# A production-recycling-reuse model for plastic beverages bottles 

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# A Production-Recycling-Reuse Model for Plastic Beverages Bottles 

By

## Nouri Dawood Matar

B.Eng., Ryerson University, 2002

A thesis presented to RyERSON UnIVERSITY
in partial fulfillment of the
requirements for the degree of

# Master of Applied Science 

in the Program of

# Mechanical Engineering 

Toronto, Ontario, Canada, 2011

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# Abstract <br> A Production-Recycling-Reuse Model for Plastic Beverages Bottles 

Nouri Dawood Matar<br>Master of Applied Science<br>2011<br>Mechanical Engineering<br>Ryerson University

In this thesis a recycling-reuse model is developed and analyzed. Discarded 2L plastic PET (polyethylene terephthalate) bottles are collected from the market. The non-contaminated PET bottles are either remanufactured or used as regrind mixed with virgin PET to produce new bottles to satisfy varying demand. Contaminated bottles are sold to industries using low grade plastic and only badly contaminated bottles go to landfill. Cost of land use and associated environmental damage is calculated as a present worth and charged to the manufacture. Analyses conducted on this model found that the amount of bottles collected had the largest influence on the outcome of the total system unit time cost. Alternative materials to PET that degrade faster are surveyed and used to demonstrate significant reduction in the cost of landfill disposal. Analysis using a minimal market price for remanufactured and newly produced bottles resulted in profit.

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## Nomenclature

A Accumulation quantity of collected bottles during a cycle.
$B_{L f} \quad$ The amount of bottles that are to be disposed of into a Landfill after sorting during a cycle
$C_{L S} \quad$ Labour cost for sorting bottles.
$C_{C o} \quad$ Labour cost for bottle collection.
$C_{L r} \quad$ Labour cost for remanufacturing.
$C_{\text {Mat }} \quad$ Material cost for remanufacturing.
$C_{v m} \quad$ Cost for virgin material per cycle.
$C_{L p} \quad$ Labour cost for bottle production.
$C_{L S c} \quad$ Labour cost for contaminated bottle sort.
$C_{L F} \quad$ Labour cost for disposing bottles in the landfill.
$C_{r e} \quad$ The real-estate rental cost per bottle.
$C_{r h} \quad$ The rehabilitation penalty cost per bottle.
$D \quad$ The demand rate.
$h_{n s s} \quad$ Carrying cost per bottle from collection and sorting per cycle-period.
$h_{r} \quad$ Carrying cost per remanufactured bottle per cycle-period.
$h_{r m} \quad$ Carrying cost per bottle from virgin material and regrind mixing per cycle-period.
$h_{p} \quad$ Carrying cost per newly produced bottle per cycle-period.
$h_{S c} \quad$ Carrying cost per contaminated bottle per cycle-period.
$K_{n s s} \quad$ Setup cost for bottle collection.
$K_{r} \quad$ Setup cost for remanufacturing.
$K_{r m} \quad$ Setup cost for virgin Material and regrind mixing.
$K_{P} \quad$ Setup cost for production/manufacturing of new bottles.
$K_{S C} \quad$ Setup cost for contaminated bottle sort.
$N_{0} \quad$ The amount of bottles disposed into the landfill during a cycle.
$P \quad$ Production rate for producing new bottles.
$Q_{P} \quad$ Replenishment order quantity in each cycle.
$Q_{r} \quad$ The quantity of bottles remanufactured.
$Q_{v m} \quad$ The material needed to be mixed with the regrind in order to produce the number of bottles required to supplement the number of remanufactured bottles in order to meet the demand rate.
$R M \quad$ The quantity of regrind and virgin material mix in a cycle.
$T \quad$ Cycle Time.
$T C_{L F} \quad$ Process cost of landfill disposal for badly contaminated bottles.
$T C_{n s s} \quad$ Process cost of collection and sorting of bottles (non-contaminated) and contaminated).
$T C_{p} \quad$ Process cost for production (newly produced bottles).
$T C_{r} \quad$ Process cost for remanufacturing bottles.
$T C_{r m} \quad$ Process cost for raw material available for production (recycled and virgin material mix).
$T C_{S c} \quad$ Process cost for sorting contaminated bottles.
TCU Total system unit time cost.
$T C U_{L F} \quad$ Process unit time cost of landfill disposal for badly contaminated bottles.
$T C U_{\text {nss }}$ Process unit time cost of collection and sorting of bottles (non-contaminated and contaminated).
$T C U_{p} \quad$ Process unit time cost for production (newly produced bottles).
$T C U_{r} \quad$ Process unit time cost for remanufacturing bottles.
$T C U_{r m}$ Process unit time cost for raw material available for production (recycled and virgin material mix).
$T C U_{S c} \quad$ Process unit time cost for sorting contaminated bottles.

## Greek Symbols

$\theta \quad$ The percentage of bottles collected based on the amount put out for collection.
$\alpha \quad$ The percentage of non-contaminated bottles that can be used for remanufacturing and regrind.
$\beta \quad$ The percentage of whole non-contaminated bottles that can be used for remanufacturing.
$\phi \quad$ The percentage of contaminated bottles that can be sold as low grade material.
$\lambda \quad$ The decay rate of PET material.
$\lambda_{r} \quad$ Slope of depleting demand rate for remanufactured bottles.
$\lambda_{r m} \quad$ Slope of depleting demand rate for re-grind and virgin material mix.

| Abbreviations |  |
| ---: | :--- |
| PET | Polyethylene terephthalate |
| BPA | Bisphenol A |
| SSP | Solid State Polycondensation |
| CGM | Corn Gluten Meal |
| SPSP | Soy Protein Starch Plastic |
| v.m | virgin material |
| T. Inc | Total Income |

## Chapter 1: INTRODUCTION

In the past decades, the focus has been on air and water pollution, as well as on the decreasing availability of landfills for waste disposal. Waste disposal in landfills causes pollution of not only the land but also of the water tables, resulting in hazards and damages to the environment, wildlife and humans. Various proposals and schemes have been made to reduce the disposal of waste in landfills. Perhaps the most significant system for waste reduction is the one that promotes recycling and reuse of discarded items.

Plastic bottles are one of the largest components of waste discarded by human beings. According to the Container Recycling Institute (CRI), "Americans throw away 200 billion beverage containers including plastic bottles and aluminum cans each year. Beverage containers make up about 15 percent of all packaging waste in the US, and in 2000 , only 31 percent were recycled" ${ }^{1}$. This suggests that $69 \%$ were disposed into the environment, which is alarming! Customers' consumption behaviour and disposal habits have not changed significantly enough to reduce disposal. Therefore, the assumption can be made that plastic bottle waste worldwide is exceedingly high.

Large soft drink companies such as Coca Cola Company, PepsiCo Incorporated and Dr. Pepper/Seven-Up Incorporated use plastic bottles to contain their product that is sold on the market. The Container Recycling Institute (CRI) has long been pressuring beverage companies to be accountable for their actions and stop the waste. "In 2002, Pepsi responded to the CRI campaign by stating that it would begin by using ten percent recycled content in its bottles, which Coca Cola was already doing. Since then, Coke has promised to up that recycled content to 25 percent in the US - Coca Cola already used bottles with 25 percent recycled PET in other countries" . In 2009 Coca Cola "announced the launch of a multi-million dollar marketing effort supporting recycling called "Give it Back""2. Together with the United Resource Recovery Corporation they opened the "world's largest plastic bottle-to-bottle recycling plant in Spartanburg, S.C." ${ }^{2}$. Their goal is to "recycle and reuse 100 percent of (our) bottles and cans in the U.S. ${ }^{, 2}$. However, neither PepsiCo, nor the other companies that use Plastic (PET) bottles have started any similar initiative.

Local government units and municipalities implement laws pertaining to plastic bottle recycling and "largely see the fiscal benefits of recycling plastic bottles because of the savings in landfill space and reduced landfill costs. ${ }^{33}$ However, more stringent laws are required by governments in order to protect the eco-system from plastic bottles decomposing in a landfill that leach harmful chemicals into the environmental water table. This in turn would increase the amount of plastic bottle recycling when compared to the amount of plastic bottle recycling happening now. Also, stringent laws would encourage the development of more advanced technologies and systems for plastic bottle recycling.

A large portion of plastic bottles that contain/house carbonated beverages are commonly made from a material known as "polyethylene terephthalate" (PET). This is a thermoplastic recyclable material that is strong and impact-resistant and has a resin identification code (a set of symbols placed on plastics to identify the polymer type) of 1 . When placed in a landfill a chemical known as BPA (bisphenol A) leaches into the environmental water table. When consumed by humans and wildlife BPA acts as a endocrine disruptor ${ }^{4}$ that "interferes with the synthesis, secretion, transport, binding, action, or elimination of natural hormones that are responsible for the maintenance of homeostasis (normal cell metabolism), reproduction, development, and/or behaviour" ${ }^{4}$. It can also create neurological issues ${ }^{5}$ and increases the risk of prostate cancer ${ }^{6}$ in males and breast cancer ${ }^{7}$ in females. Since it takes approximately 450 years ${ }^{8}$ for plastic beverage bottles to fully biodegrade in a landfill, real-estate/land availability is an issue because increased amount of landfills are required to house plastic beverage bottles which reduces the amount of inhabitable land space.

This thesis deals with the remanufacturing of discarded 2 L plastic beverage bottles and the use of recycled material in the production of new bottles based on seasonal demand requirements of the market so as to demonstrate how a reuse and recycling system can work. The aim of this system is to reduce the amount of bottles going into a landfill. The supply chain system in this thesis has two separate generic entities, which are the Bottle Collection Company and the Bottle Manufacturing Company. The Bottle Collection Company collects the bottles produced by the manufacturer and discarded by the users, and sorts the bottles into three streams: noncontaminated whole bottles, non-contaminated damaged bottles and contaminated bottles. The contaminated bottles are then sorted into two streams. The non-useable badly contaminated
bottles are sent to landfill, while the remaining ones are sold by the Bottle Collection Company to industries that use low grade plastic material (such as the construction industry). The noncontaminated whole and damaged bottles are then transported to the Bottle Manufacturing Company. In this system the whole bottles are remanufactured where they are de-labelled, cleaned and sanitized, polished and relabelled. The damaged bottles are de-labelled, cleaned, grounded into flakes and processed through a "Recycling line- recoSTAR PET" ${ }^{9}$ machine which produces improved quality PET pelletized material due to "Solid state polycondensation" ${ }^{10}$ (SSP), which takes place during processing (The SSP process increases the intrinsic viscosity of the PET pelletized material and also "decontaminates the material so effectively that is suitable for direct food contact applications" ${ }^{10}$.). The pellets/material produced from the Recycling linerecoSTAR PET machine are then mixed with the virgin PET pellets (material of origin) to generate new bottles. New bottles are produced by a two-step moulding process which requires two separate machines; one to make the pre-form of the bottle and the second to inflate the shape of the bottle by using stretch blow moulding. The remanufactured bottles plus the newly produced bottles will achieve the demand requirements dictated by the market.

A mathematical model will be developed in the thesis to capture net-profit for the seasonal cyclic demand. A diagram showing the flow of bottles collected from the market and distributed to the bottle manufacturer will be used as a guide in the development of the mathematical cost function model. The costs incurred are the bottle collection and sorting, the bottle remanufacturing, the recycling and mixing process (recycling bottles into new pelletized material and mixing them with virgin pellets), the production of new bottles and a Landfill disposal fee. After the "introduction of PET containers in the late 1970 's" ${ }^{11}$ it became known that it takes a very longtime for plastic (PET) bottles to decompose in a landfill. However, so far there is no exact model for the decomposition process. As mentioned earlier it takes approximately 450 years ${ }^{8}$ or more for plastic (PET) bottles to decompose in a landfill, it is therefore reasonable to assume that the decomposition process is exponential ${ }^{12}$. To develop the Landfill disposal fee, it is clear that the disposal cost varies with time as bottles placed in the landfill decompose. This cost will be charged at the end of each period using present worth cost. The profits gained within the math model are from the remanufactured and newly produced bottles sold to the market. Also, all assumptions related to the model will be made clear and exact calculations of the profits and
costs will be presented later. In order to determine the optimal amount of bottles produced and remanufactured for the different seasons (High, Medium and Low season) Excel will be used.

Alternative materials that could be used to produce 2 L plastic beverage bottles will also be discussed and scrutinized within the thesis. It is becoming more and more important and ethical to protect the environment and humans. Therefore materials that can be used to create bottles that can contain beverages and are safe for human consumption, biodegrade at faster rate than PET and do not leach harmful chemicals into the environmental water table when disposed in a landfill will be examined.

The practice of reusing and recycling materials/components has been around for years. Many studies pertaining to this line of research have been covered in an attempt to reduce the adverse effects of waste created by humans on the eco-system. Studies that are relevant to the thesis will be studied, analyzed, compared and summarized in a following chapter.

The cost driven by the nature of the mathematical model within the thesis may seem too high. However, minimising the current negative impact that man-made waste has on the environment justifies such cost. Alternate ways to reduce cost will also be looked at and discussed within the thesis.

In the next chapter, the literature survey will be summarized, analyzed and discussed. Chapter 3 deals with the mathematical model development that includes not only the cost of remanufactured and producing new bottles but also the cost of disposing badly contaminated bottles in a landfill. This cost includes not only the long term use of the real-estate but also rehabilitation and penalty brought back to the present-worth to be charged to the bottle manufacturer. In chapter 4, numerical calculations are presented together with the results. Sensitivity analysis is also carried and the results discussed. Further discussions, recommendations and conclusions are in chapter 5.

## Chapter 2: LITERATURE SURVEY

Currently, there is an increasing focus on the importance of recycling and reuse in an effort to save the environment from the harmful substances that result from waste disposal activities in landfill locations, which are becoming less available with time. Many cities have created a new system for waste collection where recyclables go in one bin, non-recyclables in another and food scraps go in a third. Also, in an effort to reduce the disposal of plastic bags in landfills the city of Toronto, for example, requested all retailers to charge customers a fee for these bags and have been encouraging retailers to use bags made from biodegradable material and customers to use reusable bags. However, some of those recyclable items are plastic bottles. When disposed of in landfills they take hundreds of years to degrade rendering such lands unusable for those many years. In addition, as these bottles degrade they admit harmful chemicals into the environment. Thus, recycling and reuse is an important effort to reduce the amount of bottles being disposed in landfills. In recent years, several studies that deal with recycling and reuse have been published. Also there have been several other studies that suggest the use of alternative materials such as biodegradable plastics.

Several studies that investigate production-recycling systems (or similar models) can be found in the literature. These studies will be examined and compared to the supply chain model of this thesis. Also, articles that demonstrate the use of recycled PET (polyethylene terephthalate) material in different industries (other than the Bottle Manufacturing industry) and the use of alternative materials that are environmental friendly will similarly be discussed and compared. The review of these articles will begin with production-recycling systems.

### 2.1 Production-Recycling Models

Dobos and Richter ${ }^{13}$ developed a production-recycling model/system with a "predetermined production-inventory policy" ${ }^{13}$. In their model; "a producer serves a stationary product demand occurring at a rate $D>0$. This demand is served by producing or procuring new items as well as by recycling some part $\delta$ where $0 \leq \delta \leq 1$ of the used products coming back to the producer at a constant return rate $d=\alpha D(0 \leq \alpha \leq 1)$ It is assumed that that the producer is willing to buy back $\delta$ of all the used products to recycle and/or to dispose them off" ${ }^{13}$ and "there is no difference between newly produced and recycled items" "' "The parameters $\delta$ and $\alpha$ are called
marginal use rate and marginal buyback (return) rate, respectively. The remaining part of the non-serviceable products $(1-\delta) d^{13}$ are disposed off (where $(1-\delta)$ is called the marginal disposal rate). Dobos and Richter ${ }^{13}$ analyzed their earlier model to determine if whether pure (either to produce or to recycle all products) or mixed (both produce and recycle) strategies are optimal. They found that the mixed strategies are dominated by the pure strategies and therefore the pure strategies are optimal.

Dobos and Richter ${ }^{14}$ extended their production-recycling model/system (Dobos and Richter ${ }^{13}$ ) where they account for the quality of whether a collected and returned product is suitable for recycling. By adding a constraint for the portion of serviceable items $\psi(0 \leq \psi \leq 1)$ the "maximal reusable products is equal to $\psi D{ }^{" 14}$. This allows the producer two choices: "either to repurchase only reusable products, or to buy back all the items and investigate the serviceability of the products" ${ }^{14}$. Based on further analysis of the production-recycling model with quality considerations Dobos and Richter ${ }^{14}$ show that "by minimizing the inventory holding costs it is optimal to carry out a quality control by the producer and repurchase all units" ${ }^{14}$. By minimizing the total EOQ and non-EOQ related costs they "have shown that it is better to "outsource" the quality control and repurchase only reusable products. In such cases a mixed strategy (both produce and recycle) would be economical compared to pure strategies" ${ }^{14}$.

Maity et al. ${ }^{15}$ developed an "optimal control recycling production inventory system" ${ }^{15}$ in a fuzzy environment. Where items are either produced or recycled (from used items) to satisfy demand. Within this model the "used items are bought back and then either put on recycling or disposal" ${ }^{15}$ and the recycled products are used for "new products which are sold again" 15 ; the "rate of production, recycling and disposal are assumed to be a function of time and considered as control variables" ${ }^{15}$. They assumed in their model that demand is price dependent and that price is dependent on the inventory level. They also assumed the holding costs (for serviceable and nonserviceable items) to be imprecise and decried by fuzzy variables. In this paper an "optimal control approach is proposed to optimize the production, recycling and disposal strategy with respect so that the expected value of total profit" ${ }^{15}$ is maximal. Therefore since "total profit is maximized formulating the problem as an optimal control problem" ${ }^{15}$; it is solved by the general expected value model (EVMS), calculus method and generalized reduced gradient (GRG) technique" ${ }^{15}$ where the "optimum production and stock levels are determined for known price
dependent demand function" ${ }^{15}$. Maity et al. ${ }^{15}$ found that since "production is serviceable stock dependent and unknown. The rate of production decreases as serviceable stock increases." ${ }^{\text {¹5 }}$

Li et al. ${ }^{17}$ developed an "extended EOQ model with recycling used product and producing finished products to satisfy fixed demands" ${ }^{17}$. This "model involves the repair and the continuous collection of used products and allows multiple production lot-sizes problems" ${ }^{\prime 17}$. Based on their model, they present a "joint policy" ${ }^{17}$ associated "with the collection of used products and the production of finished products under the minimization of total costs of the inventory systems" ${ }^{17}$. They were able to "deduce simultaneously the optimal economic order quantity of finished products and the optimal inventory of repairable products models" ${ }^{17}$. By conducting analysis on their production-recycling system/model Li et al. ${ }^{17}$ found that by minimizing inventory holding costs one of the pure strategies (to produce or recycle all products) is optimal.

Oh and Hwang ${ }^{18}$ developed a "deterministic inventory model" ${ }^{18}$ for a recycling system; where "the system is associated with reverse logistics" ${ }^{18}$, in which "returned items are served as raw materials" ${ }^{18}$. In this model the demand is satisfied by the production of new products/items which is created from recycled material (returned products/items collected from customers) that is mixed with raw (virgin) material plus remanufactured products (returned products/items that are collected from customers and are refurbished to "good as new ones" ${ }^{18}$ ).

Maity et al. ${ }^{19}$ developed a "production recycling model with learning effect"", where the demand rate is "time dependent and known" 19 and is "satisfied by production and recycling" ${ }^{19}$. Within this model the demand increases with time, but the rate of increase decreases with time. Used units are "bought back and then either recycled or disposed of" ". "The production function, recycling function and disposal are functions of time and unknown" ${ }^{19}$; and they are "taken as control variables" ${ }^{19}$. The "production cost is greater than the recycling cost" ${ }^{19}$ and the "non-serviceable holding cost is less than the serviceable holding cost" ${ }^{19}$. The "set-up cost is not fixed for each cycle" ${ }^{19}$, therefore a learning curve is defined for the "set-up cost of production cycles and recycling cycles" ${ }^{19}$ where the set-up cost reduces over time due to the "Learning Curve" effect. The total profit of the model/system is "maximized formulating the problem as an optimal control problem" ${ }^{19}$. It is solved by the "calculus method and generalized reduced gradient (GRG) technique" ${ }^{19}$. Where the "optimum production, recycling and stock levels are determined" ${ }^{19}$ for the "known dynamic demand function" ${ }^{19}$. Maity et al. ${ }^{19}$ found that the
"increasing demand rate is very small" ${ }^{19}$ and that the "demand is approximately constant" ${ }^{19}$, since the "manufacturing cost is higher than the recycling cost" ${ }^{19}$ the "manufacturer wants more remanufacturing as possible" ${ }^{19}$, "production goes on for a short period of time" ${ }^{19}$ and on the "other hand production and recycling occur for a long period of time" ${ }^{19}$ and lastly at the time of production "non-serviceable stock gradually increases as products are continuously collected from the market" ${ }^{19}$. However, "when recycling starts then non-serviceable stock decreases" ${ }^{19}$.

Currently 2 L plastic beverage bottles are being produced from a material known as "Polyethylene terephthalate" (PET) which contains a harmful chemical known as "Bisphenol A" (BPA). As plastic bottles decompose/break-down in a landfill BPA is admitted into the surroundings of the landfill and contaminates the environmental water table. Through the consumption of water, BPA has proven to have adverse effects on human beings and wildlife. Therefore it is fundamentally ethical that alternative materials be explored for the use of creating 2 L plastic bottles.

### 2.2 Alternative Materials

Kinoshita et al. ${ }^{20}$ developed and tested a "green composite" ${ }^{20}$ which "consists of woodchips, bamboo fibers and biodegradable adhesive" ${ }^{20}$. However in order to develop a durable composite they first experimented and developed a composite from only woodchips through "compression moulding" ${ }^{20}$. They found "the composite which is produced by solidifying only the woodchips" ${ }^{20}$ (through "compression moulding") without a binder "does not have a high strength" ${ }^{20}$; also, "it is brittle partially and its water resistance is bad" ${ }^{20}$. In order to "improve the strength and water resistance for the composite made only from woodchips" ${ }^{20}$ a "composite composed of wood chips as the matrix material, bamboo fibers as the reinforced fiber and the biodegradable resin as the adhesive" ${ }^{20}$ was developed. Kinoshita et al. ${ }^{20}$ found mixing woodchips with bamboo fibers and biodegradable adhesive created a strong and water resistant composite. The addition of biodegradable adhesive in combination with bamboo fibers (mixed with wood chips) increased the bending and impact strength (making the composite strong) and remarkably increased the resistance to water.

Samarasinghe et al. ${ }^{21}$ created and tested "Biodegradable plastic composites from corn gluten meal" ${ }^{21}$ where, "plasticised CGM (Corn Gluten Meal) can be blended at a relatively low
temperature $\left(150^{\circ} \mathrm{C}\right)$ with a synthetic biodegradable polyester and wood fibre" that is used to produce "injection-mouldable composites that degrade in soil on a timescale of months" ${ }^{21}$. In their analysis Samarasinghe et al. ${ }^{21}$ found that "when the proportion of synthetic polyester exceeds about $70 \mathrm{wt} \%$ (meaning synthetic polyester is more than $70 \%$ of the material content used to create the composite)" ${ }^{, 21}$ composites are produced that have "moderately high tensile strength, elongation at break and water resistance" ${ }^{21}$. When materials produced contain a high content of plasticised CGM (approximately 80wt \%) they have a "high tensile modulus and more rapid biodegradation" ${ }^{21}$. However, the "composites are more porous and less resistant to water" ${ }^{21}$. Therefore, they concluded that "the optimum composite formulation consequently depends on the intended applications of material ${ }^{21}$ and the formulated composites potentially "can be used to manufacture a range of 'disposable' products such as food trays, food and beverage containers and cutlery ${ }^{, 21}$.

Rouilly et al. ${ }^{23}$ developed and tested a "natural injection-mouldable composite (plastic) material from sunflower oil cake" ${ }^{23}$ by transforming the "native structure of sunflower oil cake" ${ }^{23}$ through a "thermo-mechanical -chemical treatment" processes; which causes the "defibrillation of husk fragments and denaturation / coagulation and reduction of proteic fractions" ${ }^{23}$ within the sunflower oil cake that results in a composite which has flow properties and can thus be "shaped by injection-moulding, ${ }^{23}$. After developing different injection-mouldable composites which were sunflower oil cake (SFOC) based, extruded sunflower oil cake (ESFOC) and extruded sunflower oil cake treated with $5 \%$ sodium sulphite (ESTOC) were compared and ESTOC composite was found to be optimal. After analyzing and testing the optimal composite Rouilly et al. ${ }^{23}$ found the "tensile and flexural stress at break values of the optimal composite were slightly lower than those of commercial starch-based composite material" ${ }^{23}$. However, the optimal composite proved to be water resistant and this "property can be improved further by thermal treatment" ${ }^{23}$.

Schilling et al. ${ }^{24}$ performed preliminary studies on converting sawdust into biodegradable plastics; where "hardwood sawdust was derivatized either by caboxymethylation, glutaration, maleiation, phthallation, or succination in order to produce anionic materials suitable for complexation with soy protein isolate" ${ }^{24}$. The results of their analysis found the "blending of all derivatized sawdust specimens with soy protein resulted in instant precipitation of gels. Infrared spectroscopy and differential scanning calorimetry suggested the formulation of complexes
between soy protein isolate and each of the derivatized sawdust specimens" ${ }^{24}$; where, the "specimens of protein" 24 reacting with "anionic sawdust exhibited tensile strengths of up to 2.4 MPa (which is reasonable good tensile strength for the intended purpose of bottle production), suggesting that these materials could be promising candidates for biodegradable structural materials" ${ }^{24}$.

Wu et al. ${ }^{25}$ developed and tested "low cost corn gluten meal/wood fiber", ${ }^{25}$ composites where "corn gluten meal (CGM)/wood fiber composites, plasticized by glycerol, water and ethanol, were extruded into pellets" ${ }^{" 25}$. The pellets were "compression-moulded into sheets for evaluation of water resistance, thermal stability and morphology" ${ }^{25}$. Also pellets were "injection-moulded to prepare plant pots for developing low cost, biodegradable containers used in agriculture" ${ }^{25}$. Through their analysis of the composites Wu et al. ${ }^{25}$ obtained the following results: "the water resistance of compression moulded sheets was not affected by WF (wood fiber) content. The flexural strengths of the sheets were increased after the addition of 10-30\% WF (wood fiber) but decreased by the addition of $40-50 \%$ WF (wood fiber). Their visual and morphology observations showed that "fracture occurred in the matrix for sheets with low fiber content but in the interface for high fiber content" ${ }^{25}$ and further testing and analysis of the "moulded sheets and pots showed medium water resistance" ${ }^{25}$. Wu et al. ${ }^{25}$ concluded the research by stating that "the successful preparation of injection-moulded pots with $50 \%$ WF (wood fiber) content and the medium water resistance of the pot show that the composites meet the requirements for disposable pots. Material cost for the composite will become lower if glycerol content was decreased or waste glycerol from food industry is used ${ }^{25}$. They also caution that even though the "extrusion and injection moulding are low cost" ${ }^{25}$ the "processing cost should be analyzed systematically" ${ }^{25}$.

Otaigbe et al. ${ }^{26}$ experimentally developed and tested a "biodegradable soy protein-starch plastic" ${ }^{26}$ that can be "extruded and injection-moulded into articles of various shapes and sizes" ${ }^{26}$; where "soy protein isolate" ${ }^{26}$ is blended with "polyphosphate fillers" ${ }^{26}$ to form composites that are biodegradable. They found "viable injection-mouldable plastics can be formulated from soy-protein isolate and corn starch for disposable plastic products" ${ }^{26}$, where the plastic biodegrades "after its useful service life in an environmentally-benign manner", Also the blending of "soy protein isolate" ${ }^{26}$ with "special bioabsorbable polyphosphate fillers" forms
composites that have "properties such as water resistance, stiffness, and strength for beneficial uses in many load bearing applications" ${ }^{26}$ (such as containing or housing soft drink beverages).

Since all of the composites/plastics discussed above have not been used for plastic beverage bottle production, the use of recycled plastic materials in many industries is necessary and vital to reduce and potentially eliminate the use of landfills for the disposal of plastic waste. Landfills require the use of land real-estate until disposed waste is fully decomposed and during the decomposition of waste, contaminates, get admitted into the environmental water table via the landfill; especially current plastic waste which contain dangerous chemicals that have negative effects on the eco-system. Although the use of recycled plastic within this thesis is limited/confined to certain industries (industries that can use contaminated/low grade plastic to develop their product) the material used to produce 2L plastic beverage bottles is "Polyethylene terephthalate" (PET) therefore the broad applications in the use of PET material within different industries will now be covered.

### 2.3 Industries that use Recycled PET Material

Tawfik and Eskander" ${ }^{27}$ created "Polymer Concrete from marble wastes and recycled Polyethylene terephthalate" ${ }^{27}$. The "unsaturated polyester (UP) used was prepared from the reaction of oligomers obtained from the depolymerization of polyethylene terephthalate (PET) soft drink bottles with maleic anhydride and adipic acid. The UP was then mixed with the styrene monomers at a ratio of $60: 40 \%$ by weight to obtain the SP (styrenated polyester) ${ }^{, 27}$. By mixing "marble waste as fillers" ${ }^{27}$ with styrenated polyester, polymer concrete (PC) was formed. The aim of the work performed by Tawfik and Eskander ${ }^{27}$ was to "study the use of PC to be used as polymer based building materials. Tawfik and Eskander ${ }^{27}$ found that "from the recycled PET soft drink bottles and marble waste materials a fast cured PC (polymer concrete), with acceptable physical properties, good mechanical integrity, enhanced chemical characterization, and providing better heat and flame resistance, can be synthesized ${ }^{\prime 27}$. Therefore they concluded that "the production of PC material can be developed for semi-industrial and industrial scales for its economical advantages, as well as environmental benefits where its main raw materials are waste" ${ }^{\text {" } 27}$.

Gurudatt et al. ${ }^{28}$ developed "dope-dyed polyesters fibers from recycled PET wastes for use in moulded automotive carpets" ${ }^{28}$; where, fibers produced through direct extrusion from PET bottle waste and the incorporation of different pigments (Dope-Dyeing) were "evaluated for color fastness properties and loss of mechanical properties due to dope addition" ${ }^{28}$. The results of their study found that "Dope- dyed fibers have excellent fastness properties and can be produced from PET bottles waste by pigment additions during fiber productions" ${ }^{28}$. Also the produced fibers "have properties comparable to those of virgin fibers and can find ready usage in applications like automobile carpets" ${ }^{28}$; and "their use in automotive applications ensures high benefit to cost ratio because of lower raw material costs" ${ }^{28}$. Gurudatt et al. ${ }^{28}$ concluded that the "recycling of PET bottles into fibers by direct extrusion is not only inevitable from an ecological point of view, but should also be seen as an opportunity to produce commercially viable products from waste materials" ${ }^{28}$.

Kawamura et al. ${ }^{29}$ created and tested "coating resins synthesized from recycled PET (polyethylene terephthalate) ${ }^{, 29}$; where, "bottles collected from the Japanese market" ${ }^{29}$ were reprocessed/recycled and "polyesters for powder coatings was synthesized from R-PET (recycled PET) ${ }^{22}$. The results of their study found that "the structure of a polyester resin synthesized from R-PET was the same as that of conventional polyester synthesized by the ordinary method" ${ }^{29}$ (conventional polyester is usually synthesized from ethylene glycol (EG) and terephthalic acid (tPA)), "the film properties of powder coatings formulated with a polyester synthesized from RPET instead of EG and t-PA were comparable to those of conventional coatings" ${ }^{" 29}$ and "an alkyd resin having the same characteristics as a conventional resin was successfully synthesized from R-PET instead of EG and t-PA by modifying its monomer composition and the reaction" ${ }^{29}$. Kawamura et al. ${ }^{29}$ concluded by stating that "recycling R-PET into alkyd resins provides a beneficial means for mass consumption of R-PET. Given the high production of alky resins in Japan of more than 100, 000 t (tons) in 2002, it is estimated that 5000-10000 t (tons) of R-PET can be consumed annually with this technology, ${ }^{29}$ of producing powder coatings synthesized from recycled PET (R-PET).

Griffin ${ }^{30}$ tested and evaluated "PET and recycled PET" ${ }^{30}$ as a replacement material for "PETG (polyethylene terephthalate glycol (copolyester)) ${ }^{30}$ in the production of packaging trays. Where, the purpose of the study within this paper "was to determine if a thermoformed packaging part (a
tray) could be made more environmentally friendly" 30 by replacing the "material originally qualified for a packaging tray-virgin polyethylene terephthalate glycol (PETG) copolyester,"30 with a recyclable PET material. Therefore extrusions of sheets from both virgin PET and recycled PET were made and "evaluated and compared" ${ }^{30}$ with a PETG sheet. The results of analysis found by Griffin ${ }^{30}$ were "thermoforming trials demonstrated that both PET and recycled PET could be formed with state-of-the-art equipment to provide adequate impact strength and other requirements (haze, gloss, tensile strength and effects of orientation) of the final part ${ }^{30}$. Also, "recycled PET had property values that were quite similar to those to those obtained with virgin PET and its impact strength was actually slightly higher" ${ }^{30}$. In her closing comments Griffin ${ }^{30}$ concluded that either virgin PET material or recycled PET material "could be used to replace PETG" ${ }^{30}$ material when "formed under the optimum conditions" ${ }^{30}$ in the production of packaging trays.

Rebeiz ${ }^{31}$ developed and tested polyester concrete made from "PET and fly ash wastes" ${ }^{31}$; where, "recycled polyethylene terephthalate (PET), mainly recovered from plastic beverage bottles"31 was used to "produce unsaturated polyester resins"31 in which the unsaturated polyester resins was mixed with fly ash waste (an "inorganic aggregate" ${ }^{31}$ ) to produce polyester concrete (PC). The results of the analysis found by Rebeiz ${ }^{31}$ showed that the polyester concrete (PC) had "very good mechanical and durability properties"31 and when reinforced with steel bars "the material is much stronger and more ductile when compared to "steel-reinforced Portland cement concrete"31 (a standard reinforced concrete used in the construction industry). Also the concrete would "require less reinforcement cover for the tensile reinforcing steel than Portland cement concrete because of its inherent high flexural strength, low permeability, and very good chemical resistance" ${ }^{31}$. Rebeiz ${ }^{31}$ concluded that the polyester concrete made from PET and fly ash wastes "may be used cost effectively in pavements, bridges, and precast components" ${ }^{31}$. However, "field applications and continuous monitoring of the PC would really determine the long term performance of the material under actual conditions" ${ }^{31}$.

### 2.4 Comparison of Models, Analysis of Alternative Materials and Use of Recycled PET Material

The models of Dobos and Richter ${ }^{13,14}$ and Maity et al. ${ }^{15}$ are quite different from the model of this thesis. The model of this thesis is developed based on 2L plastic beverage bottle manufacturing where ground recycled plastic material (collected from damaged "good" bottles)
is mixed with virgin plastic material to produce new bottles (hybrid product). However, the models of Dobos and Richter ${ }^{13,14}$ and Maity et al. ${ }^{15}$, as well as others recently surveyed in El Saadany ${ }^{16}$, do not consider that newly produced products/items are created from recycled material that is mixed with virgin material. Also the costs for waste disposal in these models do not consider the costs related to use of land and the effects on the environment. That is caused by products that cannot be recycled and are discarded into landfills. However, the model of this thesis develops and uses a present worth within the cost of waste disposal. This will capture costs associated with land use and environmental issues caused by unrecyclable product that is placed into a landfill. Similar to the models discussed above the Li et al. ${ }^{17}$ model is quite different from the model of this thesis. It does not consider that newly produced products/items are created from recycled material that is mixed with virgin material Also, since it is assumed in the model that recycled units are continuously repaired, the cost of waste disposal is not considered. This is contrary to the model of this thesis where a waste disposal cost is developed and used. The model Oh and Hwang ${ }^{18}$ is very similar to the model of this thesis. However, they assume that "all collected materials can be recycled" ${ }^{18}$ (meaning items collected can be used in the creation of new products or can be remanufactured). Non-serviceable items are not considered which means there is no waste disposal cost associated with their model; unlike the model of this thesis which considers non-serviceable bottles and thus has a waste disposal cost. The model of Maity et al. ${ }^{19}$ and the model of this thesis are structured differently. The Maity et al. ${ }^{19}$ model does not consider that newly produced products/items are created from recycled material that is mixed with virgin material. Also a present worth cost associated with use of land and environmental issues caused by product waste discarded in a landfill is not considered. Although, the structures of the models are different Maity et al. ${ }^{19}$ uses the "Learning curve" effect within their model to reduce the set-up cost overtime. The "Learning curve" effect is not covered in the model of this thesis; however, it can be explored in the future as a noble extension to the model of this thesis to lower set-up and labour costs. Furthermore, the model of this thesis attempts to reduce the quantity of bottles disposed in landfill. This is done by selling contaminated bottles to industries that use low grade plastic material. Only badly contaminated bottles are disposed off in a landfill.

Analysis of alternative materials will now be considered. In the article of Kinoshita et al. ${ }^{20}$ a "green composite" ${ }^{20}$ is considered. Since the materials used to create the composite are natural materials ("environmentally friendly materials" ${ }^{20}$ ), the composite is therefore environmentally
safe during decomposition in a landfill; and could be considered as a choice material for use in the future production of plastic beverage bottles. The use of the biodegradable plastic discussed within the article of Samarasinghe et al. ${ }^{21}$ to produce plastic beverage bottles is ideal because the material can degrade in a soil based landfill within months which in turn lowers the need for real-estate to create new landfills for the disposal of plastic beverage bottles. Also corn gluten meal, wood fibres and synthetic biodegradable polyester are natural materials that are environmentally safe. It is to be noted that synthetic biodegradable polyester becomes biodegradable waste when placed in a landfill. Since "biodegradable waste is a type of waste, typically originating from plant or animal sources, which may be degraded by other living organisms" ${ }^{22}$. Therefore, synthetic biodegradable polyester can be deemed environmentally friendly upon degrading in a landfill. The plastic composite discussed in the article of Rouilly et al. ${ }^{23}$ has the potential to be used in the production of plastic beverage bottles due to the fact that the material of this composite is based on sunflower oil cake (a natural material not man-made) that can easily biodegrade in a landfill and is environmentally friendly. However to make the material more impact-resistant in the applicable use of plastic beverage bottle production external plasticisers, strengthening and flex agents would have to be mixed with the material; which may not be environmentally friendly. Although only preliminary studies were performed by Schilling et al. ${ }^{24}$ the contents which make up the plastic/composite discussed with in their paper is "green material" and would biodegrade in landfill without causing harm to the environment. However, further development of the composite is required to make it strong and water resistant which is necessary to house the contents of carbonated beverages. The Composite discussed in the paper of Wu et al. ${ }^{25}$ showed week results for strength and water resistance which means currently this material would not be ideal to use in the production of plastic beverage bottles. If the material structure of this composite is reformulated by adding a biodegradable adhesive to strengthen the composite and increase its water resistance, it could possibly be used in the future production of bottles. However, by adding biodegradable adhesive to the current material mix, creates a new composite, which would have to be tested and satisfactory results demonstrated in the consideration of future use in bottle production. The "biodegradable soy protein-starch plastic" ${ }^{26}$ proposed by Otaigbe et al. ${ }^{26}$ would biodegrade in a landfill, in an "environmentally-benign manner" ${ }^{26}$. Therefore the future use of this material would be ideal in the production of plastic beverage bottles.

As mentioned earlier, recycled PET (Polyethylene terephthalate) can be used in production of various products. Polymer Concrete (Tawfik and Eskander ${ }^{27}$ ), moulded automotive carpets (Gurudatt et al. ${ }^{28}$ ), coating resins (Kawamura et al. ${ }^{29}$ ), Packaging Trays (Griffin ${ }^{30}$ ) and Polyester Concrete (Rebeiz ${ }^{31}$ ) are a few of these products that use recycled PET. Therefore the selling of contaminated bottles to industries that use low grade material as considered in this thesis is well justified.

### 2.5 Chapter Summary

In this chapter various production-recycling models have been reviewed. Also reviewed are several biodegradable materials with a potential of replacing PET material in the making of plastic bottles. The use of disposed plastic PET bottles in other industries in an effort to reduce the amount of bottles going to landfill has been explored. However the product-recycling models reviewed in this chapter are different from the model of this thesis. Most of these models do not consider that newly produced products/items are created from recycled material that is mixed with virgin material, but the model of this thesis does. Furthermore some of these models do not account for the cost of waste disposal, and those that do, do not consider the present worth cost of land use and environmental costs. The model of thesis not only recycles whole noncontaminated bottles but also uses non-contaminated damaged bottles is the production of new bottles. Also the cost of the long land use by the slow degrading plastics is brought back to the present to be charged to the bottle manufacturer. This cost includes real-estate rental, land rehabilitation and penalty for damage to the environment.

## CHAPTER 3: THE MATHEMATICAL MODEL

Before developing the mathematical model of this thesis, the problem of this thesis will be briefly described. It is known that disposing plastic bottles into a landfill causes harm to human and wildlife health, as well as the environment. Reducing these harmful effects could be achieved through recycling of these plastics. A model will be developed where the recycling of 2L plastic PET bottles is considered. However, the model of this thesis with small modifications can be applied to other types of bottles. Another problem arises from the fact that plastic bottles take approximately hundreds of years to degrade in a landfill. This makes real-estate/land availability an issue because increased amount of landfills are required to house plastic bottles thus reducing the amount of useful land space. Therefore, the present worth of real-estate use, land rehabilitation and the cost of damage to the environment will be included in the total cost of this model.

The development of the model and its assumptions will now be introduced.

### 3.1 Notations and Assumptions

### 3.1.1 Input Parameters and Decision Variables:

$\theta=$ The percentage of bottles collected based on the amount put out for collection, where $0<\theta<1$.
$\alpha=$ The percentage of non-contaminated bottles that can be used for remanufacturing and regrind, where $0<\alpha<1$. (if for example $\theta=0.75$ and $\alpha=0.25$, then the percentage of non-contaminated bottles collected is $\theta \alpha=0.1875$ )
$\beta=$ The percentage of whole non-contaminated bottles that can be used for remanufacturing, where $0<\beta<1$. (if $\beta$ is $25 \%$ of $\alpha$, then the percentage of non contaminated bottles that can be remanufactured is $\theta \alpha \beta=0.046875$ )
$\phi=$ The percentage of contaminated bottles that can be sold as low grade material, where $0<\phi<1$. (If $\phi=25 \%$, then the percentage of contaminated bottles that can be sold as low grade is $\theta(1-\alpha) \phi=0.140625)$
$D=$ The demand rate (unit/unit of time)
$K_{n s s}=$ Setup cost for bottle collection (Relating to preparation and maintenance on the collection and for the sorting process of the bottles, overhead, etc....) (\$)
$K_{r}=$ Setup cost for remanufacturing (Including machine repair, maintenance, overhead, etc....) (\$)
$K_{r m}=$ Setup cost for virgin Material and regrind mixing (Including machine repair, maintenance, overhead, etc....) (\$)
$K_{P}=$ Setup cost for production/manufacturing of new bottles (Including machine repair, maintenance, overhead, etc....) (\$)
$K_{S c}=$ Setup cost for contaminated bottle sort (Including machine repair, maintenance, overhead, etc....) (\$)
$K_{L F}=$ Setup cost to prepare the ground for bottle disposal. (\$)
$C_{L S}=$ Labour cost for sorting bottles. (\$/unit of time)
$C_{C o}=$ Labour cost for bottle collection (Which includes the involved labour, the driver's fuel and another variable costs). (\$/unit of time)
$C_{L r}=$ Labour cost for remanufacturing. (\$/unit of time)
$C_{\text {Mat }}=$ Material cost for remanufacturing (which includes cleaning, clear coat polish and relabelling). (\$/unit of time)
$C_{v m}=$ Cost for virgin material per cycle. (\$/unit of time)
$C_{L p}=$ Labour cost for bottle production. (\$/unit of time)
$C_{L S c}=$ Labour cost for contaminated bottle sort. (\$/unit of time)
$C_{L F}=$ Labour cost for disposing bottles in the landfill. (\$/time)
$C_{r e}=$ The real-estate rental cost per bottle. (\$/time)
$C_{r h}=$ The rehabilitation penalty cost per bottle. (\$/time)
$h_{n s s}=$ Carrying cost per bottle from collection and sorting per cycle-period. (\$/unit/unit of time).
$h_{r}=$ Carrying cost per remanufactured bottle per cycle-period. (\$/unit/unit of time)
$h_{r m}=$ Carrying cost per bottle from virgin material and regrind mixing per cycle-period. (\$/unit/unit of time)
$h_{p}=$ Carrying cost per newly produced bottle per cycle-period. (\$/unit/unit of time)
$h_{S c}=$ Carrying cost per contaminated bottle per cycle-period. (\$/unit/unit of time)
$\lambda_{r}=$ Slope of depleting demand rate for remanufactured bottles. (\$/unit of time)
$\lambda_{r m}=$ Slope of depleting demand rate for re-grind and virgin material mix. (\$/unit of time)
$\lambda=$ The decay rate of PET material. (percentage/unit of time)

$$
\begin{aligned}
& P=\text { Production rate for producing new bottles. (unit) } \\
& A=\text { Accumulation quantity of collected bottles during a cycle. (unit) } \\
& Q_{r}=\text { The quantity of bottles remanufactured. (unit) } \\
& Q_{v m}=\text { The material needed to be mixed with the regrind in order to produce the number of } \\
& \text { bottles required to supplement the number of remanufactured bottles in order to meet } \\
& \text { the demand rate. (unit/unit of time) } \\
& R M=\text { The quantity of regrind and virgin material mix in a cycle. (unit) } \\
& Q_{P}=\text { Replenishment order quantity in each cycle. (unit) } \\
& B_{L f}=\text { The amount of bottles that are to be disposed of into a Landfill after sorting during a } \\
& \text { cycle. (unit) } \\
& N_{0}=\text { The amount of bottles disposed into the landfill during a cycle. (unit) } \\
& T=\text { Cycle Time. (unit of time) } \\
& T C_{n s s}=\text { Process cost of collection and sorting of bottles (non-contaminated and } \\
& \text { contaminated). (\$) } \\
& T C_{r}=\text { Process cost for remanufacturing bottles. (\$) } \\
& T C_{r m}=\text { Process cost for raw material available for production (recycled and v. m mix). (\$) } \\
& T C_{p}=\text { Process cost for production (newly produced bottles). (\$) } \\
& T C_{S c}=\text { Process cost for sorting contaminated bottles. (\$) } \\
& T C_{L F}=\text { Process cost of landfill disposal for badly contaminated bottles. (\$) } \\
& T C U_{n s s}=\text { Process unit time cost of collection and sorting of bottles (non-contaminated and } \\
& \text { contaminated). (\$) } \\
& T C U_{r}=\text { Process unit time cost for remanufacturing bottles. (\$/unit of time) } \\
& T C U_{r m}=\text { Process unit time cost for raw material available for production (recycled and v. } \mathrm{m} \\
& \text { mix). (\$/unit of time) } \\
& T C U_{p}=\text { Process unit time cost for production (newly produced bottles). (\$/unit of time) } \\
& T C U_{S c}=\text { Process unit time cost for sorting contaminated bottles. (\$/unit of time) } \\
& T C U_{S c}=\text { Process unit time cost for sorting contaminated bottles. (\$/unit of time) } \\
& T C U_{L F}=\text { Process unit time cost of landfill disposal for badly contaminated bottles. (\$/unit of } \\
& \text { time) } \\
& T C U=\text { Total system unit time cost. (\$/unit of time) }
\end{aligned}
$$

### 3.1.2 Assumptions:

1. In this model the percentages of $\theta, \alpha$ and $\beta$ cannot assume the extreme values of $0 \%$ or $100 \%$. It is readily clear for $\theta$ to be 0 percent means no bottles are collected and none can be recycled. If $100 \%$ are collected it means no bottles are lost or transferred to other locations by travelers and all bottles used are put out for collection. Clearly this is unrealistic. Similarly $\alpha$ cannot assume these extreme percentages at a value of $0 \%$ for $\alpha$ means all the bottles are contaminated and therefore cannot be used for remanufacturing or regrinding. On the other hand if $\alpha$ is $100 \%$ means there are no non-contaminated bottles which is also an unrealistic assumption. Likewise, $\beta$ too cannot assume these extreme values. At a $\beta$ value of $0 \%$ all bottles are damaged and are to be used for regrinding, and at $100 \%$ all bottles are to be remanufactured. This highly un-likely to occur because there will always be damaged bottles at the point of use or in transportation. Likewise also having all bottles undamaged is unrealistic and equally unlikely. Similar reasoning applies to the extreme limits of $\phi$. Thus, in the analysis of this model values of $0.25,0.5$ and 0.75 will be used to determine the effects of varying these decision variables.
2. The demand for the bottles is assumed to be constant for each cycle but may vary from one cycle to another.
3. It is assumed that demand is always met from the producing and remanufactured. Therefore, this model assumes neither shortages nor excess inventory of bottles.
4. In this model if the contaminated bottles are sold to the low grade plastic industries, any profit generated goes to the collection company. These bottles are given to the bottle collection company in exchange for transporting the non-contaminated bottles to the manufacturing company.

### 3.2 Flow Diagram for a Plastic Bottle Recycling System:

The production, remanufacturing, recycling and waste disposal system for plastic bottle investigated in this thesis is depicted in Figure 3.1. Bottles are collected from the market at a rate $\theta D$; and then sorted by the Bottle Collection Company into non-contaminated bottles at
rate $\alpha \theta D$ and contaminated bottles at a rate $(1-\alpha) \theta D$. The non-contaminated bottles are sorted furthermore by the Bottles Collection Company into whole non-contaminated bottles and damaged non-contaminated bottles. The non-contaminated bottles are then transported to the Bottle Manufacturing Company where the non-contaminated whole bottles are remanufactured at a rate $\beta \alpha \theta D$ and the damaged non-contaminated bottles are used as regrind at a rate (1$\beta) \alpha \theta D$ in the production of new bottles. The regrind is mixed with virgin material to produce the new bottles, $Q_{v m}$ to produce the new bottles. The production of new bottles, $Q_{p}$, plus the remanufactured bottles, $Q_{r}$, divided by the cycle time $T$ equals the required demand rate $D$ in one cycle. The contaminated bottles are sorted into two streams: badly contaminated bottles which are disposed of in landfill at a rate $(1-\phi)(1-\alpha) \theta D$ and bottles that can be sold to industries (such as various industries within construction) that use low grade plastic material at rate $\phi(1-\alpha) \theta D$. Processes A to F are indicated in Figure 3.1. The operation costs of these processes will be discussed in details in section 3.3 of this chapter.


Contaminated bottle sort and Landfill disposal
Figure 3.1 - Flow diagram for plastic bottle recycling-reuse system

### 3.3 The Model:

Six processes were considered in the development of the mathematical model. It is to be noted that the cycle time used in these processes is the same even though the bottle collection will begin earlier than the production. However, for coordination purposes (e.g., Jaber and Zolfaghari ${ }^{32}$ ), the length of the cycle for collection is the same as that for production. This is done in order to correctly calculate cost allocation. The six processes are described next.

### 3.3.1 Process A: Non-Serviceable Stock (Bottle collection Company)

As displayed in Figure 3.2 below, the Bottle Collection Company collects the bottles produced by the manufacturer and discarded by the users at a rate $\theta D$, where the accumulated quantity during a cycle is $A$. The Bottle Collection Company sorts the bottles into three streams noncontaminated whole bottles and damaged bottles at a rate $\alpha \theta D$ and contaminated bottles at a rate $(1-\alpha) \theta D$. During the sorting process a vision system is used to select the required bottles that are sent to the Bottle manufacturing company. Figure 3.2 will also be used to develop the holding cost of the cost equation for bottle collection and sorting.


Figure 3.2 - Accumulated quantity of bottles during a cycle $T$
$A=\theta D T$

Using Figure 3.2 for calculating the inventory total cost, the process cost of collection and sorting for the cycle $T$ is:

$$
\begin{equation*}
T C_{n s s}=K_{n s s}+C_{L S} T+C_{C O} T+h_{n s s}\left(\frac{A}{2}\right) T \tag{3.2}
\end{equation*}
$$

Dividing equation (3.2) by the cycle time $T$ the process unit time cost function is given as:

$$
\begin{equation*}
T C U_{n s s}=\frac{K_{n s s}}{T}+C_{L S}+C_{C O}+h_{n s s}\left(\frac{A}{2}\right) \tag{3.3}
\end{equation*}
$$

### 3.3.2 Process B: Remanufacturing (Bottle Producer)

As shown in Figure 3.3 the quantity of bottles remanufactured in one cycle is $Q_{r}$ and the depletion rate is $\lambda_{r}$. These bottles maybe held prior to being remanufactured, which happens when other processes are being performed. The time for holding and the time for remanufacturing the bottles are represented in Figure 3.3 by $t_{1}$ and $t_{2}$ respectively. The non contaminated whole bottles are remanufactured by a customized automated or semi-automated process where they are de-labelled, cleaned and sanitized, polished and relabelled. The technology to perform this process does exist, however its details are beyond the scope of this thesis. Figure 3.3 will also be used to develop the holding cost of the cost equation for remanufacturing bottles.


Figure 3.3 - The quantity of bottles remanufactured during a cycle $T$

$$
\begin{equation*}
Q_{r}=\beta \alpha \theta D T \tag{3.4}
\end{equation*}
$$

Using Figure 3.3 for calculating the inventory total cost for the remanufacturing process is:

$$
\begin{align*}
T C_{r} & =K_{r}+C_{L r} T+C_{M a t} T+h_{r}\left[Q_{r} t_{1}+\frac{Q_{r} t_{2}}{2}\right] \\
& =K_{r}+C_{L r} T+C_{M a t} T+h_{r}\left[\frac{Q_{r} Q_{p}}{D}+\frac{Q_{r}^{2}}{D}-\frac{Q_{r}^{2}}{2 \lambda_{r}}\right] \tag{3.5}
\end{align*}
$$

$$
\begin{equation*}
\text { Where } T=t_{1}+t_{2} ; \text { and } t_{1}=\frac{Q_{r}+Q_{p}}{D}-t_{2} ; \text { and } t_{2}=\frac{Q_{r}}{\lambda_{r}} \tag{3.6}
\end{equation*}
$$

Dividing equation (3.5) by the cycle time $T$, the process unit time cost function is as follows:

$$
\begin{equation*}
T C U_{r}=\frac{K_{r}}{T}+C_{L r}+C_{M a t}+\frac{h_{r}}{T}\left[\frac{Q_{r} Q_{p}}{D}+\frac{Q_{r}{ }^{2}}{D}-\frac{Q_{r}{ }^{2}}{2 \lambda_{r}}\right] \tag{3.7}
\end{equation*}
$$

### 3.3.3 Process C: Regrind and Virgin Material Mix (Bottle Producer)

As shown in Figure 3.4 the quantity of regrind $((1-\beta) \alpha \theta D T)$ and virgin $\left(Q_{v m} T\right)$ material mix in one cycle is $R M$ and the depletion rate is $\lambda_{r m}$. This material maybe held prior to mixing, which happens when other processes are being performed. The time for holding and mixing the material is represented in Figure 3.4 by $t_{1}$ and $t_{2}$ respectively. The damaged non-contaminated bottles are recycled where they are de-labelled, cleaned, ground into flakes and processed through a "Recycling line- recoSTAR PET" ${ }^{9}$ machine which produces new and improved quality Polyethylene terephthalate (PET) pelletized material during processing due to "Solid state polycondensation" ${ }^{10}$ (SSP) which increases the intrinsic viscosity of the PET pelletized material and also "decontaminates the material so effectively that is suitable for direct food contact applications" ${ }^{" 10}$. The pellets/material produced from the Recycling line- recoSTAR PET machine are then mixed with the virgin PET pellets (material of origin) to generate new bottles. Figure 3.4 will also be used to develop the holding cost of the cost equation for material mixing.


Figure 3.4 - The material available for production during cycle $T$
$Q_{v m}=D-\beta \alpha \theta D-(1-\beta) \alpha \theta D$
and

$$
\begin{equation*}
R M=Q_{v m} T+(1-\beta) \alpha \theta D T \tag{3.9}
\end{equation*}
$$

Using Figure 3.4 for calculating the inventory, the total cost for the raw material process that is available for production in a cycle of length $T$ is:

$$
\begin{align*}
T C_{r m} & =K_{r m}+C_{L r m} T+C_{v m} T+h_{r m}\left(R M t_{1}+\frac{R M t_{2}}{2}\right) \\
& =K_{r m}+C_{L r m} T+C_{v m} T+h_{r m}\left[\frac{R M Q_{p}}{D}+\frac{R M Q_{r}}{D}-\frac{R M^{2}}{2 \lambda_{r m}}\right] \tag{3.10}
\end{align*}
$$

$$
\begin{equation*}
\text { Where } T=t_{1}+t_{2} ; \text { and } t_{1}=\frac{Q_{r}+Q_{p}}{D}-t_{2} ; \text { and } t_{2}=\frac{R M}{\lambda_{r}} \tag{3.11}
\end{equation*}
$$

Dividing equation (3.10) by the cycle time $T$ the raw material process unit time cost function is as follows:

$$
\begin{equation*}
T C U_{r m}=\frac{K_{r m}}{T}+C_{L r m}+C_{v m}+\frac{h_{r m}}{T}\left[\frac{R M Q_{p}}{D}+\frac{R M Q_{r}}{D}-\frac{R M^{2}}{2 \lambda_{r m}}\right] \tag{3.12}
\end{equation*}
$$

### 3.3.4 Process D: Bottle Manufacturing (Bottle Producer)

Figure 3.5 shows $Q_{P}$ as the replenishment order quantity in one cycle and $P$ is the production rate. New bottles are produced by a two-step moulding process which requires two separate machines; one to make the pre-form of the bottle and the second to inflate the shape of the bottle by using stretch blow moulding. Therefore once the pre-form is made it is then transferred to the stretch blow moulding stage to create the final shape of the bottle. Figure 3.5 will also be used to develop the holding cost of the cost equation for material mixing.


Figure 3.5 - The amount of bottles produced during cycle $T$
$P=\frac{Q_{p}}{T}$
Using Figure 3.5 for calculating the inventory, the total cost for the production process is:

$$
\begin{equation*}
T C_{p}=K_{p}+C_{L p} T+h_{p}\left(\frac{Q_{p} T}{2}\right) \tag{3.14}
\end{equation*}
$$

Therefore dividing equation (3.14) by the cycle time $T$ the process unit time cost function is as follows:

$$
\begin{equation*}
T C U_{p}=\frac{K_{p}}{T}+C_{L p}+h p\left(\frac{Q_{p}}{2}\right) \tag{3.15}
\end{equation*}
$$

### 3.3.5 Process E: Contaminated Bottle Sort (Bottle collection Company)

The contaminated bottles are sorted into two streams where a percentage of the bottles will go to the landfill and the other percentage will be sold to industries that use a lower grade of plastic material. However in our case we will only be concerned with the cost of the bottles that go to the landfill. As depicted in Figure 3.6 contaminated bottles for disposal are sorted at a rate $L$; and the amount of contaminated bottles to be disposed of in landfill is $B_{L f}$. This Figure will also be used to develop the holding cost of the cost equation for material mixing.


Figure 3.6 - The amount of badly contaminated bottles during cycle $T$
$L=(1-\phi)(1-\alpha) \theta D$ and $B_{L f}=L T$
Using Figure 3.6 for calculating the inventory, the total cost for the sorting process, where unrecoverable bottles are disposed in the landfill is:

$$
\begin{equation*}
T C_{S c}=K_{S c}+C_{L S c} T+h_{S c}\left(\frac{B_{L f}}{2}\right) T \tag{3.17}
\end{equation*}
$$

Dividing equation (3.17) by the cycle time $T$ the process unit time cost function is as follows:

$$
\begin{equation*}
T C U_{S c}=\frac{K_{S c}}{T}+C_{L S c}+h_{S c}\left(\frac{B_{L f}}{2}\right) \tag{3.18}
\end{equation*}
$$

### 3.3.6 Process F: Landfill Disposal Cost

Since the bottles placed into the landfill is assumed to decay exponentially, a cost equation needs to be developed for each time quantities of bottles are placed into the landfill. Quantities of Bottles that are placed into the landfill take time to decompose to zero (quantity) therefore as the bottles decompose in the ground the cost must be captured for the use of the real-estate and for the admitting of chemicals into the environment (As bottles decay chemicals are released into the environment). Therefore the Present-Worth is used to bring back the total cost at the end of every cycle ( $T$ ).

Since exponential decay can describe the decomposition of material in ecological systems ${ }^{12}$ it will then be applied in this thesis to capture the decay of 2 L plastic bottles when disposed in landfill (This may not apply to all other plastic materials of bervaerge bottles.) . Figure 3.7 shown below is the standard exponential decay diagram where $N(t)$ is the quantity at time $t$ and $\lambda$ is the decay rate.


Figure 3.7 - Exponential decay

The exponential decay equation described in Figure 3.7 is of the form $N(t)=N_{0} e^{-\lambda t}$, where $N_{0}$ is the initial disposed quantity (i.e. the quantity at time $t=0$ ). Figure 3.8 shown below displays the decomposition of bottles placed into the landfill for three different cycles/periods.


Figure 3.8 - Exponential decomposition of bottles
Now, the use of the land to dispose bottles until they fully decompose has a real-estate rental cost per bottle which is denoted as $C_{r e}$ and a rehabilitation and penalty cost per bottle which is denoted as $C_{r h}$. As depicted in Figure 3.8 bottles are disposed in a landfill at the end of every cycle. By, using the present worth approach, the cost for land use, rehabilitation and environmental damage of disposing bottles into a landfill are brought back at the end of every cycle. As shown below, by the use of equation (3.19):

Solving the P-W cost function for bottles placed in the landfill
The formula for the Present worth is denoted as:

$$
\begin{equation*}
P=F e^{-i t} \tag{3.19}
\end{equation*}
$$

Where $P$ is the Present worth, $F$ is a single-amount future-value and $i$ is a continuously compounding interest rate.

As defined $N(t)=N_{0} e^{-\lambda t}, C_{r e}$ is the real-estate rental cost per bottle and $C_{r h}$ is the rehabilitation and penalty cost per bottle. The single amount future-value $F=\left(C_{r e}+\right.$ $\left.C_{r h}\right) N(t)=\left(C_{r e}+C_{r h}\right) N_{0} e^{-\lambda t}$. Also, the amount of plastic bottles disposed in landfill by the end of cycle $T$ is $N_{0}$, where $N(t)=N_{0} e^{-\lambda(t-T)} \Rightarrow N(T)=N_{0} e^{-\lambda(T-T)}=N_{0}$. So, $N(t)=$ $N_{0} e^{-\lambda(t-T)}, t \in[T, \infty)$ is equivalent to $N(t)=N_{0} e^{-\lambda t}, t \in[0, \infty)$. Therefore, the process cost for landfill disposal can be written as follows:

$$
\begin{align*}
T C_{L F} & =K_{L F}+C_{L F} T+\int_{0}^{\infty}\left(C_{r e}+C_{r h}\right) N_{0} e^{-i t} e^{-\lambda t} d t  \tag{3.20}\\
& =K_{L F}+C_{L F} T+\int_{0}^{\infty}\left(C_{r e}+C_{r h}\right) N_{0} e^{-(i+\lambda) t} d t \\
& =K_{L F}+C_{L F} T+\frac{\left(C_{r e}+C_{r h}\right) N_{0}}{(i+\lambda)}
\end{align*}
$$

where,

$$
\begin{equation*}
T C_{L F}=K_{L F}+C_{L F} T+\frac{C_{r e} N_{0}}{(i+\lambda)}+\frac{C_{r h} N_{0}}{(i+\lambda)} \tag{3.21}
\end{equation*}
$$

Dividing equation (3.21) by the cycle time $T$ the process unit time cost function is as follows:

$$
\begin{equation*}
T C U_{L F}=\frac{K_{L F}}{T}+C_{L F}+\frac{C_{r e} N_{0}}{(i+\lambda) T}+\frac{C_{r h} N_{0}}{(i+\lambda) T} \tag{3.22}
\end{equation*}
$$

The total system unit time cost is the sum of equations (3.3), (3.7), (3.12), (3.15), (3.18) and (3.22):

$$
\begin{equation*}
T C U=T C U_{n s s}+T C U_{r}+T C U_{r m}+T C U_{p}+T C U_{S c}+T C U_{L F} \tag{3.23}
\end{equation*}
$$

This can also be written as:

$$
\begin{align*}
T C U= & \frac{K_{n s s}}{T}+C_{L S}+C_{C O}+h_{n s s}\left(\frac{A}{2}\right)  \tag{3.24}\\
& +\frac{K_{r}}{T}+C_{L r}+C_{M a t}+\frac{h_{r}}{T}\left[\frac{Q_{r} Q_{p}}{D}+\frac{Q_{r}{ }^{2}}{D}-\frac{Q_{r}{ }^{2}}{2 \lambda_{r}}\right]
\end{align*}
$$

$$
\begin{aligned}
& +\frac{K_{r m}}{T}+C_{L r m}+C_{v m}+\frac{h_{r m}}{T}\left[\frac{R M Q_{p}}{D}+\frac{R M Q_{r}}{D}-\frac{R M^{2}}{2 \lambda_{r m}}\right] \\
& +\frac{K_{p}}{T}+C_{L p}+h_{p}\left(\frac{Q_{p}}{2}\right) \\
& +\frac{K_{S c}}{T}+C_{L S c}+h_{S c}\left(\frac{B_{L f}}{2}\right) \\
& +\frac{K_{L F}}{T}+C_{L F}+\frac{C_{r e} N_{0}}{(i+\lambda) T}+\frac{C_{r h} N_{0}}{(i+\lambda) T}
\end{aligned}
$$

### 3.3.7 Deriving an Expression for the Order Replenishment Quantity

In order to develop an expression for the minimum of the order replenishment quantity $Q_{p}$ an expression for the cycle time $T$ in terms $Q_{p}$ is substituted into equation (3.24). The resulting equation can then be differentiated and the derivative set equal to zero, which will allow an expression for $Q_{p}$ (EOQ) to be obtained.

Now, as shown in Figure 3.1

$$
T=\frac{Q_{p}+Q_{r}}{D}
$$

Knowing that

$$
\begin{equation*}
Q_{r}=\beta \alpha \theta D T, \text { if } \Upsilon=\beta \alpha \theta, \text { then } Q_{r}=\gamma D \tag{3.25}
\end{equation*}
$$

Therefore, an equation for the cycle time $T$ can be developed as follows:

$$
\begin{align*}
& \frac{Q_{p}+Q_{r}}{D}=T \rightarrow Q_{p}+Q_{r}=D T \rightarrow Q_{p}+\Upsilon D T=D T \\
& \rightarrow Q_{p}=D T-\Upsilon D T \rightarrow Q_{p}=(D-\Upsilon D) T \rightarrow T=\frac{Q_{p}}{D(1-\Upsilon)} \tag{3.26}
\end{align*}
$$

Where $0<\Upsilon<1$

By substituting equation (3.26) into equation (3.24) the total system process unit time cost is expressed in terms of $Q_{p}$. By differentiating this expression and setting it equal to zero $\left(\frac{d^{2} T C U}{d Q_{p}^{2}}>0 \forall Q_{p}>0\right)$ the following equation is obtained for the order replenishment quantity $\left(Q_{p}\right)$ :

$$
\begin{equation*}
Q_{p}=\sqrt{\frac{D v K}{\left[h_{r}\left[\gamma+\frac{r^{2}}{v}-\frac{(\gamma D)^{2}}{2 \lambda_{r} D v}\right]+h_{r m}\left[Q_{v m}\left[\frac{1}{D}+\frac{\gamma}{D v}+\frac{Q_{v m}+2 \eta D}{2 \lambda_{r m} D v}\right]+\eta+\frac{\eta \gamma D}{D v}-\frac{(\eta D)^{2}}{2 \lambda_{r m} D v}\right]+\frac{h_{p}}{2}\right]}} \tag{3.27}
\end{equation*}
$$

Where, $\eta=(1-\beta) \alpha \theta$ and $v=(1-\Upsilon)$ and $K=K_{n s s}+K_{r}+K_{r m}+K_{p}+K_{S c}+K_{L F}$

### 3.4 Chapter Summary

In this chapter, the mathematical model for the production, remanufacturing, recycling and waste disposal system for plastic bottles is introduced and developed. The input parameters and decision variables were defined and a full description of the assumptions made. The material flow in the system was illustrated to show each process in the system. Cost equations for these processes were developed and their unit cost functions obtained. Also developed is the present worth cost function for disposing badly contaminated bottles in a landfill. After developing the equation for the cycle time, the equation for order replenishment quantity was obtained.

## CHAPTER 4: RESULTS

This chapter provides numerical examples to illustrate the behaviour of the mathematical model developed in Chapter 3. It also deals with the effect of varying the values of the decision variables on the process costs and the total system process cost. A regression analysis is carried out and its results will be used in the sensitivity analysis that would determine the decision variable(s) that has (have) the most significant effect on total system cost.

### 4.1 Suggested Parameter Values:

In the following cost estimates, all assumed values are based on the author's industrial experience. Although other cost values maybe used it is felt that the assumed values here are more realistic at the present time.
$D$ - Demand rate: Three demand rates will be considered for low, medium and high seasons. These demand rates are 25000,50000 and 75000 bottles per month. These numbers were chosen for comparative purposes only. Close estimates can be obtained only after the system is in operation.
$K_{n s s}-\underline{\text { Setup cost for bottle collection: All setup costs include are aggregates of mainly three }}$ costs: preparation, equipment maintenance (collection trucks and machinery) and overhead. It is assumed that the prep-process takes 2 hours labour at $\$ 20 / \mathrm{hr}$, maintenance $\$ 80$ per set-up and $\$ 180$ per set-up for space rental to house the equipment and administration. This gives a total of $\$ 300$ per set-up.
 $\$ 20 / \mathrm{hr}$, maintenance $\$ 30$ per set-up and space rental to house the equipment and administration at $\$ 75$ per set-up to a total of $\$ 125$ per set-up.
$K_{r m}$ - Setup cost for Virgin Material and Regrind mixing: It is considered to take 2 hours labour at $\$ 20 / \mathrm{hr}$, maintenance $\$ 40$ per set-up and space rental to house the equipment and administration at $\$ 120$ per set-up. Therefore the total of this set-up is $\$ 200$.
$K_{P}-\underline{\text { Setup cost for production/manufacturing of new bottles: Itis considered the preparation }}$ process takes 1 hour labour at $\$ 20 / \mathrm{hr}$, maintenance $\$ 30$ per set-up and space rental to house the equipment and administration at $\$ 75$ per cycle. This gives a total of $\$ 125$ per set-up.
$K_{S c}-\underline{\text { Setup cost for contaminated bottle sort (for bottles that are to be disposed in the landfill): }}$ For this set-up cost it is assumed the prep-process takes 1 hour labour at $\$ 20 / \mathrm{hr}$, maintenance $\$ 20$ per set-up and space rental to house the equipment and administration at $\$ 60$ per set-up to a total of $\$ 100$ per set-up.
$K_{L F}$ - Setup cost to prepare the ground for bottle disposal: This setup cost includes machine rental and their operators. It is assumed the labour is 3 hours at $\$ 40$ per set-up; and machine rental $\$ 80$ per set-up. Therefore, this total cost is $\$ 200$ per set-up.
$C_{L s}$ - Labour cost for sorting bottles after collection: This labour cost includes loading collected items on to a conveyor belt and collection of the sorted bottles and the operation of the vision system. Here again it is assumed in one hour 15000 units can be loaded on a conveyor belt and only $10 \%$ are manufacturers bottles. This means in one hour 1500 bottles are collected. Since this operation requires 2 operators one at the loading and one at the receiving end. Therefore the labour cost is $(2 \times \$ 10) / 1500=\$ 0.013$ per bottle and the cost for the vision system is $\$ 0.002$ per bottle. This gives a total of $\$ 0.015$ per bottle; when multiplied by $\theta D$ this gives the labour cost for sorting bottles after collection per cycle.
$C_{C o}$ - Labour cost for bottle collection: This labour cost includes a truck and two labourers. It is considered the cost of the truck and two labourers are $\$ 60$ per hour and the truck collects 60,000 items in an hour in which $10 \%$ are the required brand of bottles, this cost would be $\$ 0.01$ per bottle; when multiplied by $\theta D$ this the gives labour cost for bottle collection per cycle.
$C_{L r}-\underline{\text { Labour cost for remanufacturing (Labour involved to operate the machinery to }}$ remanufacture the bottles): Here too it is considered that 2 labour hours at $\$ 10$ dollars per hour are required to clean, sanitize and pack 200 bottles per hour. Therefore the labour cost is $\$ 0.1$ dollars per bottle. Add to this cost $\$ 0.03$ per bottle for the cost to run the machinery. This gives a running cost of $\$ 0.13 /$ bottle; when multiplied by $\beta \alpha \theta D$ which equals $\lambda_{r}$ this gives the labour cost for remanufacturing per cycle.
$C_{M a t}-\underline{\text { Material cost for remanufacturing (Cleaning, clear coat polish, re-labelling): This process }}$ involves cleaning agent/solvent, clear coat polish and labels. It is assumed $\$ 2$ is required for the solvent and clear coat for 200 bottles and the label is $\$ 0.01$ per bottle. Therefore, the material cost is $\$ 0.02$ per bottle; when multiplied by $\beta \alpha \theta D$ this gives the material cost for remanufacturing per cycle.
$Q_{v m}$ - Is the material needed to be mixed with the regrind in order to produce the number of bottles required to supplement the number of remanufactured bottles in order to meet demand.
$C_{L r m}$ - Labour cost for regrind and mixing (Labour involved to operate the machinery to regrind the bottles and add the virgin material that is mixed with the regrind): This process involves delabelling the bottles, cleaning and grinding the material into flakes and processing it through a "Recycling line- recoSTAR PET machine which produces new and improved quality PET pelletized material due to solid state polycondensation. Consider 1.5 labour hours at 10 dollars per hour required to process 1000 bottles; and 5 dollars per hour for the cost to run the machinery for the 1000 bottles per hour. Then the total cost is $\$ 0.02$ per bottle; when multiplied by [ $Q_{v m}$ $+(1-\beta) \alpha \theta D]$ which equals $\lambda_{r m}$. This gives the labour cost for regrind and mixing per cycle.
$C_{v m}$ - Cost for virgin material per cycle: This is a variable amount and is based on the demand. However, the material required for supplementing the re-grind material to produce new bottles is considered to be $\$ 0.03 /$ bottle. For the cycle this cost is obtained from $\$ 0.03$ multiplied by $Q_{v m}$.
$C_{L p}$ - Labour cost for bottle production/manufacturing: This involves a two-step moulding process which requires two separate machines; one to make the pre-form of the bottle and the second to inflate the shape of the bottle by using stretch blow moulding. Each machine is operated by a skilled operator at $\$ 15.00$ per hour. Assuming this process can produce 240 bottles per hour which gives $\$ 0.125 /$ bottle, plus $\$ 0.025 /$ bottle for the cost to run the machines and labelling. Then the total cost is $\$ 0.15$ per bottle; when multiplied by $\left[Q_{v m}+(1-\beta) \alpha \theta D\right]$ this gives the labour cost for bottle production/manufacturing.
$C_{L S c}$ - Labour cost for contaminated bottle sort: This process requires more labour work than the prior sorting process because it requires more time to determine if a bottle is badly contaminated and will be disposed in the landfill or if a bottle is contaminated but can be sold to industries that
require a lower grade of material. Although this process does not require a vision system, its longer inspection time justifies charging the same as the other sorting process i.e. $\$ 0.015$ per bottle; when multiplied by $(1-\phi)(1-\alpha) \theta D$ this gives the labour cost for contaminated bottle sort.
$C_{L F}-\underline{\text { Labour cost for disposing bottles in the landfill: This process involves an operator placing }}$ these contaminated bottles together with other disposables of different unrelated processes into a landfill. Although the number of badly contaminated bottles to be disposed of in the landfill is determined, it is difficult to estimate how many bottles will be part of each process. Therefore a flat fee of only $\$ 25$ dollars is charged per disposal.
$C_{r e}$ - The real-estate rental cost per bottle: It is assumed that a square foot of rural land can be rented for $\$ 0.5$. It is also estimated that four bottles occupy approximately one cubic foot and if bottles are piled on the average 10 feet high, then one square foot of land carries 400 bottles. Therefore the real-estate rental cost is $\$ 0.00125 /$ bottle/month
$C_{r h}$ - The rehabilitation penalty cost per bottle: The cost of the rehabilitation of the land per bottle will be similar to the real-estate rental cost per bottle. However, a penalty for the effect on the environment is added and assumed to be three times the cost of rehabilitation. Therefore the cost of rehabilitation and the penalty will be $\$ 0.005$ bottle/month. It is to be noted that a good approximation of the cost of the effect of disposing bottles in a landfill on the environment is extremely difficult to evaluate therefore we have assumed three times the cost of rehabilitation (1 for cleaning the water, 1 for cleaning soil, 1 for protecting wildlife).

Carrying costs per cycle-period - There are two kinds of carrying costs. One is the carrying cost used by the Bottle production company and the other used by the collection company. For the Bottle production company racks maybe used for storage of the bottles, while for the collection company the bottles might be in a high pile of open space. In either case, it is assumed that one square foot can store 200 bottles piled high on it and one square foot can be rented for 4 dollars a month. Therefore the cost per bottle per month for the bottle producing company is $\$ 0.02$ per bottle. This applies for the three different carrying costs of the Bottle production company which are $h_{r}, h_{r m}$ and $h_{p}$. For the carrying cost $h_{n s s}$ charged by the collection company it is estimated that $\$ 0.01$ will be added as a profit and therefore the collection company will charge $\$ 0.03 \mathrm{a}$
bottle. For $h_{S c}$ the collection company will charge $\$ 0.02$ a bottle because a profit can accrue to the company from selling the bottles to industries that require low grade material.
$\lambda$ - The decay rate of Polyethylene terephthalate (PET) material (bottle material) in landfill: It is estimated that it takes approximately 450 years ${ }^{8}$ for plastic beverage bottles to fully biodegrade in a landfill. Using the exponential decay half life where $\lambda=\frac{\ln (2)}{t}$, the decay rate $(\lambda)$ is found to be $0.0256 \%$ per month.
$i$ - The interest rate is considered to be $10 \%$ per annum. Although this rate fluctuates depending on the economic situations where it can be much higher when there is high demand for borrowing and much lower when there is low demand, a $10 \%$ rate is a reasonable average. At the present a credit line is about $6 \%$ while a credit card is $17 \%$. Also a short term mortgage rate is about $5 \%$, while a 5 year long mortgage rate is $6 \%$. Indicating interest rates for longer terms would be higher. Therefore over the life of bottle decay the interest rate would be much higher. To obtain the present worth of the landfill costs that will be charged to manufacturer continuous compounding is used.

### 4.2 Analysis

Using the input parameters established above in section 4.1, an excel program resulted in the data tabulated in Tables 4.1, 4.2 and 4.3 below for a low, medium and high demand rate of 25000, 50000 and 75000 bottles/month, respectively. In these tables the analyses start with all the decision variables at level value of 0.25 . Each parameter is then increased by a value of 0.25 holding the others constant. Next all the parameters were held at a level value of 0.5 and then each parameter is given a value of 0.25 then a value of 0.75 while holding the others constant. Next all the parameters are held at a level value of 0.75 and the process is repeated as is previously described. These processes are carried out for all three demand rates to analyze the effects on the process unit time costs and total system unit time cost.

Table 4.1 -The effect on costing for a demand rate of $25000 \mathrm{bottles} / \mathrm{month}$

| Runs | $\theta$ | $\alpha$ | $\beta$ | $\phi$ | $Q_{p}$ | $T C U_{n s s}$ | $T C U_{r}$ | $T C U_{r m}$ | $T C U_{P}$ | $T C U_{S c}$ | $T C U_{L F}$ | TCU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 0.25 | 0.25 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 185.95 | 2787.02 | 9410.08 |
| 2 | 0.25 | 0.25 | 0.25 | 0.5 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 156.29 | 1930.99 | 8524.38 |
| 3 | 0.25 | 0.25 | 0.25 | 0.75 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 126.63 | 1074.96 | 7638.68 |
| 4 | 0.25 | 0.25 | 0.25 | 0.25 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 185.95 | 2787.02 | 9410.08 |
| 5 | 0.25 | 0.25 | 0.5 | 0.25 | 25119 | 542.73 | 245.81 | 1631.52 | 4004.52 | 185.61 | 2785.93 | 9396.13 |
| 6 | 0.25 | 0.25 | 0.75 | 0.25 | 24998 | 545.53 | 310.54 | 1635.71 | 3984.86 | 186.60 | 2814.74 | 9477.99 |
| 7 | 0.25 | 0.25 | 0.25 | 0.25 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 185.95 | 2787.02 | 9410.08 |
| 8 | 0.25 | 0.5 | 0.25 | 0.25 | 25171 | 542.33 | 245.58 | 1584.77 | 4004.80 | 155.73 | 1929.50 | 8462.70 |
| 9 | 0.25 | 0.75 | 0.25 | 0.25 | 24992 | 540.61 | 307.25 | 1526.54 | 3943.32 | 125.21 | 1071.72 | 7514.65 |
| 10 | 0.25 | 0.25 | 0.25 | 0.25 | 25381 | 543.82 | 183.82 | 1643.04 | 4066.41 | 185.95 | 2787.02 | 9410.08 |
| 11 | 0.5 | 0.25 | 0.25 | 0.25 | 25171 | 796.02 | 245.58 | 1584.77 | 4004.80 | 274.76 | 5353.63 | 12259.55 |
| 12 | 0.75 | 0.25 | 0.25 | 0.25 | 24992 | 1049.77 | 307.25 | 1526.54 | 3943.32 | 364.17 | 7919.98 | 15111.03 |
| 13 | 0.5 | 0.5 | 0.5 | 0.25 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 213.46 | 3634.02 | 10313.20 |
| 14 | 0.5 | 0.5 | 0.5 | 0.5 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 173.12 | 2492.64 | 9131.49 |
| 15 | 0.5 | 0.5 | 0.5 | 0.75 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 132.78 | 1351.27 | 7949.77 |
| 16 | 0.5 | 0.5 | 0.25 | 0.5 | 24845 | 794.26 | 368.86 | 1468.37 | 3882.00 | 174.34 | 2496.42 | 9184.24 |
| 17 | 0.5 | 0.5 | 0.5 | 0.5 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 173.12 | 2492.64 | 9131.49 |
| 18 | 0.5 | 0.5 | 0.75 | 0.5 | 22484 | 791.07 | 867.94 | 1374.27 | 3384.64 | 171.81 | 2488.44 | 9078.17 |
| 19 | 0.5 | 0.25 | 0.5 | 0.5 | 24627 | 795.03 | 369.76 | 1561.61 | 3880.86 | 214.74 | 3639.47 | 10461.46 |
| 20 | 0.5 | 0.5 | 0.5 | 0.5 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 173.12 | 2492.64 | 9131.49 |
| 21 | 0.5 | 0.75 | 0.5 | 0.5 | 22741 | 790.38 | 867.26 | 1281.05 | 3385.93 | 130.25 | 1345.02 | 7799.90 |
| 22 | 0.25 | 0.5 | 0.5 | 0.5 | 24627 | 540.27 | 369.76 | 1561.61 | 3880.86 | 135.03 | 1356.72 | 7844.24 |
| 23 | 0.5 | 0.5 | 0.5 | 0.5 | 23663 | 792.66 | 618.11 | 1421.52 | 3633.43 | 173.12 | 2492.64 | 9131.49 |
| 24 | 0.75 | 0.5 | 0.5 | 0.5 | 22741 | 1051.59 | 867.26 | 1281.05 | 3385.93 | 212.11 | 3627.78 | 10425.72 |
| 25 | 0.75 | 0.75 | 0.75 | 0.25 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 176.53 | 2752.44 | 9210.66 |
| 26 | 0.75 | 0.75 | 0.75 | 0.5 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 144.25 | 1896.41 | 8322.34 |
| 27 | 0.75 | 0.75 | 0.75 | 0.75 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 111.96 | 1040.38 | 7434.02 |
| 28 | 0.75 | 0.75 | 0.25 | 0.75 | 24753 | 1053.18 | 676.34 | 1178.93 | 3578.68 | 117.87 | 1054.62 | 7659.63 |
| 29 | 0.75 | 0.75 | 0.5 | 0.75 | 21491 | 1055.96 | 1243.30 | 1069.63 | 3014.74 | 115.20 | 1048.26 | 7547.08 |
| 30 | 0.75 | 0.75 | 0.75 | 0.75 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 111.96 | 1040.38 | 7434.02 |
| 31 | 0.75 | 0.25 | 0.75 | 0.75 | 23293 | 1050.38 | 680.75 | 1456.46 | 3570.88 | 183.08 | 2777.57 | 9719.13 |
| 32 | 0.75 | 0.5 | 0.75 | 0.75 | 20809 | 1053.51 | 1244.05 | 1208.92 | 3011.34 | 148.65 | 1909.77 | 8576.24 |
| 33 | 0.75 | 0.75 | 0.75 | 0.75 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 111.96 | 1040.38 | 7434.02 |
| 34 | 0.25 | 0.75 | 0.75 | 0.75 | 23293 | 534.60 | 680.75 | 1456.46 | 3570.88 | 102.33 | 494.81 | 6839.84 |
| 35 | 0.5 | 0.75 | 0.75 | 0.75 | 20809 | 788.69 | 1244.05 | 1208.92 | 3011.34 | 107.12 | 768.39 | 7128.51 |
| 36 | 0.75 | 0.75 | 0.75 | 0.75 | 18141 | 1060.78 | 1814.00 | 957.94 | 2448.97 | 111.96 | 1040.38 | 7434.02 |

Table 4.2 -The effect on costing for a demand rate of 50000bottles/month

| Runs | $\theta$ | $\alpha$ | $\beta$ | $\phi$ | $Q_{p}$ | $T C U_{n s s}$ | $T C U_{r}$ | $T C U_{r m}$ | $T C U_{P}$ | $T C U_{S c}$ | $T C U_{L F}$ | TCU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 0.25 | 0.25 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 293.87 | 5435.45 | 17821.18 |
| 2 | 0.25 | 0.25 | 0.25 | 0.5 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 241.62 | 3723.38 | 16056.87 |
| 3 | 0.25 | 0.25 | 0.25 | 0.75 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 189.37 | 2011.32 | 14292.55 |
| 4 | 0.25 | 0.25 | 0.25 | 0.25 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 293.87 | 5435.45 | 17821.18 |
| 5 | 0.25 | 0.25 | 0.5 | 0.25 | 35524 | 859.07 | 416.27 | 3002.94 | 7791.31 | 293.39 | 5433.90 | 17796.87 |
| 6 | 0.25 | 0.25 | 0.75 | 0.25 | 35156 | 857.49 | 538.30 | 2982.05 | 7669.44 | 292.89 | 5432.31 | 17772.48 |
| 7 | 0.25 | 0.25 | 0.25 | 0.25 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 293.87 | 5435.45 | 17821.18 |
| 8 | 0.25 | 0.5 | 0.25 | 0.25 | 35598 | 858.50 | 415.94 | 2909.37 | 7791.69 | 240.83 | 3721.27 | 15937.60 |
| 9 | 0.25 | 0.75 | 0.25 | 0.25 | 35344 | 856.06 | 537.49 | 2794.99 | 7670.42 | 187.37 | 2006.74 | 14053.07 |
| 10 | 0.25 | 0.25 | 0.25 | 0.25 | 35893 | 860.61 | 294.29 | 3023.81 | 7913.15 | 293.87 | 5435.45 | 17821.18 |
| 11 | 0.5 | 0.25 | 0.25 | 0.25 | 35598 | 1308.80 | 415.94 | 2909.37 | 7791.69 | 450.35 | 10569.53 | 23445.69 |
| 12 | 0.75 | 0.25 | 0.25 | 0.25 | 35344 | 1759.18 | 537.49 | 2794.99 | 7670.42 | 607.68 | 15703.26 | 29073.03 |
| 13 | 0.5 | 0.5 | 0.5 | 0.25 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 343.07 | 7134.74 | 19587.26 |
| 14 | 0.5 | 0.5 | 0.5 | 0.5 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 272.29 | 4851.98 | 17233.72 |
| 15 | 0.5 | 0.5 | 0.5 | 0.75 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 201.51 | 2569.23 | 14880.19 |
| 16 | 0.5 | 0.5 | 0.25 | 0.5 | 35137 | 1306.32 | 658.93 | 2680.68 | 7549.38 | 274.01 | 4857.33 | 17326.64 |
| 17 | 0.5 | 0.5 | 0.5 | 0.5 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 272.29 | 4851.98 | 17233.72 |
| 18 | 0.5 | 0.5 | 0.75 | 0.5 | 31797 | 1301.80 | 1639.33 | 2511.00 | 6571.43 | 270.43 | 4846.03 | 17140.02 |
| 19 | 0.5 | 0.25 | 0.5 | 0.5 | 34827 | 1307.40 | 660.21 | 2867.46 | 7547.76 | 344.87 | 7142.45 | 19870.15 |
| 20 | 0.5 | 0.5 | 0.5 | 0.5 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 272.29 | 4851.98 | 17233.72 |
| 21 | 0.5 | 0.75 | 0.5 | 0.5 | 32160 | 1300.83 | 1638.37 | 2324.24 | 6573.25 | 197.93 | 2560.40 | 14595.02 |
| 22 | 0.25 | 0.5 | 0.5 | 0.5 | 34827 | 855.59 | 660.21 | 2867.46 | 7547.76 | 204.69 | 2576.94 | 14712.64 |
| 23 | 0.5 | 0.5 | 0.5 | 0.5 | 33464 | 1304.05 | 1148.73 | 2596.12 | 7060.56 | 272.29 | 4851.98 | 17233.72 |
| 24 | 0.75 | 0.5 | 0.5 | 0.5 | 32160 | 1761.76 | 1638.37 | 2324.24 | 6573.25 | 341.16 | 7125.91 | 19764.69 |
| 25 | 0.75 | 0.75 | 0.75 | 0.25 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 280.55 | 5386.54 | 17383.56 |
| 26 | 0.75 | 0.75 | 0.75 | 0.5 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 224.59 | 3674.48 | 15615.53 |
| 27 | 0.75 | 0.75 | 0.75 | 0.75 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 168.63 | 1962.41 | 13847.51 |
| 28 | 0.75 | 0.75 | 0.25 | 0.75 | 35007 | 1764.00 | 1265.40 | 2111.18 | 6948.81 | 177.00 | 1982.56 | 14248.95 |
| 29 | 0.75 | 0.75 | 0.5 | 0.75 | 30393 | 1767.94 | 2376.11 | 1915.41 | 5842.36 | 173.22 | 1973.55 | 14048.60 |
| 30 | 0.75 | 0.75 | 0.75 | 0.75 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 168.63 | 1962.41 | 13847.51 |
| 31 | 0.75 | 0.25 | 0.75 | 0.75 | 32942 | 1760.05 | 1271.64 | 2668.42 | 6937.78 | 289.81 | 5422.08 | 18349.78 |
| 32 | 0.75 | 0.5 | 0.75 | 0.75 | 29428 | 1764.47 | 2377.18 | 2194.77 | 5837.56 | 230.82 | 3693.37 | 16098.17 |
| 33 | 0.75 | 0.75 | 0.75 | 0.75 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 168.63 | 1962.41 | 13847.51 |
| 34 | 0.25 | 0.75 | 0.75 | 0.75 | 32942 | 847.56 | 1271.64 | 2668.42 | 6937.78 | 148.15 | 856.57 | 12730.11 |
| 35 | 0.5 | 0.75 | 0.75 | 0.75 | 29428 | 1298.44 | 2377.18 | 2194.77 | 5837.56 | 158.35 | 1410.62 | 13276.91 |
| 36 | 0.75 | 0.75 | 0.75 | 0.75 | 25655 | 1774.75 | 3492.12 | 1716.27 | 4733.33 | 168.63 | 1962.41 | 13847.51 |

Table 4.3 -The effect on costing for a demand rate of 75000 bottles/month

| Runs | $\theta$ | $\alpha$ | $\beta$ | $\phi$ | $Q_{p}$ | $T C U_{n s s}$ | $T C U_{r}$ | $T C U_{r m}$ | $T C U_{P}$ | $T C U_{S c}$ | $T C U_{L F}$ | TCU |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 0.25 | 0.25 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 388.95 | 8065.18 | 26072.04 |
| 2 | 0.25 | 0.25 | 0.25 | 0.5 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 315.28 | 5497.08 | 23430.27 |
| 3 | 0.25 | 0.25 | 0.25 | 0.75 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 241.61 | 2928.98 | 20788.50 |
| 4 | 0.25 | 0.25 | 0.25 | 0.25 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 388.95 | 8065.18 | 26072.04 |
| 5 | 0.25 | 0.25 | 0.5 | 0.25 | 43508 | 1138.15 | 574.34 | 4331.57 | 11542.26 | 388.36 | 8063.29 | 26037.97 |
| 6 | 0.25 | 0.25 | 0.75 | 0.25 | 43057 | 1136.22 | 756.05 | 4301.68 | 11360.76 | 387.75 | 8061.34 | 26003.80 |
| 7 | 0.25 | 0.25 | 0.25 | 0.25 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 388.95 | 8065.18 | 26072.04 |
| 8 | 0.25 | 0.5 | 0.25 | 0.25 | 43598 | 1137.47 | 573.94 | 4191.16 | 11542.73 | 314.31 | 5494.50 | 23254.10 |
| 9 | 0.25 | 0.75 | 0.25 | 0.25 | 43288 | 1134.48 | 755.06 | 4020.97 | 11361.96 | 239.16 | 2923.38 | 20434.99 |
| 10 | 0.25 | 0.25 | 0.25 | 0.25 | 43960 | 1140.05 | 392.69 | 4361.43 | 11723.75 | 388.95 | 8065.18 | 26072.04 |
| 11 | 0.5 | 0.25 | 0.25 | 0.25 | 43598 | 1774.98 | 573.94 | 4191.16 | 11542.73 | 609.63 | 15766.89 | 34459.34 |
| 12 | 0.75 | 0.25 | 0.25 | 0.25 | 43288 | 2412.60 | 755.06 | 4020.97 | 11361.96 | 831.35 | 23468.17 | 42850.09 |
| 13 | 0.5 | 0.5 | 0.5 | 0.25 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 458.88 | 10617.63 | 28694.46 |
| 14 | 0.5 | 0.5 | 0.5 | 0.5 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 359.29 | 7193.50 | 25170.74 |
| 15 | 0.5 | 0.5 | 0.5 | 0.75 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 259.71 | 3769.37 | 21647.02 |
| 16 | 0.5 | 0.5 | 0.25 | 0.5 | 43033 | 1771.94 | 936.05 | 3850.87 | 11181.45 | 361.39 | 7200.05 | 25301.74 |
| 17 | 0.5 | 0.5 | 0.5 | 0.5 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 359.29 | 7193.50 | 25170.74 |
| 18 | 0.5 | 0.5 | 0.75 | 0.5 | 38944 | 1766.41 | 2394.84 | 3608.64 | 9725.66 | 357.01 | 7186.22 | 25038.78 |
| 19 | 0.5 | 0.25 | 0.5 | 0.5 | 42655 | 1773.26 | 937.61 | 4131.23 | 11179.47 | 461.09 | 10627.08 | 29109.74 |
| 20 | 0.5 | 0.5 | 0.5 | 0.5 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 359.29 | 7193.50 | 25170.74 |
| 21 | 0.5 | 0.75 | 0.5 | 0.5 | 39388 | 1765.21 | 2393.66 | 3328.30 | 9727.89 | 255.32 | 3758.55 | 21228.94 |
| 22 | 0.25 | 0.5 | 0.5 | 0.5 | 42655 | 1133.89 | 937.61 | 4131.23 | 11179.47 | 263.59 | 3778.81 | 21424.61 |
| 23 | 0.5 | 0.5 | 0.5 | 0.5 | 40985 | 1769.16 | 1664.95 | 3730.09 | 10453.75 | 359.29 | 7193.50 | 25170.74 |
| 24 | 0.75 | 0.5 | 0.5 | 0.5 | 39388 | 2415.75 | 2393.66 | 3328.30 | 9727.89 | 456.54 | 10606.82 | 28928.97 |
| 25 | 0.75 | 0.75 | 0.75 | 0.25 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 372.63 | 8005.29 | 25389.83 |
| 26 | 0.75 | 0.75 | 0.75 | 0.5 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 294.42 | 5437.19 | 22743.52 |
| 27 | 0.75 | 0.75 | 0.75 | 0.75 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 216.21 | 2869.09 | 20097.21 |
| 28 | 0.75 | 0.75 | 0.25 | 0.75 | 42874 | 2418.50 | 1840.10 | 3002.84 | 10284.62 | 226.45 | 2893.76 | 20666.28 |
| 29 | 0.75 | 0.75 | 0.5 | 0.75 | 37223 | 2423.33 | 3490.74 | 2724.37 | 8639.19 | 221.83 | 2882.74 | 20382.20 |
| 30 | 0.75 | 0.75 | 0.75 | 0.75 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 216.21 | 2869.09 | 20097.21 |
| 31 | 0.75 | 0.25 | 0.75 | 0.75 | 40345 | 2413.67 | 1847.74 | 3840.15 | 10271.11 | 383.98 | 8048.81 | 26805.46 |
| 32 | 0.75 | 0.5 | 0.75 | 0.75 | 36042 | 2419.08 | 3492.05 | 3143.92 | 8633.31 | 302.05 | 5460.33 | 23450.75 |
| 33 | 0.75 | 0.75 | 0.75 | 0.75 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 216.21 | 2869.09 | 20097.21 |
| 34 | 0.25 | 0.75 | 0.75 | 0.75 | 40345 | 1124.06 | 1847.74 | 3840.15 | 10271.11 | 184.67 | 1200.54 | 18468.28 |
| 35 | 0.5 | 0.75 | 0.75 | 0.75 | 36042 | 1762.29 | 3492.05 | 3143.92 | 8633.31 | 200.39 | 2036.20 | 19268.17 |
| 36 | 0.75 | 0.75 | 0.75 | 0.75 | 31421 | 2431.67 | 5147.88 | 2441.76 | 6990.61 | 216.21 | 2869.09 | 20097.21 |

First, the effects of changing the percentage of contaminated bottles for use in industries that require low grade material will be examined. From the above tables it can be seen that when the percentage of contaminated bottles to be used for low grade material ( $\phi$ ) increases, the total system unit time cost (TCU) decreases. The process unit time costs that affect TCU are sorting contaminated bottles $\left(T C U_{S c}\right)$ and landfill disposal $\left(T C U_{L F}\right)$, both of which decrease as $\phi$ increases. The process unit time costs for bottle collection and sorting contaminated and noncontaminated bottles $\left(T C U_{n s s}\right.$ ), remanufacturing $\left(T C U_{r}\right)$, regrind and virgin material mixing $\left(T C U_{r m}\right)$ and producing new bottles $\left(T C U_{p}\right)$ remain the same and have no effect on $T C U$. This is expected and is seen in all the levels of analysis carried above because as $\phi$ increases fewer bottles will go to the landfill.

Next, when the percentage of bottles used for remanufacturing ( $\beta$ ) increases, the process unit time costs for bottle collection and sorting $\left(T C U_{n s s}\right)$, sorting contaminated bottles $\left(T C U_{S c}\right)$ and landfill disposal ( $T C U_{L F}$ ) do not significantly change. However, the process unit time cost for remanufacturing $\left(T C U_{r}\right)$ increases because more bottles are remanufactured. The process unit time cost for regrind and virgin material mixing $\left(T C U_{r m}\right)$ decreases as less material needs to be mixed for the production of new bottles. The process unit time cost of producing new bottles $\left(T C U_{p}\right)$ decreases as less bottles need to be newly produced to meet the demand rate which explains the decrease in $Q_{p}$ (replenishment order quantity). Therefore, the resultant effect is the total system unit time cost (TCU) decreases because as $\beta$ increases, the increasing cost of remanufacturing bottles $\left(T C U_{r}\right)$ cannot supersede the decreasing costs of regrind and virgin material mixing $\left(T C U_{r m}\right)$ and the production of new bottles $\left(T C U_{p}\right)$. Again, this is seen in all levels of the analysis carried above.

Changing the percentage of non-contaminated bottles $(\alpha)$ shows no significant change in the process unit time cost for bottle collection and sorting ( $T C U_{n s s}$ ). However, as $\alpha$ increases, the process unit time cost of remanufacturing bottles increases. This is because more noncontaminated bottles are available to remanufacture. The process unit time costs for sorting contaminated bottles and landfill disposal decrease because there is a reduction in the amount of badly contaminated bottles. Now, an increase in $\alpha$ also renders a larger amount of regrind material available, and thus a smaller quantity of virgin material is required in the material mix for the production of new bottles which in turn lowers the cost of purchasing virgin material
$\left(C_{v m}\right)$. Since an increase in $\alpha$ ultimately causes an increase in the rate of remanufacturing ( $\beta \alpha \theta D$ ) the amount of new bottles that need to be produced to meet the demand rate decreases, which clearly explains the decrease in the replenishment order quantity $Q_{p}$. Also, labour costs associated with regrind and virgin material mixing and in the production of new bottles decrease. This clearly causes $T C U_{p}$ to decrease; and the combined effect of a decreasing labour cost coupled with a decreasing virgin material cost $C_{v m}$ causes $T C U_{r m}$ to decrease. Similarly the process unit time costs for sorting contaminated bottles and landfill disposal decrease because there is a reduction in the amount of badly contaminated bottles. In the end result the total system unit time cost decreases as $\alpha$ increases. Here again the increasing cost of remanufacturing bottles cannot supersede the combined decrease in the costs of regrind and virgin material mixing, the production of new bottles, sorting contaminated bottles and landfill disposal; as seen in all levels of the analysis carried above.

Now, as the percentage of bottles collected $\theta$ increases the process unit time cost of bottle collection and sorting ( $T C U_{n s s}$ ) increases as expected. Also, the rate of bottles that can be remanufactured and the rate of contaminated bottles sorted and disposed of in landfill similarly increases, and consequently $T C U_{r,} T C U_{S c}$ and $T C U_{L F}$ increase. Again, when $\theta$ increases, $Q_{p}$ decreases and the rate of regrind increases which causes the cost of purchasing virgin material $\left(C_{v m}\right)$ to decrease. Also, the labour costs associated with regrind and virgin material mixing and in the production of new bottles decrease because less new bottles need to be produced. Therefore this causes $Q_{p}$ and $T C U_{p}$ to decrease; and the decrease in the labour cost in combination with a decreasing cost of virgin material causes $T C U_{r m}$ to decrease. Now, since the increase in the process unit time costs of collection and sorting bottles, remanufacturing bottles, sorting contaminated bottles and the disposal of bottles in a landfill supersedes the combined decrease in the process unit time costs of regrind and virgin material mixing and the production of new bottles the total system unit time cost increases as this is shown in all levels of analysis carried above.

The above tables also show that when the demand rate increases, the replenishment order quantity $\left(Q_{p}\right)$, process unit time costs $\left(T C U_{n s s}, T C U_{r}, T C U_{r m}, T C U_{p}, T C U_{S c}\right.$ and $\left.T C U_{L F}\right)$ and the total system unit time cost increase. This result is expected.

### 4.3 Regression Analysis

In this section, regression analysis will be performed on the independent decision variables $\theta, \alpha$, $\beta$ and $\phi$ to study their effect on the dependent total system unit time cost (TCU) for a low demand rate of 25000 bottles/month, a medium demand rate of 50000 bottles/month and high demand rate of 75000 bottles/month. Tables 4.4, 4.5 and 4.6 shown below obtain the outputs of the regression analysis at a confidence level of $95 \%$ for each of the different demand rates (low, medium and high).

Table 4.4 - Regression Analysis output for a Demand rate of 25000bottles/month

|  | Coefficients | Standard Error | t Stat | $P$-value | Lower 95\% | Upper 95\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 11165.16 | 377.12 | 29.61 | $2.68577 \mathrm{E}-24$ | 10396.02 | 11934.30 |
| $\theta$ | 5522.89 | 717.78 | 7.69 | $1.11607 \mathrm{E}-08$ | 4058.97 | 6986.82 |
| $\alpha$ | -4956.76 | 717.78 | -6.91 | $9.61235 \mathrm{E}-08$ | -6420.69 | -3492.84 |
| $\beta$ | -768.84 | 717.78 | -1.07 | 0.292377069 | -2232.77 | 695.08 |
| $\phi$ | -4330.52 | 717.78 | -6.03 | $1.11655 \mathrm{E}-06$ | -5794.45 | -2866.59 |

Table 4.5 - Regression Analysis output for a Demand rate of 50000bottles/month

|  | Coefficients | Standard Error | t Stat | $P$-value | Lower 95\% | Upper 95\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 21310.05 | 750.20 | 28.41 | $9.26989 \mathrm{E}-24$ | 19780.01 | 22840.08 |
| $\theta$ | 10861.57 | 1427.87 | 7.61 | $1.4124 \mathrm{E}-08$ | 7949.41 | 13773.72 |
| $\alpha$ | -9783.69 | 1427.87 | -6.85 | $1.11568 \mathrm{E}-07$ | -12695.84 | -6871.54 |
| $\beta$ | -1571.07 | 1427.87 | -1.10 | 0.279674847 | -4483.23 | 1341.08 |
| $\phi$ | -8589.90 | 1427.87 | -6.02 | $1.17283 \mathrm{E}-06$ | -11502.05 | -5677.75 |

Table 4.6 - Regression Analysis output for a Demand rate of 75000 bottles/month

|  | Coefficients | Standard Error | t Stat | P-value | Lower 95\% | Upper 95\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 31303.22 | 1121.96 | 27.90 | $1.585 \mathrm{E}-23$ | 29014.96 | 33591.48 |
| $\theta$ | 16144.44 | 2135.46 | 7.56 | $1.60199 \mathrm{E}-08$ | 11789.14 | 20499.74 |
| $\alpha$ | -14614.73 | 2135.46 | -6.84 | $1.14117 \mathrm{E}-07$ | -18970.03 | -10259.43 |
| $\beta$ | -2319.33 | 2135.46 | -1.09 | 0.285806371 | -6674.63 | 2035.97 |
| $\phi$ | -12863.06 | 2135.46 | -6.02 | $1.14757 \mathrm{E}-06$ | -17218.36 | -8507.76 |

From the coefficients column of Tables 4.4, 4.5 and 4.6 the estimated regression functions can be written as follows for the demand rates of 25000,50000 and 75000 (low, medium and high) respectively:

$$
\begin{align*}
& T C U=11165.16+5522.89 \theta-4956.76 \alpha-768.84 \beta-4330.52 \phi(\text { low })  \tag{4.1}\\
& T C U=21310.05+10861.57 \theta-9738.69 \alpha-1571.07 \beta-8589.90 \phi(\text { medium })  \tag{4.2}\\
& T C U=31303.22+16144.44 \theta-14614.73 \alpha-2319.33 \beta-12863.06 \phi(\text { high }) \tag{4.3}
\end{align*}
$$

Equations (4.1), (4.2) and (4.3) show an increase in the percentage of bottles collected $\theta$ will cause the total system unit time cost (TCU) to increase, and clearly a decrease in $\theta$ will cause the total system unit time cost to decrease. However, an increase in the percentage of noncontaminated bottles $\alpha$, the percentage of remanufactured bottles $\beta$ or the percentage of contaminated bottles sold to industries that acquire low grade material $\phi$ will cause the total system unit time cost to decrease, while a decrease in $\alpha, \beta$ or $\phi$ will cause the total system unit time cost to increase. Where, the amount by which $\theta, \alpha, \beta$ and $\phi$ increases or decreases will generate different outcomes of the total system unit time cost. Since the decision variables $\theta, \alpha$, and $\phi$ all have p-values less than 0.05 (regression analyses performed at $95 \%$ level of confidence) as shown in Tables 4.4, 4.5 and 4.6; an increase or decrease in these decision variables has a greater effect on the outcome of the total system unit time cost than $\beta$ which has a p -value greater than 0.05 . This is obviously reflected in the larger values of the coefficients for $\theta$, $\alpha$, and $\phi$ when compared with the coefficient for $\beta$ as seen in the above estimated regression functions. Also, in all of the above equations ((4.1), (4.2) and (4.3)) the decision variables that have the highest values for the coefficients are $\theta$ and $\alpha$ which means they have the most effect on determining the outcome of the total system unit time cost. However, since $\theta$ has a larger coefficient than $\alpha$, therefore it can be concluded that although $\theta, \alpha$, and $\phi$ all can significantly affect the total system unit time cost, an increase or decrease in the percentage of bottles collected $\theta$ has greatest effect on the outcome of the total system unit time cost.

### 4.4 Chapter Summary

In this chapter cost parameters were suggested and rationalized. Together with the decision variables these parameters were used to obtain the results in Tables 4.1, 4.2 and 4.3 for three different demand rates of 25000 , 50000 and 75000 which represent low, medium and high demand rates. Analysis was conducted using these tables to study the effect on the different process unit time costs and their effect on the total system unit time cost by varying one decision variable between $0.25,0.5$ and 0.75 and holding the others constant at a level of $0.25,0.5$ or 0.75. Regression Analysis was performed on the data of Tables 4.1, 4.2 and 4.3 and using the decision variables $\theta, \alpha, \beta$ and $\phi$ as in the independent variables and the total system unit time cost as the dependent variable (TCU). The outputs of the analyses are captured in Tables 4.4, 4.5 and 4.6. From these tables the estimated regression functions (Equations (4.1), (4.2) and (4.3)) were obtained corresponding to the low, medium and high demand rates used. Analysis of Equations (4.1), (4.2) and (4.3) shows the percentage of bottles remanufactured $\beta$ does not affect the total system unit time cost as significantly as the percentage of bottles collected $\theta$, the percentage of non-contaminated bottles $\alpha$ and the percentage of contaminated bottles sold to industries that use low grade material $\phi$. Also, the decision variable $\theta$ which has the highest coefficient has the most effect on the outcome of the total system unit time cost.

## Chapter 5: DISCUSSION, RECOMMENDATIONS AND CONCLUSION

### 5.1 Discussion

Upon further analysis of Tables 4.1, 4.2 and 4.3, the cost of landfill disposal $\left(T C U_{L F}\right)$ can have a significant effect on the total system unit time cost (TCU). As the amount of bottles placed in the landfill increases the cost of landfill disposal increases. By examining the process unit time cost equation of landfill disposal (3.22), an increase in the decay rate $\lambda$ causes a decrease in $T C U_{L F}$. The current material being used to produce 2L plastic beverage bottles is PET (polyethylene terephthalate) which takes approximately 450 years ${ }^{8}$ to fully biodegrade in a landfill. As a result this yields a very low value for the decay rate $\lambda$ which is $0.000256 /$ month. Therefore, in order to increase the decay rate and reduce the process unit time cost of landfill disposal ( $T C U_{L F}$ ), alternative materials must be used which take less time to biodegrade in a landfill. As discussed in the literature survey of this thesis, Samarasinghe et al. ${ }^{21}$ created and tested "biodegradable plastic composites from corn gluten meal (CGM) ${ }^{21}$. Also, Otaigbe et al. ${ }^{26}$ experimentally developed and tested a "biodegradable soy protein-starch plastic (SPSP)" ${ }^{26}$. Both of these plastic materials are ideal even though further testing would be required for applicable use in the production of 2 L beverage bottles because they can be injection moulded. These materials have high tensile strength, high elongation at break and high water resistance. Also, since these plastic materials are biodegradable, and mostly consist of natural substances, they can degrade in a landfill in an environmentally-benign manner within months (As stated by Samarasinghe et al. ${ }^{21}$ the use of CGM material to produce injection mouldable composites "degrade in soil on a timescale of months ${ }^{221}$ ). By taking this into account, the decay rate will dramatically increase, and the penalty can be eliminated from the rehabilitation penalty cost per bottle. Now, the value for $C_{r h}$ is $\$ 0.00125 /$ bottle/month which is only a rehabilitation cost per bottle. This will result in a decrease of $T C U_{L F}$, hence, decreasing the total system unit time cost (TCU) as seen in Table 5.1. This because the current decay rate of PET plastic material is significantly smaller when compared to the decay rate of biodegradable plastic materials made from CGM or SPSP.

Table 5.1 - Decay rate comparison of PET versus CGM or SPSP material

| $\theta$ | $\alpha$ | $\beta$ | $\phi$ | Demand Rate | Material | Time | $\lambda$ (decay rate) | $T_{C O}{ }_{L F}$ | $T C U$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.5 | 0.5 | 0.5 | 0.5 | 50000 | PET | 450 years | $0.000256 / \mathrm{month}$ | 4851.98 | 17233.72 |
| 0.5 | 0.5 | 0.5 | 0.5 | 50000 | CGM or SPSP | 5 months | $0.277 /$ month | 423.39 | 12805.13 |
| 0.5 | 0.5 | 0.5 | 0.5 | 50000 | CGM or SPSP | 4 months | $0.347 /$ month | 396.41 | 12778.16 |
| 0.5 | 0.5 | 0.5 | 0.5 | 50000 | CGM or SPSP | 3 months | $0.462 /$ month | 369.53 | 12751.27 |
| 0.5 | 0.5 | 0.5 | 0.5 | 50000 | CGM or SPSP | 2 months | $0.693 /$ month | 342.17 | 12723.92 |

The use of biodegradable plastic materials made from CGM and SPS or similar biodegradable injection-mouldable materials in the production of 2 L plastic beverage bottles has immense benefits because they can fully degrade in a landfill within months and they are environmentally safe. In turn, this provides tremendous cost savings because it lowers the landfill disposal cost $\left(T C U_{L F}\right)$ which ultimately lowers the total system unit time cost (TCU), as shown in Table 5.1. Therefore, the uses of these types of plastic materials are advantageous, as opposed to PET plastic material which takes an enormous amount of time to decay in a landfill and upon degradation admits dangerous chemicals such as BPA (bisphenol A) into the environmental water table.

Now, within the analyses of sections 4.1 and 4.2 the decay rate used in the process unit time cost for landfill disposal ( $T C U_{L F}$ ) of badly contaminated bottles is based on the decay of PET material. Since a minimal purchase price of 2 L plastic PET bottles is " $\$ 0.65$ per bottle" ${ }^{33}$ if the remanufactured $\left(Q_{r}\right)$ and newly produced $\left(Q_{p}\right)$ bottles are sold at the same price within a cycle, then the equations for total income and net profit are as follows:

$$
\begin{equation*}
\text { Total Income }=0.65\left(\frac{Q_{p}}{T}\right)+0.65\left(\frac{Q_{r}}{T}\right) \tag{5.1}
\end{equation*}
$$

And

$$
\begin{equation*}
\text { Net Profit }=\text { Total Income }-T C U \tag{5.2}
\end{equation*}
$$

By using the above equations in combination with the mathematical model of this thesis, it will be seen that recycling-reuse can be profitable as displayed in Table 5.2 shown below.

Table 5.2 - Net- Profit obtained for the low, medium and high demand

| Runs | $\theta$ | $\alpha$ | $\beta$ | $\phi$ | Demand Rates |  |  | Demand Rates |  |  | Demand Rates |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | 25000 | 50000 | 75000 | 25000 | 50000 | 75000 | 25000 | 50000 | 75000 |
|  |  |  |  |  | TCU | $T C U$ | TCU | T. Inc | T. Inc | T. Inc | Net- <br> Profit | NetProfit | Net- <br> Profit |
| 1 | 0.25 | 0.25 | 0.25 | 0.25 | 9410.08 | 17821.18 | 26072.04 | 16250 | 32500 | 48750 | 6839.92 | 14678.82 | 22677.96 |
| 2 | 0.25 | 0.25 | 0.25 | 0.5 | 8524.38 | 16056.87 | 23430.27 | 16250 | 32500 | 48750 | 7725.62 | 16443.13 | 25319.73 |
| 3 | 0.25 | 0.25 | 0.25 | 0.75 | 7638.68 | 14292.55 | 20788.50 | 16250 | 32500 | 48750 | 8611.32 | 18207.45 | 27961.50 |
| 4 | 0.25 | 0.25 | 0.25 | 0.25 | 9410.08 | 17821.18 | 26072.04 | 16250 | 32500 | 48750 | 6839.92 | 14678.82 | 22677.96 |
| 5 | 0.25 | 0.25 | 0.5 | 0.25 | 9396.13 | 17796.87 | 26037.97 | 16250 | 32500 | 48750 | 6853.87 | 14703.13 | 22712.03 |
| 6 | 0.25 | 0.25 | 0.75 | 0.25 | 9477.99 | 17772.48 | 26003.80 | 16250 | 32500 | 48750 | 6772.01 | 14727.52 | 22746.20 |
| 7 | 0.25 | 0.25 | 0.25 | 0.25 | 9410.08 | 17821.18 | 26072.04 | 16250 | 32500 | 48750 | 6839.92 | 14678.82 | 22677.96 |
| 8 | 0.25 | 0.5 | 0.25 | 0.25 | 8462.70 | 15937.60 | 23254.10 | 16250 | 32500 | 48750 | 7787.30 | 16562.40 | 25495.90 |
| 9 | 0.25 | 0.75 | 0.25 | 0.25 | 7514.65 | 14053.07 | 20434.99 | 16250 | 32500 | 48750 | 8735.35 | 18446.93 | 28315.01 |
| 10 | 0.25 | 0.25 | 0.25 | 0.25 | 9410.08 | 17821.18 | 26072.04 | 16250 | 32500 | 48750 | 6839.92 | 14678.82 | 22677.96 |
| 11 | 0.5 | 0.25 | 0.25 | 0.25 | 12259.55 | 23445.69 | 34459.34 | 16250 | 32500 | 48750 | 3990.45 | 9054.31 | 14290.66 |
| 12 | 0.75 | 0.25 | 0.25 | 0.25 | 15111.03 | 29073.03 | 42850.09 | 16250 | 32500 | 48750 | 1138.97 | 3426.97 | 5899.91 |
| 13 | 0.5 | 0.5 | 0.5 | 0.25 | 10313.20 | 19587.26 | 28694.46 | 16250 | 32500 | 48750 | 5936.80 | 12912.74 | 20055.54 |
| 14 | 0.5 | 0.5 | 0.5 | 0.5 | 9131.49 | 17233.72 | 25170.74 | 16250 | 32500 | 48750 | 7118.51 | 15266.28 | 23579.26 |
| 15 | 0.5 | 0.5 | 0.5 | 0.75 | 7949.77 | 14880.19 | 21647.02 | 16250 | 32500 | 48750 | 8300.23 | 17619.81 | 27102.98 |
| 16 | 0.5 | 0.5 | 0.25 | 0.5 | 9184.24 | 17326.64 | 25301.74 | 16250 | 32500 | 48750 | 7065.76 | 15173.36 | 23448.26 |
| 17 | 0.5 | 0.5 | 0.5 | 0.5 | 9131.49 | 17233.72 | 25170.74 | 16250 | 32500 | 48750 | 7118.51 | 15266.28 | 23579.26 |
| 18 | 0.5 | 0.5 | 0.75 | 0.5 | 9078.17 | 17140.02 | 25038.78 | 16250 | 32500 | 48750 | 7171.83 | 15359.98 | 23711.22 |
| 19 | 0.5 | 0.25 | 0.5 | 0.5 | 10461.46 | 19870.15 | 29109.74 | 16250 | 32500 | 48750 | 5788.54 | 12629.85 | 19640.26 |
| 20 | 0.5 | 0.5 | 0.5 | 0.5 | 9131.49 | 17233.72 | 25170.74 | 16250 | 32500 | 48750 | 7118.51 | 15266.28 | 23579.26 |
| 21 | 0.5 | 0.75 | 0.5 | 0.5 | 7799.90 | 14595.02 | 21228.94 | 16250 | 32500 | 48750 | 8450.10 | 17904.98 | 27521.06 |
| 22 | 0.25 | 0.5 | 0.5 | 0.5 | 7844.24 | 14712.64 | 21424.61 | 16250 | 32500 | 48750 | 8405.76 | 17787.36 | 27325.39 |
| 23 | 0.5 | 0.5 | 0.5 | 0.5 | 9131.49 | 17233.72 | 25170.74 | 16250 | 32500 | 48750 | 7118.51 | 15266.28 | 23579.26 |
| 24 | 0.75 | 0.5 | 0.5 | 0.5 | 10425.72 | 19764.69 | 28928.97 | 16250 | 32500 | 48750 | 5824.28 | 12735.31 | 19821.03 |
| 25 | 0.75 | 0.75 | 0.75 | 0.25 | 9210.66 | 17383.56 | 25389.83 | 16250 | 32500 | 48750 | 7039.34 | 15116.44 | 23360.17 |
| 26 | 0.75 | 0.75 | 0.75 | 0.5 | 8322.34 | 15615.53 | 22743.52 | 16250 | 32500 | 48750 | 7927.66 | 16884.47 | 26006.48 |
| 27 | 0.75 | 0.75 | 0.75 | 0.75 | 7434.02 | 13847.51 | 20097.21 | 16250 | 32500 | 48750 | 8815.98 | 18652.49 | 28652.79 |
| 28 | 0.75 | 0.75 | 0.25 | 0.75 | 7659.63 | 14248.95 | 20666.28 | 16250 | 32500 | 48750 | 8590.37 | 18251.05 | 28083.72 |
| 29 | 0.75 | 0.75 | 0.5 | 0.75 | 7547.08 | 14048.60 | 20382.20 | 16250 | 32500 | 48750 | 8702.92 | 18451.40 | 28367.80 |
| 30 | 0.75 | 0.75 | 0.75 | 0.75 | 7434.02 | 13847.51 | 20097.21 | 16250 | 32500 | 48750 | 8815.98 | 18652.49 | 28652.79 |
| 31 | 0.75 | 0.25 | 0.75 | 0.75 | 9719.13 | 18349.78 | 26805.46 | 16250 | 32500 | 48750 | 6530.87 | 14150.22 | 21944.54 |
| 32 | 0.75 | 0.5 | 0.75 | 0.75 | 8576.24 | 16098.17 | 23450.75 | 16250 | 32500 | 48750 | 7673.76 | 16401.83 | 25299.25 |
| 33 | 0.75 | 0.75 | 0.75 | 0.75 | 7434.02 | 13847.51 | 20097.21 | 16250 | 32500 | 48750 | 8815.98 | 18652.49 | 28652.79 |
| 34 | 0.25 | 0.75 | 0.75 | 0.75 | 6839.84 | 12730.11 | 18468.28 | 16250 | 32500 | 48750 | 9410.16 | 19769.89 | 30281.72 |
| 35 | 0.5 | 0.75 | 0.75 | 0.75 | 7128.51 | 13276.91 | 19268.17 | 16250 | 32500 | 48750 | 9121.49 | 19223.09 | 29481.83 |
| 36 | 0.75 | 0.75 | 0.75 | 0.75 | 7434.02 | 13847.51 | 20097.21 | 16250 | 32500 | 48750 | 8815.98 | 18652.49 | 28652.79 |

Therefore, based on the results provided in Table 5.2 it is shown that as the demand rate increases the net-profit increases. Also, on the average, profit of $\$ 7351.46$ per month, $\$ 15731.50$ per month and $\$ 24273.12$ per month can be obtained for the low demand rate of 25000 bottles per month, medium demand rate of 50000 bottles per month and high demand rate of 75000 bottles per month, respectively.

### 5.2 Recommendations

The results of the process unit time cost for landfill disposal $\left(T C U_{L F}\right)$ within the analysis of this thesis is based on PET material which is currently being used in industry for the production of 2 L plastic bottles. In section 5.1, it was shown that the use of biodegradable plastic materials made from natural contents such as corn gluten meal (CGM) or soy protein-starch (SPS) are good candidates as alternative materials to be tested and possibly used in plastic bottle production as opposed to PET material because they degrade in a landfill in a significantly much shorter period of time and are made from environmentally safe substances that degrade in a benign manner. This in turn provides cost savings because the process unit time cost for landfill disposal $\left(T C U_{L F}\right)$ decreases due to an increase in the decay rate $(\lambda)$ and the elimination of penalty from the rehabilitation penalty cost per bottle $\left(C_{r h}\right)$. This decreases the total system unit time cost (TCU). Therefore, it is necessary that additional and extensive research be conducted on different materials that are cost effective and environmentally friendly as this will not only provide cost savings but is ethical in terms of protecting the environment. However, replacing bottles made from PET material by alternative biodegradable materials could be so expensive as to offset the cost savings from landfill disposal found earlier in this thesis. Therefore, this research should focus not only on the production of these bottles on a large scale but also their production at a low cost. Now, based on the fact that PET is the material being used to produce 2L plastic beverage bottles and the minimal purchase price for a 2 L PET plastic beverage bottle is " $\$ 0.65$ per bottle""33 , then as displayed in the results of Table 5.2 plastic bottle recycling-reuse can be profitable. However, the model of this thesis may be used for a further study attempting to reduce this price. This can be a topic for future research. Further research could also be carried out in processes of bottle sorting with the aim of reducing cost. Sorting carried out in Process A and Process E could be combined to produce four streams: whole non-contaminated bottles, damaged non-contaminated bottles, contaminated bottles to be sold to industries that use low
grade plastic PET material and badly contaminated bottles to be disposed of in a landfill. This research should focus on cost savings from the combination of these two processes.

Finally, the learning curve effect could be applied to the model of this thesis in order to reduce the different processing costs over time.

### 5.3 Conclusion

In this thesis, a recycling-reuse model that remanufactures non-contaminated PET plastic bottles and uses regrind from damaged non-contaminated PET bottles mixed with virgin PET material in the production of new bottles was developed and analyzed in order to reduce the amount of plastic PET bottles that are disposed of in landfill. The model is assumed to have no shortages and the different percentages regarding the classes of bottles are taken to be deterministic. In this model, a present worth cost is charged for landfill disposal. This cost included the use of realestate, cost of land rehabilitation and penalty for contaminating the water and harming wildlife and the environment. In the analyses conducted on this recycling model, it was found that the percentage of bottles collected from the market $\theta$, had the largest influence on the outcome of the total system unit time cost $(T C U)$. This is because an increase in $\theta$ caused the process unit time costs of bottle collection and sorting $\left(T C U_{n s s}\right)$, remanufacturing bottles $\left(T C U_{r}\right)$, contaminated bottle sorting $\left(T C U_{S c}\right)$ and landfill disposal of bottles $\left(T C U_{L F}\right)$ to increase. This supersedes the decreasing process unit time costs of regrind and virgin material mixing ( $T C U_{r m}$ ) and the production of new bottles $\left(T C U_{p}\right)$. The use of alternative biodegradable plastics that degrade in a landfill in a significantly much shorter period of time and are made from environmentally safe substances that degrade in a benign manner was also examined. It was found that these plastic materials are ideal to be tested and possibly used in the production of 2L plastic bottles because they decrease the landfill disposal cost $\left(T C U_{L F}\right)$, which in turn causes the total system unit time cost (TCU) to decrease. Finally, if remanufactured $\left(Q_{r}\right)$ and newly produced $\left(Q_{p}\right)$ PET plastic bottles are sold at a minimal price of " $\$ 0.65$ per bottle" ${ }^{33}$, recycling-reuse can be profitable because, on the average, a profit of $\$ 7351.46$ per month, $\$ 15731.50$ per month and $\$ 24273.12$ per month can be obtained for a low, medium and high demand rate, respectively.

It is now appropriate to make a few remarks regarding the uniqueness of this model and its limitations.

In order to reduce the disposal of bottles in landfill this model uses reuse and recycled processes, but unlike other models it includes the selling of contaminated plastic PET bottles to industries that use low grade plastic materials. Also, the model charges a present worth cost for land use, land rehabilitation and penalty. This cost was arrived at by considering exponential decay and continuous compounding of interest. This cost is also unique to this model.

This model is limited to the recycling- reuse of PET plastic bottles, it does not apply to metal bottles, metal cans, glass bottles or bottles made of other plastic materials. In the case of glass or bottles made of other plastic, the model has to be modified. For example, other plastic will need a different machine than the Recycling line - recoSTAR PET machine because this machine is used to process only PET material. Furthermore, the production of new bottles is limited to the characteristics of the Recycling line - recoSTAR PET machine. This is because this machine processes the crushed flake of the PET material in batches and does not provide for continuous processing.

Profits calculated earlier within in this thesis were based on the sale price of " $\$ 0.65$ per bottle" ${ }^{33}$ if however bottles are sold at a much cheaper price than this, these profits will not only erode but the model may result in losses.

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