flow characteristics that could be used to explain particular traffic conditions: flow, speed, and density.

Flow (q) is the number of vehicles passing a specific point in a given period of time in a single lane. The flow is expressed as an hourly rate (veh/hr/lane). A unique flow parameter is the maximum flow (q_m). Speed, commonly taken as the space-mean speed and it represents the average rate of motions (mi/hr). Two unique speed parameters are the free-flow speed (u_f) and the optimum speed (u_o). The optimum speed occurs under maximum flow condition while free-flow occurs when the driver has the liberty of driving at the desired speed without being hindered by other vehicles. Density (k) is the number of vehicles occupying a section of roadway in a single lane. Density is expressed per mile and per lane (veh/mi/lane). Two unique density parameters are the jam density (k_{jam}) and the optimum density (k_o). The jam density occurs when both the flow and the speed approach zero, while the optimum density occurs under the maximum flow.

A number of traffic flow models have been proposed in a continuous effort to represent the relationship between the key traffic flow characteristics. They are generally classified between single-regime models and multi-regime-models.

3.2.1.1 Single-Regime Models

Daiheng (2015) and May (1990) explain single-regime models that consider one mathematical expression to cover the complete range of flow conditions which includes free-flow and congested flow conditions. In this part, these models are briefly reviewed, while the next section discusses multi-regime models.

Greenshields proposed a linear speed-density relationship and used a decreasing function expression to indicate this relation. The intercepts were intuitively identified at both ends, when the density approaches zero the speed approaches the free-flow speed *(u_f)*. And as the density increases, speeds are reduced until in reaches a standstill situation where the speed is equal to zero, and this occurs when the density reaches the maximum value (k_{jam}) (Daiheng, 2015).

$$u = u_f - \left(\frac{u_f}{k_{jam}}\right)k$$

Using the identity rule of traffic flow that is $q = k \times u$, one could derive the flow-density relationship from Greenshields model as:

$$q = u_f \left(k - \frac{k^2}{k_{jam}} \right)$$

The above relation indicates a parabola in which under very low density conditions the flow approaches zero and the speed approaches free flow speed. And as the flow increases, density increases while the speed is decreasing. When optimum density is reached, flow becomes maximum (May, 1990).

Similarly the flow-speed relationship could be obtained using Greenshields model as follows:

$$q = k_f \left(u - \frac{u^2}{u_f} \right)$$

While Greenshields model is ideal for illustration purpose, field observations made by transportation study centers has shown that the observed optimum density values are not compatible with the model which suggests that $k_o = \frac{k_{jam}}{2}$. It was also found that the speed does not decrease in a linear manner and that the free flow speed is sustainable for some time even as the density continue to increase (Daiheng, 2015).

Other single-regime models are proposed, for example Greenberg assumed a nonlinear speed-density model of the following form, however, a disadvantage of this model is that the free-flow speed is infinity:

$$u = u_o \ln \left(\frac{k_{jam}}{k} \right)$$

Underwood proposed a model of the following form, however while the free-flow speed is not infinity, the speed never reaches zero and jam density is infinity (May, 1990):

$$u = u_f e^{-k/k_o}$$

Northwestern University researchers proposed the following s-curve speed-density curve, however the speed still does not go to zero as the density approaches the jam density:

$$u = u_f e^{\frac{-1}{2\left(\frac{k}{k_o}\right)^2}}$$

Drew proposed a model based on Greenshields model but with an additional parameter n. Varying the parameter n, creates a family of models that includes Greenshields model when n=1:

$$u = u_f (1 - \left(\frac{k}{k_{jam}}\right)^{n+1/2}$$

When compared to data collected from the field by transportation study centers, each of these models showed some variances at different portions of the data set. For example while some models showed good fit with the free-flow side, it was not in harmony with the capacity part.

3.2.1.2 Multi-Regime Models

Some discontinuity was evident in the traffic relationship data sets collected from the field and therefore several researchers have proposed multi-regime models with separate mathematical formulations representing different portions of the density data set, and additional parameters to distinguish different traffic conditions.

Daiheng (2015) has presented some of the most prominent multi-regime model as compared in the following table:

Multi-regime Model	Free-flow Regime	Transitional Regime	Congested-Flow Regime
Edie model	$u = 54.9e^{-k/163.9}$ $(k \leq 50)$	-	$u = 26.8 \ln\left(\frac{162.5}{k}\right)$ $(k \geq 50)$
2-regime linear model	$u = 60.9 - 0.515k$ $(k \leq 65)$	-	$u = 40 - 0.265k$ $(k \geq 65)$
Modified Greenberg model	$u = 48$ $(k \leq 35)$	-	$u = 32 \ln\left(\frac{145.5}{k}\right)$ $(k > 35)$
3-regime linear model	$u = 50 - 0.098k$ $(k \leq 40)$	$u = 81.4 - 0.913k$ $40 < k \leq 65$	$u = 40 - 0.265k$ $(k > 65)$

Multi-regime models generally provide better fit to empirical observations, however, as noted by May (1990) there is some difficulty in determining the breakpoint between regimes.

Daiheng (2015) notes that a further advanced step in representing the speed-density relationship and the associated fundamental traffic flow relationships is to consider the speed as a distribution at each density level. By doing so it is possible to account for the scattering effect that occurs in real-world data and effectively providing a stochastic relationship. As such the speed-density relationship could be written as per the following generic form where omega (ω) is a distribution parameter and is dependent on the value of the density k at each point:

$$u = f(k, \omega(k))$$

3.2.2 Travel Congestion in the Model

Greenshields model is ideal for illustration purpose and it was adopted in the study to represent the traffic flow relationship. The relationships are applied to determine the traffic state but with some modifications to account for travelers' types discussed earlier.

Consider the set of variables in Table 2 below which describe the traffic state.

Table 2 Set of variables describing traffic state

Notation	Description
v	Travel speed (mi/hr)
v_0	Free flow speed (mi/hr)
t	Travel time per unit distance (hr/mi)
t_0	The free flow travel time (hr/mi)
k	Density per unit area (veh/mi^2)
k_j	Jam density per unit area (veh/mi^2)
Ω	Jam density in the absence of curbside parking (veh/mi^2)
$P_{\max}$	Maximum number of parking spaces that could be accommodated by the street per unit area (space/mi2)
α	Equivalency factor for converting the stock of cruising cars C to an equivalent in-transit passenger cars T_p
β	Equivalency factor for converting the stock of commercial vehicles T_c to an equivalent in-transit
γ	Equivalency factor for converting the stock of double-parked vehicles H to an equivalent stock of in-transit

The travel speed v can be expressed as $v = v_0\,(1 - k/k_j)$, where v_0 is the free flow travel speed, k is the traffic density per unit area, and k_j is the jam density. Accordingly, the travel time per unit distance t (which is the reciprocal of the travel speed $v = 1/t$) can be expressed as Eq. (1).

$$t = \frac{t_o}{1 - \frac{k}{k_j}} \tag{1}$$

The density of cars per unit area k can be expressed as the sum of densities of the four types of travelers occupying the street space T_p, C, T_c, and H. However, before we sum them we need to convert these different types of densities to an equivalent in-transit passenger car density T_p using equivalency factors α, β, and γ. The first factor α is used to account for the effect of cruising vehicles. The second factor β is used to account for the effect of the stock of in-transit commercial vehicles. Finally, the third factor γ accounts for the effect of double-parking and I discuss it in the next section. k can be expressed as Eq. (2).

$$k = T_p + \propto C + \beta T_c + \gamma H \tag{2}$$

The jam density k_j is affected by the proportion of the street area assigned to parking spaces. The more road space that is allocated to parking, the less is the available street area for travelling vehicles. This relation can be modified from Arnott and Inci (2006) as follows, where $P_{\max}$ is the number of available parking spaces if all the street area was allocated to parking (with no area left for travelling cars).

$$k_j = \Omega \left[1 - \frac{P_p + \theta P_c}{P_{max}} \right] \tag{3}$$

It can be seen from Eq. (1), (2), & (3) that the travel time per unit distance $t(T_p, T_c, C, P_p, P_c, H)$ is an increasing function of the vehicles' densities per unit area T_p, T_c, C, H, and is also an increasing function of the stock of assigned parking spaces P_c, P_p.

3.3 Modeling the effect of double-parking

In Eq. (2), I introduced γ as a new factor to convert the stock of double-parked vehicles to equivalent in-transit vehicles. To estimate the value of gamma, I contemplate the effect of double-parking as a temporary lane drop that creates a bottleneck in traffic flow and carry a standard bottleneck analysis such as explained in May (1990). The outcome of the analysis helps estimate the impact to the traffic density in the congested area upstream of the bottleneck.

Consider for example a three-lane road section, where at one location a vehicle stops and double-parks occupying a travelling lane. This incident reduces the capacity of the road at this section to two lanes. Consider locations $A, B, C,$ and D shown in Figure 3, and assume that the flow at location A is equivalent to 2.5 lanes of flow. Assuming Greenshield's relationship holds, the traffic state at location A must be on the low-density leg of the flow-density curve as shown on Figure 3. At location C, the flow-density curve is different from the three-lane section because it is only two lanes. Therefore, the capacity and the jam density are two-thirds of their corresponding values on the bigger curve. The traffic flow at this section must drop from 2.5 lanes flow to the maximum capacity of the two lanes section. Location B just upstream of the bottleneck is on the three-lane section. However, it is influenced by the bottleneck so the flow on location B is equal to the flow on location C. Since this section represents a congested zone it necessarily falls on the right arm of the flow-density curve as shown in Figure 3. Location D represents the section just downstream of the bottleneck where the flow of traffic is still equal to the flow of the bottleneck, but now the traffic is travelling again on the three-lane section. Finally, gamma is considered as the ratio between the traffic densities at location B and location A as shown in Eq. (4).

$$\gamma = \frac{d_B}{d_A} \tag{4}$$

where:

d_B = density at location B veh/mi

d_A= density at location A veh/mi

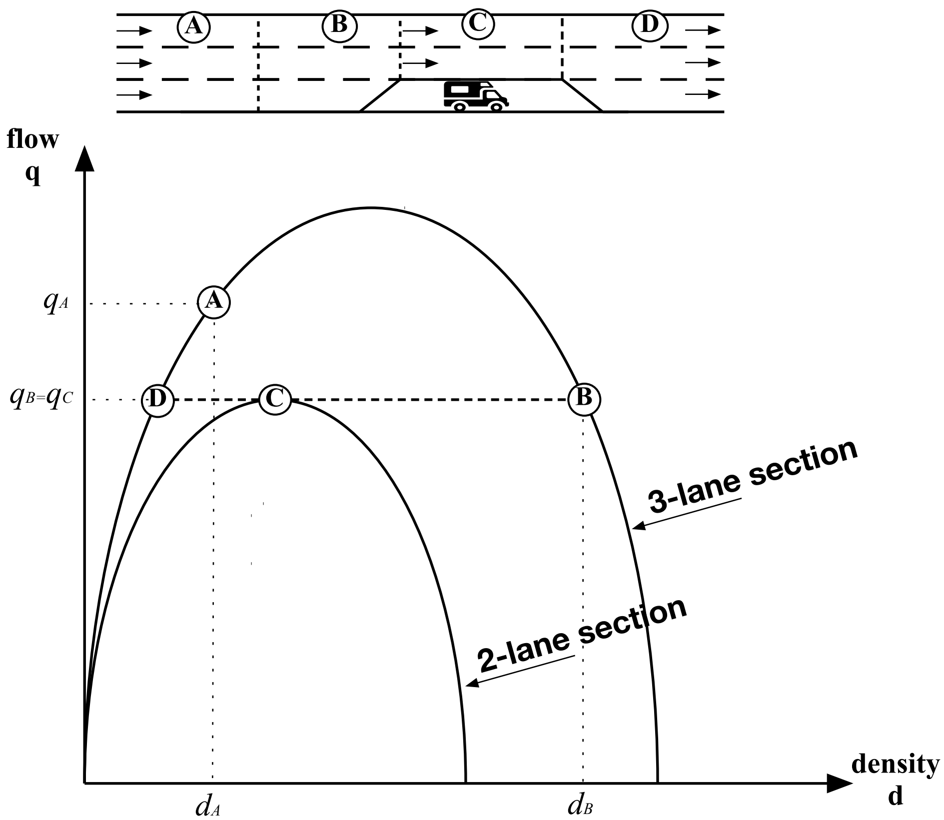


Figure 3 Modelling double-parking effect

For illustration, assume the following numbers are applicable for an urban three-lane road section in downtown:

Lane capacity q_m = 660 vph/lane (or 1980 vph for 3 lanes)
Free flow speed u_f = 20 mph
Jam density d_{jam} = 176 veh/mi-lane (or 528 veh/mi for 3 lanes)

Assume the Greenshield flow-density relationship in Eq. (5) applies to this setting. As described, this example would result in $\gamma = \frac{450.67}{102.33} = 4.40$. Other lane drop scenarios such as 2 lanes to 1 lane lead to $\gamma = \frac{315.14}{59.59} = 5.29$, and for 4 lanes to 3 lanes it is $\gamma = \frac{584.83}{146.48} = 3.99$. This gives a fair range for selecting an appropriate factor to apply to double-parking vehicle densities.

$$q = u_f d - \left(\frac{u_f}{d_{jam}}\right) d^2 \tag{5}$$

where:
d = traffic density
u_f = free flow speed
d_{jam} = jam density

3.4 Demand function

Passenger car demand per unit area D_p is assumed to be elastic (price sensitive). I use the same formula presented in Arnott and Inci (2006) except for the assumptions related to the travel time t, which now incorporates the effect of commercial vehicles as well as passenger cars as explained in Section 3.2. A Cobb-Douglas formula of the form $D_p = D_o F^e$ is used to define the relation between the commuter trip demand and the trip price, where F is the full trip price and e is the demand elasticity with respect to price.

F consists of three cost components: the in-transit travel time $m_p t$, the cruising for parking travel time $C\frac{l_p}{P_p}$, and the parking fee $f l_p$.Cruising time is set to be the cruising stock C times the reciprocal of the rate of vacating a parking spot per unit area $\frac{P_p}{l_p}$.

The value of time ρ_p is used to convert the time components to equivalent dollar cost. The demand function could therefore be written as Eq. (6).

$$D_p = D_o \left[\rho_p m_p t + \rho_p C \left(\frac{l_p}{P_p} \right) + f l_p \right]^e \tag{6}$$

where:

D_p = passenger car trip demand per unit time-area (veh/hr-mi^2)

D_o = constant calibrated depending based on actual demand in study area

m_p = distance travelled by passenger cars in the downtown area to destination (mi)

C = stock of cruising passenger cars per unit area (veh/mi^2)

P_p = parking spaces allocated to passenger cars per unit area = stock of cars parked (veh/mi^2)

l_p = parking duration of passenger cars (hr)

f = on-street parking fee per unit time ($/hr)

e = elasticity of demand with respect to trip price

While we have assumed an elastic demand function for passenger cars, the same is not valid for freight. It is far less sensitive to the price of parking, as discussed in Section 3.1.

3.5 Equilibrium conditions

The parking equilibrium is described by a system of equations that considers saturated parking in a steady-state traffic flow in an increment of time, as summarized in Figure 4. Saturated parking indicates a demand that is high enough such that parking spaces remain 100% occupied during the study period, so as soon as one spot is vacated it is taken by another cruising car. It has a more explicit effect on the model: since 100% of spaces are assumed to be occupied, the terms P_p and P_c refer to both number of spaces occupied as well as number of spaces available. As mentioned by Arnott and

Inci (2006), unsaturated conditions would not have a cruising problem, so we restrict ourselves to the saturated setting.

The steady state flow is defined as a stationary point in a dynamic setting where the traffic inflow into the system equals the traffic outflow. The steady state saturated parking equilibrium is demostrated in Figure 4 and can be described by four equilibrium conditions described in Eq. (7) – Eq. (10), a pair for each type of vehicle.

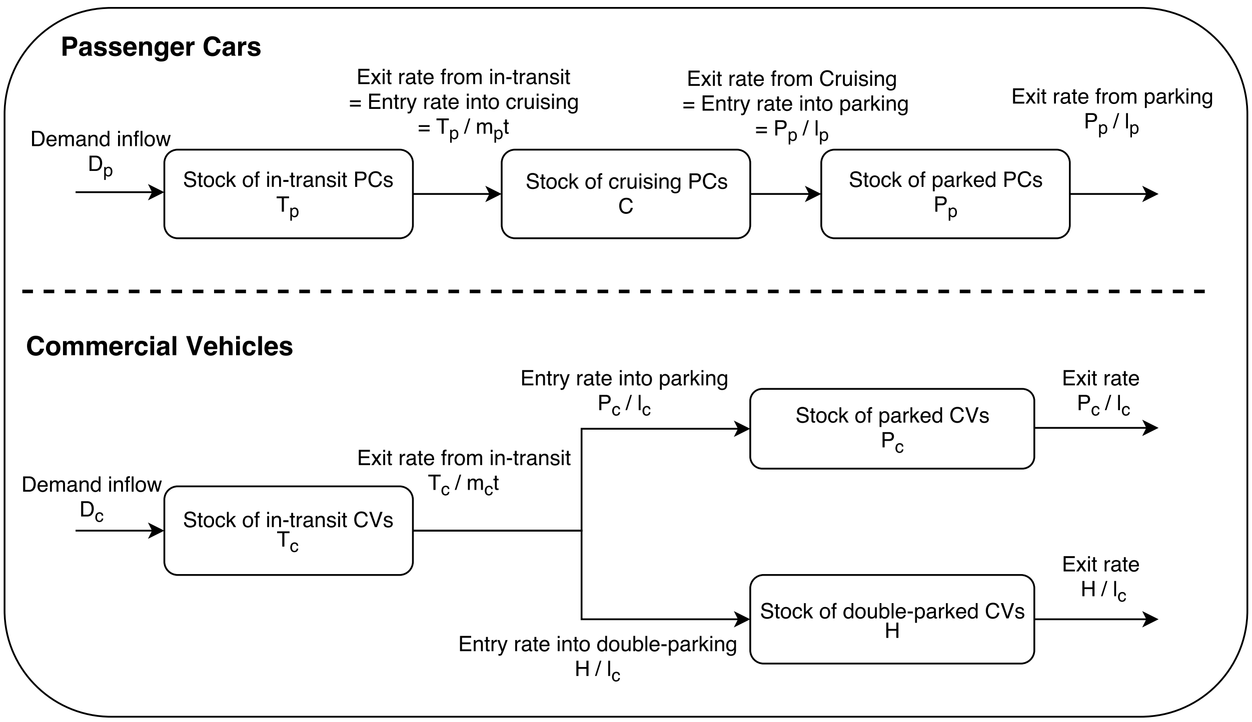


Figure 4 Saturated parking in a steady state flow

For passenger cars

$$D_p = \frac{T_p}{m_p t\left(T_p, T_c, C, P_p, P_c, H\right)} \tag{7}$$

$$\frac{T_p}{m_p t\left(T_p, T_c, C, P_p, P_c, H\right)} = \frac{P_p}{l_p} \tag{8}$$

For commercial vehicles

$$D_c = \frac{T_c}{m_c t\left(T_p, T_c, C, P_p, P_c, H\right)} \tag{9}$$

$$\frac{T_c}{m_c t\left(T_p, T_c, C, P_p, P_c, H\right)} = \frac{P_c}{l_c} + \frac{H}{l_c} \tag{10}$$

Eq. (7) and Eq. (9) are state transition conservation conditions. Eq. (7) for passenger cars and Eq. (9) for commercial vehicles require the flow of vehicles entering the in-transit pool per unit area (D_p and D_c) as equal to the flow of vehicles exiting the in-transit pool per unit area $\frac{T}{mt}$. Since t is the travel time per unit distance, we multiply it by the travel distance m to get the total time spent in-transit.

Eq. (8) and Eq. (10) describe the dynamic equilibrium. For passenger cars, Eq. (8) states that the exit rate from the in-transit pool $\frac{T_p}{m_p t}$ is now considered to be entry rate into the cruising for parking pool. And as mentioned earlier, cars will continue to cruise until a space is open, so the exit rate from crusing from parking could be defined in terms of parking spaces per unit area and parking duration as $\frac{P_p}{l_p}$ (Arnott and Inci, 2006).

As shown in Figure 4, $\frac{P_p}{l_p}$ in this case also defines the entry and exit rates from the parking pool.

Eq. (10) defines the dynamic equilibrium condition for commercial vehicles in terms of the double-parking behavior of commercial vehicles. The vehicles exiting the in-transit pool are ones that have arrived to destination and would require to park, so the exit rate from the in-transit pool is $\frac{T_c}{m_c t}$. If parking spaces are not available then they double-park near the destination. P_c is the stock of parking spaces assigned to commercial vehicles per unit area and H is the stock of double-parking commercial vehicles per unit area. Accordingly, the entry rate into the parking pool is $\frac{P_c}{l_c}$, where l_c is the average parking duration of commercial vehicles, and the remaining stock for the entry rate into double parking is $\frac{H}{l_c}$.

3.6 Analysis of social optimum

Under this model framework, the social optimum can be obtained under either a first-best allocation (where the number of parking spaces $P = P_p + P_c$ can be set under saturated conditions) or in a second-best allocation setting where a policymaker is restricted to allocating between P_p and P_c based on only the current set of total parking spaces P. In both scenarios, policymakers can set a parking fee.

The average cost of a passenger car trip is the sum of four components as shown in (Arnott and Inci 2006): the cost of in-transit travel time, the cruising for parking time, the cost of parking fee, and the opportunity cost of time at the destination, as shown in Eq. (11).

$$Avergage\ cost\ of\ passenger\ car\ trip = \rho_p m_p t + \rho_p C\left(\frac{l_p}{P_p}\right) + f l_p + \rho_p l_p \quad (11)$$

The total cost is calculated by aggregating the average cost of all passenger car trips. This is achieved by multiplying the average cost by the flow per unit area $\frac{P_p}{l_p}$. The total cost, or *social cost*, of passenger cars can be written as Eq. (12).

$$Total\ cost\ of\ passenger\ car\ trip = \rho_p T_p + \rho_p C + f P_p + \rho_p P_p \tag{12}$$

Likewise, the average cost of commercial vehicles is broken-down to four components. There is the cost of in-transit travel time, the cost of parking fee applied only to the proportion of vehicles that park $\frac{P_c}{H+P_c}$, the cost of the double-parking fine which applies only to the proportion of vehicles that double-park $\frac{H}{H+P_c}$, and finally the opportunity cost of time at destination, as shown in Eq. (13).

$$Average\ cost\ of\ commercial\ vehicle\ trip \\ = \rho_c m_c t + f l_c \left(\frac{P_c}{H + P_c}\right) + q l_c \left(\frac{H}{H + P_c}\right) + \rho_c l_c \tag{13}$$

where:

q = double-parking fine per unit time

In a steady-state environment the constant flow of commercial vehicles per unit area D_c is equal to the combined exit rates of parked vehicles $\frac{P_c}{l_c}$ and double-parked vehicles $\frac{H}{l_c}$, as illustrated in Figure 4. Taking the flow as $\frac{P_c+H}{l_c}$ and multiplying it by the average cost in Eq. (13), the total cost of commercial vehicles trips is shown in Eq. (14).

$$Total\ cost\ of\ commercial\ vehicles\ trip = \rho_c T_c + f P_c + qH + \rho_c (P_c + H) \tag{14}$$

Finally, the total social cost of both passenger cars and commercial vehicles is the sum of Eq. (12) and Eq. (14), as shown in Eq. (15).

$$Total\ cost = \rho_p T_p + \rho_p C + f P_p + \rho_p P_p + \rho_c T_c + f P_c + qH + \rho_c (P_c + H) \tag{15}$$

The total social benefit B is likewise the sum of passenger cars and commercial vehicles benefits $B_p = \int_0^{P_p/l_p} D_P^{-1}(x)dx$, where $B_c = 0$ since D_c is a constant.

The social surplus equals the social benefit minus the social cost as shown in Eq. (16).

$$SS = \int_0^{P_p/l_p} D_P^{-1}(x)dx - \left[\rho_p T_p + \rho_p C + f P_p + \rho_p P_p + \rho_c T_c + f P_c + qH + \rho_c (P_c + H)\right] \tag{16}$$

In other words, the social optimum in the first-best allocation is the set $(D_p, T_p, C, P_p, T_c, H, P_c, f)$ that maximizes the social surplus subject to the equilibrium conditions. For second-best allocation, one of the two variables (P_P, P_C) is taken out since the constraint $P = P_P + P_C$ for an exogenous P must be satisfied. The optimization problem is written in full in Eq. (17).

$$\max_{D_p,T_p,C,P_p,T_c,H,P_c,f} \int_0^{P_p/l_p} D_P^{-1}(x)dx - \left[\rho_p T_p + \rho_p C + fP_p + \rho_p P_p + \rho_c T_c + fP_c + qH + \rho_c(P_c + H)\right] \quad (17a)$$

Subject to

$$D_p = D_o * \left[\rho_p m_p t + \rho_p C\left(\frac{l_p}{P_p}\right) + fl_p\right]^e \quad (17b)$$

$$D_p = \frac{T_p}{m_p t(T_p, T_c, C, P_p, P_c, H)} \quad (17c)$$

$$\frac{T_p}{m_p t(T_p, T_c, C, P_p, P_c, H)} = \frac{P_p}{l_p} \quad (17d)$$

$$D_c = \frac{T_c}{m_c t(T_p, T_c, C, P_p, P_c, H)} \quad (17e)$$

$$\frac{T_c}{m_c t(T_p, T_c, C, P_p, P_c, H)} = \frac{P_c}{l_c} + \frac{H}{l_c} \quad (17f)$$

$$t = \frac{t_o}{1 - \frac{k}{k_j}} \quad (17g)$$

Eq. (17) is a nonlinear optimization problem with nonlinear equality constraints. Due to the strict equality constraints, the problem is solved by finding a vertex that maximizes the objective function (17a). With a concave inverse demand function, the objective is also concave since the costs are all linear. As a result, the model should converge to a global optimum. I employ an interior point method using a commercial solver (fmincon) in MATLAB for convenience (further details of solver and hardware settings in Section 5). The unique optimum is empirically verified in the Toronto case study in Section 5 (and encourage users of this model to do so as well) by initiating the solver with numerous initial guesses to demonstrate that they do indeed all reach the same optimum.

An examination of the objective presented in Eq. (17) indicates that the optimization process should tend to clear the stock of double-parking vehicles followed by the cruising vehicles as they produce the highest costs. One way they incur higher costs compared to in-transit vehicles is through their effect on the traffic density k as defined in Eq. (2), which adversely affects the travel time t.

4 MODEL VERIFICATION

I demonstrate that the proposed model is indeed a generalization of the Arnott and Inci (2006) model by replicating it as a special case of the proposed model. The benchmark example used in Arnott and Inci (2006) is input into this model. I then proceed to show that when commercial vehicles are considered, the proposed model is capable of incorporating their behavior and provide a more comprehensive view of traffic and parking in the downtown.

4.1 Input parameters

For comparison purpose I use the same calibration values from Arnott and Inci (2006) (Table 2 in their study). It was based on a study-area featuring 64 blocks per square mile, and an assumed 58 parking spaces per block, so the total available parking spaces per square mile is 3712 spaces. If hypothetically all the street area was assigned to parking (with no street area left to travelling cars) then this would yield a max number of parking spaces $P_{max} = 11{,}136$. Therefore the ratio of the allocated parking area to the total street area is $\frac{P}{P_{max}} = \frac{3712}{11136} = 0.33$, which is used in Eq. (3) to estimate k_j.

Other parameters used in the model include the demand function constant, D_0, which is set to 3190.04 when the base trip price is $F = 15$ and $\Omega = 2667.2$ with 30 percent of cars cruising for parking. The free flow travel speed is assumed to be 20 mph, which corresponds to a free flow travel time per unit distance of 0.05 hr/mi. And finally, the elasticity of trip demand with respect to trip price is $e = -0.2$. In all scenarios I hold the parking fee fixed at $f = \$1/hr$ to enable comparison of these equilibrium cases.

4.2 Results

Table 3 displays the outcome of the (Arnott and Inci 2006) model in their base scenario and the corresponding outcome of the proposed generalized model in the same scenario in addition to two other scenarios.

The base scenario represents a case where no flow of commercial vehicles are allowed, i.e. $D_c = 0$. In such case, the outcomes of both models are shown to be identical, and this could be seen by comparing the first two columns of Table 3. This test empirically proves that the model is capable of representing the (Arnott and Inci 2006) as a special case by setting the truck demand to zero.

In Scenario 1, I now show what happens to the equilibrium if we do model the truck delivery behavior. The commercial vehicles are introduced with $D_c = 250\, veh/hr/mi^2$ and parking period $l_c = 0.15\, hr$ (9 min) and an in-transit travel distance between stops of $m_c = 0.181\, mile$. In this Scenario 1, I assume no parking spaces are allocated to trucks. The proposed model shows that the corresponding density of commercial vehicles in-transit per unit area is $T_c = 13.34$ and the density of double-parked vehicles per unit area $H = 37.5\, veh/mi^2$. Due to the presence of trucks and double-parking behavior, the model shows that the new T_p has increased while C has reduced. This is a direct reflection of the increased trip price from $t(T_p, T_c, C, P_p, P_c, H)$ and makes sense intuitively. The results of Scenario 1 suggest that ignoring trucks when they are operating at 250 truck demand compared to 1856 passenger car demand (~ 12% of traffic demand) can overestimate cruising by 223% and underestimate travel times by 22.8%.

Table 3 Comparing Equilibrium Outcome- Base Model vs. Proposed Model

	A&I 06 Model	Proposed Model		
	Base Scenario No commercial vehicles (CVs) considered	Base Scenario No commercial vehicles (CVs) considered	Scenario 1 CVs considered, but no parking assigned	Scenario 2 CVs considered, and parking assigned to it
Inputs				
m_p *(mi)*		2		
l_p *(hr)*		2		
ρ_p *($/hr)*		20		
t_0 *(hr/mi)*		0.05		
D_0 *(constant)*		3190.04		
P_{max} *(space/mi^2)*		11136		
Ω *(veh/mi^2)*		2667.2		
K_j *(veh/mi^2)*		1778.2		
e *(unitless)*		-0.2		
f *($/hr)*		1		
α *(uniless)*		1.5		
β *(unitless)*	*n/a*	1.8	1.8	1.8
γ *(uniless)*	*n/a*	5.07	5.07	5.07
m_c *(mi)*	*n/a*	0.181	0.181	0.181
l_c *(hr)*	*n/a*	0.15	0.15	0.15
P_p *(space/mi^2)*	3712	3712	3712	3692
P_c *(space/mi^2)*	*n/a*	0	0	20
D_c *(veh/hr/mi^2)*	*n/a*	0	250	250
Resulting Equilibrium				
D_p *(veh/hr/mi^2)*	1856	1856	1856	1846
t *(hr/mi)*	0.2275	0.2275	0.2948	0.2768
T_p *(veh/mi^2)*	844.5	844.5	1094.34	1022.03
C *(veh/mi^2)*	361.89	361.89	112.05	215.77
T_c *(veh/mi^2)*	*n/a*	0	13.34	12.53
H *(veh/mi^2)*	*n/a*	0	37.5	17.5

In Scenario 2, I now assign 20 of the 3712 parking spaces to commercial vehicles. The corresponding equilibrium shown in the last column of Table 3 shows a 6.1% reduction travel time t, 53.3% reduction in truck double-parking, and 92.6% increase in passenger car cruising. The above examination shows how the proposed model provides a new set of tools for policy makers to evaluate the full effect of parking polices on road users and traffic congestion taking into account truck delivery behavior.

In order to exercise policies that can effectively clear the double-parking vehicles and cruising passenger cars, I demonstrate in a case study of Toronto how the model in Eq. (17) is applied to optimize the social surplus.

5 CASE STUDY OF TORONTO PARKING PRICING AND ALLOCATION

Toronto is Canada’s largest economic center and most populous city. Haider et al. (2010) carried an extensive analysis for a segment in downtown Toronto and found that around 80,000 packages and parcels are delivered to that part of the downtown in a given day. The study points to the inadequate supply of infrastructure necessary for the freight industry to deliver packages and parcels to consignees in an efficient manner without disrupting the traffic.

In this section I consider part of downtown Toronto shown in Figure 5 to demonstrate the application of the model and how useful it could be in creating significant gains in social surplus.

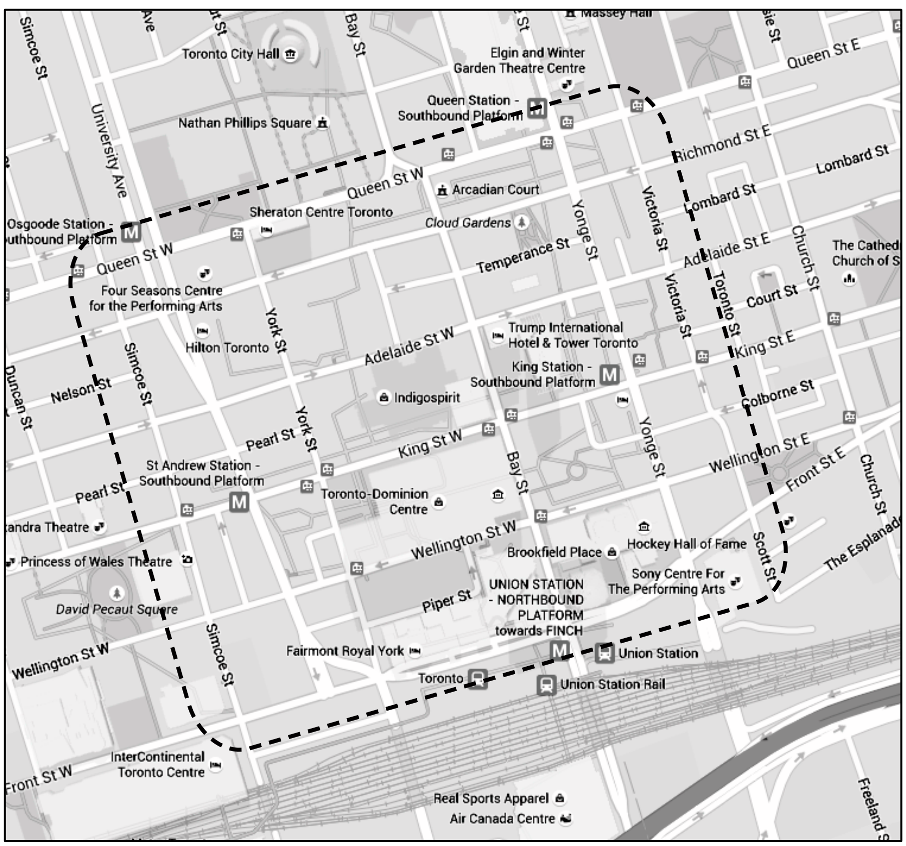


Figure 5 Study area in Downtown Toronto

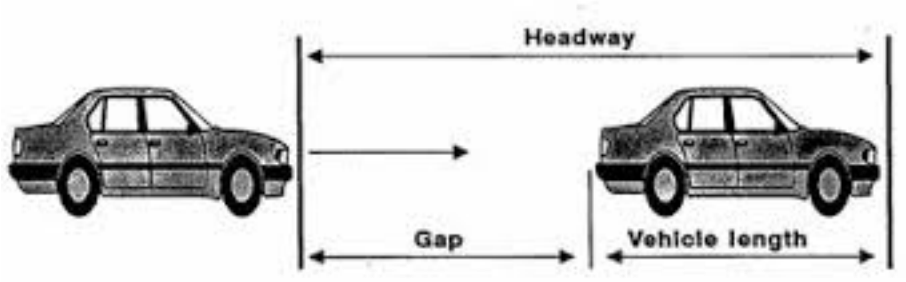
5.1 Field data and assumptions

The following data is collected from the City of Toronto (either from field surveys or from online resources) in June through July of 2015. The chosen area is bound by Simcoe St. and Victoria St. from east and west, and Queen St. and Front St. from north and south.

The study area is almost 0.2923 square miles. Within this perimeter, we can measure a total lane length of 19.836 lane miles and if we remove the parts occupied by the intersections from this number it becomes 18.845 lane miles. To find the total street area multiply this length by the typical lane width in the downtown, which is 0.00211 miles (3.4m), the total street area is therefore almost 0.0398 square miles (102,890 square meters).

According to the city *by-law no. 569-2013*, the minimum length of a passenger car parking space considering parallel parking is 6.7m. I assume that the width of a parking space is that of a typical street lane in Toronto downtown, which is 3.4 m. With the above information we can find P_{max} by dividing the total street area by the area of a single parking space, which generates almost 4,517 parking spaces in the study area and when normalized per square mile we get $P_{max} = 15{,}452\, space/mi^2$. Assume that one-fourth of the street area is allocated to on-street parking so $P = 15452/4 = 3863\, spaces/mi^2$.

Ω is calibrated assuming that at jam density the headway distance between vehicles is $30\, ft$ and therefore the jam density per lane per mile stands at 176 vehicles. Headway distance is the spacing between the fronts of successive vehicles, usually in one lane of a roadway as demonstrated in the image below. The jam headway distance is therefore the headway distance that is expected at jam density, the maximum density that takes place when the speed of vehicles is zero.



(FHWA, 2005)

In this study, the headway distance is considered 30 feet which is inline with default values used in a number of references. For example The Highway Capacity Manual HCM 2010 applies a default jam density value of 190 veh/mi/ln, this could be translated to a jam headway distance equal to 28.8 feet using the formula:

$$k_{jam} = \frac{5280}{d}$$

Ω can then be estimated as the product of the total lane miles 18.845 and 176, and divided by the study area 0.2923, which gives $\Omega = 11{,}346.97$.

The demand function constant D_0 is calibrated at 3319.8 for the study area assuming a base trip price $F = 15$ (following Arnott and Inci 2006). For the on-street parking fee in this area I considered the 2015 rate, which is metered using pay-and-display devices, and charges $4/hr (TPA, 2015).

For data related to the commercial vehicles, I referred to the Cordon Count Data Retrieval System (CCDRS) (DMG, 2015) managed by regional municipalities in the Greater Toronto Region and the University of Toronto to obtain an estimated truck flow in the study area of $D_c = 865\ veh/hr/mi^2$.

For the value of time of commercial vehicles (VOTCV), there are a number of studies providing estimates. Kawamura (2000) estimated the VOTCV to be US $23.4/hr based on stated preference data collected in California. However, this value represents a trucking industry prior to the year 2000, and primarily for long haul trucks. Ismail et al. (2009) estimated a value from the Canadian border crossings to be within CAD \$100/hr to \$125/hr. Miao et al. (2014) looked at value of delay and found that the fleet operator's value is in the range of US \$94 to \$121/hr. Lastly, de Jong et al. (2014) estimated the value in The Netherlands and found EUR 59/hr for small trucks. To be on the conservative end, I assume a value of CAD \$100/hr from Ismail et al. (2009), which is in Canadian dollars and coincides with Miao et al. (2014), and on the more conservative end.

In January 2014 new parking enforcement rules went into effect in Toronto, with a minimum fixed parking fine of $q = \$150$ at busy streets in Toronto according to the media (Citynews, 2014). More recently in January 2015, Toronto police have introduced a zero tolerance rush hour policy in the downtown, in which police will not just issue fines but will also tow every vehicle that parks illegally in downtown during rush hours. In response to that the Ontario trucking Association released a statement asking the mayor of Toronto to reconsider the enforcement of the policy for delivery trucks (CBCnews, 2015). For our case study I assume the cost of a fine is simply the \$150.

The average parking duration for commercial vehicles is assumed to be 9 min (0.15 hr) based on my observation of 10 sampled cases in the study area. Finally, I consider the commercial vehicle parking space dimension to follow the requirements of Type B of the city *by-law no. 569-2013*, which is 11 m in length. Using this information we can estimate θ, the ratio between the parking space dimensions of commercial vehicle and that of passenger cars, as $\frac{11}{6.7} = 1.64$.

A factor $\alpha = 1.5$ is assumed similar to Arnott and Inci (2006), and for β, there are ample references to estimate this value in accordance with prevailing site conditions in the study area, such as the Highway Capacity Manual (HCM, 2010). I have assumed a default value of 1.8 for this factor.

A number of data sources were used in the case study. For each value the relevant source(s) was mentioned when it was first cited in the study. Below is a table of the main data sources used in the study:

Data	Data Sources
Parking spaces and total lane miles in the study area	Field surveys
Curbside parking charge fee in the study area.	Toronto Parking Authority (TPA, 2015)
Parking space requirements in the city of Toronto	City by-law no. 569-2013
Data related to commercial in the study area	Cordon Count Data Retrieval System (CCDRS) (DMG, 2015) and
Value of Time	(Kawamura, 2000), (Miao et al., 2014), (Ismail et al., 2009), (de Jong et al., 2014)
Fines as per new parking enforcement rules in Toronto	(Citynews, 2014)

5.2 Results

The results of the equilibrium scenario and the two social optimal scenarios are presented. For the optimization using Eq. (17), I used MATLAB R2015a on a PC with 1.7 GHz Intel Core i5 processor and 4GB RAM. The *fmincon* solver employed an interior-point algorithm, with a maximum number of iterations set to1000. To verify that the model reaches a global optimum, 10 different initial guesses are used for each of the two social optimum scenarios and they all reached the same optimum solutions. These are shown in Table 4, along with their run times and convergence details. All runs reached the same global optimum solutions.

Table 4 Sets of initial guesses examined

(4a) Initial guesses for the second-best allocation policy

Variable	Starting Points									
	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
T_p	10	50	50	0	200	400	200	400	800	900
T_c	10	10	50	0	10	200	200	200	500	800
H	10	10	50	0	0	150	50	150	200	50
C	10	10	50	0	0	200	100	200	200	100
t	0.05	0.05	0.3	0.2	0.06	0.3	0.3	0.3	0.4	0.3
P_p	10	300	700	1000	3800	600	500	600	2000	4000
P_c	10	100	400	500	120	1000	40	80	50	50
f	0	1	3	3	0	5	4	5	6	6
Iterations	43	16	15	13	8	15	14	16	12	13
Runtime (sec)	0.31	0.11	0.10	0.09	0.08	0.10	0.10	0.12	0.09	0.09

(4b) Initial guesses for the first-best allocation policy

Variable	Starting Points									
	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
T_p	10	10	0	10	10	50	50	300	800	0
T_c	10	10	0	10	10	0	50	200	400	0
H	10	10	0	10	10	0	50	0	0	0
C	10	10	0	10	10	0	50	0	0	0
t	0.05	0.05	0.3	0.05	0.05	0.05	0.1	0.3	0.3	0.1
P_p	10	200	500	1000	2000	1000	2000	4000	6000	5000
P_c	10	200	0	1000	2000	0	2000	500	500	2000
f	1	1	0	2	2	0	1	0	0	0
Iterations	175	101	38	25	18	50	19	11	21	14
Runtime (sec)	1.75	0.84	0.28	0.15	0.12	0.39	0.13	0.08	0.12	0.10

Table 5 compares the outcome of three cases applied to the study area. In the first case, I consider the equilibrium that takes place with current parking rates and the allocation of all parking spaces to passenger cars only. Based on our field survey in the study area, trucks currently do not have dedicated on-street parking priority in Toronto.

In the second scenario I apply the model to optimize the social surplus while holding the parking spaces fixed. The outcome of such optimization shows how best to allocate the available parking spaces between commercial and passenger cars and the corresponding optimum parking fees that clears cruising for parking.

In the last case I discuss a first-best allocation scenario where on-street parking spaces are variable and the model is applied to find the optimum allocation of passenger car and commercial vehicles parking spaces, as well as the optimum parking fees.

Table 5 Optimization Results – Social Optimum vs. Equilibrium – Case study

	Equilibrium	Social Optimum Parking fixed	Social Optimum Parking variable
Inputs			
m_p *(mi)*		2	
l_p *(hr)*		2	
ρ_p *($/hr)*		20	
t_0 *(hr/mi)*		0.05	
D_0 *(constant)*		3319.8	
P_{max} *(space/mi²)*		15452	
Ω *(veh/mi²)*		11346.97	
e *(dimensionless)*		-0.2	
α *(dimensionless)*		1.5	
β *(dimensionless)*		1.8	
γ *(dimensionless)*		4.4	
q *($/hr)*		150	
m_c *(mi)*		0.181	
l_c *(hr)*		0.15	
ρ_c *($/hr)*		110	
D_c *(veh/hr/mi²)*		865	
θ *(dimensionless)*		1.64	
$P = P_p + \theta P_c$ *(space/mi²)*	3863	3863	-
P_p *(space/mi²)*	3863	-	-
P_c *(space/mi²)*	0	-	-
f *($/hr)*	4	-	-
Solution			
P_p^* *(space/mi²)*	-	3650	4406
P_c^* *(space/mi²)*	-	130	130
f^* *($/hr)*	-	8.93	2.86
D_p *(veh/hr/mi²)*	1932	1825	2203
T_p *(veh/mi²)*	233.99	186.93	227.19
C *(veh/mi²)*	442.02	0	0
T_c *(veh/mi²)*	9.48	8.02	8.07
H *(veh/mi²)*	129.75	0	0
t *(hr/mi)*	0.0606	0.0512	0.0516
v *(mi/hr)*	16.5	19.5	19.4
ΔSS *($/hr-mi²)*		+$13,502	+$23,204

5.3 Discussion

In the first case, the equilibrium results show that despite a realized passenger car demand of 1932 $veh/hr/mi^2$ compared to truck delivery demand of 865 $veh/hr/mi^2$, the resulting stock of road traffic is $T_p = 233.99\, veh/mi^2$ compared with $T_c = 9.48\, veh/mi^2$. This result is interesting because a typical road traffic count would indeed show about 5% truck traffic typically, and in this case we obtain 3.9%. However, it turns out that much of the truck demand is being allocated to double-parking for deliveries.

The implication of ignoring commercial vehicle on-street parking is evident on the relatively high density of double-parking vehicles on the streets with $H = 129.75\, veh/mi^2$. On the other hand, the applied parking fee did not eliminate all the cruising for parking as $C = 442.02\, veh/mi^2$, which is about 54% of the total road users in the road space allocated to travelling. The higher cruising stock in this case is a result of ignoring the traffic entering the downtown looking to park in garage parking. Incorporating this traffic would reduce the relative proportion of cruising vehicles, for example if street parking is comprised of only half the traffic coming to downtown, then the actual cruising proportion should be closer to 27%. Since this data is unavailable, the actual cruising rate is not considered a valid output of the model.

The relative proportion of each segment of road users with respect to total road users is best demonstrated in Figure 6 below. The first stacked bar represents the first scenario, while the other bars represent the optimized scenarios. The height of each bar reflects the total number of vehicles that result from each policy. The first bar shows the resulting densities of four segments of road users; these are the stocks of in-transit passenger and commercial vehicles T_p and T_c as well as the stocks of cruising and double-parking vehicles C and H. The congestion in this scenario is evident in the total height of the bar which indicates the highest aggregate stock of vehicles on the street among the three scenarios, and this in turn has resulted in the lowest travelling speed among the scenarios at 16.5 mi/hr.

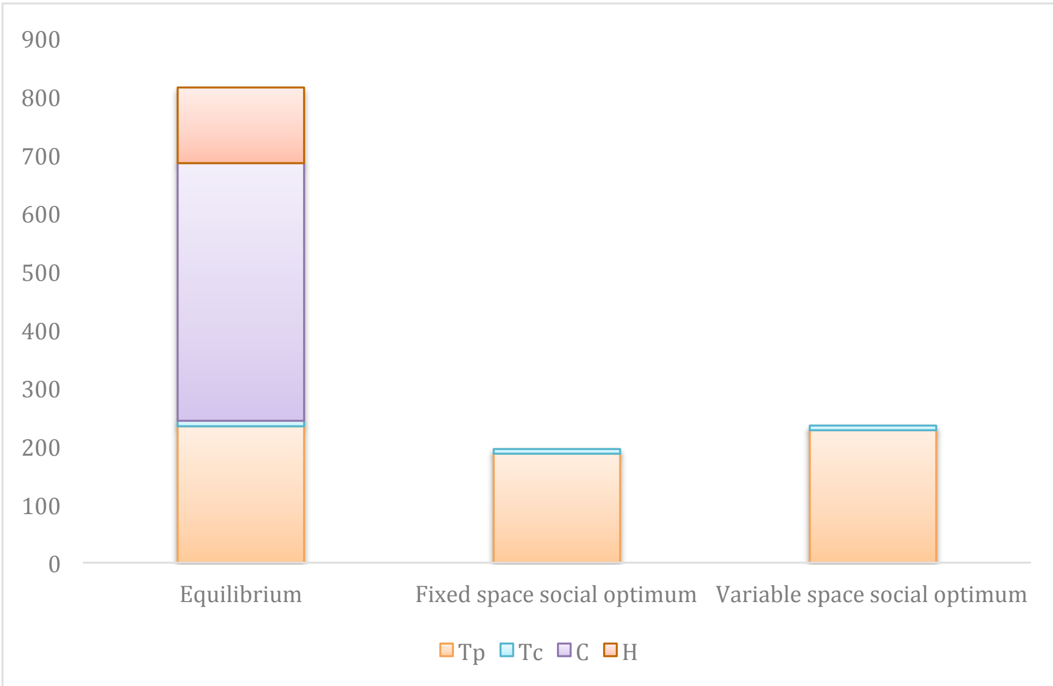


Figure 6 Mixture of road space usage across three scenarios in Toronto.

In the second scenario, the allocation of the fixed parking spaces is optimized, resulting in 130 spaces/mi^2 assigned to commercial vehicles and 3650 spaces/mi^2 to passenger cars. Parking fees are also optimized at $8.9/hr. With this policy the travel speeds rise to an average of 19.5 mi/hr and the total gain in social surplus compared to the initial equilibrium is $13,502 per hour per square mile. Both the cruising and double-parking are eliminated.

The last case demonstrates the social optimum under first-best allocation, where parking spaces are not fixed anymore. Applying the proposed model, the optimum passenger car parking spaces is $P_p = 4406\, spaces/mi^2$ and for commercial vehicles $P_c = 130\ spaces/mi^2$. By allowing total parking spaces to increase by 20%, the parking fee can be reduced down to $f = \$2.86/hr$. This trade-off between parking fee and space availability fits with Arnott and Inci (2006). The total gain in social surplus from this policy compared to the initial case is $23,204 per square mile per hour.

5.3.1 Sensitivity Analysis

In this section I carryout three cases of sensitivity analyses to examine the effect that different factors could have on the results of the model.

5.3.1.1 Parking Duration

The duration of 0.15 hr found from sampling on the field is increased to 0.2 hr and 0.25 hr to examine the effect on the first best and second best allocation policies, as shown in Table 6. Increasing the duration from 0.15 hr to 0.25 hr leads to a larger effect on the first-best policy than to the second best policy.

Table 6 Sensitivity analysis of Toronto parking space allocation policies with respect to the truck parking duration

Truck Parking Duration lc (hr)	0.15	0.2	0.25
Tp	233.99	240.41	247.18
Tc	9.48	9.74	10.02
H	129.75	173	216.25
C	443.02	435.6	428.83
t	0.0606	0.0622	0.064
Speed	16.5	16.1	15.63
Δ SS compared to 2nd best SO	$13,502	$16,174	$19,330
Δ SS compared to 1st best SO	$23,204	$28,600	$34,990

5.3.1.2 Lane Flow Sensitivity

Gamma *(ϒ)* depends on the prevailing traffic flow conditions, and on the number of lanes on the street. Below is a sensitivity analysis that considers the effect of different values of gamma by changing the traffic flow from 2.5 lanes of flow to 2.9 and 2.1 lanes of flow. This is almost 15% change in the traffic flow; the corresponding values of gamma are shown in the table.

The outcome reflects the changes that take place in the traffic composition in each case, which appears to be minor changes, more importantly the optimization results remains unaffected in terms of parking space allocation.

Flow	2.5 lanes	2.9 lanes	2.1 lanes
Gamma	4.4	3.6	5.4
Tp	233.99	230.58	238.72
Tc	9.48	9.35	9.68
H	129.75	129.75	129.75
C	443.02	445.43	437.28
t	0.0606	0.0597	0.0618
Speed	16.5	16.75	16.18
Δ SS compared to 2nd best SO	$13,502	$13,492	$13,522
Δ SS compared to 1st best SO	$23,204	$23,194	$23,224

5.3.1.3 The Effect of Number of Lanes in the Bottleneck

A lane drop from 4-lanes to 3-lanes would create different effect than a lane drop from 3-lanes to 2-lanes. The effect in each case could be estimated using analysis similar to the one explained in section 3.3 in this study. Below is a sensitivity analysis outcome of three cases of lane drops applied while fixing all the values used in the case study except the number of lanes.

Number of lanes before Number of lanes After	4 lanes 3lanes	3 lanes 2 lanes	2 lanes 1 lane
Gamma	3.99	4.4	5.29
Tp	232.25	233.99	237.84
Tc	9.41	9.48	9.64
H	129.75	129.75	129.75
C	443.76	443.02	4338.17
t	0.0601	0.0606	0.0616
Speed	16.65	16.5	16.24

6 CONCLUSION

The above case study demonstrate that the current practice of disregarding the effect of commercial vehicles and their parking behaviour on congested downtown street networks has inevitably lead to devising inefficient solutions to meet the congestion. For policy makers to be able to best respond to congestion problems it is necessary to capture the effect of all road users including commercial vehicles. The case study demonstrates how developing an inclusive policy leads to considerable efficiency gains, which is much needed on the streets of the busy downtown centers.

It is well established that urban truck deliveries make a big impact on commuter parking, because of the shared use of parking spaces, the inelasticity of freight demand, and the need to double-park when no spaces are available due to need for proximity. Nonetheless, the literature on downtown on-street parking generally continues to exclude truck delivery behavior. The few studies of truck deliveries are simulation based or do not integrate with commuter parking.

In this study, I present an analytical equilibrium model that evaluates the effects of different parking policies in urban centers with respect to network congestion, cruising, double-parking, and the travel behavior of commercial and passenger vehicles. It is the first such model, and also the first analytical evaluation of downtown Toronto parking pricing and space allocation policies. The parking model is shown to be a generalization of the commuter equilibrium model from Arnott and Inci (2006), one that can also capture a truck delivery fleet class that is inelastic to traffic conditions and double-parks when no spaces are available.

The case study makes several key findings. First, I measured and estimated parameters of the model for downtown Toronto such that a baseline scenario is defined. This baseline scenario can serve as a benchmark for policymakers to consider different policies. From the baseline, I considered two policy tools. The first is to price and allocate the existing parking spaces to trucks to optimize social welfare. In this case the proposed model shows that increasing the parking fee from \$4/hr to nearly \$9/hr and assigning 3.4% of parking spaces to truck parking would eliminate cruising and truck double-parking, resulting in a social surplus gain of over \$13,500/hr/mi^2.

Under a first-best allocation policy where the total number of parking spaces can also change, the proposed model shows that it is optimal to increase number of parking spaces by 20% (of which truck parking spaces would constitute 2.9% of spaces), and this could reduce parking fees to under \$3/hr, eliminate cruising and truck double-parking, and increase social surplus to \$23,200/hr/mi^2.

The model helps policy makers develop strategies to improve urban parking policies by being able to plan and optimize trade-offs in parking spaces, prices, and network congestion.

Commercial vehicles serve financial and commercial institutions in the downtown and it constitute a segment of road users that is frequently ignored by both policy makers and researchers. However, efficient solutions to congestion problems must capture all segments of road users to be able to respond with proper polices. The continued double-parking behaviour of commercial vehicles shows that ignoring this segment and resorting to traffic fines might not provide the sought efficiency in the network. It is therefore necessary to incorporate this segment with other road users and devise inclusive policies. In the case study I have demonstrated how the developed model captures all the segments of roads users and optimizes the road space accordingly, allowing the most efficient allocation of on-street parking and the optimum corresponding parking fees.

The main objective of the model is to provide an analytical means for policy makers to understand the congestion effects that result from neglecting the parking behavior of trucks in their policies. However, several features incorporated in the model make it suitable for evaluating other situations that resemble comparable scenarios.

For example construction and maintenance activities might require temporary lane closures; in such case the analysis provided for modeling the effect of double-parking could be used to explain the resulting congestion effect on the traffic flow. In the study, it was considered that trucks are more likely to double-park near their destinations occupying a travelling street lane. The effect of double-parking was considered as a temporary lane drop that creates a bottleneck in the traffic flow. The same bottleneck effect could be expected in temporary construction lane closures and thus it is possible to apply the same analysis. May (1990) first applied similar analysis to estimate the traffic flow conditions just upstream of the bottleneck, and at the bottleneck location, and finally downstream of the bottleneck.

This model has shortcomings and opportunities for future city logistics research. First, the model currently ignores off-street parking, transit mode access, heterogeneous population and parking durations, and truck fleet operating characteristics like fleet size and number of stops. These modifications are needed should a policymaker have an interest in analyzing those particular scenarios. Many have been incorporated into Arnot and Inci (2006) (see Arnott, 2006; Arnott and Rowse, 2009, 2013), so it's just a matter of taking the model generalization in this study and applying it to those cases.

One of the long term objectives of this research is to provide a dynamic (per unit time) snapshot model of downtown parking for designing parking informatics systems. Unlike the ones proposed in the literature, there is a need for parking and delivery information systems for both commuters and commercial vehicle drivers. There is also an opportunity to relate the parking and deliveries to commercial vehicle tours (You et al., 2015) and commuter activity planning (Chow and Djavadian, 2015).

7 REFERENCES

Anderson, S.P., de Palma, A., 2004. The economics of pricing parking. *Journal of Urban Economics* 55, 1-20.

Arnott, R., 2006. Spatial competition between parking garages and downtown parking policy. *Transport Policy* 13, 458-469.

Arnott, R., de Palma, A., Lindsey, R., 1991. A temporal and spatial equilibrium analysis of commuter parking. *Journal of Public Economics* 45, 301-335.

Arnott, R., Inci, E., 2006. An integrated model of downtown parking and traffic congestion. *Journal of Urban Economics* 60, 418-442.

Arnott, R., Rowse, J., 1999. Modeling parking. *Journal of Urban Economics* 45, 97-124.

Arnott, R., Rowse, J., 2009. Downtown parking in auto city. *Regional Science and Urban Economics* 39, 1-14.

Arnott, R., Rowse, J., 2013. Curbside parking time limits. *Transportation Research Part A* 55, 89-110.

Cao, J., Menendez, M., 2015. A parking-state-based transition matrix of traffic on urban networks. ISTTT 21, Transportation Research Procedia 7, 149-169, doi:10.1016/j.trpro.2015.06.009.

Calthrop, E., Proost, S., 2006. Regulating on-street parking. *Regional Science and Urban Economics* 36, 29-48.

CBCnews, 2015. Zero tolerance rush hour policy introduced to downtown drivers. *CBCnews* Retrieved 08 September 2015, from http://www.cbc.ca/news/canada/toronto/zero-tolerance-rush-hour-policy-introduced-to-downtown-drivers-1.2888820

CNBC, 2015. New York man sues UPS. *CNBC* Retrieved 04 October 2015, from http://www.cnbc.com/2015/10/02/ups-sued-for-repeatedly-blocking-new-york-bike-lanes.html

Chow, J.Y.J., Yang, C.H., Regan, A.C., 2010. State-of-the-art of freight forecast modeling: lessons learned and the road ahead. *Transportation* 37(6), 1011-1030.

Chow, J.Y.J., Djavadian, S., 2015. Activity-based market equilibrium for capacitated multimodal transport systems. *Transportation Research Part C*, Special Issue from ISTTT 21, in press, doi:10.1016/j.trc.2015.04.028.

CityNews, 2014. New Toronto parking rules kick in today. *CityNews*. Retrieved 08 September 2015, from http://www.citynews.ca/2014/01/17/new-toronto-parking-rules-kick-in-today

Daily News, 2014. Police warn city-wide double-parking crackdown is coming. *Dailynews* Retrieved 04 October 2015, from http://www.nydailynews.com/new-york/uptown/double-parking-crackdown-article-1.1714638

DNA info, 2015. Harlem Bicyclist sues UPS. *DNAinfo* Retrieved 04 October 2015, from https://www.dnainfo.com/new-york/20151002/central-harlem/harlem-bicyclist-sues-ups-for-routinely-parking-trucks-bike-lanes.

de Jong, G., Kouwenhoven, M., Bates, J., Koster, P., Verhoef, E., Tavasszy, L., Warffemius, P., 2014. New SP-values of time and reliability for freight transport in the Netherlands. *Transportation Research Part E* 64, 71-87.

DMG, 2015. Cordon Count Data Retrieval System Overview, https://www.jpint.utoronto.ca/ccdrs/new_index.html, last accessed Aug 24, 2015.

Federal Highway Administration, 2005. Traffic Control Systems Handbook. Dunn Engineering Associates.

Fosgerau, M., de Palma, A., 2013. The dynamics of urban traffic congestion and the price of parking. *Journal of Public Economics* 105, 106-115.

Geroliminis, N., Daganzo, C., 2008. Existence of urban-scale macroscopic fundamental diagrams. Some experimental findings. *Transportation Research Part B* 42, 759-770.

Glazer, A., Niskanen, E., 1992. Parking fees and congestion. *Regional Science and Urban Economics* 22, 123-132.

Haider, M., Lalonde, L., Mehboubi, M., Livett, C., Spenard, D., 2009. Challenges facing express delivery services in Canada's urban centres. Institute of Housing & Mobility, Ryerson University, https://canadiancourier.org/uploads/Challenges_Facing_Express_Delivery_Services_in_Canada_s_Urban_Centres.pdf.

HCM, 2010. Highway Capacity Manual, http://hcm2010.org/.

Holguín-Veras, J., Ozbay, K., Kornhauser, A., Brom, M.A., Iyer, S., Yushimito, W.F., Ukkusuri, S., Allen, B., Silas, M.A., 2011. Overall impacts of off-hour delivery programs in New York City metropolitan area. *Transportation Research Record* 2238, 68-76.

Ismail, K., Sayed, T., Lim, C., 2009. A study of the commercial vehicle value of time for operation at border crossings. In the Annual Conference of the Transportation Association of Canada.

Jaller, M., Holguín-Veras, J., Hodge, S.D., 2013. Parking in the city: challenges for freight traffic. *Transportation Research Record* 2379, 46-56.

Kawamura, K., 2000. Perceived value of time for truck operators. *Transportation Research Record* 1725, 31-36.

Kobus, M.B.W., Gutiérrez-i-Puigarnau, E., Rietveld, P., Van Ommeren, J.N., 2013. The on-street parking premium and car drivers' choice between street and garage parking. *Regional Science and Urban Economics* 43, 395-403.

Lam, W.H.K., Li, Z.C., Huang, H.J., Wong, S.C., 2006. Modeling time-dependent travel choice problems in road networks with multiple user classes and multiple parking facilities. *Transportation Research Part B* 40, 368-395.

Mackowski, D., Bai, Y., Ouyang, Y., 2015. Parking space management via dynamic performance-based pricing. ISTTT 21, Transportation Research Procedia 7, 170-191, doi:10.1016/j.trpro.2015.06.010.

May, Adolf 1990. Traffic Flow Fundamentals. Prentice-Hall NJ,US.

Miao, Q., Wang, X. B., Adams, T. M., 2014. Assessing the value of delay to short-haul carriers. *Journal of Applied Research and Technology* 12(4), 642-653.

Morris, A.G., Kornhauser, A.L., Kay, M.J., 1998. Urban freight mobility: collection of data on time, costs, and barriers related to moving product into the central business district. *Transportation Research Record* 1613, 27-32.

Ni, Daiheng, 2015. Traffic Flow Theory: Characteristics, Experimental Methods, and Numerical Techniques. *Butterworth-Heinemann*

Nourinejad, M., Wenneman, A., Habib, H.N., Roorda, M.J., 2014. Truck parking in urban areas: application of choice modelling within traffic microsimulation. *Transportation Research Part A* 64, 54-64.

Qian, Z.S., Xiao, F.E., Zhang, H.M., 2012. Managing morning commute traffic with parking. *Transportation Research Part B* 46, 894-916.

Qian, Z.S., Rajagopal, R., 2013. Data-driven active parking management. *Computing in Civil Engineering*, 395-402.

Shoup, D., 2005. The high cost of free parking. *American Planning Association*.

Tipagornwong, C., Figliozzi, M., 2015. A study of the impacts of commercial vehicle parking availability on service costs and double-parking behavior**.** Transportation Research Board 94th Annual Meeting.

TPA, 2015. Toronto Parking Authority, http://www.toronto.ca/311/knowledgebase/98/101000048098.html, last accessed Aug 24, 2015.

Vickrey, W.S., 1954. The economizing of curb parking space. *Traffic Engineering* 25, 62-67.

Vickrey, W.S., 1969. Congestion theory and transport investment. *American Economic Review* 59, 251-261.

Yang, H., Liu, W., Wang, X., Zhang, X., 2013. On the morning commute problem with bottleneck congestion and parking space constraints. *Transportation Research Part B* 58, 106-118.

You, S.I., Chow, J.Y.J., Ritchie, S.G., 2015. Inverse vehicle routing for activity-based urban freight forecast modeling and city logistics, working paper.

Zhang, X., Huang, H.J., Zhang, H.M., 2008. Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city. *Transportation Research Part B* 42(1), 38-56.

Zhang, X., Yang, H., Huang, H.J., 2011. Improving travel efficiency by parking permits distribution and trading. *Transportation Research Part B* 45(7), 1018-1034.