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SUBOPTIMAL RATE ADAPTIVE RESOURCE ALLOCATION IN MULTIUSER OFDM COMMUNICATION SYSTEMS

by

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Suboptimal Rate Adaptive Resource Allocation in Multiuser OFDM Communication Systems

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This thesis aims to study the performance of adaptive resource allocation in the downlink of multiuser OFDM systems with fixed or variable rate requirements (with fairness consideration) as well as low complexity algorithms for real-time implementations in practical systems.

We first verify the simplifying assumption of flat transmit power over the entire bandwidth. Two different optimal and suboptimal power allocation schemes are applied in a single-user system and the decrease in the total throughput due to the presence of the power mask on subcarriers is measured. Based on the comparison of the achieved data rates, a flat transmit power is then assumed in the proposed suboptimal multiuser resource allocation algorithms. Two suboptimal resource allocation algorithms are then proposed using this simplifying assumption. The objective of the first algorithm is to maximize the total throughput while maintaining rate proportionality among the users. The proposed suboptimal algorithm prioritizes the user with the highest sensitivity to the subcarrier allocation and the variance over the subchannel gains is used to define the sensitivity of each user. The second algorithm concerns rate adaptive resource allocation in multiuser OFDM systems with fixed rate constraints for each user. We propose a suboptimal joint subchannel and power allocation algorithm which attempts to maximize the total throughput while supporting the users with their minimum rate requirements. The main feature of this algorithm is its low complexity while achieving close to optimum capacity.

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List of Variables

A_m	real random variable representing the amplitude of individual waves in a multipath
B	the total bandwidth of the system
$c_{k,n}$	the assignment index of the nth subcarrier to the kth user
E_0	real constant representing the local average E-field in a multipath
F	real number representing fairness in the system
F_p	real number representing proportional fairness in the system
f_c	the carrier frequency
f_d	maximum Doppler frequency
f_m	frequency shift of the mth wave in a multipath due to Doppler effect
K	total number of users
k	user index
$H_{k,mean}$	the mean of subchannel gains for the kth user
$H_{k,n}$	the real magnitude of the channel-to-noise ratio of the nth subcarrier seen by the kth user
$h_{k,n}$	the real magnitude of channel gain for the nth subcarrier seen by the kth user
l	multipath index
M	total number of waves in a multipath
m	wave index in each multipath
N	total number of subcarriers in the system
N_k	the set of subcarriers assigned to the kth user
N_0	power spectral density of one-sided AWGN
n	subcarrier index
P_{total}	total transmit power constraint
p_k	the total power allocated to the kth user
p_n	the power allocated to the nth subchannel
$p_{k,n}$	the transmit power in the nth subcarrier of the kth user
R_k	the achieved data rate of the kth user
$R_{k,min}$	the minimum required data rate of the kth user
R_T	total achieved data rate of the system
r_n	number of bits assigned to the nth subcarrier
s_k	the variance of subchannel gains for the kth user
$T_c(t)$	in-phase component of E-field in a multipath
$T_s(t)$	quadrature component of the E-field in a multipath
α_k	proportional rate constraint for the kth user
β	the bit error rate
Γ	power gap for the required bit error rate
$\gamma_{k,n}$	signal-to-noise ratio of the nth subcarrier seen by the kth user
λ	Lagrange multiplier
ϕ_m	phase angle of the mth wave in a multipath

Chapter 1

Introduction

1.1 Overview of Future Wireless Communications Systems

The future wireless communication systems should support a large number of users with flexibility in their quality of service (QoS) requirements. The challenges to ensure the fulfillment of these requirements arise from the nature of the wireless channel as well as the limited availability of radio resources i.e., the frequency spectrum and the total transmit power. As the data rate requirements get higher and higher, the transmission bandwidth increases significantly. In broad-band applications, the wireless channel encounters frequency selective-multipath fading which means that the transmitted signal (considered as an electromagnetic wave) is scattered, diffracted and reflected, and reaches the antenna as an incoherent superposition of many signals each as a poorly synchronized echo component of the desired signal [1]. This phenomenon leads to severe intersymbol interference (ISI) both in time and frequency. To solve this issue, intelligent radio resource management algorithms in both the physical and the media access control layers are essential with the ability to combat ISI.

1.2 Multicarrier Modulation (OFDM)

Orthogonal frequency division multiplexing (OFDM) is one of the promising solutions to provide a high performance physical layer which is based on the concept of multicarrier transmission. The idea is to divide the broadband channel into N narrowband parallel subchannels each with a bandwidth much smaller than the coherence bandwidth of the channel. The main building blocks of an OFDM wireless communication system are shown in Fig. 1.1 [2].

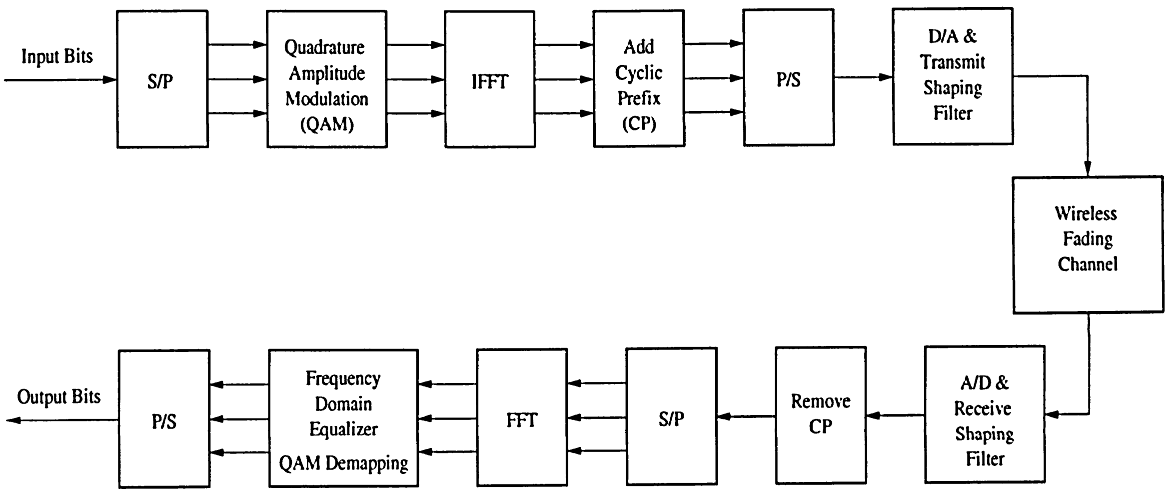

Figure 1.1: Block diagram of an OFDM wireless communication system.

The high rate data stream is split into N parallel substreams of lower rate data which are modulated into N OFDM M-level quadrature amplitude (QAM) symbols and transmitted simultaneously on N orthogonal subcarriers [1]. Each of these parallel complex subchannels can be treated as an ISI-free QAM subchannel. Therefore, the performance of the system with multicarrier modulation can be analyzed as an aggregate of N ISI-free QAM subchannels [3]. If the number of subchannels is sufficiently large (i.e., the bandwidth of each subchannel is sufficiently small), the frequency response in each subchannel is close to be flat. Further, the frequency spacing between the subchannels ensures the orthogonality of the subcarriers. The complex symbols at the output of the modulators are then transformed into OFDM symbols in time domain by the inverse fast Fourier transform (IFFT) in the transmitter. Before transmission, a cyclic prefix (CP) which is the copy of the last IFFT samples is added to the front of the OFDM symbol. CP is sized appropriately to serve as a guard interval to maintain orthogonality between the subcarriers in the multipath wireless channel. Therefore, the ISI could be eliminated provided that the amount of time dispersion from the channel is smaller than the duration of the guard interval. In the receiver, the guard interval is removed and the time samples are transformed into modulated symbols by means of fast Fourier transform (FFT). The rest of the receiver blocks essentially invert the operations at the transmitter.

In a single user system, the user can use all N subcarriers; however, wireless communication systems are essentially multiuser systems. In a downlink wireless system, a centralized basestation needs to communicate to multiple users and there should be a multiple access scheme to allocate the limited number of subcarriers and the power to the users. How the resources are allocated among the users is very critical to the system performance.

1.3 Spectrum Sharing Technologies

Four typical spectrum sharing technologies are time division multiple access (TDMA), frequency division multiple access (FDMA), code division multiple access (CDMA), and spatial division multiple access (SDMA). In TDMA, the transmit time is divided into a serial number of time slots. Each user is assigned a predetermined time slot and may be able to occupy the entire bandwidth throughout that interval. In FDMA, each user is assigned predetermined frequency bands all the time. CDMA distinguishes the users in the code domain; each active user is allocated a specific sequence and all the users share the entire bandwidth all the time without causing significant interference to each other. In SDMA, multiple transmit and receive antennas are utilized to separate users in the spatial domain, also allowing the users to access the same bandwidth simultaneously. These multiple access technologies are usually used in combination. In WCDMA, TDD employs CDMA with TDMA where the transmission time is divided into a number of time slots and within each time slot, multiple users employ CDMA to access the whole bandwidth. FDMA is used in almost all cellular systems.

Each of these multiple access schemes could be combined with OFDM in an OFDM-based multiuser communication system. When TDMA and FDMA are combined with OFDM, the subcarrier allocation is referred to as static or non-adaptive, since each user is assigned a time slot or frequency band respectively regardless of the channel status. In other words, in non-adaptive fixed subcarrier allocation schemes, an independent dimension is allocated to each user without considering the channel status. The wireless channel is however, time-varying and frequency selective. The channels experienced by different users are largely independent because of users' different locations. Since the fading parameters for different users are mutually independent, the probability that a subcarrier is in deep fade for all users is very low. Each subcarrier is likely to be in good condition for some users in the system and could be allocated to the user with the best channel gain on it. By exploiting the multiuser diversity, channel aware adaptive resource allocation outperforms the static resource allocation such as TDMA or FDMA in terms of system throughput.

1.4 Adaptive Resource Allocation in Multiuser OFDM Systems

In a multiuser OFDM system with adaptive resource allocation, multiple users may be scheduled for transmission on different subchannels within an OFDM symbol. In the downlink of an OFDM communication system, the resources are the total transmit power and the bandwidth (subchannels). Due to users' different locations and their independent fading characteristics, the subchannels can be allocated to the users based on their channel conditions. The power is then allocated to the subchannels assigned to each user. Fig. 1.2 shows the concept of adaptive resource allocation in multiuser OFDM systems.

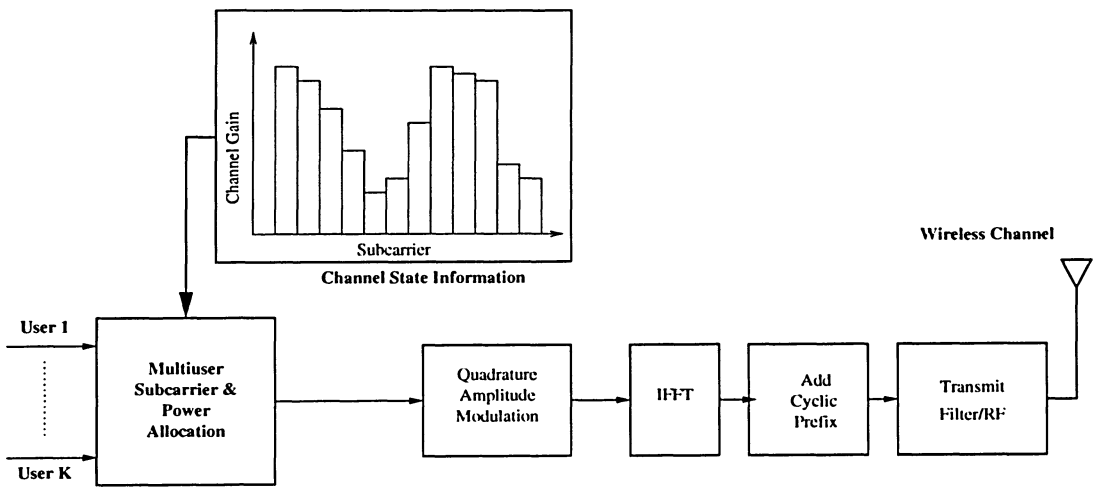

Figure 1.2: Block diagram of a multiuser OFDM transceiver with adaptive subcarrier and power allocation.

The problem of resource allocation in a multiuser OFDM system with N subcarriers and K users is basically to determine the elements of the matrix $\mathbf{C}=[c_{k,n}]_{K\times N}$ specifying which subcarriers should be assigned to which user and the vector $\mathbf{p}=[p_n]_{N\times 1}$ specifying how much power should be allocated to each subcarrier. To determine the elements of $\mathbf{C}$ and $\mathbf{p}$, the problem is formulated in the form of an optimization problem with one or more constraints according to the objective in the system. An overview of the problem is shown in Fig. 1.3.

Adaptive resource allocation in multiuser OFDM systems can be formulated as an optimization problem to minimize or maximize a parameter of the system with certain conditions specified in one or more constraints. Two major classes of dynamic resource allocation

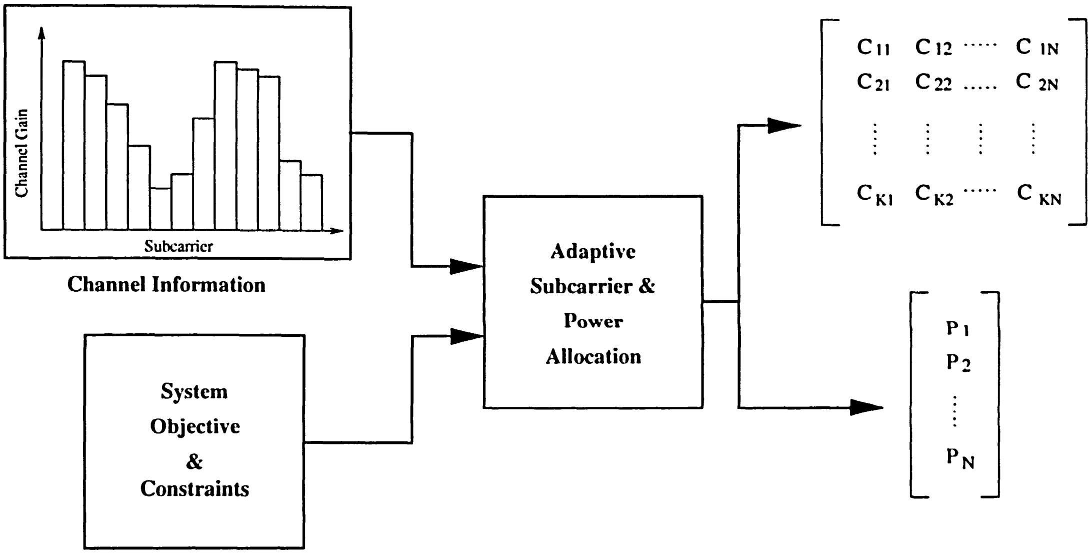

Figure 1.3: Overview of the problem of resource allocation in multiuser OFDM systems.

schemes have been reported in the literature: 1) Margin Adaptive (MA) [4-6] and 2) Rate adaptive (RA) [7–16]. The optimization problem in margin adaptive allocation schemes is formulated with the objective of minimizing the total transmit power while providing each user with its required quality of service in terms of data rate and BER. The objective of the class of rate adaptive algorithms is to maximize the total throughput of the system with the constraint on the transmit power. While the sum capacity of a system provides a good measurement of the spectral efficiency, it is not a valid indication of each user's satisfaction in a multipath fading channel. Therefore, rate adaptive algorithms are divided into two major groups based on the user rate constraints; in the first group, there is a fixed rate requirement for each user. The algorithms in this group (e.g., [7]) attempt to maximize the total throughput of the system while supporting each user with its fixed rate requirement. A large number of RA algorithms [8–17] fall into the second group where they consider the concept of fairness or constrained-fairness by introducing rate proportional constraints among the users.

The formulated optimization problems are often very difficult to solve and low complexity algorithms have been proposed to reduce the complexity. It was proved in [8] and [12], that the total throughput of a multiuser OFDM system is maximized if each subchannel is assigned to the user with the best channel gain on it and the power is distributed with water-filling policy. However, when the path loss difference among users are large, the users with higher channel gain will be allocated most of the resources while leaving less for the users with low channel gain. Therefore, a major category of the rate adaptive resource allocation algorithms considered fairness among users.

In the group of RA algorithms, Song et al. [8] used the concept of utility function to formulate the problem of maximizing the total data rate with the constraint on the power. Utility function maps the network resources a user utilizes into a real number and is a function of the user's data rate. An extreme case of the problem is obtained when there are infinite number of orthogonal subcarriers each with an infinitesimal bandwidth within the total available bandwidth. In this extreme case, the whole bandwidth is divided into several non-overlapping frequency sets that are assigned to the users. Song et al. investigated the extreme case in two theorems [9, theorem I and II] (also see [10] for the proof of theorem I). Combining the results of the theorems, they derived the optimal frequency set and the power allocation for the extreme case and showed the necessary and sufficient conditions for the global optimality of the solution.

The case of resource allocation in practical OFDM wireless networks with finite number of subcarriers using utility functions is investigated in [11] where it is shown that the optimality conditions for the extreme (continuous) case also hold for discrete case with finite number of subcarriers.

The most important decision to make in a utility-based optimization problem is to choose the utility function properly according to the objective of the system. Since in almost all wireless applications the most important factor to determine a user's satisfaction is its reliable data transmission rate, the utility function is chosen to be a non-decreasing function of the rate. If the utility function is chosen to be the data rate, for instance, the spectral efficiency is maximized and the optimal solution derived in [8] becomes the same as the one obtained in [12] by Jang et al. In this case, the first theorem gives the optimal subcarrier allocation necessitating each subcarrier to be assigned to the user with the best channel gain on it. The second theorem provides the optimal power allocation among the assigned subcarriers. Although the total rate is maximized, the users with poor channel conditions are penalized. To maintain fairness, the utility function should be chosen to prioritize the users with low data rate.

One way to accomplish both efficiency and fairness is to use utility functions that are both increasing and marginally decreasing. As a result, the slope of the utility curve decreases with an increase in the data rate. Choosing a marginally decreasing utility function also guarantees its strictly concavity which ensures the global optimality as well as uniqueness of the optimal solution. A logarithmic utility function is both increasing and marginally decreasing. Therefore, a resource allocation policy using a logarithmic utility function is said to be proportionally fair [18]. Different types of utility functions have been proposed in [10, 11, 18, 19] depending on the type of application. Choosing the proper utility function which ensures both efficiency and fairness is better obtained through subjective survey rather than pure theoretical derivation.

The problem of maximizing the total throughput with fairness was formulated differently in [13] and [14]. Rhee and Cioffi [13] studied the max-min problem, whereby maximizing the worst user's capacity, it is assured that all users achieve the same data rate. Shen et al. [14] considered this problem by introducing proportional constraints among the users' data rates. their proposed algorithms were further modified in [16] and [17]. In all these algorithms, while the objective is to maximize the total throughput within the power budget, the goal is to maintain the proportionality among the users' rates according to proportional constraints rather than reaching a specific requested data rate.

The performance of each of the algorithms mentioned above highly depends on the formulation of the problem, the validity of the assumptions and the optimization method they have applied. Fig. 1.4 gives a summary of different classes of resource allocation in multiuser OFDM systems.

The ultimate goal of all these adaptive resource allocation algorithms is to achieve the highest possible data rate with the minimum transmit power while supporting the users with variable or fixed rate constraints of the system. Various optimization and numeric methods can be applied to obtain the optimal solution for the optimization problem formulated in each group. The objective however, is not just reaching the optimal solution but to consider the trade-off between the optimality of the solution and the complexity of the algorithm and choose a method which not only has an acceptable performance but is also fast and practical in real time applications. In this thesis, two low complexity suboptimal algorithms are proposed with the objective of maximizing the total throughput with fixed and variable rate constraints. The low complexity of the proposed algorithms is due to the simplifying assumption of equal transmit power on all the subcarriers in the system, which will be verified first.

1.5 Assumptions in the Thesis

There are certain assumptions regarding the system and the channel under consideration. All these assumptions except the last one apply to both in referred existing algorithms as well as the proposed algorithms in this thesis. The flat power spectrum density mask has been applied in the proposed algorithms in this thesis and has been assumed in the referred literature wherever indicated.

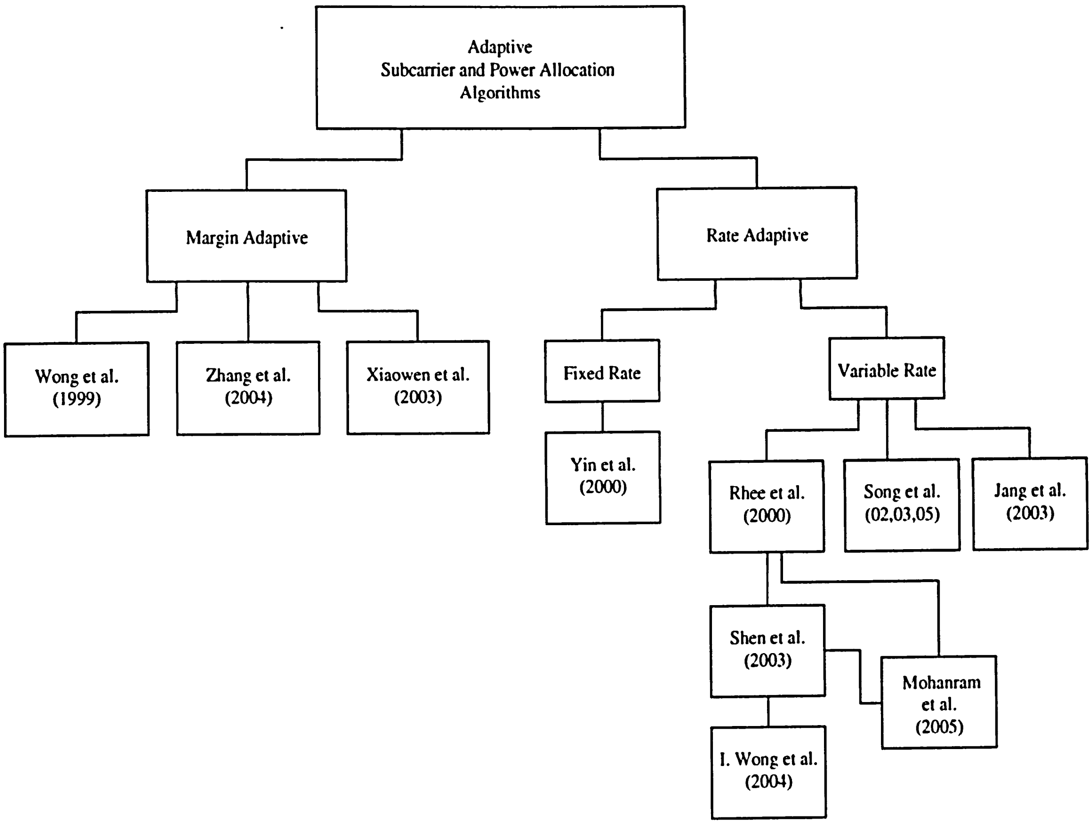

Figure 1.4: Summary of algorithms developed for resource allocation in multiuser OFDM systems.

- *Perfect channel state information for all the users available at the base station before any resource allocation*

  The advantages of adaptive resource allocation in multiuser OFDM systems are partially due to multiuser diversity which is based on assigning each subchannel to the user that has good channel gain on it. To do so, user channel information should be known at the basestation. It is assumed that users perfectly estimate and feedback their channel information to the basestation and the channel condition is always available to the basestation in the beginning of each transmission block although it increases the system overhead due to the feedback information. Also, it is assumed that the channel is quasi-static and that the channel condition does not change within each OFDM transmission block. Otherwise, the resources will be assigned to the users while the channel state has changed which makes the expected performance invalid. Several authors have investigated channel prediction [20, 21] to reduce the amount of feedback overhead and the performance of adaptive OFDM systems with imperfect channel state information is an on-going research area, e.g. [22].

- *Frequency-selective fading channel with additive white Gaussian noise*

  The mathematical model used for the channel under consideration should be able to characterize many of the physical phenomenon of the wireless channel that we encounter in practice. The simplest mathematical model for a communication channel is the additive noise channel. In this model, the transmitted signal $s(t)$ is corrupted by an additive random noise process $n(t)$. The additive noise in wireless channels may be due to a variety of causes each modelled as a continuous, discrete or mixed random variable. However, by the central limit theorem [23], the cumulative effect of a large number of random variables will be approximately normal, regardless of the nature or probability distribution of each random variable. This type of noise is characterized statistically as a Gaussian noise process and the resulting mathematical model for the channel with such noise is called the additive Gaussian noise channel. If the noise has a flat (constant) power spectral density over the entire bandwidth, it is called white noise. Since this channel-noise model applies to a broad class of physical communication channels and because of its mathematically tractability, a channel with additive while Gaussian noise (AWGN) is assumed in this thesis.

  Furthermore, the wideband channel is assumed to be frequency selective. The broadband channel is divided into N narrowband subchannels. The bandwidth of each subchannel is assumed to be much smaller than the coherence bandwidth of the channel. Therefore, it is assumed that each subchannel undergoes approximately flat fading.

- *Continuous Shannon channel capacity as a measure of achieved data rate*

  The Shannon capacity formula $C = W \log_2 \left(1 + \frac{P}{N_0 W}\right)$ [24] is used to measure each subchannel's data rate in bits/second, where W and P denote the subchannel's bandwidth and allocated power, and $N_0/2$ denotes the power spectral density of the noise in watts/Hz. In practical systems, the achieved data rates are discrete values due to the type of modulation and different coding schemes. This formula however, being a continuous function, simplifies the analysis of adaptive resource allocation and provides an upper bound on the achievable throughput. To model the signal-to-noise degradation in the continuous Shannon capacity function due to bit error rate, a power gap [25] can be included in the formula based on the modulation scheme.

- *Single cell environment*

  In this thesis, only resource allocation in a single cell is considered. Hence, other-cell interference is not modelled. In multi-cell OFDM systems, the resource allocation problem becomes more complicated even if the assignment of sub-channels is predetermined. This is because users in different cells reuse the same sub-channels and cause interference to each other. For users at the cell edges, other-cell interference is not negligible as it greatly impacts the user channel-to-interference-plus-noise ratio. The resource allocation algorithms discussed in this thesis can be applied to users for whom other-cell interference does not dominate the amount of additive white Gaussian noise. The problem of resource allocation in multi-cell OFDM networks have been investigated by several researchers, [26–28].

- *K active users always present*

  In this thesis, it is assumed that there are K active users all the time requiring an opportunity to communication. Also, when one user is scheduled for transmission, he/she always has some information data to transmit. The algorithms proposed in this thesis can be applied for those active users.

1.6 Contribution and Organization of the Thesis

In Chapter 2, the problem of rate adaptive resource allocation in multiuser OFDM systems is formulated with: 1) fixed, and 2) variable data rate constraints. The problem formulation of each group is followed by the discussion of the existing algorithms.

Chapter 3 presents the first contribution of this thesis: Measuring the decrease in the total throughput of the system when there is a power mask on all the subchannels. We apply two different power allocation methods in a singe-user system. The first method [14] is optimal and consists of water-filling in frequency domain, while the second [3] is suboptimal with flat transmit power on all the subchannels. Comparing the achieved data rate in two cases, a simplifying assumption is verified for deriving the multiuser suboptimal algorithms in Chapter 4.

The main contribution of this thesis, presented in Chapter 4, is on the sum capacity of a multiuser OFDM system with proportional rate constraints. Since the optimal solution to the problem of constrained fairness is extremely computationally complex to obtain, we propose a low-complexity suboptimal algorithm consisting of only subcarrier allocation where the power spectral density is assumed to be flat on the entire bandwidth. This assumption is based on the results of the single-user power allocation obtained in Chapter 3. In performing the subcarrier allocation, users' variance on subchannel gains is used to prioritize the users to choose their best available subchannel.

The third contribution of this thesis, also presented in Chapter 4, includes a very low complexity algorithm for rate adaptive resource allocation with fixed rate constraints. The simulation results show that this algorithm achieves the predetermined required data rate of all the users in the system while its total data rate is close to capacity.

The simulation results and performance comparison of all the proposed methods are presented in Chapter 5.

In Chapter 6, we summarize the contributions of this thesis and discuss the future research topics.

Chapter 2

Rate Adaptive Resource Allocation in Multiuser OFDM Systems

2.1 Introduction

The block diagram of a multiuser OFDM system is shown in Fig. 2.1 [15]. In the downlink of a multiuser OFDM system, the basestation should communicate with multiple users with limited resources i.e., bandwidth and power. Using the channel information, the basestation applies the combined subcarrier and power allocation algorithm to assign subcarriers to different users and the number of bits/OFDM symbol from each user to be transmitted on each subcarrier. The power allocated to each subcarrier is then determined by the number of assigned bits as well as the corresponding modulation scheme. Along with each OFDM symbol, the subcarrier and bit allocation information is sent to the receivers via a separate control channel; therefore, each user needs only to decode the bits on its respective assigned subcarriers. The channel information is updated as fast as possible and the resource allocation is carried out as soon as the channel information is collected.

In this chapter, we describe and formulate the optimization problems under consideration in this thesis. First, we introduce the parameters and basic assumptions used in modelling the wireless channel. The concepts of efficiency and fairness are described in Section 2.3. The problem of rate adaptive resource allocation with variable rate constraints (constrained-fairness) is formulated in Section 2.4 followed by the discussion of the existing algorithms. The optimization problem with the same objective but with fixed user data rate constraints is then formulated in Section 2.5. Finally, the advantages and practical limitations of adaptive modulation will be discussed in Section 2.6.

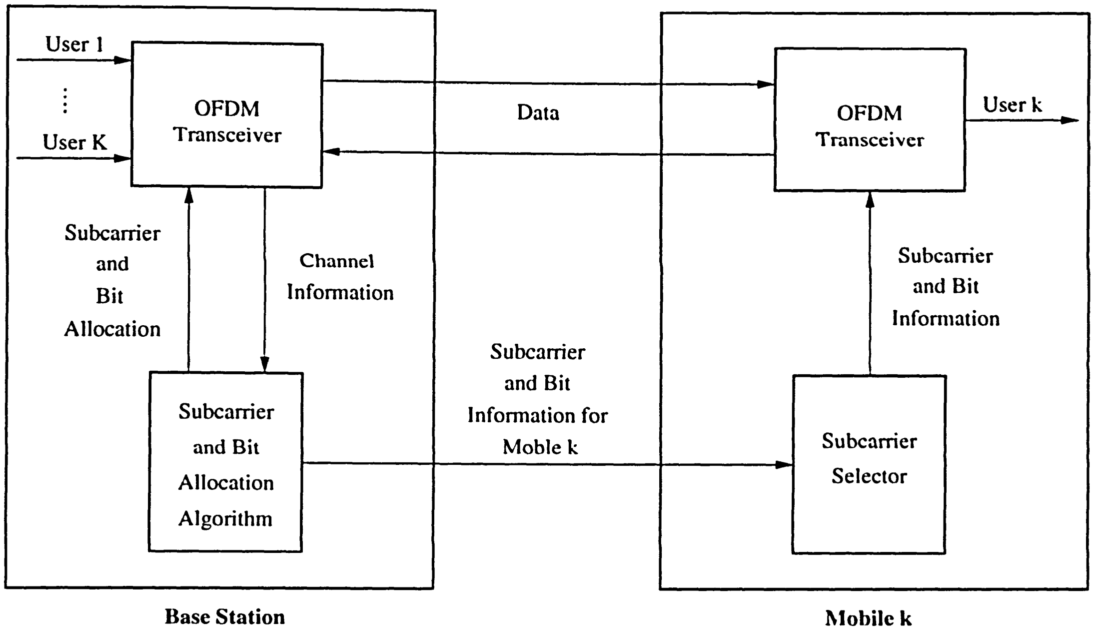

Figure 2.1: Block diagram of a multiuser OFDM system.

2.2 Channel Characteristics

It is assumed that there are K users and N subcarriers in a single cell system. In those areas where there is no direct line-of-sight path between the transmitter and the receiver, multiple reflections occur from different objects which result in the electromagnetic wave travelling along different paths of varying length. The interaction between these waves causes multipath fading with frequency selectivity where the fading parameter changes with frequency. Therefore, the wireless channel is assumed to be wide-band time-varying frequency-selective multipath fading.

One of the parameters to characterize such channels in frequency domain is the coherence bandwidth defined as the range of frequencies over which the channel can be considered flat [29]. In other words, the coherence bandwidth is the range of frequencies which are passed by the channel with approximately equal gain and linear phase. By choosing the bandwidth of the subchannels much smaller than the coherence bandwidth of the channel, each subchannel is assumed to undergo flat fading.

There are several multipath models to explain the statistical nature of the wireless channel. The first model was suggested by Ossana [29] which was based on interference of waves incident and reflected from the flat sides of randomly located buildings. Ossana's model assumes the existence of a direct path between the transmitter and receiver and is therefore inflexible and inappropriate for urban areas where the direct path is almost always blocked by the buildings and other obstacles. The other widely used model for flat fading channels (also used in this thesis) is Clark's model based on scattering [29]. Based on this model, the fading parameters of the channel is considered to be a random variable with Rayleigh distribution. It is generally assumed that the fading rate is slow enough such that the time-varying channel can be considered quasi-static, where the channel status does not vary within each transmission block. Finally, it is assumed that the additive while Gaussian noise (AWGN) is present for all subcarriers of all users and that the basestation has full knowledge of the instantaneous channel transfer functions at the beginning of each transmission block.

In an adaptive multiuser subcarrier allocation scheme, the subcarriers are assigned to the users based on instantaneous channel information and the constraints and objectives of the system. Fig. 2.2 shows a snapshot of a wireless channel with eight subcarriers and four users. The snapshot shows two important properties of subchannel gains in a multiuser frequency-selective fading channel: Firstly, different subcarriers of each user suffer from different fading levels due to frequency selectivity of the channel; Secondly, the subchannels of different users vary independently due to different locations of the users. Using the channel information, the transmitter performs the subcarrier and power allocation to achieve the best performance in the system.

2.3 Efficiency and Fairness

Efficiency and fairness are two crucial parameters in resource allocation for wireless communication systems. Spectral efficiency is defined as the data rate per unit bandwidth and is calculated by dividing the total throughput of a system by its total bandwidth. Therefore, it takes into account the total data rate rather than each user's achieved data rate. A system might achieve the highest throughput hence the highest spectral efficiency while being unfair to those users far away from the basestation or with bad channel conditions. Fairness on the other hand, indicates how equally the resources are distributed among the users. There is always a trade off between efficiency and fairness in wireless resource allocation.

Fairness could be defined in terms of different parameters of the system. It could be defined in terms of bandwidth, where each user is assigned an equal number of subcarriers [30] or it could be in terms of power where each user is allocated equal portion of the power from the budget. It could also be in terms of data rate where the objective is to allocate the resources to the users such that all the users achieve the same data rate [13]. When the objective is to ensure rate proportionality among the users, it is called optimization problem with constrained-fairness [15]. The problem and suboptimal solutions proposed in [13] is a

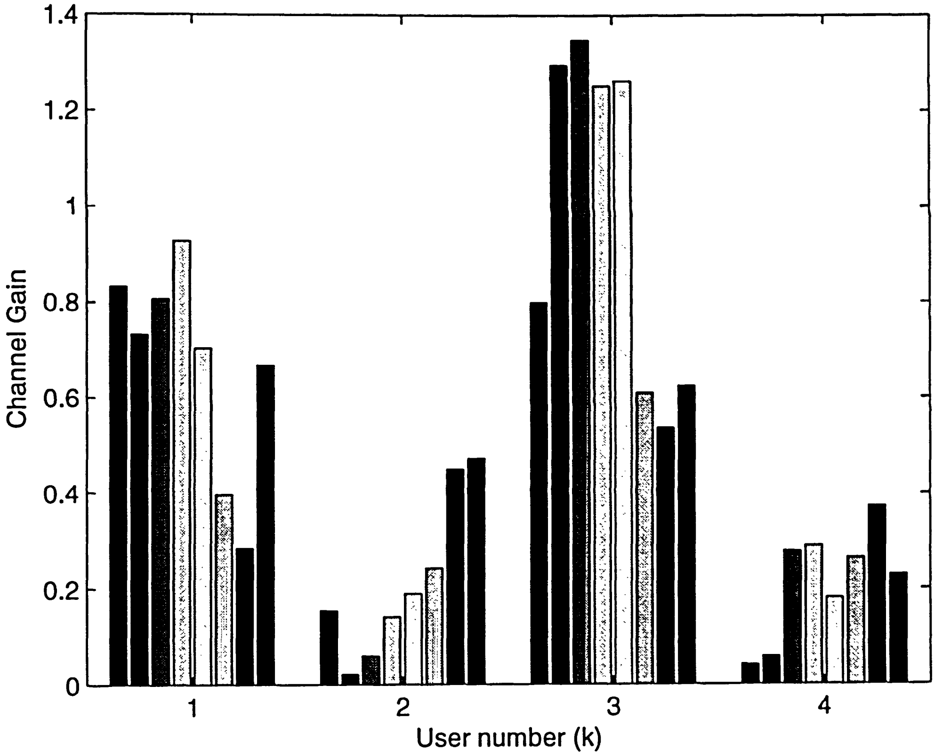

Figure 2.2: Snapshot of a wireless channel with eight subcarriers and four users, ($N = 8, K = 4$).

special case of constrained-fairness with equal proportional constraints.

2.4 Problem of Rate Adaptive Resource Allocation with Fairness

In order to formulate the problem, it is assumed that $U=\{1, 2, ..., K\}$ and $A=\{1, 2, ...N\}$ are the sets of users and subcarriers respectively. The data rate of the kth user R_k in bits/s is given by:

$$R_k = \frac{B}{N} \sum_{n=1}^{N} c_{k,n} \log_2 (1 + \gamma_{k,n}), \tag{2.1}$$

where B is the total bandwidth of the system and $c_{k,n}$ is the subcarrier assignment index indicating whether the kth user occupies the nth subcarrier. $c_{k,n} = 1$ if the subcarrier n is allocated to user k; otherwise it is zero. The bandwidth of each subchannel is $\frac{B}{N} = \frac{1}{T}$, where T is the OFDM symbol duration. Note that as the symbol duration is increased, the relative amount of dispersion in time caused by multipath delay spread decreases [12]. $\gamma_{k,n}$ is the signal-to-noise ratio (SNR) of the nth subcarrier for the kth user and is given by:

$$\gamma_{k,n} = p_{k,n} H_{k,n} = \frac{p_{k,n} h_{k,n}^2}{N_0 \frac{B}{N}}, \tag{2.2}$$

where $p_{k,n}$ is the power allocated for user k in subchannel n and $h_{k,n}$ and $H_{k,n}$ denote the real magnitude of the channel gain and channel-to-noise ratio for user k in subchannel n respectively. $N_0 \frac{B}{N}$ is the noise power on each subcarrier with $N_0/2$ as the power spectral density of AWGN.

(2.1) gives the data rate achieved by the kth user in a zero margin system. In practical modulation schemes however, the effective SNR has to be adjusted according to the modulation scheme for the desired BER, β. The difference between the SNR needed to achieve a certain data transmission rate for a practical system and the theoretical limit is the power loss called SNR gap. The SNR gap for MQAM modulation has been calculated in [25, 31] for certain cases as follows: If coherence phase detection is used, the BER for an AWGN channel with MQAM modulation is bounded by [31]:

$$\beta \leq 2e^{-1.5\gamma/(M-1)}, \tag{2.3}$$

where $M = 2^r$ and r denotes the number of bits. γ is the SNR defined as in (2.2). If $r \geq 2$ and $0 \leq \gamma \leq 30$ dB, BER could be better approximated within 1 dB by [25]:

$$\beta \leq 0.2^{-1.5\gamma/(M-1)}. \tag{2.4}$$

Using (2.4), the number of bits r is given by:

$$r = \log_2 \left(1 + \frac{\gamma}{\Gamma}\right), \tag{2.5}$$

where Γ is the SNR gap and a function of BER:

$$\Gamma = \frac{-\ln(5\beta)}{1.5}. \tag{2.6}$$

From (2.1), the total data rate R_T of the system is thus given by:

$$R_T = \frac{B}{N}\sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n}\log_2\left(1+\frac{\gamma_{k,n}}{\Gamma}\right), \tag{2.7}$$

where the required BER determines Γ.

The optimization problem with proportional rate constraints is then formulated as:

$$\begin{aligned}
&\text{Objective :}\\
&\max_{c_{k,n},p_{k,n}} \quad R_T = \frac{B}{N}\sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n}\log_2\left(1+\frac{p_{k,n}h_{k,n}^2}{N_0\frac{B}{N}\Gamma}\right),\\
&\text{subject to :}\\
&\qquad C1 : c_{k,n} \in \{0,1\}, \quad \forall k,n\\
&\qquad C2 : \sum_{k=1}^{K} c_{k,n} = 1, \quad \forall n\\
&\qquad C3 : p_{k,n} \geq 0, \quad \forall k,n\\
&\qquad C4 : \sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n}p_{k,n} \leq P_{total},\\
&\qquad C5 : R_1 : R_2 : ... : R_K = \alpha_1 : \alpha_2 : ... : \alpha_K,
\end{aligned} \tag{2.8}$$

In the formulation of the problem, the constraints are denoted by C1-C5. The first two constraints are on subcarrier allocation to ensure that each subchannel is assigned to only one user. The next two constraints are on power allocation where P_{total} is the total transmit power of the system. $\{\alpha_1, \alpha_2, ..., \alpha_K\}$ is the set of predetermined proportional constraints where α_k is a positive real number with α_{min}=1 for the user with the least required proportional rate.

2.4.1 Existing Solutions

The optimization problem given in (2.8) is generally very difficult to solve. It involves binary variables $c_{k,n}$ for subcarrier assignment and continuous variables $p_{k,n}$ for power allocation. Such an optimization problem is called a mixed binary integer programming problem. The nonlinear constraints in C5 further increase the difficulty in finding the optimal solution because the feasible set is not convex.

It was shown in [12] that the data rate of a multiuser OFDM system is maximized when each subcarrier is assigned to *only* one user with the best channel gain for that subcarrier and the total transmit power is distributed over the assigned subcarriers by the water-filling policy. Therefore, to determine $c_{k,n}$ for subcarrier allocation, it is assumed that subcarriers are not shared by different users. However, a common approach (e.g. in [4, 13]) in order to make the problem tractable (by converting it into a convex optimization problem) is to relax the constraint on $c_{k,n}$ to allow it to take any real value on the half-open interval (0,1]. Consequently, the subcarriers could be shared and the original maximization problem is transformed into a standard minimization problem. $c_{k,n}$ is not allowed to be zero since the objective is not defined for $c_{k,n}=0$. After obtaining the optimal solution for the problem, subcarrier n is assigned to the user that has the largest sharing factor $c_{k,n}$. If the number of subcarriers is much larger than the number of users, i.e. $N \gg K$, the sharing factors will be mostly zero or one, and the performance degradation is negligible.

Although the objective function becomes convex, the feasible set remains non-convex due to the nonlinear constraints in C5. If the constraints are linearized, the achieved solution would become slightly off the feasible set defined by the nonlinear constraints. Furthermore, deriving the optimal solution may still not be practical in real time applications due to high computational complexity. As a result, suboptimal algorithms have been developed which differ mostly in:

1. the approach they choose to split the procedure into several (preferably independent) steps to make the problem tractable and,

2. their simplifying assumptions to reduce the complexity of the allocation process.

The performance of each algorithm highly depends on the formulation of the problem and the validity of these simplifying assumptions.

In a system with K users and N subcarriers, each of the N subcarriers is to be allocated to one of K users. In addition, the power allocated to each of the K users should be optimized. Therefore, there are $K+N$ parameters to optimize to achieve the optimal solution and there are K^N possible subcarrier allocations assuming no subcarrier can be used by more than one user. Ideally, the subcarrier and power allocation should be carried out jointly which leads to high computational complexity necessitating suboptimal algorithms.

To solve this problem, a very simple but highly efficient algorithm was proposed in [13]. It was suggested in [3] that in a single-user system, the total data rate of the zero margin system is close to capacity even with flat transmit power spectral density (PSD) as long as the energy is poured only into subchannels with good channel gains. This is a very important result since it completely eliminates the major step of power allocation concentrating mainly on subcarrier allocation. Based on this assumption that each subchannel is assigned to the user whose channel gain is good for it, a flat transmit PSD was used in [13] indicating that the power allocated to each subcarrier is constant and equal to $\frac{P_{total}}{N}$. Therefore, the resource allocation reduces to only subcarrier allocation with N optimization parameters. In the process of subcarrier allocation, two goals take place alternatively: 1) maintaining fairness among the users by giving priority to the user with the least achieved rate to choose the next subcarrier; 2) maximizing the total data rate by allocating the best available subcarrier to that user. The suboptimal algorithm proposed in [13], simulated with equal proportional rate constraints, showed 50 $\sim$ 130 % of capacity gain over a non-adaptive TDMA resource allocation scheme. This algorithm achieves acceptable fairness as long as the number of subcarriers is much larger than the number of users i.e. $N \gg K$ [13].

Though proportional fairness amongst users in achieved in [13], the frequency selectivity of the channel is not fully considered by allocating power uniformly across all subcarriers belonging to a particular user. To improve its performance, Shen et al. [15] added a second step of adaptive power allocation to further enforce the rate proportionality among the users. They adopted a two step approach as follows: in the first step, the modified version of the algorithm outlined in [13] is employed for subcarrier allocation to achieve coarse proportional fairness. Hence, instead of giving priority to the user with the least achieved data rate R_k, priority is given to the user with the least achieved proportional data rate i.e., $\frac{R_k}{\alpha_k}$. In this step, the rate is calculated considering equal power on all the subcarriers. After subcarrier allocation is carried out, the problem is simplified into a maximization over continuous variables of power. In the second step, the power is reallocated between the users and then among the subcarriers through the use of water-filling to enforce the rate proportionality among the users. To find the kth user's power p_k, Lagrange multiplier techniques [32] are used to formulate and then solve the optimization problem resulting in K nonlinear equations with K unknowns. These equations can not be solved directly and numerical methods such as Newton-Raphson and its variants are used. Two special cases were analyzed in [15] which are described below. In each case, the computational complexity of the algorithm is calculated to be $O(K)$.

1) High channel-to-noise ratio case: Based on the fact that adaptive subcarrier allocation was used in the first step, it could be assumed that the best subchannels were chosen for each user and that they have relatively small channel gain differences among them. Furthermore, assuming the basestation can provide a large amount of power and the channel-to-noise ratio is high, the signal-to-noise ratio is much larger than 1. With these approximations, the system of K equations is transformed into a single nonlinear equation in one variable which could be solved using Newton's root finding method.

2) Linear case: In this case, it is assumed that the proportion of subcarriers assigned to each user is approximately the same as the rate constraints (also assumed in [7]). In other words:

$$N_1 : N_2 : ... : N_K = \alpha_1 : \alpha_2 : ... : \alpha_K. \tag{2.9}$$

The linear case was further investigated in [16]. In the proposed algorithm, the subcarrier proportionality in (2.9) is not just an assumption; on the contrary, this proportionality is enforced through the subcarrier allocation in the first step. In this algorithm, although the user with the least proportional capacity is still getting priority to choose its best available subchannel, the number of subchannels to be assigned to each user, N_k, is determined by its rate constraints given by $N_k = \lfloor \frac{\alpha_k N}{\sum_{k=1}^{K} \alpha_k} \rfloor$. Once the kth user gets the allotment of N_k subcarriers, it will be assigned no more subchannels until all the users are assigned their pre-determined proportion of subcarriers. With this approximation, it is shown in [15] that the system of K nonlinear equations (in the second step) turns into a K linear simultaneous equations which could be written in matrix form. The total power for each user, p_k, is obtained solving these K linear equations and the water-filling is applied to allocate the power to the assigned subcarriers of each user.

The suboptimal algorithms described above, either use fixed power allocation and perform only subcarrier allocation [13], or handle subcarrier and power allocation separately as in [15,16] to reduce the complexity of the algorithm. However, subcarrier and power allocation have to be carried out jointly to achieve the optimal solution. A very subtle but effective change in Rhee's algorithm [13] was made by Mohanram et al. [17] to perform joint subcarrier and power allocation thereby avoiding the second step of power allocation outlined in [15] or [16]. In the algorithm proposed in [17], optimization of $N + K$ parameters is carried out by alternating between subcarrier and power allocation. The allocation procedure is the same as [13] but differs in updating user's achieved data rate to find the user with the minimum achieved rate. When a subcarrier is allocated to a user, the power allocated to that user is incremented by $\frac{P_{total}}{N}$, i.e. the power allocated to each user is proportional to the number of subcarriers currently allocated to the user. The total power allocated to the user is then distributed among the assigned subcarriers with water-filling policy resulting in higher user rate. This updated rate information is then used in giving priority to the user with the minimum achieved rate to choose the next available subcarrier. Since the power redistribution is needed when there are more than one subcarrier assigned to a user, the water-filling should be performed $N - K$ times.

Interesting observations can be made from the simulation results: 1) The achieved total throughput [17, Fig. 1] is slightly higher compared to Shen's algorithm [15] with up to 25% gain (for 12 users) compared to Rhee's algorithm [13]. It even achieves up to 28% gain (for 12 users) when combined with the power allocation algorithm proposed in [15]. 2) Combining the additional step of optimal power allocation proposed in [15] with the joint resource allocation in [17] does not improve the data rate of the worst user (the user with the minimum achieved data rate) [17, Fig. 2]. 3) Finally, the algorithm shows higher achieved gain in total throughput compared to [13] when the PSD of AWGN is higher. This could be explained by the fact that applying water-filling versus fixed power allocation yields larger gains at low SNRs [33].

An overview of the algorithms described above is shown in Fig. 2.3. It also includes their main approaches and simplifying assumptions in formulating the problem and obtaining the solution.

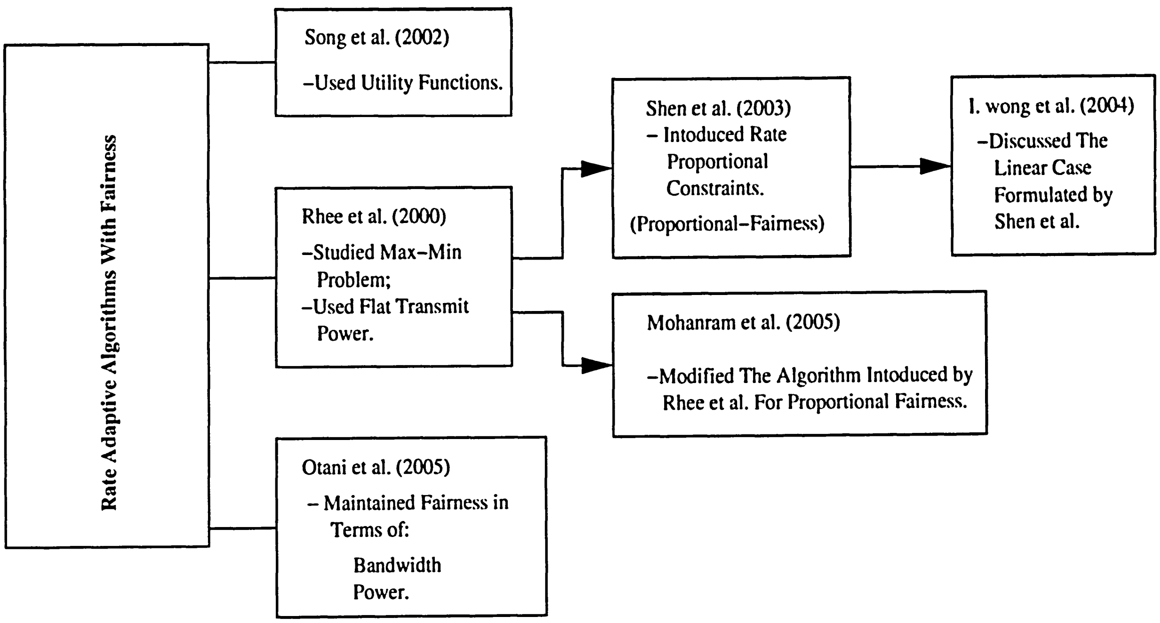

Figure 2.3: An overview of the existing solutions in the class of rate adaptive algorithms with fairness.

In Chapter 4, we propose a suboptimal subcarrier allocation algorithm which considers a new parameter in order to find the best user for each subcarrier. The objective is still to maximize the sum capacity while maintaining rate proportionality among the users.

2.5 Problem of Rate Adaptive Resource Allocation with Fixed Rate Constraints

The optimization problem of rate adaptive resource allocation with fixed user rate constraints is formulated the same as in (2.8). The only difference is in C5 which indicates the rate constraints.

$$
\begin{aligned}
&\text{Objective :}\\
&\max_{c_{k,n},p_{k,n}} \quad R_T = \frac{B}{N}\sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n}\log_2\left(1+\frac{p_{k,n}h_{k,n}^2}{N_0\frac{B}{N}\Gamma}\right),\\
&\text{subject to :}\\
&\qquad \mathrm{C1}: c_{k,n}\in\{0,1\},\ \ \forall k,n\\
&\qquad \mathrm{C2}: \sum_{k=1}^{K} c_{k,n}=1,\ \ \forall n\\
&\qquad \mathrm{C3}: p_{k,n}\geq 0,\ \ \ \forall k,n\\
&\qquad \mathrm{C4}: \sum_{k=1}^{K}\sum_{n=1}^{N} c_{k,n}p_{k,n}\leq P_{total},\\
&\qquad \mathrm{C5}: R_k\geq R_{k,min}\ \ \forall k.
\end{aligned}
\tag{2.10}
$$

C5 is to ensure that the achieved data rate of each user, R_k, is equal to or larger than its minimum required data rate denoted by $R_{k,min}$.

If there is a mixture of users with fixed required rates and users with variable rates, the objective remains the same and only C5 slightly changes to:

$$
\mathrm{C5}: \begin{array}{l} R_1 : R_2 : \ldots : R_L = \alpha_1 : \alpha_2 : \ldots : \alpha_L, \\ R_k \geq R_{k,min} \quad k = L+1, L+2, \ldots, K \end{array}
\tag{2.11}
$$

where it is assumed that among the first L users, rate proportionality should be maintained while the rest of the users require fixed minimum data rates.

2.5.1 Existing Solutions

In the optimization problem formulated in (2.10), the maximum rate could be still achieved by assigning each subcarrier to the user with the largest channel gain. However, although the total rate is maximized, the rate constraint of each user would not be satisfied. This problem was addressed by Yin et al. [7]. In [7], the problem has been partitioned into three steps: 1) The first step determines how many subcarriers N_k and how much power p_k are needed for each user; 2) The second step includes the subcarrier allocation which determines the particular set of subcarriers for each user; 3) The third step is the bit loading which determines the number of bits for each subcarrier or in other words, the power allocation on each user's assigned subcarriers.

The complexity of this method arises from the fact that N_k and p_k are not independent. Therefore, certain simplifying assumptions have been considered in each step. First, it is assumed that the number of the subcarriers and the power allocated to a particular user depend on its rate requirement and its average channel condition. Second, the amount of power assigned to each user is assumed to be proportional to the number of allocated subcarriers. There is no power allocation to the subcarriers in the first step, and only the total power of each user is determined. It is interesting to note that theses two assumptions constitute the very basic assumptions of the algorithms in [16] where a flat transmit power were used on all the subcarriers and the proportion of subcarriers for each user was assumed to be approximately the same as the eventual data rate after power allocation. To calculate N_k and p_k in [7], each user's channel condition is assumed to be flat on all subcarriers represented only by its average channel-to-noise ratio hence neglecting the frequency diversity in the first step.

Once N_k and p_k are determined, the exact subcarrier assignment is solved applying Hungarian algorithm [34]. Finally, the bits are loaded to the subcarriers knowing the total power of each user which is done by the known single user bit loading algorithm. It is in the second and the final step that each user's channel condition on each subcarrier is considered while being neglected in the first step.

The problem of rate adaptive resource allocation with a mixture of variable and fixed data rate constraints was introduced in [13] and was further discussed by Suh et al. [35] where it is assumed that there are K users and only one of them requires a fixed data rate called priority user.

In Chapter 4, we propose a suboptimal subcarrier allocation for the problem of rate adaptive resource allocation with fixed user data rates. The proposed algorithm consists of N major steps to assign N subcarriers to the users. The simulation results will be presented in Chapter 5.

2.6 Adaptive vs Fixed Modulation

Each subcarrier in a multiuser OFDM system, can potentially have a different modulation scheme and each modulation scheme provides a trade-off between spectral efficiency and BER. In OFDM systems with a fixed modulation scheme over all subcarriers, the carrier modulation is designed such that it maintains acceptable performance when the channel quality is poor. Thus, these systems are effectively designed for the worst channel condition. This results in such systems using BPSK or QPSK with poor spectral efficiency of 1 or 2 bits/s/Hz respectively. However, each subcarrier in a multipath frequency selective channel experiences different fade which might result in received power variation of as much as 30dB [36]. Consequently for a subcarrier with good channel gain, the modulation can be increased to 16-64 QAM significantly increasing the spectral efficiency of the overall system to 4-6 bits/s/Hz. In other words, using adaptive modulation, the subcarrier modulation is matched to its channel signal to noise ratio, maximizing the overall spectral efficiency.

There are several limitations with adaptive modulation. Adaptive modulation requires accurate channel estimates at the receiver and a reliable feedback path between the receiver and transmitter [25]. If the channel is changing faster than it can be estimated and fed back to the transmitter, adaptive techniques will perform poorly. Also, overhead information needs to be exchanged and updated regularly, as both the transmitter and receiver must know what modulation is being used which further increases the overhead with the mobility of the receiver.

Chapter 3

Resource Allocation in a Single User OFDM System

Although the existing wireless communication systems should support more than one user, the allocation algorithms for a single user OFDM system are important in the sense that they give a better understanding of the issues involved. Also, most of the algorithms for multiuser systems use the single user power allocation in one or more steps during their allocation process. In this chapter, several algorithms developed for subcarrier and power allocation to optimize the performance of a single user OFDM system are introduced. The results will be used in the suboptimal resource allocation proposed in the next chapter.

3.1 Water-filling

The process of water-filling [24] is the optimal solution to the problem of adaptively distributing the power among various channels with the objective of maximizing the total capacity. Basically, it is assumed that there are N independent Gaussian channels in parallel where each parallel component represents a different frequency. The channel models a non-white additive Gaussian noise channel and the noise level is assumed to be independent from channel to channel. Furthermore, it is assumed that there is a power constraint on the total transmit power. The objective is to distribute the total power among the channels so as to maximize the capacity.

It is shown in [24] that this problem could be reduced to a standard optimization problem with a total power constraint and can be solved using Lagrange multipliers. The solution indicates that more power should be allocated to the channels with the lowest noise level. When the available power is increased further, some of the power is put into noisier channels. The process by which the power is distributed among the various channels is identical to the way the water distributes itself in a vessel, hence the process is referred to as water-filing. The process is illustrated graphically in Fig. 3.1 where N_i and P_i , $i = 1, 2, 3$, denote the noise and the assigned power of each channel respectively..

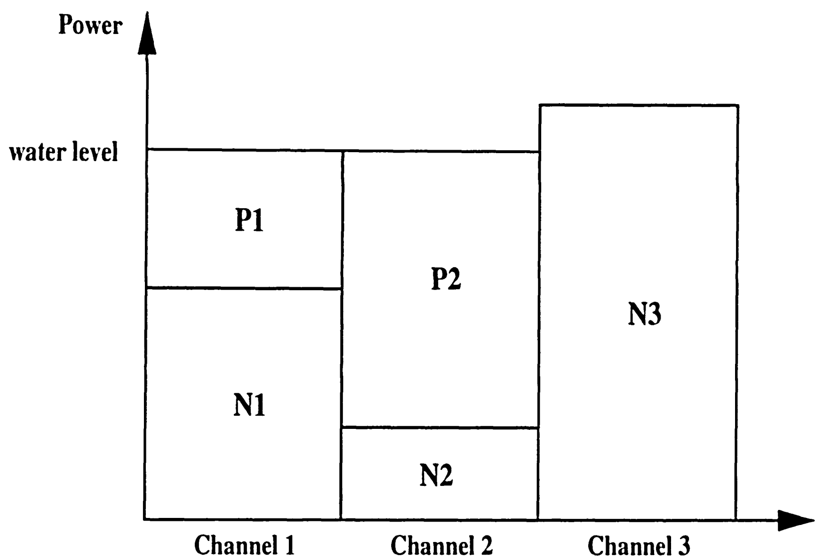

Figure 3.1: Water-filing for parallel channels.

The problem of adaptive power allocation for a single user with a total power constraint could be formulated as follows:

$$\begin{aligned} &\max_{p_n} \ \frac{B}{N}\sum_{n=1}^{N}\log_2\left(1 + p_n H_n\right) \\ \text{subject to:} & \\ &\sum_{n=1}^{N} p_n \leq P_{total}, \end{aligned} \tag{3.1}$$

where p_n is the power allocated to subcarrier n and H_n is the amplitude of channel-to-noise ratio for subcarrier n. P_{total} is the total power constraint. This problem assumes that there are N subcarriers available for a single user. The same problem applies to a multiuser OFDM system where the subcarrier allocation has been carried out and the total power for each user p_k is known (e.g., the second step in [14] or joint resource allocation in [17]). In such cases, the problem of resource allocation is reduced to K single user power allocation with fixed subcarrier assignment each formulated as in (3.1). In each problem, N indicates the total number of subcarriers assigned to the user.

Using Lagrange method, the above optimization problem with only one constraint (total power) is reformulated as:

$$L(p_n, \lambda) = \frac{B}{N}\sum_{n=1}^{N}\log_2(1 + p_n H_n) - \lambda\left[\sum_{n=1}^{N} p_n - P_{total}\right], \tag{3.2}$$

where L is the Lagrangian function and λ is the Lagrangian multiplier corresponding to the total power constraint. Since $\log_2(.)$ is convex and continuously differentiable over the interval of $[0, P_{total}]$, from the Kuhn-Tucker conditions [32], we will have:

$$\log_2(1 + p_n H_n)' - \lambda = 0, \tag{3.3}$$

where the derivative is calculated with respect to p_n. Hence, it becomes:

$$\frac{H_n}{1 + p_n H_n} - \lambda = 0, \quad \forall n. \tag{3.4}$$

The system of $N + 1$ simultaneous equations for $N + 1$ variables will then become:

$$\begin{cases} p_n = \left[\frac{1}{\lambda} - \frac{1}{H_n}\right]^+, \quad n = 1, 2, ..., N \\ \sum_{n=1}^{N} p_n = P_{total}, \end{cases} \tag{3.5}$$

where

$$[x]^+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}.$$

The operator shown by $[x]^+$ is necessary to ensure that the power allocated to each subcarrier is non-negative i.e., $p_n \geq 0$. The Lagrange multiplier λ is referred to as the level of water. Usually, the optimal power allocation could not be obtained directly from (3.5) and iterative algorithms are needed to obtain an appropriate water level λ.

Shen et al. [14] derived another optimal strategy for power distribution among the subcarriers of a single user in resource allocation for multiple users with proportional rate constraints. They addressed the problem by separating the subcarrier allocation from the power allocation. After the subcarrier allocation is carried out using Rhee's algorithm [13], the problem of power allocation among the users is formulated as:

$$
\begin{aligned}
&\text{Objective :} \\
&\max_{c_{k,n},p_{k,n}} \quad R_T = \frac{B}{N}\sum_{k=1}^{K}\sum_{n\in\Omega_k}\log_2\left(1+\frac{p_{k,n}h_{k,n}^2}{N_0\frac{B}{N}}\right), \\
&\text{subject to :} \\
&\qquad \text{C1} : \Omega_k \text{are disjoint for all k}, \\
&\qquad \text{C2} : \Omega_1 \cup \Omega_2 \cup ... \cup \Omega_K \subseteq \{1,2,...,N\}, \\
&\qquad \text{C3} : p_{k,n} \geq 0, \quad \forall k,n \\
&\qquad \text{C4} : \sum_{k=1}^{K}\sum_{n\in\Omega_k} p_{k,n} \leq P_{total}, \\
&\qquad \text{C5} : R_1 : R_2 : ... : R_K = \alpha_1 : \alpha_2 : ... : \alpha_K,
\end{aligned}
\tag{3.6}
$$

where N is the total number of subcarrier in the system, Ω_k is the set of subchannels for user k and Ω_k and Ω_l are disjoint when $k \neq l$ to ensure that each subchannel is assigned to only one user.

Using Lagrange method, the optimization problem is equivalent to finding the maximum of the cost function:

$$
\begin{aligned}
L &= \sum_{k=1}^{K}\sum_{n\in\Omega_k}\frac{B}{N}\log_2(1+p_{k,n}H_{k,n}) + \lambda_1\left(\sum_{k=1}^{K}\sum_{n\in\Omega_k}p_{k,n} - P_{total}\right) \\
&+\sum_{k=2}^{K}\lambda_k\left(\sum_{n\in\Omega_1}\frac{B}{N}\log_2(1+p_{k,n}H_{k,n}) - \frac{\alpha_1}{\alpha_k}\sum_{n\in\Omega_k}\frac{B}{N}\log_2(1+p_{k,n}H_{k,n})\right)
\end{aligned}
\tag{3.7}
$$

where $\{\lambda_k\}_{k=1}^{K}$ are the set of Lagrange multipliers. Differentiating (3.7) with respect to $p_{k,n}$ and setting each derivative to zero, the Kuhn-tucker conditions become:

$$
\begin{aligned}
\frac{\partial L}{\partial p_{1,n}} &= \frac{B}{N\ln(2)}\frac{H_{1,n}}{1+p_{1,n}H_{1,n}} + \lambda_1 + \sum_{k=2}^{K}\lambda_k\frac{B}{N\ln(2)}\frac{H_{1,n}}{1+p_{1,n}H_{1,n}} = 0 \\
\frac{\partial L}{\partial p_{k,n}} &= \frac{B}{N\ln(2)}\frac{H_{k,n}}{1+p_{k,n}H_{k,n}} + \lambda_1 - \lambda_k\frac{\alpha_1}{\alpha_k}\frac{B}{N\ln(2)}\frac{H_{k,n}}{1+p_{1,n}H_{1,n}} = 0
\end{aligned}
\tag{3.8}
$$

for $k = 2,3,...,K$ and $n \in \Omega_k$.

Using either of the equations in (3.8), it can be concluded that:

$$\frac{H_{k,n}}{1+p_{k,n}H_{k,n}} = \frac{H_{k,m}}{1+p_{k,m}H_{k,m}} \tag{3.9}$$

for any two subcarriers $n, m \in \Omega_k$. If the subcarriers of each users are sorted, (i.e., $H_{k,1} \leq H_{k,2} \leq ... \leq H_{k,N_k}$), the optimal power distribution for a single user k on its nth assigned subchannel can be rewritten as:

$$p_{k,n} = p_{k,1} + \frac{H_{k,n} - H_{k,1}}{H_{k,n}H_{k,1}}, \quad n = 1, 2, ..., N_k. \tag{3.10}$$

Note that N_k is the total number of subcarriers assigned to the kth user. As (3.10) shows, for each user (e.g., kth user), more power is put into the subchannels with higher channel-to-noise ratio. Shen referred to this as water-filling in frequency domain.

The total power of the kth user p_k is then given by:

$$p_k = \sum_{n=1}^{N_k} p_{k,n} = N_k p_{k,1} + \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,1}H_{k,n}}. \tag{3.11}$$

In a system where the total power for each user p_k and the subcarriers for each user N_k are known, the power distribution among the subcarriers is carried out as follows: First, the subchannel gains are sorted in ascending order. Using (3.11), the power of the subcarrier with the lowest channel-to-noise ratio $p_{k,1}$ is then calculated as:

$$p_{k,1} = \frac{p_k - \sum_{n=2}^{N_k} \frac{H_{k,n} - H_{k,1}}{H_{k,1}H_{k,n}}}{N_k}. \tag{3.12}$$

The power allocated to the rest of the subchannels $p_{k,n}, n = 2, 3, ..., N_k$ is then calculated using (3.10). As shown in (3.10), the power of the nth subcarrier consists of two parts: The first portion is the power allocated to the first subcarrier and the second portion depends on the difference of nth and the first (worst) subcarriers' channel gains.

3.2 Greedy Algorithm: Bit Allocation

One of the well-known single user bit allocation solutions is the greedy algorithm [4]. When the subcarriers assigned to a user are known, the optimization problem reduces to bit loading for a single user. The greedy algorithm then assigns bits to the subcarriers one bit at a time and in each iteration, the subcarrier that requires the least additional power is selected. The bit allocation process continues till all the required number of bits are assigned (e.g., in a margin adaptive system with a fixed rate requirement R) or when the total allocated power is equal or more than the transmit power constraint (e.g., in a rate adaptive system with total power constraint P_{total}). The basic structure of the greedy algorithm is as follows:

- Initialization
  $r_n = 0, \forall n$
  $R_{tot} = 0,$
  $\Delta P_n = \frac{f_n(1)-f_n(0)}{h_n^2};$
- Bit Assignment Iteration:
  $n = \arg\min_n \Delta P_n,$
  $r_n = r_n + 1,$
  $R_{tot} = R_{tot} + 1,$
  $P_{tot} = P_{tot} + \Delta P_n;$
  $\Delta P_n = \frac{f_n(r_n 1)-f_n(r_n)}{h_n^2};$
  end

In the algorithm outlined above, r_n and P_n are the number of bits and the power allocated to subcarrier n respectively and $f_n(r)$ is the corresponding receive power (in energy per symbol) required for reliable reception of r bits/symbol which depends on the minimum bit error rate and the modulation scheme. R_{tot} and P_{tot} are the total number of bits and the total transmit power respectively.

The initialization stage computes for each subcarrier the additional power to transmit an additional bit. The additional bit is assigned to the subcarrier that needs the minimum additional power and then the new additional power for that subcarrier is updated. In a system with total power constraint P_{total}, the algorithm stops when $P_{tot} \geq P_{total}$ whereas in a margin adaptive system with fixed rate requirement R, the minimum required transmit power is obtained when the addition of the last bit leads to $R_{tot} \geq R$. Different modulation schemes will lead to different $f_n(r)$, thus different bit allocation and total transmit power.

This algorithm is optimal since $f_n(r)$ is an increasing function of r and that the power needed to transmit a certain number of bits in a subcarrier is independent of the numbers of bits allocated to other subcarriers.

3.3 Suboptimal Power Allocation: Flat Transmit Power

In a multiuser system with K users and N subcarriers, there are N parameters to optimize in order to assign the subcarriers to the users. To each subcarrier, a portion of the total transmit power should be allocated hence, there are N parameters in order to allocate the power to all the subcarriers. However, if the total power allocated to a user is known, it will be optimally distributed among its assigned subcarriers with water-filling policy. Therefore, the problem is formulated to have a total of $N + K$ parameters to optimize to allocate subcarriers and distribute power among the users.

These $N+K$ parameters should be optimized jointly using iterative algorithms which still leads to high computational complexity. An approach to gain a significant computational advantage is to use a flat energy distribution over the entire bandwidth. A flat transmit power spectral density would be necessary in case there is a power mask constraint on each subcarrier and it is tighter than the total power constraint [13]. This approach would significantly reduce the complexity of the allocation problem as it reduces the problem to only subcarrier allocation with only N parameters to optimize. However, this complexity reduction is only worthy if the performance degradation is negligible compared to optimal water-filling method.

One way to gain insight about the validity of this simplifying approach is to examine the performance of optimal and suboptimal power allocation (with flat power spectral density) in a single user OFDM system. In [3], a very simple algorithm is proposed with the objective of finding the optimal transmission bandwidth in terms of maximizing the overall data throughput under the total power constraint. The system has one user with N available subcarriers and P_{total} as the total power constraint. The objective is to find the optimum number of subcarriers N_{opt} from the available subchannels N such that $N_{opt} \leq N$ and the power on each subchannel is $\frac{P_{total}}{N_{opt}}$. The algorithm is summarized as follows:

- Step 1: Initialization
  $r_{max} = 0$,
  $N_{opt} = 0, m = 1$;
  $p = \frac{P_{total}}{m}$;

- Step 2: Sort the subcarriers in descending order such that
  $H_n \geq H_{n+1}, \forall n < N$

- Step 3: Subcarrier Assignment Iteration:

$r_{ach} = \sum_{n=1}^{m} \log_2 (1 + pH_n)$,
if $r_{ach} > r_{max} : r_{max} = r_{ach}, N_{opt} = m$;
if $m \neq N : m = m + 1$, go to step 3;
end

In the algorithm outlined above, r_{max} is the maximum number of bits that could be possibly obtained by equally distributing the available power P_{total} among the subcarriers. Initially, there are N subcarriers available. However, it might not be optimal to use all the N subcarriers. Therefore, the number of subcarriers used at each iteration and the optimal number of subcarriers are updated and recorded in m and N_{opt} respectively. In Step 2, the subchannels are sorted in descending order based on their channel-to-noise ratio. At each iteration, the number of used subcarriers is incremented and the achieved number of bits r_{ach} is updated. An increase in r_{ach} indicates that the added subcarrier should be used. Otherwise, if there is no increase or if there is a decrease in r_{ach}, the algorithm stops and $\{n\}_1^{N_{opt}}$ gives the set of optimal subcarriers which is the first N_{opt} subcarriers in the initial sorted set.

This algorithm yields the optimum system bandwidth with flat power spectral density for two reasons: First, $\log_2(x)$ is a non-decreasing function with non-decreasing x. By calculating the achieved number of bits at each iteration, the optimum number of usable subchannels is determined and since the subchannels are initially sorted in descending order, the best set of subchannels for the obtained number is chosen over all N possibilities.

In order to compare the performance of a system with optimal water-filling and suboptimal flat power allocation, we simulated two of the algorithms described above. The system under consideration has only one user with N subchannels. The total transmit power is P_{total}. The user can use all N subchannels. The first algorithm is based on Shen's equations for single user power allocation derived in (3.11) and (3.12) which used water-filling to distribute power among the subcarriers. The second is the suboptimal power allocation which finds the best number and set of subcarriers with flat transmit power. Both algorithms use the same channel information as the subchannel gains. The simulation results are summarized in Chapter 5.

Chapter 4

Proposed Suboptimal Resource Allocation Algorithms for Rate Adaptive Multiuser OFDM Systems

Ideally, subchannels and power should be allocated jointly to the users. However, this poses a computational burden at the basestation in order to reach the optimal solution. Furthermore, the basestation has to rapidly compute the optimal subchannel and power allocation as the wireless channel changes. Hence, low-complexity suboptimal algorithms are preferred for cost-effective and delay-sensitive implementations. To achieve this goal, it is necessary to consider certain simplifying assumptions to further reduce the complexity of the optimization problem to make it tractable. These specific assumptions distinguish each algorithm and its special approach from the rest, resulting in a different data rate and total transmit power. A way to reduce the complexity of the allocation procedure is to carry out the subcarrier assignment and the power allocation separately. In this chapter, we propose two suboptimal subcarrier algorithms: The objective of both is to maximize the total throughput of the system within a fixed total transmit power. In the first algorithm (Section 4.1), rate proportionality it to be maintained among the users while in the second (Section 4.3), the users should be supported with minimum fixed rate requirements.

4.1 Suboptimal Subcarrier Allocation Algorithm for Multiuser System with Proportional Rate Constraints

In this section, a suboptimal subcarrier allocation algorithm is proposed based on the sensitivity of the users to the subcarrier allocation. In the proposed algorithm, equal power distribution is assumed across all subchannels. The algorithm is based on prioritizing the critical (most sensitive) user in the system, and the variance of the subchannel gains for each user is used to define the sensitivity of the user to the subcarrier allocation. To describe the method, a snapshot of the channel characteristics for a system with two users and eight subchannels is shown in Fig. 4.1.

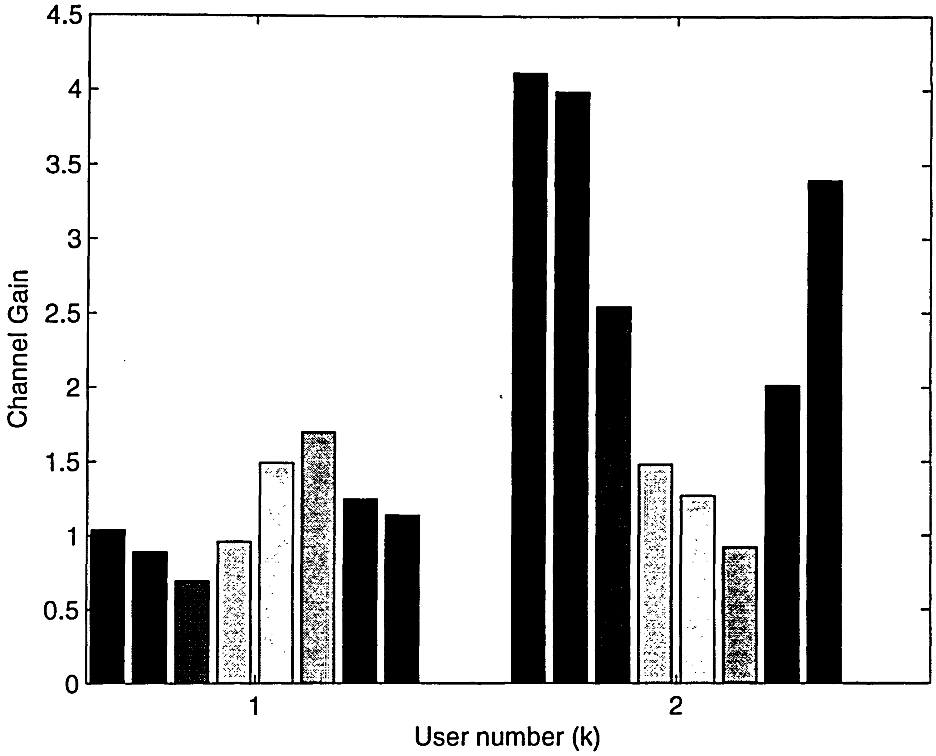

Figure 4.1: A snapshot of the channel with two users and eight subchannels.

Table 4.1: The channel characteristics of a two-user system shown in Fig. 4.1

User Number	user 1	user 2
Variance	**0.1073**	**1.5617**
Min Channel Gain	0.6929	0.9229
Max Channel Gain	1.6989	4.1256

As shown in Fig. 4.1, the channel gains of user 1, have a small variance of 0.1073 while user 2 has subchannels with channel gains changing from 1.6989 to 4.1256 and the variance of 1.5617. This information is tabulated in Table 4.1. The changes in subchannel gains result from multiuser diversity and frequency selectivity of the channel. In the wireless multipath channel under consideration, the fading parameters for the users are mutually independent. Therefore, each subchannel experiences different fades for different users. This phenomenon is referred to as multiuser diversity. It is due to multiuser diversity that a subchannel is rarely in deep fade for all the users. As a result of frequency selectivity of the channel, different subchannels of the same user experience different levels of fade. However, how different they undergo fading could be measured by their channel gain variance as indicated in Table 4.1.

Considering subcarrier i and subcarrier j for the kth user, the change in data rate of the kth user due to assigning subcarrier i instead of subcarrier j, $\Delta R_k(i,j)$ is given by:

$$\begin{aligned} \Delta R_k(i,j) &= R_k(i) - R_k(j) \\ &= \frac{B}{N}\left[\log_2(1+\gamma_{k,i}) - \log_2(1+\gamma_{k,j})\right] \\ &= \frac{B}{N}\log_2\left(\frac{1+p_{k,i}H_{k,i}}{1+p_{k,j}H_{k,j}}\right), \end{aligned} \tag{4.1}$$

where $R_k(i)$ and $R_k(j)$ are the kth user's achieved data rates on subcarriers i and j respectively. Without loss of generality, we assume that $H_{k,i} > H_{k,j}$. The maximum change in the achieved rate of the kth user is then given by:

$$\Delta R_{k,max} = \frac{B}{N}\log_2\left(\frac{1+p(H_{k,mean}+s_k)}{1+p(H_{k,mean}-s_k)}\right), \tag{4.2}$$

where $H_{k,mean}$ is the average channel-to-noise ratio for user k, s_k is the kth user's channel gain standard deviation from the mean and a flat transmit power p is assumed on all the subcarriers. Now if s_k is zero then $\Delta R_k(i,j) = 0$ implying that it results in no difference in the kth user's data rate whether it chooses subcarrier i or subcarrier j. However, as s_k increases, so does $\Delta R_k(i,j)$ increasing the sensitivity of the user's data rate to the subcarrier allocation.

In other words, from the user's point of view, if the variance of the subchannel gains for the kth user is small (e.g., user 1 in Fig. 4.1 and Table 4.1), it matters less which subchannel is allocated to user k since the subchannels experience almost the same amount of fade for this user. On the other hand, the user with high variance of subchannel gains (e.g., user 2 in Fig. 4.1 and Table 4.1) is more sensitive to the subchannel allocation. The user's point of view is not important when the objective of the resource allocation is to maximize the overall throughput of the system regardless of individual's achieved data rate. However, when fairness is an issue, it is inevitable that each user should be allocated some portion of the bandwidth.

In [13,14,16,17], the user with the least proportional data rate has the priority to choose the next subcarrier. In the proposed algorithm, priority is given to the critical (most sensitive) user with the largest variance on channel gains to choose its best subcarrier. However, if a user has a large variance and this characteristic continues to hold, then this user ends up getting all the subchannels while the others get nothing. To solve this problem, it is assumed that the proportion of the subcarriers assigned to each user is approximately the same as the rate constraints as in (2.9), which is repeated here:

$$N_1 : N_2 : \dots : N_K = \alpha_1 : \alpha_2 : \dots : \alpha_K,$$
$$N_k = \left\lfloor \frac{\alpha_k N}{\sum_{k=1}^{K} \alpha_k} \right\rfloor, \tag{4.3}$$

where N_k is the number of subcarriers to be assigned to user k and α_k is the kth user's proportional rate constraint as defined in (2.8).

The proposed algorithm is described below: In the first step, all the variables are initialized. A and A^* are the sets of available (unallocated) and allocated subcarriers respectively. N^* is the sum of minimum number of subcarriers initially required by the users derived from (4.3) and U is the set of all users.

In the second step, the variance of the subchannel gains for each user V_k is calculated. The critical user with the largest variance is then given priority to choose its best subcarrier. At each iteration, the assigned subcarrier is eliminated from the set and the gain variance of each user is updated taking into account only those subcarriers that have not yet been assigned. Once user k gets the allotment of N_k subcarriers, that user can no longer be assigned more subchannels.

- Initialization
  $c_{k,n} = 0, \ \forall k, n$
  $R_k = 0, \ \forall k$
  $A = \{1, 2, \dots, N\},$
  $A^* = \emptyset,$
  $N^* = \sum_{k=1}^{K} N_k$
  $U = \{1, 2, \dots, K\}.$

- Subcarrier Allocation

– for $m = 1$ to N^*

$V_k = \text{Var}(H_{k,n}),\ \forall n \in A$

$k = \arg\max V_k,\ (\text{if } N_k > 0)$

$n = \arg\max_{n \in A} H_{k,n},$

$c_{k,n} = 1, N_k = N_k - 1,$

$A = A - \{n\}$ and $A^* = A^* \bigcup \{n\},$

$R_k = R_k + \frac{B}{N}\log_2(1 + \gamma_{k,n}).$

– while $A \neq \emptyset$

Scenario 1:

$k = \arg\min(R_k/\alpha_k),$

$n = \arg\max_{n \in A} H_{k,n},$

$c_{k,n} = 1, N_k = N_k + 1,$

$A = A - \{n\}$ and $A^* = A^* \bigcup \{n\},$

$R_k = R_k + \frac{B}{N}\log_2(1 + \gamma_{k,n}).$

Scenario 2:

for $n = 1$ to $(N - N^*)$

$k = \arg\max_{k \in U} H_{k,n},$

$c_{k,n} = 1, N_k = N_k + 1,$

$R_k = R_k + \frac{B}{N}\log_2(1 + \gamma_{k,n}),$

$U = U - \{k\}.$

end

The remaining subchannels are allocated in the final step. We can have different scenarios based on the flexibility in the objective of the algorithm and the number of unallocated subchannels:

- Scenario 1: If adherence to the proportionality constraints needs be strictly enforced, then the user with the least achieved proportional data rate should be given priority to choose the best available subcarrier.

- Scenario 2: If a rough proportionality is acceptable, each user gets at most one subcarrier. In order to further increase the total rate, the user with the largest channel gain chooses the first share.

For both scenarios, equal power allocation among the subcarriers is assumed. In the simulation results, we refer to this algorithm as Alg. 1.

4.2 Fairness Index

The fairness index is defined as [15]:

$$F = \frac{\left(\sum_{k=1}^{K} \alpha_k\right)^2}{K \sum_{k=1}^{K} \alpha_k^2}, \tag{4.4}$$

with the maximum value of 1 to be the fairest case in which all users would achieve the same data rate. Based on the above equation, we define a new parameter F_p to examine the performance of the system in maintaining proportional fairness which is given by:

$$F_p = \frac{\left(\sum_{k=1}^{K} \frac{R_k}{\alpha_k}\right)^2}{K \sum_{k=1}^{K} (\frac{R_k}{\alpha_k})^2}, \tag{4.5}$$

where R_k and α_k are the achieved rate and the proportional rate constraint for the kth user respectively. F_p is a real number in the interval (0,1] with the maximum value of 1 for the case that the achieved rate proportions among the users are the same as the predetermined set $\{\alpha_k\}_{k=1}^{K}$.

4.3 Suboptimal Joint Subcarrier and Power Allocation Algorithm for Multiuser System with Fixed Rate Constraints

When the objective is to ensure fairness among the users, the subcarrier allocation introduced in [13] was shown to achieve close to optimal performance with very low complexity. The same but normalized version of this method was adopted in [14, 16, 17] to ensure rate proportionality where the normalization is with respect to the proportional rate constraints. However, with a slight change in this method, a very simple algorithm is obtained which could apply to a class of rate-adaptive optimization problem where the goal is to ensure each user achieves its minimum required data rate.

In this method, instead of prioritizing the user with the minimum achieved data rate (or proportional data rate in case of proportional-fairness), the user with the highest data rate to achieve is giving priority to choose the next subcarrier. The method is illustrated in Fig. 4.2.

Fig. 4.2 shows two users with different required data rates denoted by $R_{i,min}$ and $R_{j,min}$.

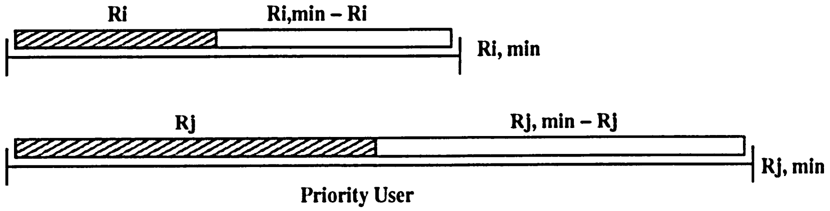

Figure 4.2: Prioritizing the user with the highest data rate to achieve.

The achieved data rates are denoted by R_i and R_j respectively. In this method, the user that requires the highest data rate, i.e. the jth user with $(R_{j,min} - R_j)$ to achieve, is given priority to choose the next subcarrier although it is the user that already has achieved a higher data rate. Note that If the rate constraints are equal for all the users, i.e. $R_{i,min} = R_{j,min} = R_{min}$ (complete fairness), then the user requiring the largest data rate, $\hat{k}$:

$$\begin{aligned} \hat{k} &= \arg\max_{k} \, (R_{min} - R_k) \\ &= \arg\min_{k} \, R_k. \end{aligned} \tag{4.6}$$

would become the user with the minimum achieved data rate. Therefore the proposed algorithm would have the same performance as that described by Rhee et al [13].

The proposed joint subcarrier and power allocation algorithm consists of N major steps to assign all N subcarriers to the users which is described below:

- Step 1: Initialization
  $c_{k,n} = 0, \ \forall k, n$
  $R_k = 0, \ \forall k$
  $A = \{1, 2, ..., N\}$,
  $U = \{1, 2, ..., K\}$.

- Step 2: Subcarrier Allocation
  while $(A \neq \emptyset$ or $U \neq \emptyset)$:
  $k = \arg\max_{k \in U}(R_{k,min} - R_k)$,
  $n = \arg\max_{n \in A} H_{k,n}$,
  $c_{k,n} = 1, A = A - \{n\}$,
  R_k(updated with water-filling policy),
  if $R_k \geq R_{k,min}$ then $U = U - \{k\}$.
  end.

In the first step, all the parameters are initialized. In the second step, the user with the highest required data rate is given priority to choose its best available subcarrier. Assuming a flat transmit power over the entire bandwidth, each subcarrier adds an equal portion of the total power ($\frac{P_{total}}{N}$) to the user it has been assigned to. The current power of each user P_k is then allocated to its subcarriers by either water-filling policy as in [17] or greedy algorithm described in Section 3.2. After each iteration, the assigned subcarrier is excluded from the set of available subcarriers A, and the difference between the original required data rate and the achieved data rate becomes the new data rate constraint for each user. The procedure continues till all the subcarriers are assigned to the users or all the users achieve their rate requirements. In case there are unused subcarriers, they would be assigned to their best user in the system to further increase the total throughput (not shown above), or may be reserved for future use.

The main characteristic of the proposed algorithm is its low complexity. Also, by reallocating the power among the subcarriers after assigning each subcarrier, we ensure that each user has achieved its maximum data rate within its allocated power. Therefore, the goal of supporting the users' required rates is combined with the goal of maximizing the total throughput of the system. In the simulation results, the proposed algorithm in this section is referred to as Alg. 2.

Chapter 5

Simulation Results

In this chapter, simulation results are presented to show the performance of the proposed resource allocation algorithms. First, the simulation parameters are introduced. Simulation results are then presented in two parts: In Section 5.2, the performance of a single user system with optimal water-filing power allocation is compared with that of a single user system with suboptimal flat transmit power (described in Chapter 3). The results will be used in developing two suboptimal subcarrier allocation algorithms for multiuser OFDM systems proposed in Chapter 4. The performance of the proposed algorithms compared with the suboptimal existing solutions in terms of total data rate and fairness will be given in Sections 5.3 and 5.4.

5.1 Simulation Parameters

In this section, the channel model and the simulation parameters are introduced. In all the simulations, the wireless channel is modelled as a frequency selective channel consisting of six independent Rayleigh fading multipaths with exponential power profile.

The Clarke's model [29] has been used to characterize each flat fading multipath. In each multipath, M scattered components arrive at the receiver each with E and H field components. The E-field can be expressed as an in-phase $T_c(t)$ and quadrature form $T_s(t)$ as:

$$E(t) = T_c(t)\cos(2\pi f_c t) - T_s(t)\sin(2\pi f_c t) \tag{5.1}$$

where f_c is the carrier frequency and the two components are given by:

$$
\begin{aligned}
T_c(t) &= E_0 \sum_{m=1}^{M} A_m \cos(2\pi f_m t + \phi_m) \\
T_s(t) &= E_0 \sum_{m=1}^{M} A_m \sin(2\pi f_m t + \phi_m)
\end{aligned}
\tag{5.2}
$$

where E_0 is a real constant, representing the local average E-field and A_m is a real random variable representing the amplitude of individual waves. The amplitude of $E(t)$ is normalized such that the ensemble average of the A_m's is given by $\sum_{m=1}^{M} \overline{A_m^2} = 1$. $f_m = f_d \cos \alpha_m$ where f_d is the maximum Doppler frequency and α_m denotes the angle between the direction of the mobile's receiver and the direction at which the mth wave arrives. ϕ_m is the phase angle of the mth wave.

If M is sufficiently large, $E(t)$, $T_c(t)$ and $T_s(t)$ will all be Gaussian random processes at any time t. T_c and T_s are uncorrelated zero-mean Gaussian random variables with an equal variance of $E_0^2/2$. The envelope of the received E-field is given by:

$$
|E(t)| = \sqrt{T_c^2 + T_s^2} = r(t) \tag{5.3}
$$

and it could be shown that the random received signal envelope r has Raleigh distribution provided that T_c and T_s are Gaussian random variables.

Modelling the channel and running the simulations have all been done in Matlab. A_m and ϕ_m are both normal random variables with zero mean and variance of one. The random numbers generated for A_m are normalized as mentioned above. The phase angles α_m are assumed to have uniform distribution on the interval $(0,2\pi]$. It is assumed that the power delay profile is exponentially decaying with e^{2l} where l is the multipath index. After calculating the in-phase and quadrature components for all six multipaths for each user, the N-point FFT is taken which results in N complex numbers where N is the total number of subcarriers. The magnitude of each complex number represents the amplitude of the subchannel gain for that specific user.

Normalized values of 1W and 1MHz are chosen for the total power and the bandwidth respectively to simplify the comparison. There are $N = 64$ subcarriers in the system and perfect knowledge of subchannel gains is assumed (The subchannel gains are obtained as described above). After each channel realization, different resource allocation algorithms were run using the same subchannel gains. A total of 1000 different channel realizations were used and the results were averaged.

5.2 Single User Resource Allocation

In this section, two single-user algorithms are compared in terms of achieved total data rate. In the first algorithm, the power is distributed among the subcarriers using water-filling policy given by (3.9)-(3.12). In the second algorithm, the optimal number of subcarriers and the best set to transmit the data is found. A flat transmit power is then assumed on all the assigned subcarriers.

Fig. 5.1 shows the performance of the suboptimal algorithm (with flat transmit power) in terms of the achieved data rate versus number of subcarriers involved in the transmission. There are 64 subcarriers in the system. Starting with one subcarrier, if increasing the number of subcarriers results in an increase in the achieved data rate, more subcarriers are used in data transmission. At each iteration, the power allocated to each subchannel is the total power divided by the number of subcarriers involved in that iteration.

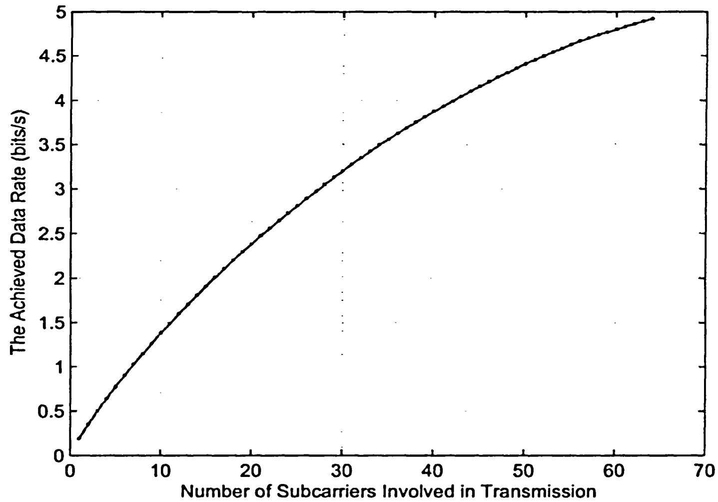

Figure 5.1: (Suboptimal algorithm) Achieved data rate versus the number of subcarriers used in transmission. Total number of subcarriers is $N = 64$ and BER $= 10^{-3}$.

Fig. 5.2, shows the achieved data rate in each algorithm versus the total number of available subcarriers, $N = 10 \sim 100$. In this figure, for any desired number of subcarriers N, the channel is realized 1000 times and the average achieved data rates are recorded. In water-filling policy, all the subcarriers are always used unless the calculated power for the subcarrier with the least channel gain (given by (3.12)) becomes negative. This happens when the gain difference between the subcarriers are so large. In this case, the first subchannel in the ascending order set (the subchannel with the least channel gain) is not allocated any power and the power distribution starts with the second subchannel.

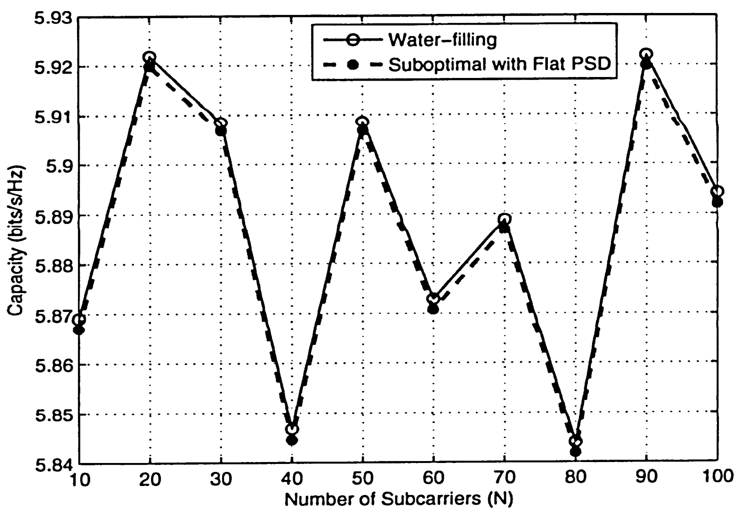

Figure 5.2: Achieved data rate versus the total number of subcarriers, N.

The portions of power allocated to the subchannels in water-filling algorithm indicate their importance or share in data transmission. In the suboptimal algorithm with flat transmit power, however, the power is equally distributed among all the subcarriers that are involved in data transmission. It is clearly seen that the suboptimal algorithm with flat transmit power achieves almost the same data rate on average, compared to the one with water-filling power distribution.

Based on the results derived above, one may conclude that in a system with N subchannels, a flat transmit power over all the subcarriers would give close to optimum performance. However, this is not always the case. This rate difference between the optimal and suboptimal power allocation is negligible only when in the suboptimal algorithm, the optimal number and set of subcarriers are chosen to transmit data and the total power is equally distributed among these subcarriers while the rest of subchannels are allocated no power. In other words, when the power is equally distributed only among the subchannels with good channel gains, then the performance is close to optimal. The reason that in multiuser OFDM systems, a flat transmit power is used over the entire bandwidth not just a group of selected subcarriers is that it is assumed that due to multiuser diversity, only subchannels with good channel gains are assigned to each user; hence almost all the subchannels involved in data transmission are in good channel condition.

5.3 Suboptimal Subcarrier Allocation in Multiuser OFDM System with Proportional Rate Constraints

In this section, the performance of Alg. 1, proposed in Section 4.1, is compared with the existing algorithms [13,15]. The objective is to reach the highest achievable data rate while maintaining the rate proportionality among the users according to the given proportional constraints.

The simulation results are presented in two parts: In the first part (Section 5.3.1), the comparison is made over the achieved total data rate as well as fairness with different rate proportionality among the users. In the second part (Section 5.3.2), the rate constraints are set to be equal. The comparison is made between Alg. 1, the algorithm proposed in [13] with flat PSD and the method proposed in [14] with adaptive power allocation.

5.3.1 Non-equal Proportional Constraints

Fig. 5.3 shows the achieved spectral efficiency in Alg. 1 and the method proposed in [14] for average SNR ranging from 10-40dB. $K = 16$ and the proportional constraints are randomly chosen from the set $\{1, 2, 4\}$ for each channel realization. The average SNR is defined as $\frac{P_{total}}{N_0 B}$.

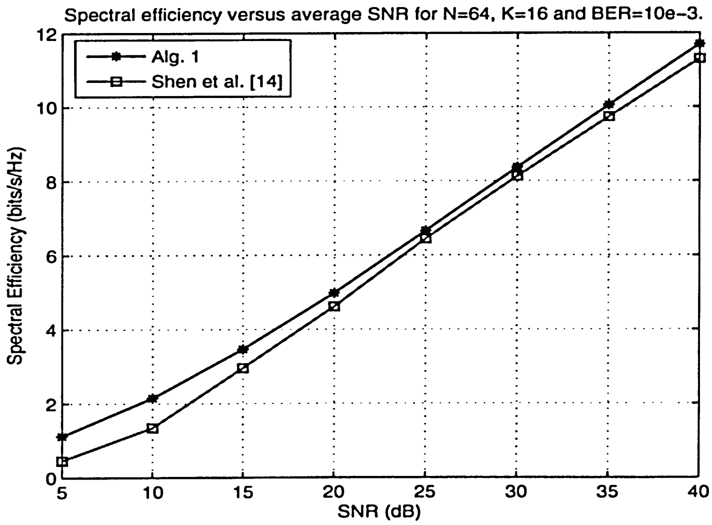

Figure 5.3: Spectral efficiency versus average SNR for $N = 64$ subcarriers and $K = 16$ users. BER $= 10^{-3}$.

It can be seen that the proposed algorithm has achieved slightly higher total data rate compared to method [14]. This advantage is due to relaxation of the proportionality constraints which were enforced in [14] through the reallocation of the power among the users and subcarriers.

The comparison of the two algorithms in terms of rate proportionality is shown in Fig. 5.4. The leftmost bars are the normalized constraints $\{\phi_k\}_{k=1}^{K}$, where $\phi_k = \frac{\alpha_k}{\sum_{k=1}^{K}\alpha_k}$. The same normalization is used for achieved data rates. It is shown that the method in [14] has better performance since it applies adaptive power allocation to enforce the proportionality among the users.

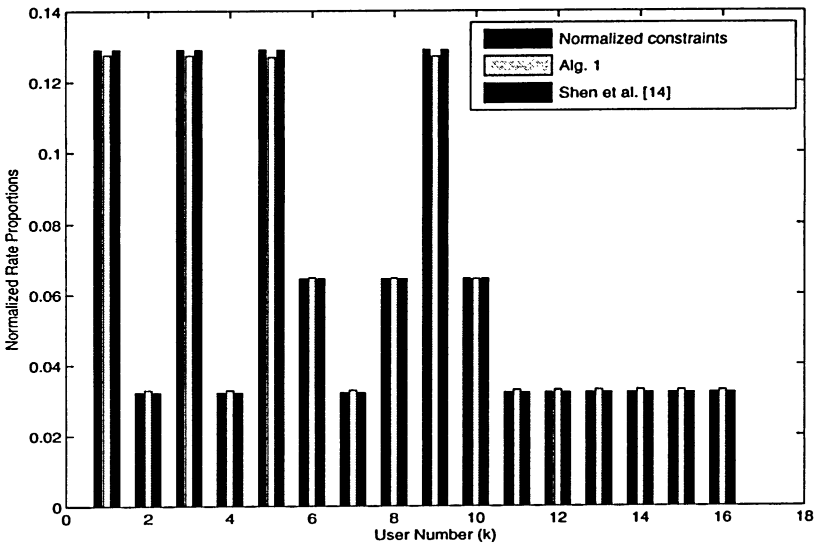

Figure 5.4: Normalized capacity ratios per user for SNR = 30dB, K = 16 and BER=10^{-3}.

The rate proportionality in Alg. 1 is enforced through the number of subcarriers assigned to each user. However, the performance of our proposed algorithm is close to the required proportional constraints with no power reallocation. To better examine the proportional fairness of these algorithms for different number of users, their performance is shown in Fig. 5.5 in terms of F_p using equation (4.5). It is shown that $0.999 < F_p < 1$ for the proposed algorithm whereas $F_p = 1$ for the method in [14].

5.3.2 Equal Proportional Constraints

In this part, $\alpha_k = 1\ \forall k$, enforcing all the users to get the same amount of data rate. Fig. 5.6 shows the comparison of the minimum user's capacity between Alg. 1 and the methods proposed in [13] and [14] for $K = 2 \sim 16$ users. Both Alg. 1 and the method in [13] apply the power mask of $p_n = \frac{P_{total}}{N}$ on the subcarriers whereas the algorithm in [14] applies a second

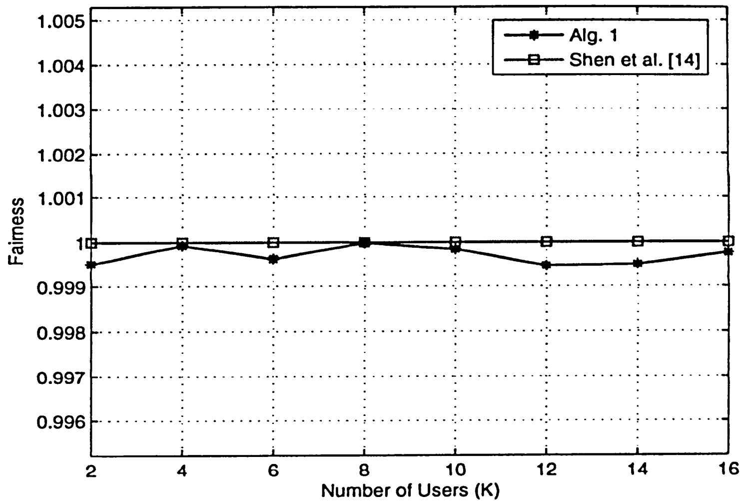

Figure 5.5: Fairness versus number of users $K = 2 \sim 16$, for SNR = 30dB and N = 64.

step of adaptive power allocation. User's average data rate for different number of users is also shown in Fig. 5.7. It is seen that both minimum user's data rate and user's average data rate in the proposed algorithm are higher than the other two methods for different number of users which shows higher achieved overall capacity by increasing each user's achieved data rate. Also, the performance of the algorithm proposed in [13] is very close to the algorithm in [14] for equal rate proportions since they both apply the same subcarrier allocation.

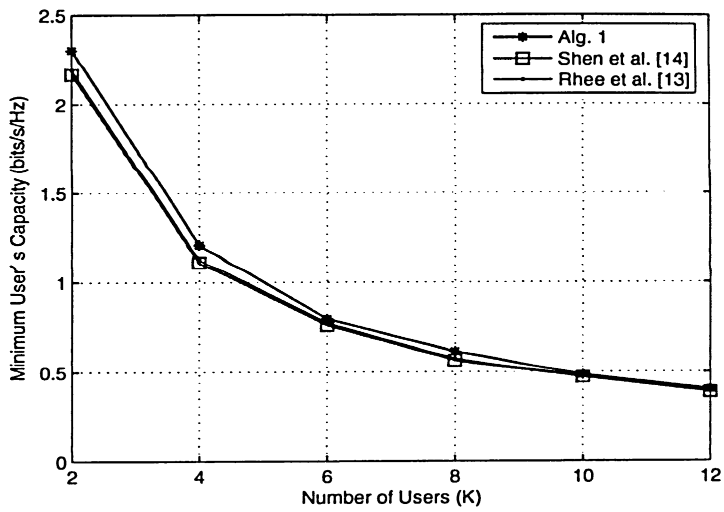

Figure 5.6: Minimum user's capacity versus number of users for $N = 64$ subcarriers and BER $= 10^{-3}$.

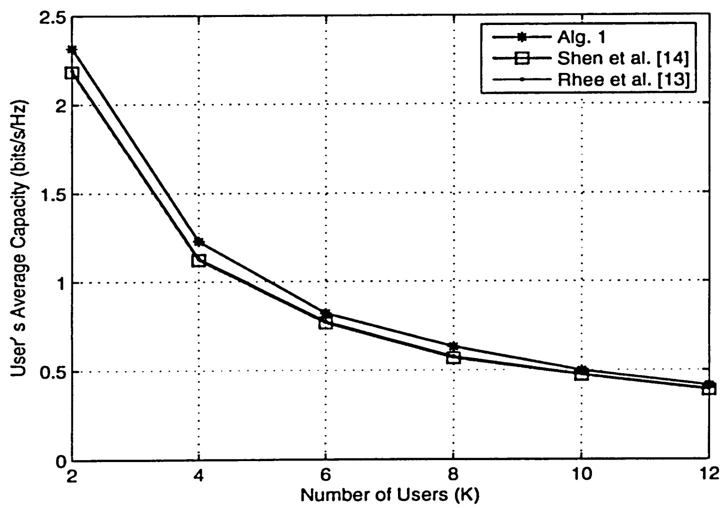

Figure 5.7: User's average capacity versus number of users. There are $N = 64$ subcarriers and BER $= 10^{-3}$.

5.4 Suboptimal Subcarrier Allocation in Multiuser OFDM System with Fixed Rate Constraints

In this section, the performance of Alg. 2, proposed in Section 4.3, is examined. As described before, the proposed algorithm prioritizes the user that has the largest rate requirement. For

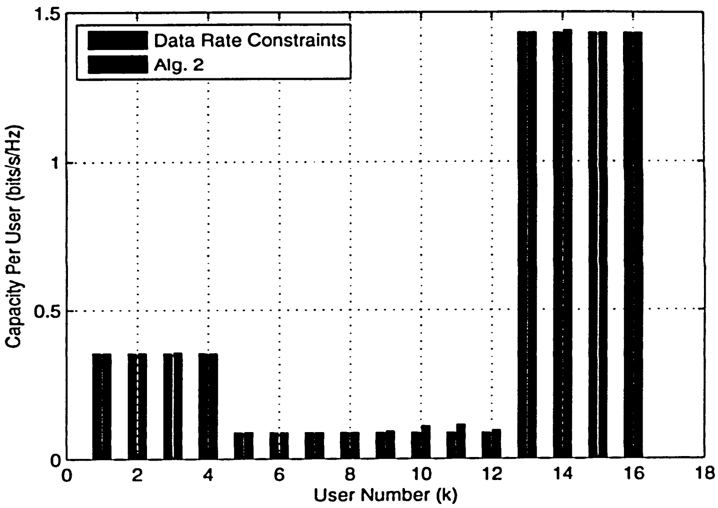

Figure 5.8: Capacity per user for SNR = 30dB, K = 16 and BER=10^{-3}. The rate requirements are such that the first 4 users achieve 4 times the rate of the next 8 users and 1/4 times the rate of the last 4 users.

instance, in the case of two users, if $R_{1,min}$ is 4 times as $R_{1,min}$, the second user would not get any subcarrier until the first user achieves at leach 0.75 of its rate requirement.

In order to examine the effect of different rate proportionalities on the performance of the algorithm, the rate requirements are set based on the results derived from simulating the algorithm proposed in [14]. Shen et al. formulated the problem of maximizing the total throughput with proportional rate constraints and derived the optimal power distribution for any fixed subcarrier allocation. Also, they showed in [14] that if they apply their method on the subcarrier allocation proposed in [13], their proposed algorithm would achieve 95% of the optimal performance in a two-user system.

The performance of Alg. 2 is examined as follows: In each channel realization, the algorithm proposed in [14] is run for a desired set of proportional rate constraints. The achieved data rates are then adopted as the fixed rate requirements for Alg. 2.

Fig. 5.8 shows a snapshot of all the users' achieved data rates as well as their corresponding rate requirements. The left bars are the rate constraints whereas the right bars are the achieved rates by Alg. 2. It is seen that all the users have achieved their minimum rate requirements.

Chapter 6

Conclusion and Future Work

6.1 Conclusions

The first contribution of this thesis, is measuring the decrease in the total throughput of the system when there is a power mask on all the subchannels. Based on Shannon's capacity formula, the achieved data rate of a subchannel is a logarithmic function of the signal-to-noise ratio in the subchannel. Hence, the achieved data rate depends both on the subchannel's gain and the power allocated to the subchannel. In a multiuser system, the channel conditions of different users are largely independent due to users' different locations. Therefore, adaptive resource allocation algorithms can improve the system performance by adaptively assigning the subcarriers to the users as well as adaptively allocating power to the assigned subchannels in the system. However, subcarrier and power allocation are not independent in the system and they should be optimized jointly to reach the optimal solution which poses a prohibitive computational burden at the basestation.

To lower the computational complexity, a flat transmit power is assumed on all the subcarriers. A flat power spectral density might also be necessary in cases where there is a power mask on the subchannels of the system. To see the validity this assumption, we compare the achieved data rates of two single-user systems with different power allocation algorithms. In the first algorithm, the power is allocated to the subcarriers with water-filling in frequency derived in [14]. In the second algorithm, referred to as the suboptimal method, the best set of subchannels is used for data transmission and the power is allocated equally among them. The simulation results showed a negligible degradation in the achieved data rate of the suboptimal method compared to the water-filling power allocation provided that the power is poured only into subchannels with good channel gains.

This result is used in the developing the proposed suboptimal subcarrier allocation algorithm with proportional rate constraints which is the second contribution of this thesis.

While adaptive power allocation can be applied to enforce the rate proportionality among the users [14], the subcarrier allocation plays a significant role in the overall performance. In Section 4.1, we propose a suboptimal subcarrier allocation algorithm to maximize the total throughput while maintaining rate proportionality among the users [37]. This algorithm, referred to as Alg. 1, is based on prioritizing the critical users in the system and the variance of subchannel gains for each user is used to define the sensitivity of the user to the subcarrier allocation. Although the system capacity is maximized when each subchannel is allocated to the user with the best channel gain on it, when considering the problem combined with fairness, the system's performance could be improved if the second best subchannel is selected for data transmission when considering a user with low subchannel gain variance. The simulation results show improvement in terms of total data rate with acceptable rate proportionality compared to the algorithms proposed in [13,14]. In Alg. 1, the power is distributed equally on all the subcarriers based on the results obtained from power allocation in single-user systems.

The third contribution of this thesis is developing a low complexity subcarrier allocation algorithm to maximize the total data rate while supporting the users with their minimum rate requirements. In the proposed algorithm, referred to as Alg. 2, the subcarrier and the power allocation are carried out jointly. The proposed algorithm consists of N major steps to assign all N subcarriers to the users. At each step, the user that has the highest data rate to achieve is given priority to choose its best available subcarrier. Each subcarrier adds a constant power proportion of P_{total}/N to the user it has been assigned to. The power is then redistributed among the assigned subcarriers with water-filling policy and the user' achieved data rate is updated. The achieved data rate subtracted from the initial rate requirement would then become the data rate constraint for the next step. Compared to the algorithm proposed in [4] and [7], Alg. 2 achieves the required rate constraints much faster without separating the assignment of bandwidth from the power or applying an extra stage or power allocation to the users and subcarriers.

6.2 Future Work

- *Semi-adaptive Resource Allocation*

  For multiuser OFDM systems discussed in this thesis, it is assumed that the channel conditions are perfectly estimated and fed back to the basestation and that the channel condition remains almost the same while the resource allocation is carried out in the basestation. In this framework, the system overhead for conveying the channel state information from the users to the basestation and the resource allocation schemes from the basestation to the users has not been incorporated into the problem formulation. One possible solution to reduce the system overhead is that at each mobile station, only the channel conditions of those subchannels that have variations more than a certain threshold be fed back to the basestation. Furthermore, the subchannel allocation could be carried out once and only the power allocation be adapted to the channel variations.

- *Effectiveness of Adaptive Power Allocation*

  Several practical constraints were mentioned in [25] which limit the applicability of adaptive modulation. A similar question addresses the applicability and the effectiveness of adaptive power allocation in multiuser OFDM systems. With the relationship between the data rate and the signal-to-noise ratio being of a logarithmic nature, it is very much beneficial to know under which circumstances the adaptive power allocation has a major effect on the performance of the system. Knowing these conditions, a time-efficient subcarrier allocation algorithm could be designed which attempts to optimize the performance of the system while highly reducing the complexity of the algorithm by meeting certain conditions accordingly and omitting the unimportant parameters from the optimization problem.

- *Resource Allocation Separability*

  It is shown in [8, 12] that with the objective of maximizing the total throughput of the system, the optimal subcarrier allocation is independent form the optimal power allocation. In this case, the subcarrier allocation is carried out independently based on the subchannel gains and then the power is distributed among the subcarriers by water-filling policy. The independence of the bandwidth from the power highly reduces the complexity of the algorithm. It can be investigated if a similar approach could apply to other optimization problems, i.e., if there are other system objectives where the optimal power allocation is independent from the optimal subcarrier allocation.

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