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Optimal inventory policy for the two-level supply chain with defective items

By

S.M. Hafiz al Mamun

A project presented to Ryerson University
In partial fulfillment of the requirements for
The degree of Master of Engineering
Mechanical Engineering
Ryerson University

Toronto, Ontario, Canada, 2009

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ABSTRACT

S.M. Hafiz al Mamun

Master of Engineering
Mechanical Engineering
Ryerson University

2009

This project focuses on two-level closed-loop supply chains with defective items. The objective of this project is to develop and design a model that minimizes the total expected cost per unit time, which includes set-up costs, holding costs, transportation/shipping costs, and screening costs of the integrated two-level close-loop supply chain. The model also finds the optimum order size and optimum number of shipments. The buyer screens the products received from the vendor to find the defective items. The holding cost of the defective items at the buyer's end is paid by the vender. After the screening process, the defective items are shipped back to the vendor and the vendor has to carry the shipping cost of the defective items. Two scenarios may arise: where both the vendor and buyer are domestic or international, where vendor and buyer are located in two different countries. In the case of an international supply chain, exchange rate between two countries has also been considered. In current world since the business growing fast, the inventory management of any business enterprise improving their performance financially by minimizing the holding cost. The analysis shows how the percentage of defective item affects the total expected cost. The project work has an important involvement for improvement in the vendor-buyer correlated high-tech supply chain industries.

Acknowledgements

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Nomenclature

h_v	Unit stock holding cost per item per year for the vendor
h_B	Unit stock holding cost per item per year for buyer
$\mathbf{E}[\mathrm{TC}(n,Q)]$	Expected annual total integrated cost of the supply chain
D	Demand
Q	Size of each shipment from the vendor to the buyer
P	Production rate
S_v	Setup cost for vendor per production run
S_B	Setup cost for buyer per order
n	The number of shipments
T	Time interval between delivery
t	Screening time
F	Fixed transportation cost per shipment
C	Variable transportation cost
Y	Percentage of defective items (random variable)
$f(y)$	Probability density function of Y
x	Screening rate (Q/t)
d	Unit screening cost
e	Exchange rate

Chapter 1

INTRODUCTION

The inventory management control model was introduced in the earliest decades and this led many researchers to work on economic order quantity (EOQ) for real life situations. So a number of literatures on inventory model have been published in different directions. In particular, a joint economic-lot-size model was first proposed by Banerjee (1986). That was special case where a vendor produces to order for a purchaser on a lot-for-lot basis. Goyal (1988) developed a more general joint economic lot size model that provided the minimum total relevant costs. Then again Goyal (1995) added shipment size with the previous model. After that, Viswanathan (1998) shows that there exists no strategy as provided in Goyal (1995) and Banerjee (1986) to obtain the best solution for all possible problem parameters. In the same line of research Hill has great numbers of research articles, for example, Hill (1999) introduced the idea of integrated production-inventory model and derived for the integrated production–inventory and shipping policy for a globally optimal batching problem. Hoque and Goyal (2000) illustrated a mathematical model and procedure for the single-vendor single-buyer production–inventory system. The system described the unequal and equal sized shipments from the vendor to the buyer under the transportation capacity constraint.

There is another stream of research on EOQ in the literature. For example, Salameh and Jaber (2000) developed an inventory model for imperfect quality items and they derived the optimal EOQ formula. In this model, the imperfect quality items were sold at discounted price at the end of the screening time. The paper shows that the economic lot size follows a concave function with respect to the average percentage of imperfect quality items. Cardenas-Barron (2000) has corrected the error in Salameh and Jaber (2000) and provided numerical examples. Then, Goyal and Cardenas-Barron (2001) developed a model to determine the economic production quantity for an item with imperfect quality. Papachristos and Konstantaras (2004) perfected the model in Salameh and Jaber (2000) by deriving: (1) the sufficient condition to ensure no shortages; and (2) optimal quantity when the imperfect items are sold at discounted price at the end of the cycles. Maddah and Jaber (2004) revisited the model presented in Salameh and Jaber (2000) and developed a model based on renewal process for the optimal economic quantity for items with imperfect quality and they analyzed the effect of screening speed and variability of the supply process on the order quantity.

The integrated closed-loop supply chain problem with a single vendor and single buyer, when the vendor is supplying a product with defective items is an interesting decision problem. The buyer has to decide how much to order in each purchase order and the vendor to decide the economic production quantity to meet the demand from the buyer, and also decides the economic number of shipment from the vendor and to the buyer. However, for a two-level opened-loop supply chain, Huang (2004) developed a model to determine the optimal integrated vendor–buyer inventory policy of imperfect items in a just-in-time manufacturing environment. Chung (2007) showed the different necessary and sufficient conditions for the Huang (2004).

In this project, a two-level closed-loop supply chain is studied while addressing several flaws from the existing models. The earlier research works assumed identical inventory costs for both good and defective items, and this project considers different inventory costs for both good and defective items. As the defective items are not usually stored in the same warehouse, where the good items are stored, the good items and defective items should have different holding costs. Wahab and Jaber (2009) developed the model for optimal lot sizes for imperfect quality item with different holding costs for the good and defective items. They also introduced the learning effects.

When each lot has defective items and percentage of defective is a random, the cycle time should be a random variable. However, in Huang (2004), the cycle time was considered as it is not a random variable, and the expected average cost per cycle is determined as follows: first the total cost per cycle is divided by the cycle time, where both are random variables, and then expectation with respected to the percentage is taken. In this project, this flaw has been perfected by considering a renewal process where the expected average cost is determined as expected cycle cost is divided by the expected cycle time. This project also assumes a closed loop supply chain. Earlier researchers did not explain about the consequence of defective items after the screening, whether they are dumped or sold at a discounted price. This project considers that the chain is closed with the return of defective items to vendor (Figure 2.1). The cost of shipping the defective items is paid by the vendor. This project considers both domestic (the vendor and buyer are at same country) and international (the vendor and buyer are at different country) supply chains. In case of international supply chain, exchange rates play a vital role. The project also analyzes its effect.

In this project, a two-level closed-loop supply chain is considered, that a single vendor and a single buyer. The buyer places his/her order quantity to the vendor. There is a fixed order cost associated with that order. The vendor manufactures the product in batches. The vendor has a setup cost for each production run. The vendor has a holding cost during this time frame. The vendor ships the product to the buyer. There is a fixed transportation cost associated with shipment from the vendor to the buyer. The buyer is paying the transportation cost. The buyer screens the products received from the vendor to find the defective items with his/her own cost. The buyer has holding costs only for the good items. The holding cost of imperfect items in the buyer's inventory is paid by the vendor. The defective items are sent back to the vendor and the vendor has to carry this shipping cost.

The objective is to design a model that minimizes the total expected cost per unit time (which includes set-up costs, holding costs, transportation/shipping costs and screening costs) and finds the optimum order size and number of shipments. The motivation behind this project is to provide the analysis to show the effect of number of defective items on the total cost of the two-level closed-loop supply chain. The analysis confirms the effect of defective items in a two-level closed-loop supply chain for a single vendor and single buyer increase the expected total cost and order size.

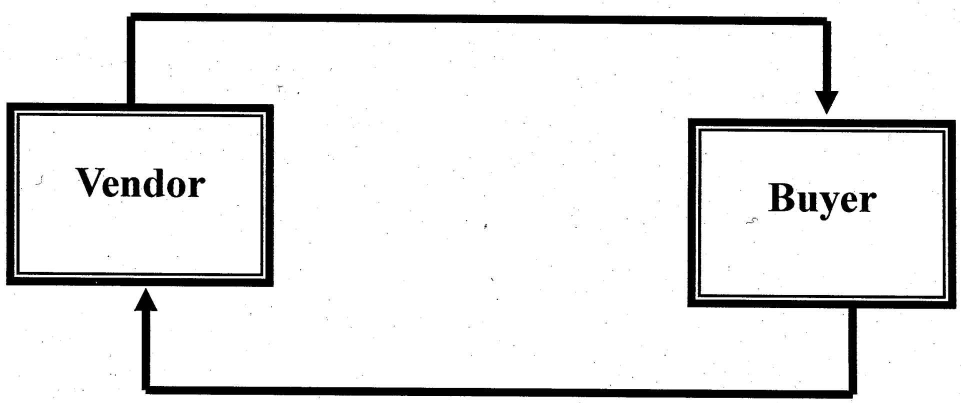
Chapter 2 illustrates the model for the two-level supply chains where the vendor and the buyer are domestic. The mathematical model and numerical example with analysis are illustrated for that domestic supply chain problem. Then chapter 3 shows the problem analysis for two-level supply chain of an international situation where the exchange rate between the two countries involves. Domestic and international both chapters show the common analysis to minimize the total expected cost per unit time with respect to the percentage of defective items.

Chapter 2

A TWO-LEVEL SUPPLY CHAIN: DOMESTIC VENDOR AND BUYER

In inventory management of the two-level closed-loop supply chain, the vendor and buyer are always affected by the defective items. However, the (classical) economic order quantity (EOQ) model assumes all the items are produced without defective items. But in real world the product quality is affected by the reliability of the production process. This effect of inventory control has been recognized by many researchers. The supply chain spends the extra money for the distribution of each defective item. The buyer and the vendor set up the long term agreement and they work together in the integrated supply chain policy to maximize their profits. The analysis derives a mathematical model with the optimal order quantity and the optimal number of shipment and finally develops an integrated total cost function for both buyer and vendor. Figure 2.1 depicts a two-level closed-loop supply chain with a single vendor and a single buyer.

Shipping the Q size with defective items



Shipping back the defective items $QE[Y]$

Figure 2.1: The two-level closed-loop supply chain

2.1 Assumptions

- Single product, single vendor and single buyer.

- Production rate is uniform and finite.
- Demand is constant and known.
- Zero lead time is applied,
- Buyer is paying the holding cost for good items.
- Buyer is paying the screening cost.
- Vendor is paying the holding cost for defective items.
- There is no shortage.

2.2 Problem Statement

The model has a single buyer and a single vendor with a known demand rate for designing a two-level closed-loop domestic supply chain, where the buyer and the vendor are located at the same county. The buyer places order, Q, to the vendor. The setup cost for buyer, S_B, is fixed. The vendor is producing one type of item with a production rate, P, which is greater than demand, D, and setup cost, S_v, for the vendor per production run. Unit stock holding cost for the vendor per year, h_v, is fixed. The vendor is shipping the finished goods with order size, Q, to the buyer. The vendor ships the lot to the buyer who is paying the fixed transportation cost F per shipment. The order size Q is assumed coming to the buyer with the percentage defective items Y, which is a random variable. The buyer screens the shipment with unit screening cost, d, to sort the good and defective items. The vendor is paying the unit stock holding cost for the defective items at the buyer's warehouse. Once the screening is completed, the buyer is sending the defective items to the vendor. The vendor is also paying the transportation cost that consists of a fixed cost, F, and a variable cost, C, for the shipment of defective items.

2.3 Mathematical model

In this section, the mathematical model for the expected total cost functions of the buyer and the vendors are formulated. The expected total cost of the integrated supply chain is determined as the sum of those costs. The objective is to determine the optimal shipment size and the optimal number of shipment that minimizes the total expected cost for the buyer and the vendor of the integrated supply chain.

2.3.1 Vendor's total expected cost per unit time

There are three major costs considered for the vendor: setup cost, holding cost, and transportation cost for the optimal policy of integrated production-inventory model. The holding cost of the vendor comes from two parts: holding cost for the average accumulation of the vendor's inventory during the production run; and the holding cost for defective items that are screened at the buyer's site.

(a) Holding cost per unit time

The vendor maintains an inventory during the production run. The accumulation of the vendor's inventory during the production run is the area GEC and the area AGX in the Figure 2.2, which shows the total inventory profile of the vendor. The vendor has to pay that holding cost.

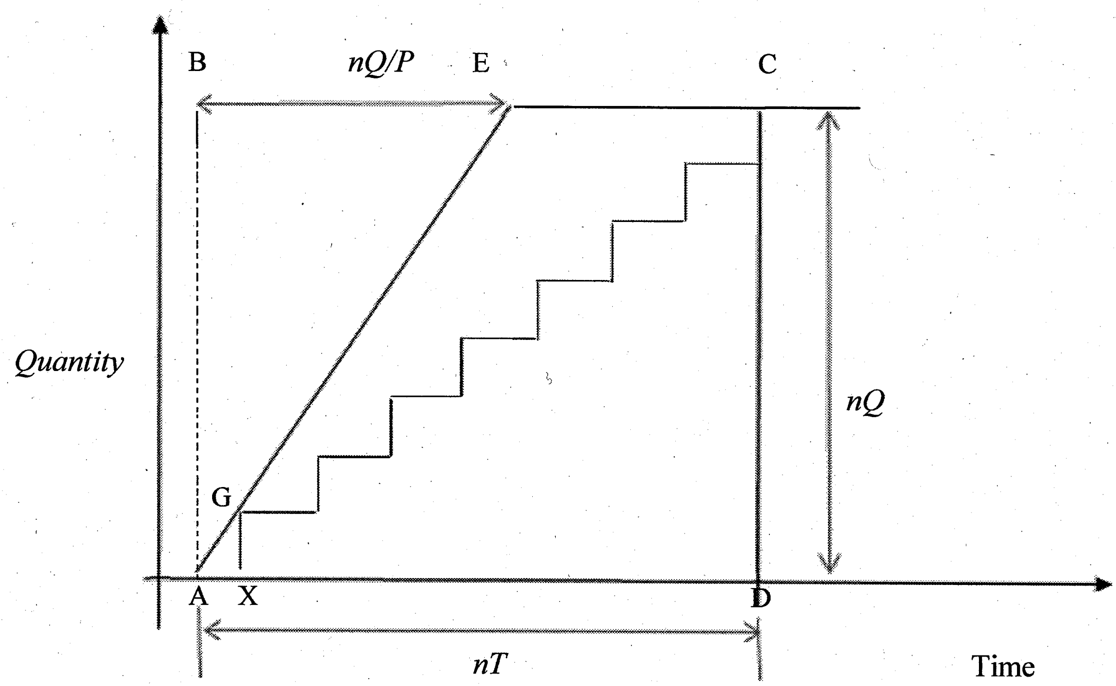


Figure 2.2: The total inventory profile of the vendor

The line AE represents the constant production rate, P. It is assumed that production exactly meets the demand (which is known and fixed) in the first cycle and therefore the line AE touches the ladder at point G. After the first cycle, production rate is more than the demand that causes to build up inventory in (n-1) cycles. To represent the term P in the area ABCD, the area is divided into two strips: one with $(nQ)(Q/P)$ in the first cycle and the other with $(n\text{-}1)TnQ$ in the rest (n-1) cycles. Area ABCD $= nQ\frac{Q}{P} + (n-1)TnQ = nQ\left[\frac{Q}{P} + (n-1)T\right]$.

$$\text{Area ABE} = \frac{1}{2}\,nQ\frac{nQ}{P} = \frac{n^2Q^2}{2P}.$$

$$\text{Area under ladder} = [Q + 2Q + 3Q + \ldots + (n\text{-}1)Q]\,T.$$

The total inventory is represented by the area = AECD - Area under ladder.

= Δ ABCD - Δ ABE - Area under ladder.

Total holding cost:

$$= h_v\left\{\left[nQ\left(\frac{Q}{P} + (n-1)T\right) - \frac{n^2Q^2}{2P}\right] - T[Q + 2Q + \cdots + (n-1)Q]\right\}$$

$$= h_v\left\{\left[nQ\left(\frac{Q}{P} + (n-1)\frac{(1-Y)Q}{D}\right) - \frac{n^2Q^2}{2P}\right] - (1-Y)\frac{Q(n-1)nQ}{2D}\right\}$$

$$= Q^2nh_v\left[\frac{1}{P}\left(1-\frac{n}{2}\right) + \frac{1}{2D}(n-1)(1-Y)\right]. \quad (2.1)$$

Since the percentage of the defective is a random variable, the expected value is taken.

$$\text{Total expected holding cost} = Q^2nh_v\left[\frac{1}{P}\left(1-\frac{n}{2}\right) + \frac{1}{2D}(n-1)(1-\mathbf{E}[Y])\right]. \quad (2.2)$$

Jaber and Salameh (2000) first introduce the defective items consideration into an economic order quantity model. But their model considers the identical inventory cost for both good and defective items. But in reality, inventory cost for good items should be different from the inventory costs of defective items. The defective items are not usually stored in the same warehouse as the good items. The good items and defective items should have different holding costs according to Wahab and Jaber (2009). The vendor delivers the order quantity to buyer. At the buyer's end, during screening, the buyer maintains two different kinds of inventory. One is for good items and the other is for the defective items. In this model vendor has to pay the holding cost for the defective items which is maintained by the buyer. Figure 2.3 shows the inventory profile for buyer for a single cycle.

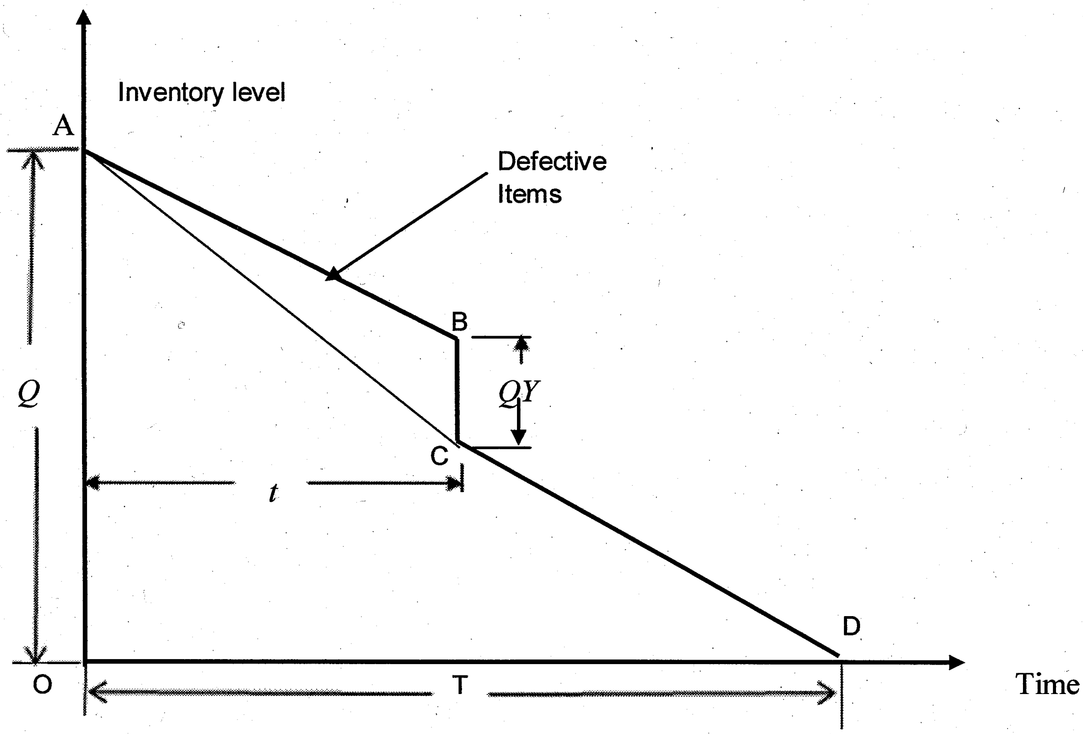


Figure 2.3: Inventory profile of the buyer per cycle

In Figure 2.3, the total inventory for defective items is shown in the triangle ABC.

The total holding cost for the defective items, $h_v = \left\{\frac{1}{2} \times QY \times t\right\} h_B = \left\{\frac{1}{2} \times QY \times \frac{Q}{x}\right\} h_B = \frac{YQ^2}{2x} h_B$,
where, $t = \frac{Q}{x}$.

For n cycle, the holding cost for defective items $= \frac{YQ^2}{2x} h_B n.$ (2.3)

Taking the expectation with respect to random variable Y, the expressions for the expected holding cost for the defective items $= nh_B \left\{\frac{\mathrm{E}[Y]Q^2}{2x}\right\}$. (2.4)

Summing the equations (2.2) and (2.4), the total expected holding cost for vendor is

$$= Q^2 nh_v \left[\frac{1}{P}\left(1 - \frac{n}{2}\right) + \frac{1}{2D}(n-1)(1 - \mathbf{E}[Y])\right] + nh_B \left\{\frac{\mathrm{E}[Y]Q^2}{2x}\right\}. \quad (2.5)$$

(b) Setup cost per unit time

The vendor has setup cost per production run= S_v. (2.6)

(c) Transportation cost

After screening, the defective items are returned to the vendor and the shipping cost is paid by the vendor. Since the quantities of defective items are random, a fixed and variable cost is considered. The consequence of defective items after the screening never explained, whether they are dumped or sold at cheaper price. Usually buyer returns the defective items, and there is a cost for shipping the defective items back to the vendor. This shipping cost of defective items is paid by the vendor.

The transportation cost per cycle $= F + CYQ$. (2.7)

Since the vendor receives n deliveries of defective items, and taking the expectation with respect to the random variable Y the transportation cost is $= n(F + C\mathbf{E}[Y]Q)$. (2.8)

The vendor's total expected cost is the sum of expected holding cost, setup cost, and transportation cost and given by,

$$\mathbf{E}[TC_V(n,Q)] = Q^2 n h_v \left[\frac{1}{P}\left(1-\frac{n}{2}\right) + \frac{1}{2D}(n-1)(1-\mathbf{E}[Y])\right] \tag{2.9}$$

$$+\left\{\frac{\mathbf{E}[Y]Q^2}{2x}\right\} h_B n + n(F + C\mathbf{E}[Y]Q) + S_v \,.$$

And the total expected cost per unit time of the vendor is given by:

$$\mathbf{E}[UTC_V(n,Q)] = \frac{\mathbf{E}TC_V(n,Q)}{n\mathbf{E}[T]}\,. \tag{2.10}$$

Since, the cycle time of the buyer is the function of defective item (which is a random variable); the cycle time is also a random variable. So the expected value of cycle time of the vendor needs to be calculated.

The expected cycle time of the vendor $= \mathbf{E}[T] = \frac{Q(1-\mathbf{E}[Y])}{D}$. (2.11)

Substituting (2.9) and (2.11), the total expected cost per unit time can be written as,

$$\mathbf{E}[UTC_V(n,Q)] = DQ\left[\frac{h_v}{P(1-\mathbf{E}[Y])}\left(1-\frac{n}{2}\right) + \frac{h_v}{2D}(n-1) + \frac{h_B\mathbf{E}[Y]}{2x(1-\mathbf{E}[Y])}\right] \tag{2.12}$$

$$+\frac{D}{Q(1-\mathbf{E}[Y])}\left[F + \frac{S_v}{n}\right] + \frac{C_v D\mathbf{E}[Y]}{(1-\mathbf{E}[Y])}\,.$$

2.3.2 Buyer's expected cost per unit time

The buyer has four types of costs: setup, holding, transportation, and screening costs. The Buyer is paying the transportation cost for the shipment from the vendor. The buyer also screens the received shipment and spends a fixed screening cost per item.

(a) Holding cost

The buyer is screening and sorting the items into good items and defective items. The holding cost of the defective items is charged from the vendor. The buyer only bears the holding cost of the good items.

From Figure 2.2, holding cost of the good items, = $h_B\left(\frac{YQ^2}{2x}+\frac{Q(1-Y)T}{2}\right)$. (2.13)

Substituting T= (1-Y)(Q/D) in Equation (2.13), holding cost of the good items can be written as,

$$= h_B\left(\frac{YQ^2}{2x}+\frac{Q^2(1-Y)^2}{2D}\right). \tag{2.14}$$

(b) Setup cost per unit time

The buyer's setup cost is considered as a fixed of Setup cost S_B per vendor's cycle. The setup cost per buyer's cycle = $\frac{S_B}{n}$. (2.15)

(c) Transportation cost

Whenever the order is shipped to the buyer, there is a transportation cost for the shipment and it is fixed. This shipment cost includes the good and bad items from vendor to buyer.
Transportation cost = F. (2.16)

(d) Screening cost

The screening is to sort the good and defective items. The buyer is paying screening cost d per unit item. The screening cost = dQ. (2.17)

The buyer's total expected cost is the sum of the holding cost, setup cost, screening cost, and transportation cost and that is given by:

$$\mathbf{E}[TC_B(n,Q)] = h_B\left(\frac{YQ^2}{2x}+\frac{Q^2(1-Y)^2}{2D}\right)+\frac{S_B}{n}+F+dQ. \tag{2.18}$$

The buyer's total expected cost per unit time = $\mathbf{E}[UTC_B(n,Q)] = \frac{\mathrm{E}[TC_B(n,Q)]}{\mathrm{E}[T]}$

$$\mathbf{E}[UTC_B(n,Q)] = \frac{D}{(1-\mathrm{E}[Y])}[\frac{Qh_B}{2}\left(\frac{\mathrm{E}[Y]}{x}+\frac{\mathrm{E}[(1-Y)^2]}{D}\right)+\frac{1}{Q}\left(\frac{S_B}{n}+F\right)+d]. \tag{2.19}$$

Total cost of the two-level closed-loop supply chain

Now the expected total cost function of the integrated two-level closed-loop supply chain is the sum of the vendor's and the buyer's total cost equation given in Equations (2.11) and (2.17), respectively. The total integrated cost is given below,

$$\mathbf{E}[UTC(n,Q)] = \mathbf{E}[UTC_V(n,Q)] + \mathbf{E}[UTC_B(n,Q)]$$

$$\mathbf{E}[UTC(n,Q)] = QD\left[\frac{1}{(1-\mathbf{E}[Y])}\left\{\frac{h_B\mathbf{E}[Y]}{x}+\frac{h_B\mathbf{E}[(1-Y)^2]}{2D}+\frac{h_V}{P}\left(1-\frac{n}{2}\right)\right\}+\frac{h_V}{2D}(n-1)\right]+$$

$$\frac{D}{Q(1-\mathrm{E}[Y])}\left[\frac{1}{n}(S_B+S_V)+2F\right]+\frac{D(d+C_V\mathrm{E}[Y])}{(1-\mathrm{E}[Y])}. \tag{2.20}$$

By simplifying the equation (2.18) it becomes,

$$\mathbf{E}[UTC(n,Q)] = QD\left[M\left\{\frac{h_BN}{x}+\frac{h_BW}{2D}+\frac{h_V}{P}\left(1-\frac{n}{2}\right)\right\}+\frac{h_V}{2D}(n-1)\right]+\frac{DM}{Q}\left[\frac{1}{n}(S_B+S_V)+2F\right]+$$

$$DM(d+CN), \tag{2.21}$$

Where, $M = \frac{1}{1-E[Y]}$, $N = \boldsymbol{E}[Y]$ and $W = \boldsymbol{E}[(1-Y)^2]$

By differentiating the total expected cost function given in Equation (2.21) with respect to Q, and solving the differential equation getting the value of order size, Q^*

$$Q^* = \sqrt{\frac{DM(\frac{S_B+S_V}{n}+2F)}{D[M\left\{\frac{h_BN}{x}+\frac{1}{2}\frac{h_BO}{D}+\frac{h_V\left(1-\frac{1}{2}n\right)}{P}\right\}+\frac{1}{2}\frac{h_V(n-1)}{D}]}}. \tag{2.22}$$

And substituting Q^* in the total cost function equation (2.21) gives the number of shipment n^*, which is,

$$n^* = \frac{1}{2}\sqrt{\frac{S_V[\{(-h_V+(1-N)h_B)x+h_V MND\}P+2h_V xMD]\left(\frac{1}{2}S_B+F\right)h_V(-P+MD)x}{xh_V(P+MD)(S_B+FW)}}. \quad (2.23)$$

For the optimal order size and the optimal number of shipment, now it is to check whether the second order differentiation is positive or negative.

For Order size Q, the second order differentiation equation is as follows:

$$\frac{d^2\mathrm{E}[UTC(n,Q)]}{dQ^2} = \frac{2DM(\frac{S_B+S_V}{n}+2F)}{Q^3} > 0. \quad (2.24)$$

For number of shipment n, the second order differentiation equation is as follows:

$$\frac{d^2\mathrm{E}[UTC(n,Q)]}{dn^2} = \frac{2DM(S_B+S_V)}{n^3 Q} > 0 \quad (2.25)$$

The equations (2.24) and (2.25) shows that the second order derivatives with respect to Q and n are positive. This indicates the Q^* and n^* are optimal and expected total cost function is minimized.

2.4 Numerical results

Table 2.1: Data sheet

Notation	Description	Value
S_v	Set up cost(vendor)	300
S_B	Set up cost(buyer)	100
D	Demand	50000
F	Transportation cost (fixed)	25
C	Variable transportation cost	3
d	Unit screening cost	0.5
P	Production rate	160000

h_v	Holding cost for vendor	2
h_B	Holding cost for buyer	5
x	Screening rate	175200

The optimal order size, the optimal number of shipment, the expected total cost and unit cost are computed based on the data from Table 2.1. The table values are obtained from the paper of Salameh and Jaber (2000) for the fixed costs, demand and production, screening rate. In the table 2.2, the percentage of defective item Y is uniformly distributed with minimum value 'a' and maximum value 'b'. So, the percentage of defective items changes by keeping zero value for 'a' and varying the 'b' value. Each percentage of defective items provides the optimal order quantity, number of shipment and total expected cost per unit time. The results are presented in Table 2.2 (a) as the percentage of defective is varied from 0.5% to 5%.

Table 2.2: Optimal order quantity, optimal shipment, and the total cost of domestic supply chain

	Y~ U(a,b)		$E(Y)$	Order Size, Q	Number of shipment, n	Total expected cost		Optimal total expected cost	Unit Cost
	a	b				When n=7	When n=8		
1	0	0.01	0.005	1137	7.01	38489.89	39340.40	38489.89	4.84
2	0	0.02	0.01	1139	7.04	39405.80	40225.01	39405.80	4.94
3	0	0.03	0.015	1141	7.07	40331.12	41119.02	40331.12	5.05
4	0	0.04	0.02	1143	7.10	41265.97	42022.57	41265.97	5.16
5	0	0.05	0.025	1144	7.14	42210.51	42935.81	42210.51	5.27
6	0	0.06	0.03	1146	7.17	43164.89	43164.89	43164.89	5.38
7	0	0.07	0.035	1148	7.21	44129.26	44791.95	44129.26	5.49
8	0	0.08	0.04	1150	7.24	45103.78	45735.15	45103.78	5.60
9	0	0.09	0.045	1152	7.27	46088.60	46688.64	46088.60	5.72
10	0	0.1	0.05	1155	7.31	47083.89	47652.59	47083.89	5.82

The number of shipment should be integer. But the solution from the Equation 2.23 is 7.01 when the expected percentage of defective item is 0.005. Hence, one needs to check for n=7 and n=8, and has to choose the value of n that gives the lowest total expected cost. From the table 2.2, the number of shipment as 7 gives the optimal total expected cost.

Several tests have been conducted to see how the model behaves with the variation of percentage of defective items. As the percentage of defective item moves from 0.5% to 5% the total expected cost per unit time for the integrated two-level supply chain increases from \$ 38490 to \$ 47084, which is almost 23% raise for the total cost per unit time. Figure 2.4 shows that increase of the expected total cost with the change of percentage of defective item.

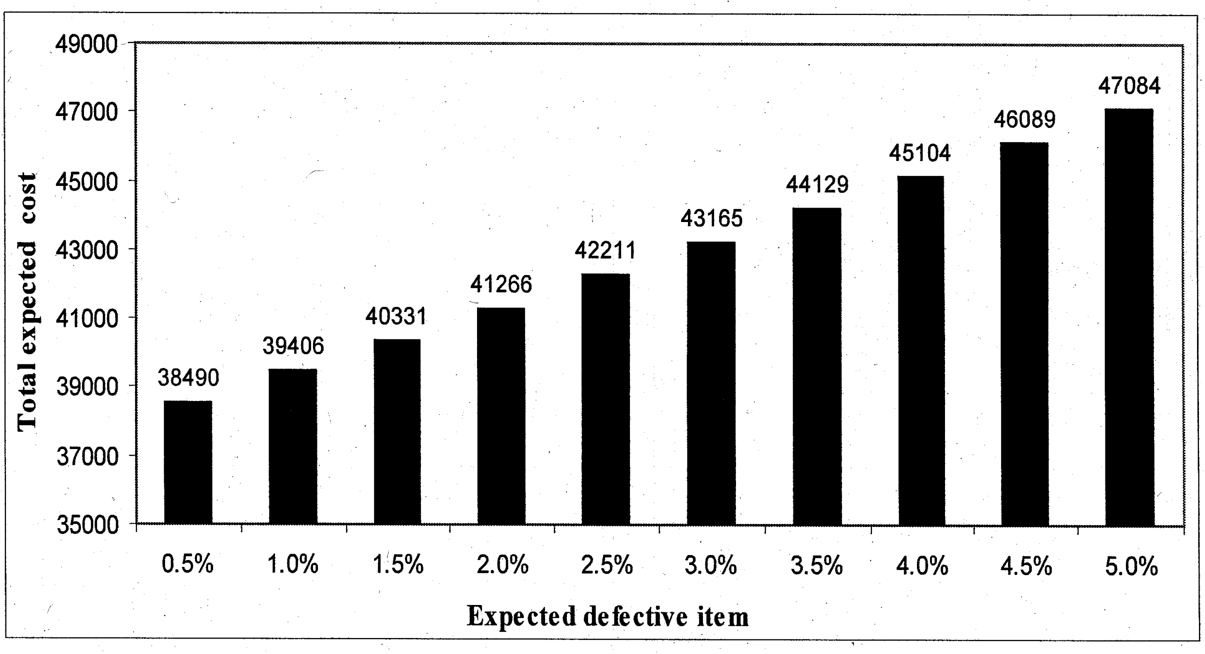


Figure 2.4: The total expected cost of supply chain vs. percentage of expected defective items (domestic supply chain)

Figure 2.5 shows the movement of the increase of unit cost with the raise of percentage of defective item. The unit cost is the average cost per item. The unit cost is calculated as a result of the expected total cost ($E[Y]$) divided by the order size (Q). Once the percentage of defective item increases from 0.5% to 5%, the average unit cost raises from \$4.84 to \$5.82. So, it is almost \$1 added to the unit cost for the percentage of defective item increase.

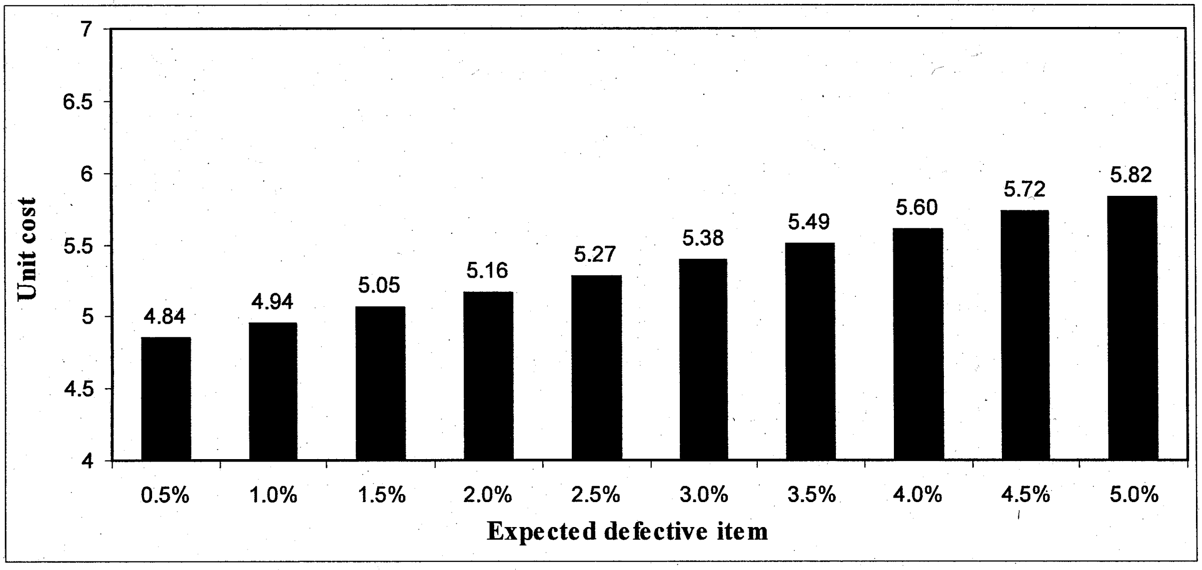


Figure 2.5: Unit cost of product vs. percentage of expected defective items (domestic supply chain)

Figure 2.6 shows the relation between the expected defective item and the order size in the integrated two-level closed-loop supply chain. The percentage of defective item increases the optimal order size. So, the buyer has to order more for keeping an allowance for the defective items in each purchase order.

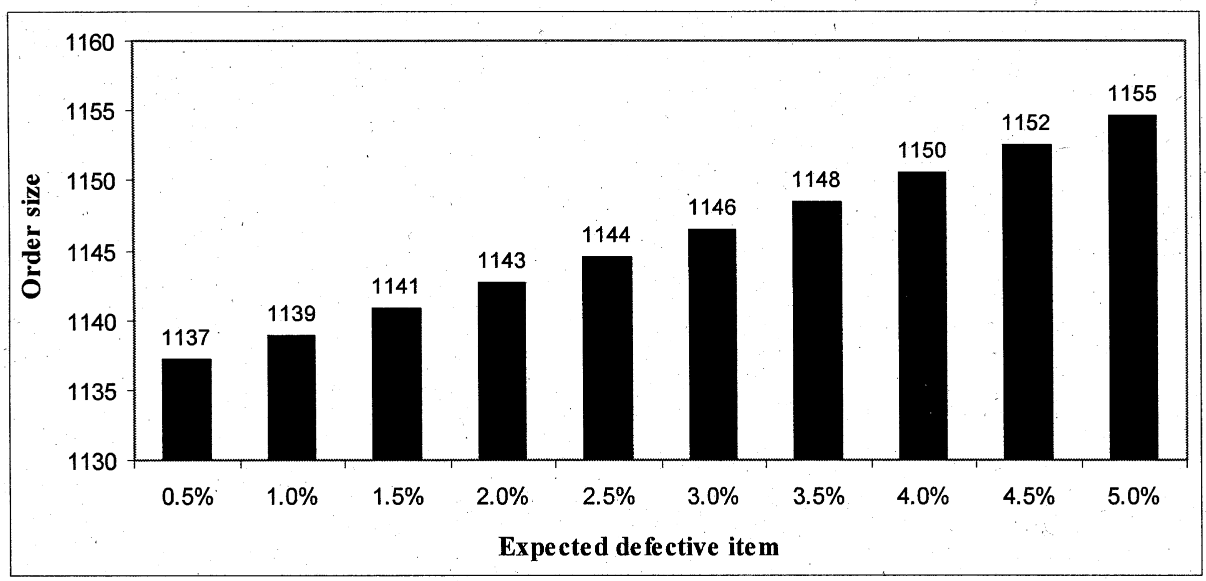


Figure 2.6: The expected defective item of supply chain vs. order size (domestic supply chain)

It is interesting to analyze the buyer's total expected cost with the change of the percentage of defective items. The cash flow takes place from the buyer to the vendor in any supply chain management. In two-level integrated supply chain, the cost analysis for buyer is very important to verify how much the buyer can save in terms of unit price by purchasing items with less defective items.

Table 2.3 displays the numerical results of the optimal order size, the number of shipment, buyer's expected total cost and unit price with the percent change of the defective item.

Table 2.3: Optimal order quantity, optimal shipment, and the total cost of the buyer

	Y~ U(a,b)		E(Y)	Order Size, Q	Number of shipment, n	Total expected cost(Buyer)		Optimal total expected cost	Unit Cost
	a	b				When n=7	When n=8		
1	0	0.01	0.005	1137	7.01	29693.87	29726.07	29693.87	3.73
2	0	0.02	0.01	1139	7.04	29818.29	29848.04	29818.29	3.74
3	0	0.03	0.015	1141	7.07	29944.14	29971.54	29944.14	3.75
4	0	0.04	0.02	1143	7.10	30071.45	30096.59	30071.45	3.76
5	0	0.05	0.025	1144	7.14	30200.23	30223.21	30200.23	3.77
6	0	0.06	0.03	1146	7.17	30330.50	30351.43	30330.50	3.78
7	0	0.07	0.035	1148	7.21	30462.30	30481.25	30462.30	3.79
8	0	0.08	0.04	1150	7.24	30595.65	30612.72	30595.65	3.80
9	0	0.09	0.045	1152	7.27	30730.56	30745.86	30730.56	3.81
10	0	0.1	0.05	1155	7.31	30867.06	30880.68	30867.06	3.82

Table 2.3 shows that when the number of shipment is 7 the total expected cost is more optimal.The buyer's expected total cost also increases with the increase of defective items. When percentage of defective item moves from 0.5% to 5% the total expected cost for the buyer increases from \$ 29694 to \$ 30868, which is 4% extra expense for buyer's expected total cost.

Figure 2.7 shows the increase of the buyer's expected total cost with change of the percentage of defective items from 0.5% to 5%.

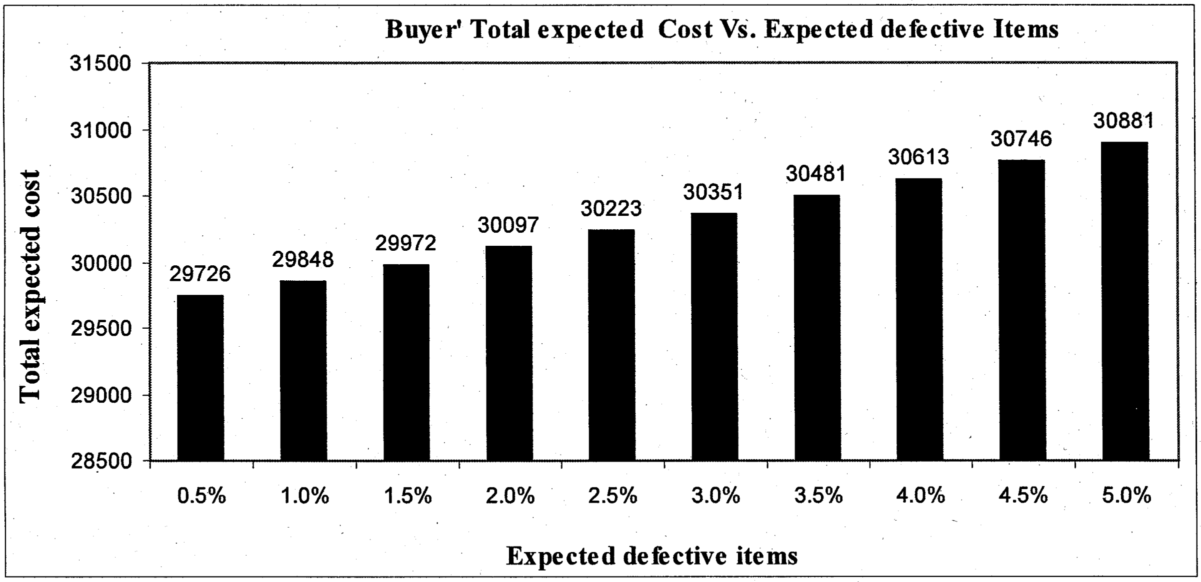


Figure 2.7: Buyer's Total expected cost vs. percentage of expected defective items

Now the average unit price for buyer gives an idea about the contribution of the buyer for each product. If the buyer allows 5% of the defective items from the vendor, the unit price increases from 3.73 to 3.82. So the buyer is paying 0.01 dollar extra for each item when the defective item increases from 0.5% to 5%. Figure 2.8 shows the picture of that increase.

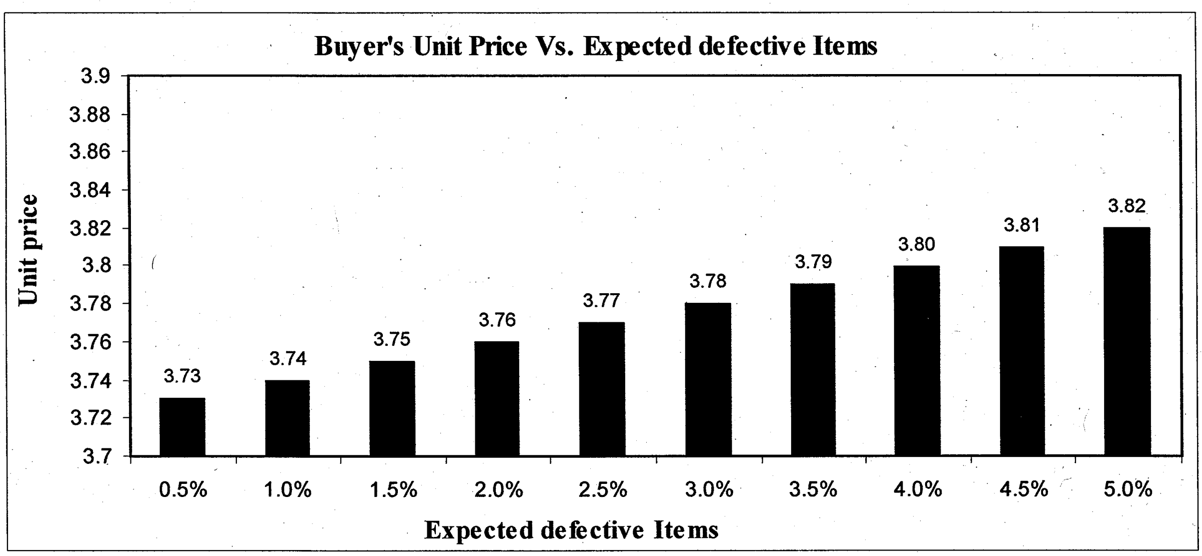


Figure 2.8: Buyer's unit price vs. percentage of expected defective items

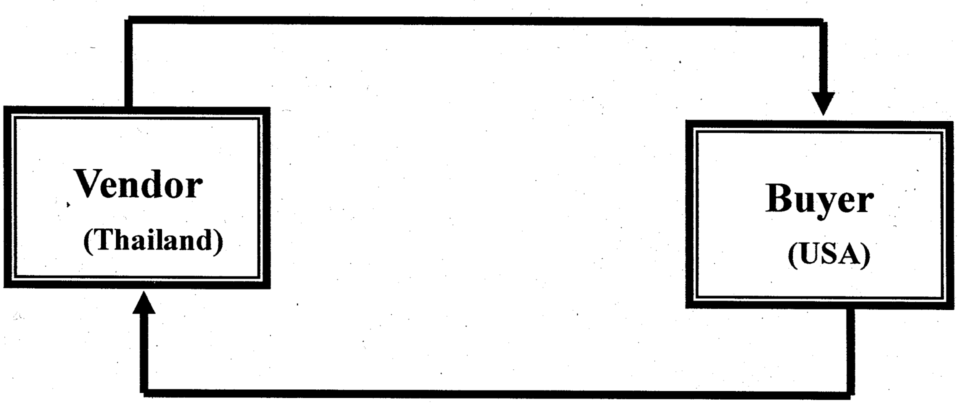
Chapter 3

TWO LEVEL SUPPLY CHAIN: INTERNATIONAL VENDOR AND BUYER

3.1 Introduction

This chapter focuses on the designing of a two-level closed-loop supply chain where the vendor and the buyer are located in two different countries. It is assumed that a particular electronic product is manufactured in one country and market located in another one country. For example, the manufacturing company (vendor) produces electronic product at Thailand and exports the electronic items to USA (buyer). Exchange rate of the currencies plays a role in such model in calculating the total cost of the integrated supply chain. So this model includes the exchange rate of currencies between the countries into the model and investigates its effect on the total cost of the integrated supply chain. Assumptions made in chapter 2 for domestic two-level supply chain are also retained here except the inclusion of exchange rate e, which is assumed to be normally distributed with mean μ and standard deviation σ, i.e., $e \sim N(\mu, \sigma)$.

Shipping the Q size with defective items



Shipping back the defective items $E[Y].Q$

Figure 3.1 Two-level supply chain with a vendor and a buyer in two different countries

3.2 Mathematical model

The mathematical model is formulated in this section for the two level international supply chain, where the derivations are similar to the domestic supply chain with the addition of exchange rate. The vendor from Thailand supplies the order quantity of the electronic goods to the buyer with the imperfect quality items. The objective of the buyer and the vendor is to minimize the total cost function of this two-level international supply chain.

3.2.1 Vendor's total expected cost per unit time (Thailand)

The electronic manufacturing vendor company in Thailand is considering three major costs for the integrated international supply chain, those are: (a) the holding cost, (b) the setup cost, and (c) the transportation cost. As the vendor is in Thailand, the expected cost reflects on the exchange rate of currencies between USA and Thailand. The Thai currency is less than US currency. The exchange rate is normally distributed with mean μ. So, dividing Equation (2.12) by μ the following expression gives the total cost for the vendor.

$$\mathbf{E}[UTC_V(n,Q)] = \frac{1}{\mu}\{DQ\left[\frac{h_v}{P(1-\mathbf{E}[Y]}\left(1-\frac{n}{2}\right)+\frac{h_v}{2D}(n-1)+\frac{h_B\mathbf{E}[Y]}{2x(1-\mathbf{E}[Y])}\right] \tag{3.1}$$

$$+\frac{D}{Q(1-\mathbf{E}[Y])}\left[F+\frac{S_v}{n}\right]+\frac{C_vD\mathbf{E}[Y]}{(1-\mathbf{E}[Y])}\}.$$

3.2.2 Buyer's annual cost (USA)

The exchange rate does not have impact in the buyer's cost as buyer is in USA. There are four major cost components of the buyer are exactly same as considered in chapter 2. Total cost is the sum of holding cost for good items, setup cost, transportation cost and screening cost. Hence, Equation (2.19) also gives the buyer's expected total cost per unit time in case of international two-level supply chain.

Total cost for two level international supply chain

Now the total cost for the two-level international supply chain can be calculated by summing up the vendor's (Equation 3.1) and buyer's (Equation 2.19) total cost.

$$\mathbf{E}[UTC(n,Q)] = \mathbf{E}[UTC_V(n,Q)] + \mathbf{E}[UTC_B(n,Q)]$$

$$\mathbf{E}[UTC(n,Q)] = QD\left[\frac{1}{(1-\mathbf{E}[Y])}\left\{\frac{h_B\mathbf{E}[Y]}{2x}\left(1+\frac{1}{\mu}\right)+\frac{h_B\mathbf{E}[(1-Y)^2]}{2D}+\frac{h_V}{\mu P}\left(1-\frac{n}{2}\right)\right\} + \frac{h_V}{2\mu D}(n-1)\right] + \frac{D}{Q(1-\mathbf{E}[Y])}\left[\frac{1}{n}\left(S_B+\frac{S_V}{\mu}\right)+F+\frac{F}{\mu}\right]+\frac{D(d+C\mu\mathbf{E}[Y])}{(1-\mathbf{E}[Y])}. \tag{3.2}$$

By differentiating the total expected cost function given in Equation (3.2) with respect to Q, and solving the differential equation getting the value of order size, Q^*

$$Q^* = \sqrt{\frac{DM\left[\frac{1}{n}\left(S_B+\frac{S_V}{\mu}\right)+F+\frac{F}{\mu}\right]}{D\left[M\left\{\frac{h_BN}{2x}\left(1+\frac{1}{\mu}\right)+\frac{1}{2}\frac{h_BW}{D}+\frac{h_V\left(1-\frac{1}{2}n\right)}{\mu P}\right\}+\frac{1}{2}\frac{h_V(n-1)}{\mu D}\right]}}, \tag{3.3}$$

where $M = \frac{1}{1-\mathbf{E}[Y]}$, $N = \mathbf{E}[Y]$ and $W = \mathbf{E}[(1-Y)^2]$

$$n^* = \frac{\sqrt{h_v x A S_v B}}{h_v x C}, \tag{3.4}$$

where

$$A = (DM\mu S_B + DM\mu F + FMD - FP - P\mu F - P\mu S_B).$$
$$B = (h_v MNDP + 2h_v xMD - h_v Px - h_B Px\mu N + h_B Px\mu)$$
$$C = (DM\mu S_B + DM\mu F + FMD - FP - P\mu F - P\mu S_B)$$

For the optimal order size and the optimal number of shipment, now it is to check whether the second order differentiation is positive or negative. For Order size Q, the second order differentiation equation is as follows:

$$\frac{d^2\mathbf{E}[UTC(n,Q)]}{dQ^2} = \frac{2MD\left(S_B+\frac{S_V}{B}+F+\frac{F}{B}\right)}{Q^3}. \tag{3.5}$$

For the number of shipment n, the second order differentiation equation is as follows:

$$\frac{d^2\mathbf{E}[UTC(n,Q)]}{dn^2} = \frac{2DM\left(S_B+\frac{S_V}{B}\right)}{Qn^3}. \tag{3.6}$$

From equations (3.5) and (3.6) it can be shown that the second order derivatives with respect to Q and n are positive. This indicates the Q^* and n^* are minimize.

3.3 Numerical results

In case of international supply chain, the order size, numbers of shipments and the total cost have been obtained by considering the exchange rate. For a particular supply chain model the exchange rate can vary time to time. Table 3.1 shows the values of unit price and expected total cost with varying exchange rate. The exchange rate fluctuates from 35 to 39.5.

TABLE 3.1: Optimal order quantity, optimal shipment, and the total cost of international supply chain

	Y~ U(a,b)		E(Y)	Order Size, Q	Number of shipment, n	Total expected cost		Optimal total expected cost	Unit Cost
	a	b				When n=7	When n=8		
1	0	0.01	0.005	4265	8.23	37389.00	37422.29	37389.00	1.25
2	0	0.02	0.01	4253	8.26	37382.58	37415.70	37382.58	1.26
3	0	0.03	0.015	4241	8.27	37376.34	37409.63	37376.34	1.26
4	0	0.04	0.02	4229	8.29	37370.27	37403.52	37370.27	1.26
5	0	0.05	0.025	4218	8.31	37364.36	39393.86	37364.36	1.27
6	0	0.06	0.03	4207	8.33	37358.61	37388.01	37358.61	1.27
7	0	0.07	0.035	4196	8.35	37353.01	37382.41	37353.01	1.27
8	0	0.08	0.04	4186	8.37	37347.56	37376.96	37347.56	1.27
9	0	0.09	0.045	4176	8.39	37342.25	37371.75	37342.25	1.28
10	0	0.1	0.05	4167	8.41	37337.07	37371.75	37337.07	1.28

From table 3.1, the number of shipment as 7 gives the optimal total expected cost. Figure 3.2 shows the change of expected total cost with respect to expected exchange rate.

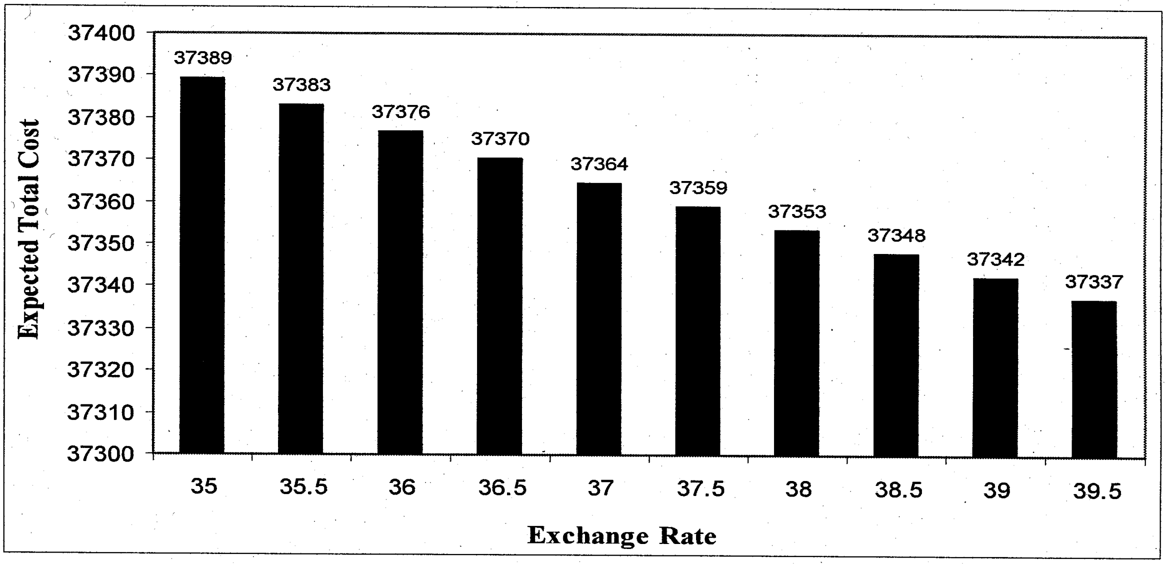


Figure 3.2: The expected total cost of supply chain vs. exchange rate (international supply chain)

The expected total cost is calculated in US dollar. So the increase in exchange rate reduces the expected total cost for the integrated two-level supply chain. As the exchange rate increases from 35 to 39.5, the expected total cost changes from 37389 to 37337. The Figure 3.3 shows the change of unit price with the change of exchange rate, where the unit price amplifies from 1.25 to 1.28.

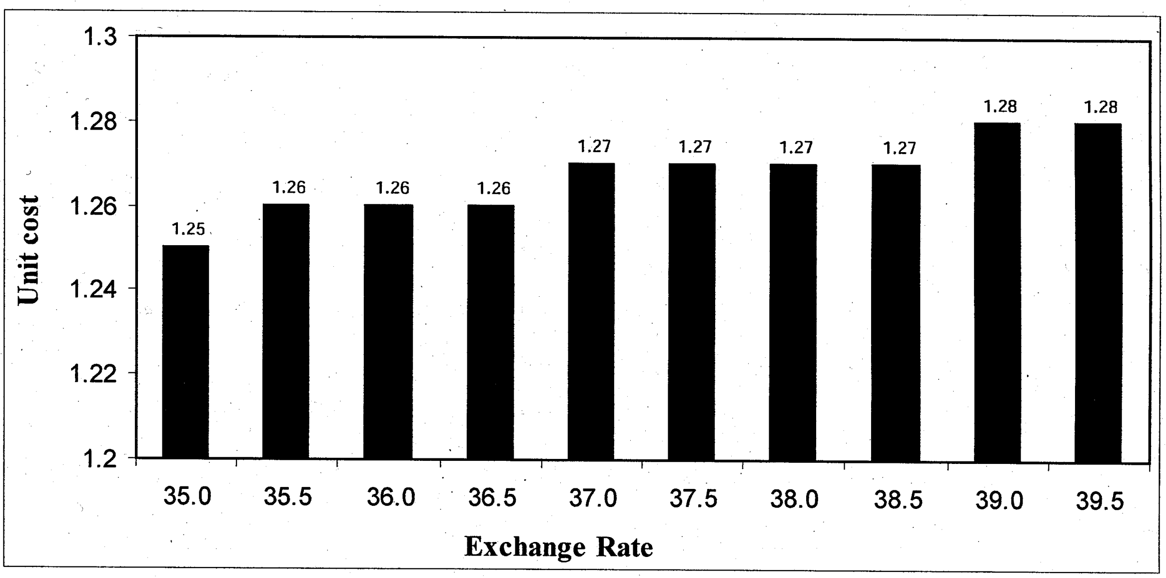


Figure 3.3: The unit price of product vs. exchange rate (international supply chain)

CHAPTER 5

CONCLUSION

In today's competitive market, an individual entity in a supply chain cannot thrive by its own. It needs to cooperate with others. In this project, an integrated two-level closed loop supply chain model consisting of a single vendor and single buyer has been considered with an aim to study the effect of defective items on the total expected cost. The objective is to minimize the total cost that includes set-up costs, holding costs, transportation costs and screening costs. The total cost that considers set-up cost and holding cost is not new. But the model considers and includes different holding costs for good and imperfect items, transportation/shipping cost for the defective items and screening cost into the total cost function. The buyer screens the products by his/her own cost received from the vendor to find out the defective items. The defective items are shipped back to the vendor and the vendor has to bear the shipping cost. This project considers both scenarios: domestic and international. The integrated supply chain can save the energy of transporting the defective items back to the vendor. In some supply chain cases, the defective items can not be sold at the discounted price and will be dumped on selected site, which is not environment friendly. If the vendor supplies items without defectives, the amount of order quantity and frequency of shipments from the vendor to the buyer decreases and this also helps the supply chain cutting the integrated cost and energy.

It is observed that, as the percentage of defective items increase from 0.5% to 5%, the total cost of the integrated supply chain increases by around 22%. Consequently, the average cost per unit product in the integrated supply chain also increases by \$ 1. So, if the percentage of defective items could be reduced, then the unit price of the product would be reduced. From the buyer's point of view, his/her cost also jumps by \$0.01 per item, as the percentage of defective items increases from 0.5% to 5%. In other word, if the buyer pays extra 1 cents per unit to the vendor, then the defective item can be eliminated from the integrated supply chain. In such a case, as the percentage of defective items becomes zero, the reverse link of this particular two-level closed loop supply chain will disappear. Only the forward link will work from the vendor to the buyer without defective items.

As inventory management issues in the business are increasing day by day, the business enterprises are facing more challenges to improve their performance financially by cutting the inventory holding cost. This model shows that minimizing defective items (i.e. better environment performance) can rather lead to an increase in profit through minimizing the total cost. Therefore, this work will have a significant contribution for the enhancement in the supply chain management of vendor-buyer related industries. Also companies in high-tech industries can successfully implement this model.

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