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# Electricity market efficiency and voltage stability

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# **ELECTRICITY MARKET EFFICIENCY AND VOLTAGE STABILITY**

by

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Bachelor of Electrical Power and Machines Engineering, Cairo University,  
2008

A thesis

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in partial fulfillment of the

requirements of the degree

Master of Applied Science

in the program

Electrical and Computer Engineering

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## **ABSTRACT**

Several electricity markets were created in the last two decades by deregulation and restructuring vertically integrated utilities. In order to serve the best interest of participating entities, it is important to operate electricity markets at their maximum efficiency.

In most cases, electricity markets were formed to operate on existing physical power systems that had evolved over several decades as vertically integrated utilities. Location of generating stations, large urban load centers and enabling transmission systems were unique to every power system and followed the ‘lay of the land’. Depending upon a power system layout, voltage stability and margin to voltage collapse are unique to it.

While an electricity market is to be operated efficiently, its optimal generation schedule to supply energy through an electric power system has to be reliable and meet the strict standards including those that relate to voltage stability. This work elicits the relationship between market efficiency and voltage stability. To this end, a formulation and a solution algorithm are presented. Two contrasting 5-bus cases illustrate how the transmission system layout influences the relationship between voltage stability and market efficiency. The IEEE 118-bus system is also used to illustrate this relationship.

## **PUBLICATIONS**

T. Zhang, A. Elkasrawy, and B. Venkatesh, “A new computational method for reactive power market clearing”, *Int J Electr Power Energ Syst* 2009; 31(6):285–93.

A. Elkasrawy and B. Venkatesh, ‘Survey of Different Economic Mechanisms for Reactive Power Services’, submitted to *IEEE Canadian Review*

A. Elkasrawy and B. Venkatesh, ‘Market Efficiency and MW Margin to Voltage Instability’, submitted to *IET Generation, Transmission and Distribution*.

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## LIST OF SYMBOLS

**NG, NS, NK** and **NM** are number of generators, var sources, segments of the active power of generators and segments of the reactive power of generators

**i, j, k, m** are indices for buses (connected generator), var sources, segment of generator real power and segment of generator reactive power

<b><math>PG_{ik}</math></b>	real power of $k^{\text{th}}$ segment of generator at $i^{\text{th}}$ bus
<b><math>QG_{im}</math></b>	reactive power of $m^{\text{th}}$ segment of generator at $i^{\text{th}}$ bus
<b><math>PD_i, QD_i</math></b>	real and reactive power load at $i^{\text{th}}$ bus
<b><math>P_i, Q_i</math></b>	net real and reactive power injected from $i^{\text{th}}$ bus into connected transmission lines
<b><math>QS_j</math></b>	reactive output of $j^{\text{th}}$ switchable capacitor
<b><math>UG_i</math></b>	status of generator at $i^{\text{th}}$ bus $\in \{0,1\}$
<b><math>US_j</math></b>	status of $j^{\text{th}}$ var source $\in \{0,1\}$
<b><math>V_i, \delta_i</math></b>	Bus voltage magnitude and phase angle at $i^{\text{th}}$ bus
<b><math>a_i, b_{ik}</math></b>	No-load and piece-wise ( $k^{\text{th}}$ segment) linear real power cost coefficients of generator at $i^{\text{th}}$ bus
<b><math>d_{im}</math></b>	Coefficient for piece-wise linear capability limit relation between real and reactive powers of generator at $i^{\text{th}}$ bus
<b><math>e_i, f_{im}</math></b>	No-load and piece-wise ( $m^{\text{th}}$ segment) linear reactive power cost coefficients of generator at $i^{\text{th}}$ bus
<b><math>g_j, h_j</math></b>	No-load and linear reactive power cost coefficients of $j^{\text{th}}$ var source
<b><math>J</math></b>	Load Flow Jacobian with sub-matrices, J1, J2, J3 and J4
<b><math>\sigma</math></b>	diagonal matrix holding singular values of Load Flow Jacobian.
<b><math>SL, SR</math></b>	left and right side singular vector matrices
<b><math>\overline{PG}_i, \overline{PG}_{ik}</math></b>	upper limits on real power output of generator at the $i^{\text{th}}$ bus and its $k^{\text{th}}$ segment
<b><math>\underline{PG}_i, \underline{PG}_{ik}</math></b>	lower limits on real power output of generator at the $i^{\text{th}}$ bus and its $k^{\text{th}}$ segment
<b><math>\overline{QG}_i, \overline{QG}_{im}</math></b>	upper limits on reactive power output of generator at the $i^{\text{th}}$ bus and its $m^{\text{th}}$ segment
<b><math>\underline{QG}_i, \underline{QG}_{im}</math></b>	lower limits on reactive power output of generator at the $i^{\text{th}}$ bus and its $m^{\text{th}}$ segment
<b><math>\underline{QS}_j, \overline{QS}_j</math></b>	lower and upper limits on the $j^{\text{th}}$ var source
<b><math>\underline{V}_i, \overline{V}_i</math></b>	lower and upper limits on the $i^{\text{th}}$ bus voltage magnitude
<b><math>\Delta x</math></b>	indicates change in the generic variable 'x'

## ABBREVIATIONS AND ACRONYMS

<b>CMC</b>	competitive market clearing
<b>LMP</b>	Locational Marginal Price
<b>OPF</b>	Optimal Power Flow
<b>MILP</b>	Mixed Integer Linear Programming

# CHAPTER 1

## INTRODUCTION

Power systems have evolved over several decades and their evolution is unique to every state/province/country. This development follows the ‘lay of the land’, availability of resources and space to build power stations, development of cities and other load centers, right of way to build transmission lines and such other constraints. Based upon these details, a power system has a level of voltage stability [1] and margin to voltage collapse.

The system voltage stability and margin to voltage collapse are parameters that are tightly controlled and specified by the regulatory body with jurisdiction in that area. One example of such a regulatory body is the North Eastern Power Coordinating Council that oversees the province of Ontario, Canada. Many studies [2-7] have been done in order to include the voltage stability maximization objective in the OPF (Optimal Power Flow). Ref. [8] examines questions of stability in interconnected power systems coupled with Market Dynamics. In other papers [9-10], the cost minimization objective and Locational Marginal Prices (LMPs) in the Power Market were discussed.

In 1990’s, through deregulation of the business of electric power systems and restructuring of monolithic utilities, competing components were created and new ones were built that compete to sell and buy electric energy in the electricity market. Every jurisdiction has its own market and a market clearing mechanism. Efficiency of the market is the cornerstone of an electricity market.

Market efficiency is directly tied to the ability of generators and loads to freely buy and sell energy across the transmission system. Congestion in transmission lines

causes evident impediments in market efficiency [11-12]. The requirement to maintain a certain margin to voltage collapse also restricts dispatch of generators in certain situations and consequently may reduce market efficiency. This thesis explores the relationship between market efficiency and MW Margin to Voltage Collapse.

In order to enable such a study, a single ended auction market is considered with generators bidding to sell energy. A formulation that minimizes generation costs and maximizes MW Margin to Voltage Collapse (or voltage stability margin) is presented. The formulation is constrained by regular constraints that limit generators and other devices to their physical limits and power balance equations of the AC transmission system.

This thesis considers two contrasting cases of a 5-bus system. The first one negatively correlates the two objectives while the second case positively correlates the two objectives. A similar exercise on the IEEE 118-bus system is completed and reported.

Chapter 2 provides a theoretical foundation for this work. It introduces basic definitions of the electricity market and the associated energy auction process.

Chapter 3 presents the proposed nonlinear mixed integer programming formulation to settle an electricity market. This model is linearized to form an incremental model. As the incremental MILP model has two objectives, it is transformed into a fuzzy MILP model and solved successively to reach the optimal solution. A Full description of the solution method is given at the end of the Chapter 3.

Chapter 4 presents results of tests on two contrasting cases of a 5-bus system and the IEEE 118-bus system. The values of MW Margin to Voltage Collapse before and after the Optimization are provided as a measure of Voltage Stability besides the

## Chapter 1 Introduction

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minimum singular value. The results demonstrate the strong relationship between market efficiency and MW Margin to Voltage Collapse (or voltage stability margin).

In Chapter 5, conclusions derived from the study and test results are listed.

Finally, in the Appendices, the data for the two 5-bus systems and the modified IEEE 118-bus system that were studied in this work are provided.

## CHAPTER 2

# THEORY

This chapter introduces basic theoretical foundation for this work. In this chapter, we choose to use the definitions of market efficiency and voltage stability as given in [12] and [1] respectively. For the work described in this chapter, we choose to use a single ended auction model for simplicity. We wish to stress that this work would apply equally to a double-ended auction model as well. For the purpose of illustration, we chose a 3-bus system with two generator busses and one load bus.

### 2.1 3-Bus System

The 3-bus system is shown in Figure 1. The line from bus 1 to bus 3 is long and the line from bus 2 to bus 3 is short. The generator 1 (at bus 1) is selling energy at a low price and generator 2 (at bus 2) is selling energy at a higher price. Tables 1 and 2 give the data for this system.

#### 2.1.1 Voltage Stability

Treating bus 1 as the slack bus, the linearized power balance equations appear to be:

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_3 \end{bmatrix} = \begin{bmatrix} J1_{22} & J1_{23} & J2_{23} \\ J1_{32} & J1_{33} & J2_{33} \\ J3_{32} & J3_{33} & J4_{33} \end{bmatrix} \begin{bmatrix} \Delta \delta_2 \\ \Delta \delta_3 \\ \Delta V_3 \end{bmatrix} \quad (1)$$

Subscripts reflect the bus numbers. Rewriting (1) in the matrix form:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J1 & J2 \\ J3 & J4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} \quad (2)$$

The submatrices [J1], [J2], [J3] and [J4] form the system Jacobian [J]. Decomposing the Jacobian matrix [J] using singular values decomposition:

$$[J] = [SL] [\sigma] [SR]^t \quad (3)$$

It has been illustrated in several works before that the minimum singular value of the load flow Jacobian will have a higher value at a higher voltage stable state [1, 7 and 16].

As mentioned before, the MW Margin to Voltage Collapse is the amount of load that the system can supply before it encounters voltage collapse. It would act as an additional indicator or measure of Voltage Stability Margin together with the minimum singular value. The MW Margin to Voltage Collapse is easily calculated in a few steps. Consider a base value of generations and loads at all the buses. The power balance equations are solved for this set of base values of generations and loads. Then, loads and generations are multiplied by a factor K and the power balance equations are resolved. The value of the factor K is gradually increased in small steps from unity until a solution for the power balance equations does not exist. The highest value of K that has a solution for the power balance equations is recorded. The MW Margin to Voltage Collapse equals: Total Base Case System Load x (K-1).

## Chapter 2 Theory

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TABLE 1 GENERATOR DATA OF THE 3-BUS SYSTEM

Bus	Fixed costs (\$)	Segment 1		Segment 2		Segment 3	
		Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW
1	0	30	100	40	100	50	100
2	0	70	100	80	100	90	100

TABLE 2 LINE DATA OF THE 3-BUS SYSTEM

Case #	From	To	R (pu)	X(pu)
1	1	3	0	0.10
2	2	3	0	0.02

Note: Line charging is neglected in this study.

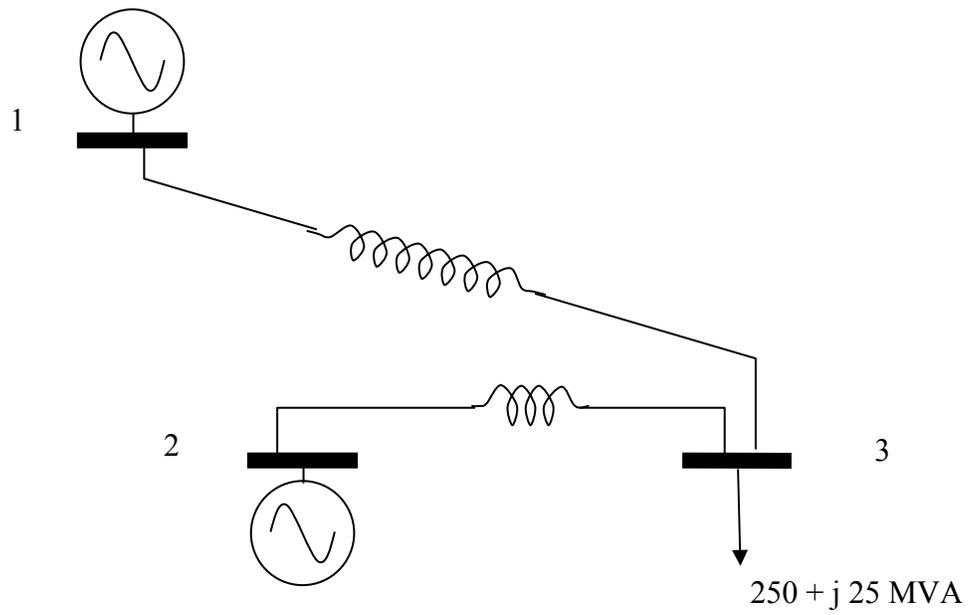


Figure 1 3-bus Sample System

### 2.1.2 Electricity Market

In a typical Double Auction Electricity Market, the Independent System Operator (ISO) manages the Auction Process as shown in Figure 2. The ISO receives the supply offers from the Generators and the buying bids from the Loads or Consumers. It stacks the supply offers from the lowest priced offers in an ascending order up to the highest and the purchase bids from the highest bid in a descending order down to the lowest to construct the supply and demand schedules. The point of intersection of the two schedules is called the Competitive Market Clearing (CMC) Point as shown in Figure 3. The quantity units lying to the left of the CMC point are called inframarginal units, while all other units are called extramarginal units. All inframarginal suppliers are paid the same market clearing price and all inframarginal buyers pay the same market clearing price. The net seller surplus is the area between the horizontal line at the CMC price level and the supply schedule while the net buyer surplus is the area between the same horizontal line and the demand schedule. The sum of the seller and buyer surpluses yields the Total Net Seller Buyer Surplus (TNSBS). The efficiency of the market (M.E) can be calculated using the following formula:

$$M.E = 100\% \times \left( \frac{\text{Actual TNSBS}}{\text{Max TNSBS}} \right)$$

M.E will be 100 % if all inframarginal units trade and no extramarginal units trade.

In case of a single Auction Electricity Market –which is the case in Ontario, Canada-, the consumers do not make purchase bids. Only Generators make supply offers to the ISO. The ISO forecasts the energy needed and settles the market as shown in Figure 4. Since there is no buyer surplus, TNSBS will be equal to the net seller surplus.

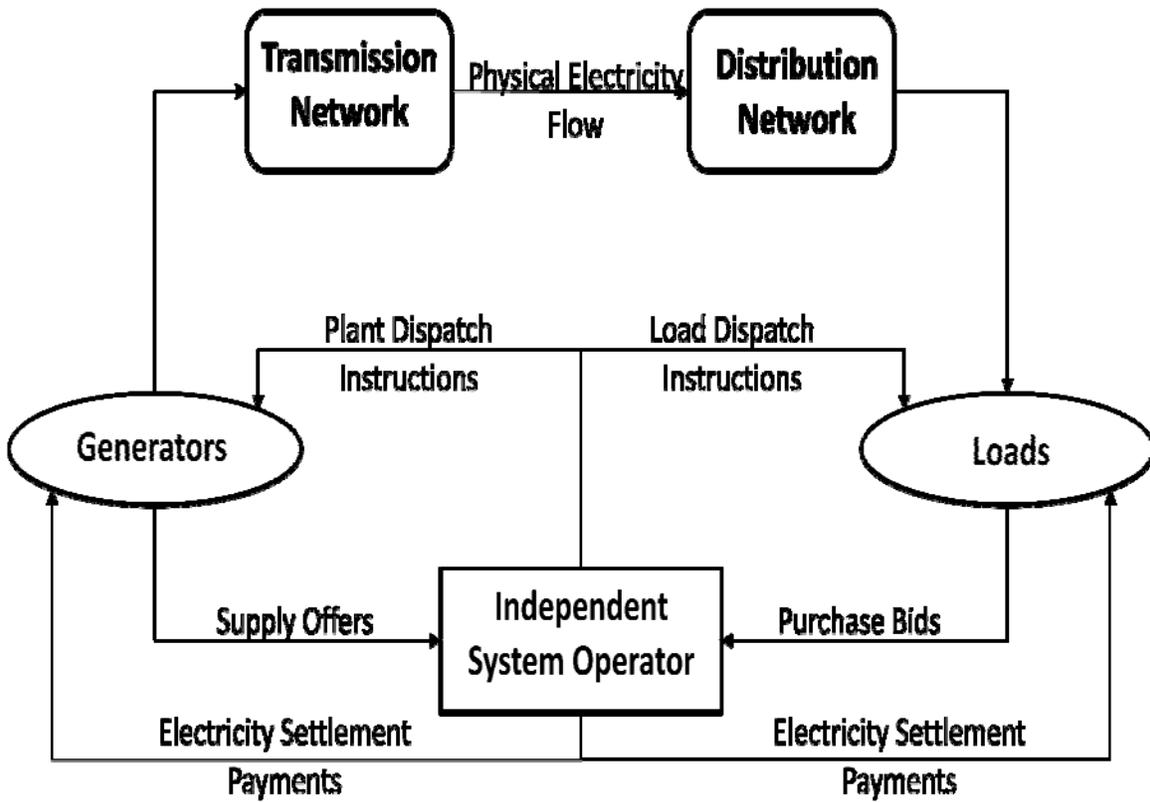


Figure 2 Structure of Double Auction Electricity Market

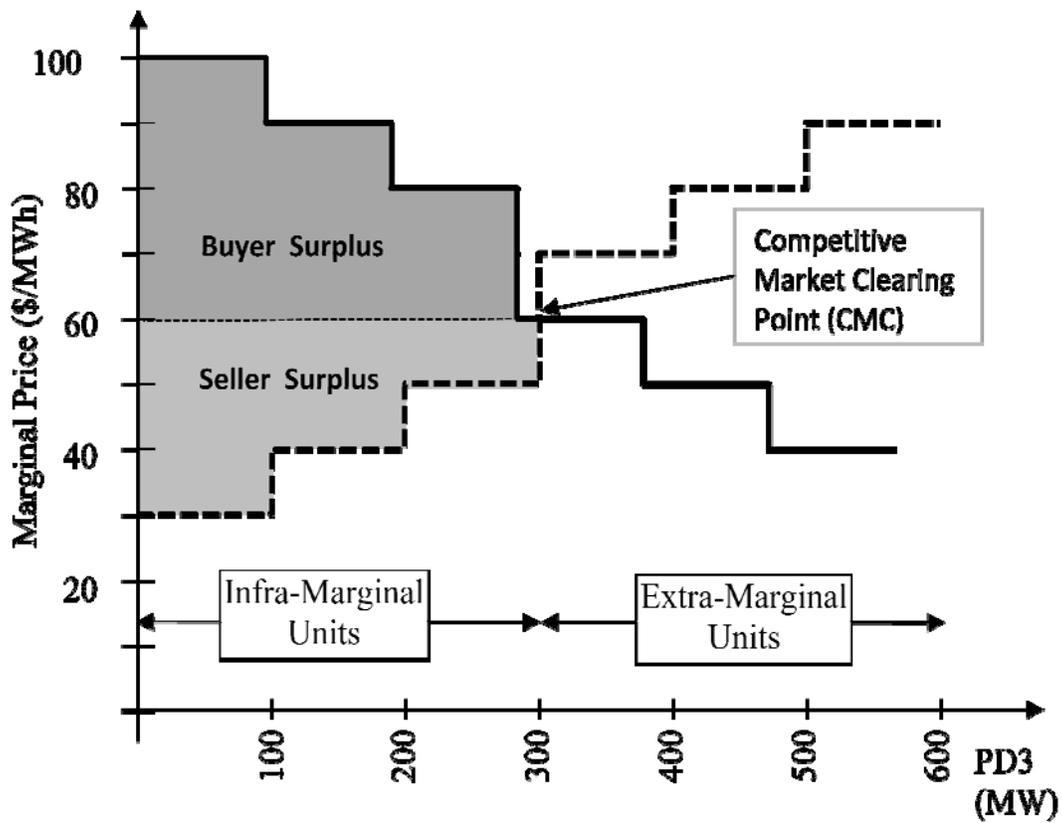


Figure 3 Settlement of Double Auction Market

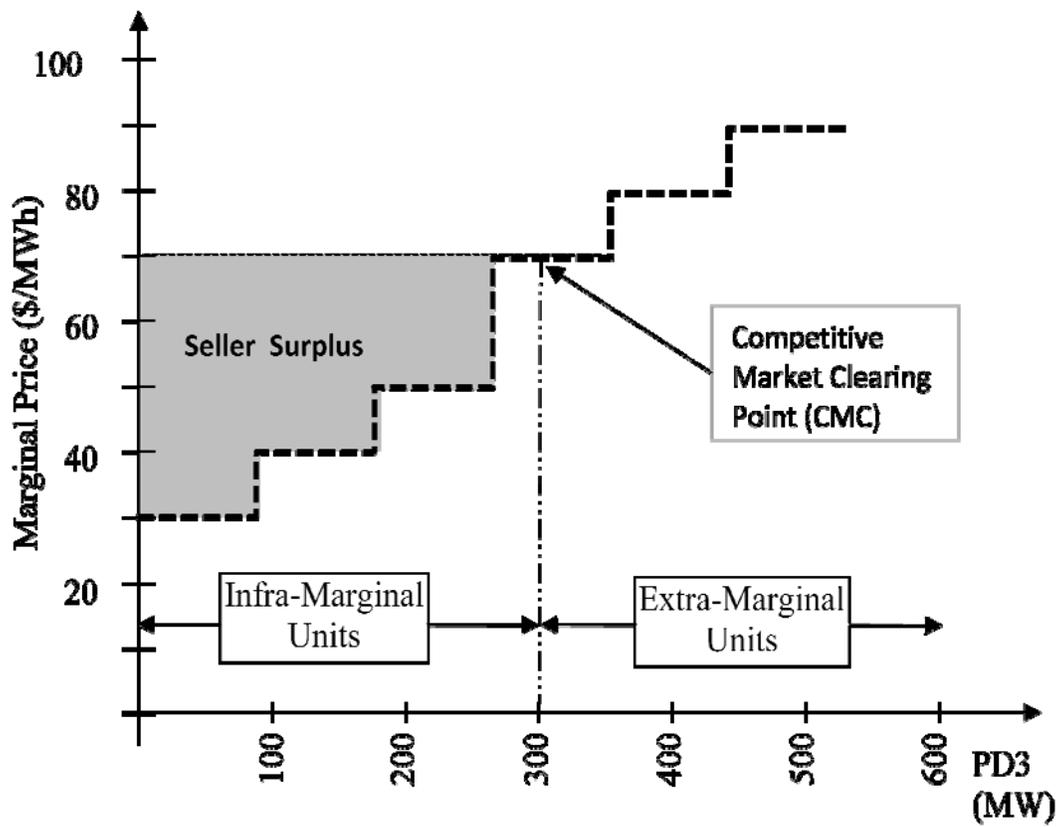


Figure 4 Settlement of Single Auction Market

## 2.2 Analysis

The 3-bus system, as can be seen from the data, is a lossless system. By virtue of being a lossless system without congestion, the price of real power is the same at all the busses of the system. The auction diagram may be created by stacking the sell bids in their increasing order as shown in Figure 5 as ‘*case 1*’. As this is a single ended auction market, the demand is a vertical line at the total system demand of 250 MW. In case 1, one may see that generator 1’s segments 1, 2 and 3 are dispatched to deliver 100 MW, 100 MW and 50 MW each respectively and the total generation cost to the system equals \$ 9,500. The *competitive market clearing* (CMC) point is at \$ 50 / MWh. The net seller surplus equals \$ 3,000 and the market efficiency equals 100%. Segments 1, 2 and 3 of generator 1 are infra-marginal units. Segments 1, 2 and 3 of generator 2 are extra-marginal units. The minimum singular value equals 0.9393 and the Load Margin before Voltage Collapse equals 39.75 MW.

Thereafter, in order to bring generation closer to the load, segments of generator 1 (infra-marginal units) are closed one at a time starting with segment 3, then 2 and finally 1. These yield cases 2, 3 and 4 as shown in Figure 5. In each case, a higher priced segments from generator 2 (extra-marginal units) are dispatched and the marginal price increases to \$ 70/ MWh, \$ 80 / MWh and \$ 90 / MWh respectively. The seller surplus progressively decreases in each case and market efficiency decreases as tabulated in Table 3.

Of interest is that when the generation moves from generator 1 to 2, the source of generation moves closer to the load and system Jacobian’s minimum singular value as well as the MW Margin to Voltage Collapse increase. It brings the system to a better voltage stable state.

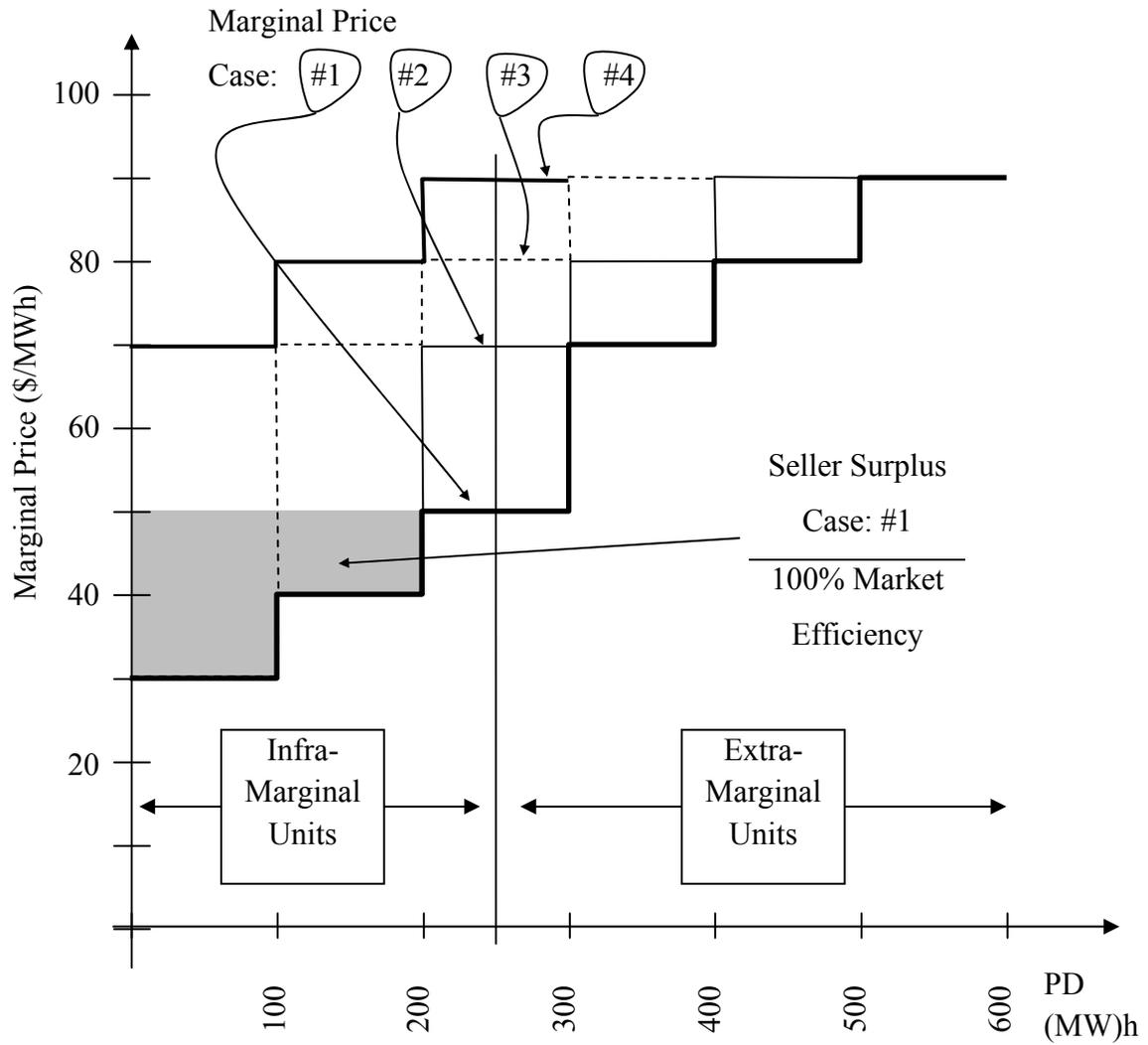


Figure 5 Single Ended Auction Model with 4 cases

Chapter 2 Theory

TABLE 3 MARKET RESULTS OF THE 3-BUS SYSTEM

Case #	PG <sub>1</sub>	PG <sub>2</sub>	$\sigma_{\min}$	Margin to Voltage Collapse (MW)	System Marginal Price (\$/MWh)	Total Generation Cost (\$)	Market Efficiency %
1	250	0	0.9393	39.75	50	9,500	100
2	200	50	1.2253	42.25	70	10,500	100
3	100	150	1.5399	45.63	80	14,000	66
4	0	250	1.6234	58.25	90	19,500	0

Note: The total real power system load in this lossless system is:  $250\text{MW} = \text{PG}_1 + \text{PG}_2$ .

From the above, three important aspects can be surmised and they are listed below:

**Inference #1:** As we move generation from cheaper to expensive units, extra-marginal units are committed. Consequently, the market surplus and market efficiency reduces.

**Inference #2:** As we move generation from generator away from load to those that are close to loads, minimum singular value of the system Jacobian and MW Margin to Voltage Collapse increases and the system becomes more voltage stable.

**Inference #3:** If more expensive generators are located close to load centers, a higher voltage stable state means more expensive generators are dispatched, a higher marginal price and a less efficient market.

While these inferences are true only for this 3-bus system, these may also be observed in general for larger systems as well. In larger systems, lesser-priced large coal / nuclear power plants are situated far away from large urban load centers and expensively priced smaller peak-load generators are located closer to them. Therefore, in order to develop a tool to determine the influence of voltage stability margin on market efficiency for larger systems, we propose a formulation and solution scheme in the next Chapter.

## CHAPTER 3

# FORMULATION

This Chapter presents a formulation to settle an electricity market. The formulation considers two objectives. The first objective considers bids by generations and minimizes the net costs to clear the market. The second objective maximizes the MW Margin to Voltage Collapse (or the minimum singular value of the system's Jacobian). The two objectives are constrained by power balance equations and limits on generators real power outputs, generators reactive power outputs, generator voltage magnitudes, capacitor outputs and load bus voltage magnitudes. As the constraining equations and objectives are nonlinear and have integer variables, the complete formulation presented below is a nonlinear mixed integer formulation with multiple objectives. Thereafter, this formulation is linearized to develop an incremental model that is of a mixed integer linear programming nature and multiple objectives. This formulation is solved using fuzzy optimization technique to handle the multiple objectives. This fuzzy optimization model and solution algorithm also presented in the last section of this Chapter.

### 3.1 Complete Nonlinear Mixed Integer Optimization Formulation

This section presents the complete formulation for settling an electricity market. It considers price bids from generators for real and reactive power and price bids for reactive power capacitors. It also relates real and reactive power generation limits using a linear approximation of the generator capability chart.

#### 3.1.1 Generator Model

The generator model has three parts. They include

- a) model of reactive power price, b) model of real power price, and,
- c) model of capability constraint that relates max output of real and reactive powers.

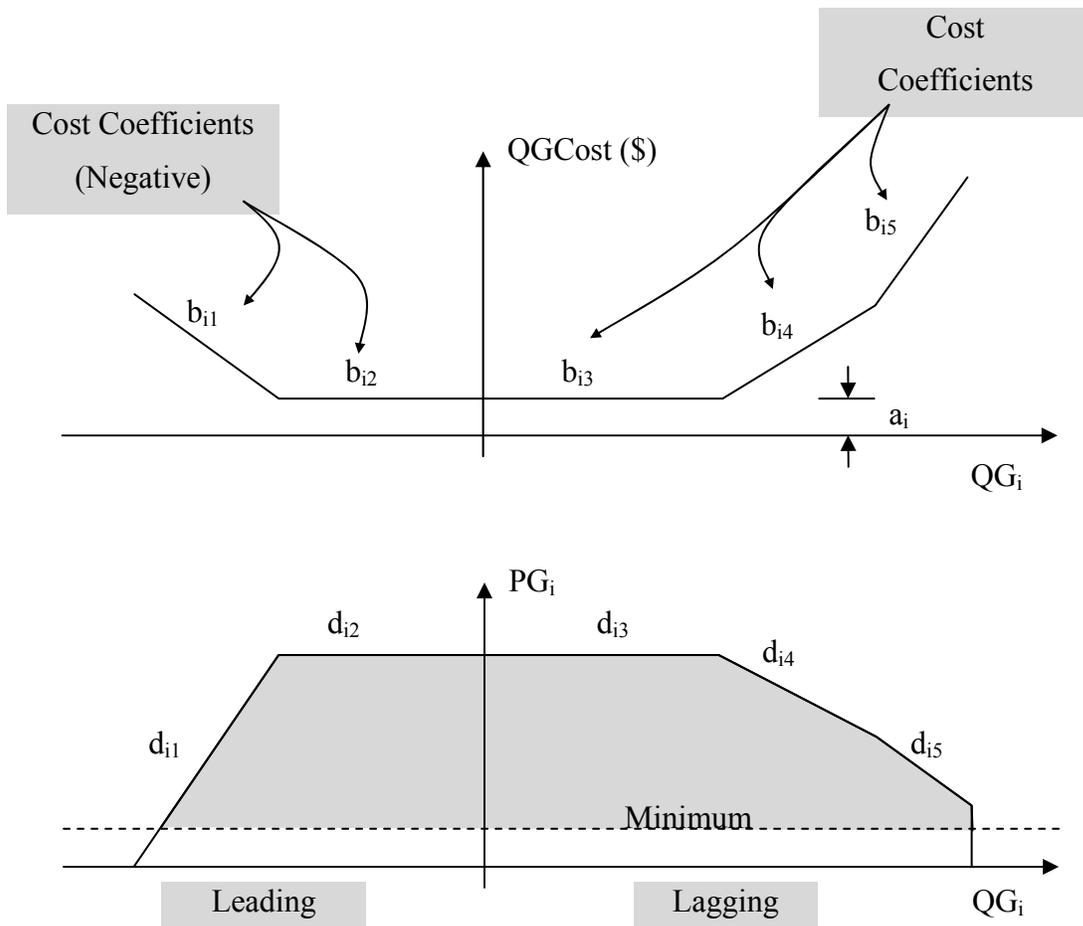


Figure 6 Combined reactive power pricing and capability charts

Note: The coefficients  $d_{i2}$  and  $d_{i3}$  are zero. As a convention,  $QG_i$  is taken positive when lagging and negative when leading.

### 3.1.1.1 Economic Model of Reactive Power Generation

The diagram as shown in Fig. 3 (upper part) models a generator's reactive power cost curve accounting for its own losses due to additional reactive power output and lost opportunity cost. It has five segments with segments 1 and 2 in the negative (leading) side and segments 3, 4 and 5 on the positive side (lagging).

Considering  $UG_i$  (a binary integer) to model the status of participation of the generator in the reactive power market, each segment has an output,  $QG_{in}$  where indices  $i$ ,  $n$  refer to the  $i^{\text{th}}$  generator's  $n^{\text{th}}$  segment.  $\underline{QG}_{in}$  and  $\overline{QG}_{in}$  refer to minimum and maximum generation by the  $n^{\text{th}}$  segment. For segments 1 and 2, minimums would be a negative value with a zero maximum. Segments 3, 4 and 5 will have a positive maximum with zero minimums.

One may write up limits on the generator and each segment as below:

$$UG_i \cdot \underline{QG}_i \leq \sum_{n=1}^{NM} QG_{in} \leq UG_i \cdot \overline{QG}_i \quad (4)$$

$$\underline{QG}_{in} \leq QG_{in} \leq \overline{QG}_{in} \quad (5)$$

In (5), segments 1 and 2 have  $QG_{in}$  which will assume only negative values and hence  $\underline{QG}_{in}$  will have negative values and  $\overline{QG}_{in}$  will be zeros. For segments 3, 4 and 5, have  $QG_{in}$  that will assume only positive values and hence  $\underline{QG}_{in}$  will be zeros and  $\overline{QG}_{in}$  will be positive values.

With  $e_i$  and  $f_{in}$  being fixed and incremental, the total system cost to procure reactive power from generators equals:

$$QG\text{Cost} = \sum_{i=1}^{NG} \left[ UG_i \cdot e_i + \sum_{n=1}^5 f_{in} \cdot QG_{in} \right] \quad (6)$$

### 3.1.1.2 Economic Model of Real Power Generation

The cost of active power consists of a fixed part and an incremental part. The total capability of a generator is divided in a number of segments. The cost of generation within each segment has an ascending fixed rate through all the segments. The total system cost to procure active power from generators equals:

$$PG_{\text{Cost}} = \sum_{i=1}^{NG} \left[ a_i \cdot UG_i + \sum_{k=1}^{NK} b_{ik} \cdot PG_{ik} \right] \quad (7)$$

The Limits on segments may be written as follows:

$$0 \leq PG_{\text{in}} \leq \overline{PG}_{\text{in}} \quad (8)$$

### 3.1.1.3 Relation Between Generator Real and Reactive Power Outputs

The diagram in the bottom of Fig. 3 depicts an approximate capability chart where line segments are used to approximate the actual diagram and generate linear relations. This diagram is used to derive limits on real power output in relation to reactive power output.

When the generator is operating in segments 2 and 3, reactive power output does not restrict real power output. In segments 1, 4 and 5, reactive power output limits real power output. Hence, limits on real power output may be related to reactive power output in those segments (1, 4 and 5) using straight line relations where  $d_{\text{in}}$  represents slope of the graph segments.

$$\underline{PG}_i \leq PG_i \leq \overline{PG}_i + \sum_n d_{\text{in}} \cdot QG_{\text{in}} \quad (9)$$

where  $d_{\text{in}}$  corresponding to segments 2 and 3 would assume a value of zero.

### 3.1.2 Economic Model of Switchable Capacitors

Other reactive power sources may participate in the transmission network and provide strategic reactive power support. This might help the system to be more efficient with lesser losses and be more voltage stable. Their services are bundled into one model for simplicity and their remuneration is computed as below:

$$QSCost = \sum_{j=1}^{NS} [US_j \cdot g_j + h_j \cdot QS_j] \quad (10)$$

where  $QS_j$  and  $US_j$  are reactive power output and status (binary integer) of the  $j^{\text{th}}$  source.  $NS$  is the number of such sources. Output of reactive power sources are limit as below:

$$US_i \cdot \underline{QS}_i \leq QS_i \leq US_i \cdot \overline{QS}_i \quad (11)$$

### 3.1.3 Total Cost Minimization – First objective

In the above, **PG**, **QS**, **UG**, **US** and **V** (generators) are control variables and others are dependent variables. The variables **QG**, **V<sub>L</sub>** and **δ** are dependent on the state of the system. It is assumed that optimal schedule is made available from the 24-hour unit commitment and short-term real power dispatch. It will form the starting point in this process. Hence, the first objective is to minimize the sum of equations 6, 7 and 10.

$$\sum_{i=1}^{NG} \left[ a_i \cdot UG_i + \sum_{k=1}^{NK} b_{ik} \cdot PG_{ik} \right] + \sum_{i=1}^{NG} \left[ e_i \cdot UG_i + \sum_{m=1}^{NM} f_{im} \cdot QG_{im} \right] + \sum_{j=1}^{NS} [g_j \cdot US_j + h_j \cdot QS_j] \quad (12)$$

### 3.1.4 Voltage Stability Maximization – Second Objective

In order to maximize the MW Margin to Voltage Collapse, the minimum singular value of the load flow Jacobian is maximized. It must be stressed here that the value of minimum singular value does not provide the margin to voltage collapse. Alternatively, the MW margin to voltage collapse is determined by continuation power flow for a given load and generation pattern. Hence, the second is to maximize  $\sigma$  (3) where  $\sigma$  is the least singular value of the load flow Jacobian. From equation (3):

$$[J] = [SL] [\sigma] [SR]^t$$

With the left and right hand side singular vector matrices being orthogonal:

$$[\sigma] = [SL]^t [J] [SR] \quad (13)$$

We will refer to it in this chapter with  $\sigma(\delta, V)$  where it is a function of bus voltage phasor  $V \angle \delta$  as the Jacobian  $[J]$  is a function of the bus voltage phasor. Hence the second objective of this nonlinear mixed integer programming problem is:

$$\text{Maximize } [\sigma(\delta, V)] = [SL]^t [J(\delta, V)] [SR] \quad (14)$$

### 3.1.5 Complete Model

Finally, using modeling of sources as outlined above, the nonlinear mixed integer programming challenge with two objectives of minimizing total market settlement cost and maximizing voltage stability margin is constructed and presented. The complete model will be as follows:

Min the Total Costs (from equation 12):

$$\sum_{i=1}^{NG} \left[ a_i \cdot UG_i + \sum_{k=1}^{NK} b_{ik} \cdot PG_{ik} \right] + \sum_{i=1}^{NG} \left[ e_i \cdot UG_i + \sum_{m=1}^{NM} f_{im} \cdot QG_{im} \right] + \sum_{j=1}^{NS} \left[ g_j \cdot US_j + h_j \cdot QS_j \right] \quad (15)$$

$$\text{Maximize } \sigma(\delta, V) = [\text{SL}]^t [\text{J}(\delta, V)] [\text{SR}] \quad (16)$$

Constraints:

a) AC Power Balance Equations [15]:

$$\text{PG}_i - \text{PD}_i - P_i(\delta, V) = 0 \quad (17)$$

$$\text{QG}_i - \text{QD}_i - Q_i(\delta, V) + \sum_{j \in i} \text{QS}_j = 0 \quad (18)$$

b) Limits on real power outputs (from equations (9) & (8))

$$\text{UG}_i \cdot \underline{\text{PG}}_i \leq \sum_{k=1}^{\text{NK}} \text{PG}_{ik} \leq \text{UG}_i \cdot \overline{\text{PG}}_i + \sum_{m=1}^{\text{NM}} d_{im} \cdot \text{QG}_{im} \quad (19)$$

$$0 \leq \text{PG}_{im} \leq \overline{\text{PG}}_{im} \quad (20)$$

c) Limits on reactive power outputs (from equations (4) & (5))

$$\text{UG}_i \cdot \underline{\text{QG}}_i \leq \sum_{m=1}^{\text{NM}} \text{QG}_{im} \leq \text{UG}_i \cdot \overline{\text{QG}}_i \quad (21)$$

$$\underline{\text{QG}}_{im} \leq \text{QG}_{im} \leq \overline{\text{QG}}_{im} \quad (22)$$

d) Limits on capacitor output (from equation (11))

$$\text{US}_j \cdot \underline{\text{QS}}_j \leq \text{QS}_j \leq \text{US}_j \cdot \overline{\text{QS}}_j \quad (23)$$

e) Limits on generator voltage (control) and load bus voltages (dependent)

$$\underline{V}_G \leq V_G \leq \overline{V}_G \quad (\text{Generator Bus voltage}) \quad (24)$$

$$\underline{V}_L \leq V_L \leq \overline{V}_L \quad (\text{Load Bus voltage}) \quad (25)$$

f) Limits on UG and US

$$0 \leq \text{UG} \leq 1 \quad (\text{Generator Status}) \quad (26)$$

$$0 \leq \text{US} \leq 1 \quad (\text{Switchable Capacitor Status}) \quad (27)$$

Additionally optimization must ensure that vectors UG and US remain binary integers.

As (17) – (18) are nonlinear equalities, this is nonlinear mixed integer optimization problem. Its solution is challenging. This formulation can be solved using several techniques. Special care must be taken while handling the multiple objectives. In this thesis, successive fuzzy MILP technique is used [13-14]. Depending upon the importance given to the objectives, they are accordingly optimized in the optimization process. The formulation is set up in the next sections and solved for three systems.

### 3.2 Incremental Model

Consider a starting point of X. Then, we might reformulate the problem (15) – (27) such that X + ΔX gives the optimal solution.

Considering a starting point, it is assumed that the values of the vector X= [UG PG US QS V<sub>G</sub>], are available. An incremental model of X+ΔX is then set up in order to find the optimal incremental changes of ΔX= [ΔUG ΔPG ΔUS ΔQS ΔV<sub>G</sub>] so that the total cost is minimized and the Voltage Stability is maximized.

Min the Total Costs from (15):

$$\sum_{i=1}^{NG} \left[ a_i \cdot (UG_i + \Delta UG_i) + \sum_{k=1}^{NK} b_{ik} \cdot (PG_{ik} + \Delta PG_{ik}) \right] + \sum_{i=1}^{NG} \left[ e_i \cdot (UG_i + \Delta UG_i) + \sum_{m=1}^{NM} f_{im} \cdot (QG_{in} + \Delta QG_{in}) \right] + \sum_{j=1}^{NS} \left[ g_j \cdot (US_j + \Delta US_j) + h_j \cdot (QS_j + \Delta QS_j) \right] \quad (28)$$

$$\text{Maximize } \sigma + \Delta\sigma (\delta + \Delta\delta, V + \Delta V) \quad (29)$$

Constraints:

a) Power Balance Equations from (17) & (18):

$$(PG_i + \Delta PG_i) - PD_i - P_i(\delta + \Delta\delta, V + \Delta V) = 0$$

$$(QG_i + \Delta QG_i) - QD_i - Q_i(\delta + \Delta\delta, V + \Delta V) + \sum_{j \in i} QS_j + \Delta QS_j = 0$$

Expanding  $P_i(V + \Delta V, \delta + \Delta\delta)$  and  $Q_i(V + \Delta V, \delta + \Delta\delta)$  using Taylor's series and retaining only the first order terms:

$$PG_i + \Delta PG_i - PD_i - P_i(\delta, V) - [J1 \ J2] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = 0 \quad (30)$$

$$QG_i + \Delta QG_i - QD_i - Q_i(\delta, V) - [J3 \ J4] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} + \sum_{j \in i} (QS_j + \Delta QS_j) = 0 \quad (31)$$

b) Limits on real power outputs from equations (19) & (20):

$$(UG_i + \Delta UG_i) \cdot \underline{PG}_i \leq \sum_{n=1}^{NK} (PG_{in} + \Delta PG_{in}) \leq (UG_i + \Delta UG_i) \cdot \overline{PG}_i + \sum_{n=1}^{NM} d_{in} \cdot (QG_{in} + \Delta QG_{in}) \quad (32)$$

$$0 \leq PG_{in} + \Delta PG_{in} \leq \overline{PG}_{in} \quad (33)$$

c) Limits on reactive power outputs from equations (21) & (22):

$$(UG_i + \Delta UG_i) \cdot \underline{QG}_i \leq \sum_{n=1}^{NM} (QG_{in} + \Delta QG_{in}) \leq (UG_i + \Delta UG_i) \cdot \overline{QG}_i \quad (34)$$

$$\underline{QG}_{in} \leq QG_{in} + \Delta QG_{in} \leq \overline{QG}_{in} \quad (35)$$

d) Limits on capacitor output from equation (23):

$$(US_i + \Delta US_i) \cdot \underline{QS}_i \leq QS_i + \Delta QS_i \leq (US_i + \Delta US_i) \cdot \overline{QS}_i \quad (36)$$

e) Limits on generator voltage (control) and load bus voltages (dependent) from equations (24) & (25):

$$\underline{V}_G \leq V_G + \Delta V_G \leq \overline{V}_G \quad (37)$$

$$\underline{V}_L \leq V_L + \Delta V_L \leq \overline{V}_L \quad (38)$$

f) Limits on UG and US from equations (26) & (27):

$$0 \leq UG + \Delta UG \leq 1 \quad (39)$$

$$0 \leq US + \Delta US \leq 1 \quad (40)$$

This incremental model can be solved using a mixed integer linear programming program. However, a few simplifications have to be done. They include combining the multiple objectives into a single objective and simplification by removing Vector X where possible. It is proposed in the revised incremental model below.

### 3.2.1 Revised Incremental Model

Since the objective is to determine optimal incremental changes, when subtracting Objective equation (15) from (28), the first Objective equation becomes as follows:

Min the Total Costs increments:  $\Delta TC (\Delta X)$

$$\sum_{i=1}^{NG} \left[ \Delta UG_i \cdot e_i + \sum_{n=1}^{NM} f_i \cdot \Delta QG_{in} \right] + \sum_{i=1}^{NG} \left[ \Delta UG_i \cdot a_i + \sum_{k=1}^{NK} b_{ik} \cdot \Delta PG_{ik} \right] + \sum_{j=1}^{NS} \left[ \Delta US_j \cdot g_j + h_j \cdot \Delta QS_j \right] \quad (41)$$

Considering a change in the operating state from  $(V, \delta)$  to  $(V+\Delta V, \delta+\Delta\delta)$ , one may write another form of (29) as below:

$$[\sigma + \Delta\sigma] = [SL + \Delta SL]^t [J + \Delta J] [SR + \Delta SR] \quad (42)$$

Ignoring terms  $[\Delta SL]$  and  $[\Delta SR]$  and relating change in singular values  $(\Delta\sigma)$  to change in Jacobian  $(\Delta J)$ , one may write:

$$[\Delta\sigma] = [SL]^t [\Delta J] [SR] \quad (43)$$

Using the hessian of the power balance equations  $[H]$  one may write:

$$[\Delta J] = [H] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (44)$$

It may be pointed out that the hessian is a three dimensional matrix and is stored as a sparse matrix. Using (43) and (44), one can write change in singular values in terms of change in state of the system as an approximate linear relation as below:

$$\Delta\sigma = [SL]^t [H] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} [SR] \quad (45)$$

So the second objective becomes:

$$\text{Maximize } \Delta\sigma(\Delta\delta, \Delta V) = [SL]^t [H] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} [SR] \quad (46)$$

A more detailed explanation of the objective of the system's Minimum singular value is given in Appendix A.

Constraints:

Subtracting the Constraints equations (17) & (18) from equations (30) & (31) yields:

a) Power Balance Equations:

$$\Delta PG_i - \Delta P_i (\Delta \delta, \Delta V) = 0 \quad \text{or} \quad \Delta PG_i - [J1 \ J2] \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = 0 \quad (47)$$

$$\Delta QG_i - \Delta Q_i (\Delta \delta, \Delta V) + \sum_{j \in i} \Delta QS_j = 0 \quad \text{or} \quad \Delta QG_i - [J3 \ J4] \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} + \sum_{j \in i} \Delta QS_j = 0 \quad (48)$$

The remaining Constraints (32) – (40) can be rewritten as follows:

b) Limits on real power outputs (32)

$$(UG_i + \Delta UG_i) \underline{PG}_i \leq \sum_{n=1}^{NK} (PG_{in} + \Delta PG_{in}) \leq (UG_i + \Delta UG_i) \overline{PG}_i + \sum_{n=1}^{NM} d_{in} \cdot (QG_{in} + \Delta QG_{in}) \quad (49)$$

$$- PG_{in} \leq \Delta PG_{in} \leq \overline{PG}_{in} - PG_{in} \quad (50)$$

We add step size limits on the  $\Delta PG_{in}$  so that the linear model is valid as below:

$$- \overline{\Delta PG}_i \leq \sum_{n=1}^{NK} \Delta PG_{in} \leq + \overline{\Delta PG}_i \quad (\text{Limits on } \Delta PG \text{ step}) \quad (51)$$

c) Limits on reactive power outputs (34)

$$(UG_i + \Delta UG_i) \underline{QG}_i \leq \sum_{n=1}^{NM} (QG_{in} + \Delta QG_{in}) \leq (UG_i + \Delta UG_i) \overline{QG}_i \quad (52)$$

$$\underline{QG}_{in} - QG_{in} \leq \Delta QG_{in} \leq \overline{QG}_{in} - QG_{in} \quad (53)$$

d) Limits on capacitor output (36)

$$(US_i + \Delta US_i) \underline{QS}_i \leq QS_i + \Delta QS_i \leq (US_i + \Delta US_i) \overline{QS}_i \quad (54)$$

We add step size limits on the  $\Delta QS_i$  so that the linear model is valid as below:

$$- \overline{\Delta QS}_i \leq \Delta QS_i \leq + \overline{\Delta QS}_i \quad (\text{Limits on } \Delta QS \text{ step}) \quad (55)$$

e) Limits on generator voltage (control) and load bus voltages (dependent) (37)-(38)

$$\underline{V}_G - V_G \leq +\Delta V_G \leq \overline{V}_G - V_G \quad (56)$$

$$\underline{V}_L - V_L \leq +\Delta V_L \leq \overline{V}_L - V_L \quad (57)$$

We add step size limits on the  $\Delta V_{Gi}$  so that the linear model is valid as below:

$$-\overline{\Delta V_{Gi}} \leq \Delta V_{Gi} \leq +\overline{\Delta V_{Gi}} \quad (\text{Limits on } \Delta VG \text{ step}) \quad (58)$$

f) Limits on UG and US (39)-(40)

$$UG \leq +\Delta UG \leq 1 - UG \quad (59)$$

$$US \leq +\Delta US \leq 1 - US \quad (60)$$

In the above problem, the solution vector equals:  $\Delta X = [\Delta UG \ \Delta US \ \Delta PG \ \Delta QG \ \Delta VG]$ . This MILP model is solved to determine  $\Delta PG$ ,  $\Delta QS$  and  $\Delta VG$ . Thereafter, we update PG, VG and QS. Then by solving power balance equations, we determine values of QG, VL and  $\delta$ .

This process of setting up the MILP model, its solution, update of control variables and solution of power balance equations is grouped as a LPMOVE. LPMOVEs are repeated successively to determine optimal settings. In the following text, we transform the above formulation into a fuzzy formulation so that we combine the two objectives and provide a single objective optimization model.

### 3.3 Fuzzy Optimization Model

Since we have to two conflicting objectives that need to be optimized simultaneously, our problem became a multiobjective optimization problem. There are many methods to solve this multiobjective problem as constructing single aggregate objective function (AOF), Normal Boundary Intersection method (NBI), Normal

Constraint method (NC), Multiobjective Optimization Evolutionary Algorithms (MOEA)...etc. One of the methods that also can be used to solve the problem and which we choose to use in this thesis is the Fuzzy method. In 1965 a new logic based on Fuzzy sets was introduced by L.A.Zadeh [17]. He defined Fuzzy set as a class of objects with a continuum of grades of membership. Such set is characterized by a membership function which assigns to each object a grade of membership between zero and one. Fuzzy method is computationally simple and very efficient.

In order to optimize the formulation presented in the preceding section, the objectives of the multiobjective formulation are transformed to fuzzy sets. In a fuzzy optimization model, two satisfaction parameters for Total Cost ( $\mu_T$ ) and Voltage stability ( $\mu_V$ ) are created. The minimum of all satisfaction parameters ( $\lambda$ ) is maximized while observing other non-fuzzy constraints. Mathematically, this is achieved by forming fuzzy functions for each objective as below.

### 3.3.1 Fuzzy Model of Total Cost Min Objective

Let the following fuzzy set define the satisfaction of a solution  $\Delta X$  with respect to the incremental objective of cost minimization (41):

$$\Delta TC = \left\{ (\mu_T(\Delta X), \Delta TC(\Delta X)) \mid \underline{\Delta TC} < \Delta TC(\Delta X) < \overline{\Delta TC} \right\} \quad (61)$$

The values of  $\Delta X$  are defined such that:  $\underline{\Delta TC} < \Delta TC(\Delta X) < \overline{\Delta TC}$ . This constraint limits the solution within a set of feasible values. The variable  $\mu_T$  is the satisfaction of the solution  $\Delta X$  with respect to the first objective. The satisfaction  $\mu_T$  can be defined as:

$$\mu_T = \frac{\overline{\Delta TC} - \Delta TC(\Delta X)}{\overline{\Delta TC} - \underline{\Delta TC}} \quad (62)$$

One can surmise from Fig. 4 that as  $\Delta TC$  moves from the maximum value ( $\overline{\Delta TC}$ ) to the minimum value ( $\underline{\Delta TC}$ ), the satisfaction increases from zero to one.

### 3.3.2 Fuzzy Model of Voltage Stability Margin Max Objective

Let the following fuzzy set define the satisfaction of a solution  $\Delta X$  with respect to the incremental objective of voltage stability margin maximization (46):

$$\Delta\sigma = \left\{ (\mu_v(\Delta X), \Delta\sigma(\Delta X)) \mid \underline{\Delta\sigma} < \Delta\sigma(\Delta X) < \overline{\Delta\sigma} \right\} \quad (63)$$

The values of  $\Delta X$  are defined such that:  $\underline{\Delta\sigma} < \Delta\sigma(\Delta X) < \overline{\Delta\sigma}$ . The variable  $\mu_v$  is the satisfaction of the solution  $\Delta X$  with respect to the second objective. The satisfaction  $\mu_v$  can be defined as:

$$\mu_v = \frac{\Delta\sigma(\Delta X) - \underline{\Delta\sigma}}{\overline{\Delta\sigma} - \underline{\Delta\sigma}} \quad (64)$$

One can surmise from Fig. 5 that as  $\Delta\sigma$  moves from the minimum value ( $\underline{\Delta\sigma}$ ) to the maximum value ( $\overline{\Delta\sigma}$ ), the satisfaction increases from zero to one.

### 3.4 Complete Fuzzy Model

Let  $\lambda$  be the intersection of fuzzy satisfaction functions of (62) and (64). The variable  $\lambda$  is maximized to maximize the satisfaction of the two objectives. Setting  $\lambda$  lesser than  $\mu_T$  and  $\mu_v$ , one gets the following relations from (62) and (64):

$$\lambda \leq \mu_{TC} = \frac{\overline{\Delta TC} - \Delta TC(\Delta X)}{\overline{\Delta TC} - \underline{\Delta TC}} \quad \text{or} \quad (\overline{\Delta TC} - \underline{\Delta TC})\lambda + \Delta TC(\Delta X) \leq \overline{\Delta TC} \quad (65)$$

$$\lambda \leq \mu_S = \frac{\Delta\sigma(\Delta X) - \underline{\Delta\sigma}}{\overline{\Delta\sigma} - \underline{\Delta\sigma}} \quad \text{or} \quad \overline{\Delta\sigma} \leq \Delta\sigma(\Delta X) - (\overline{\Delta\sigma} - \underline{\Delta\sigma})\lambda \quad (66)$$

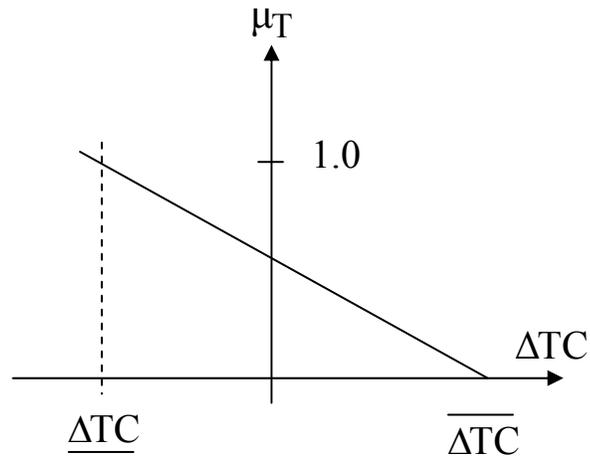


Figure 7 Satisfaction function  $\mu_T$  for the first objective of Total Cost Minimization

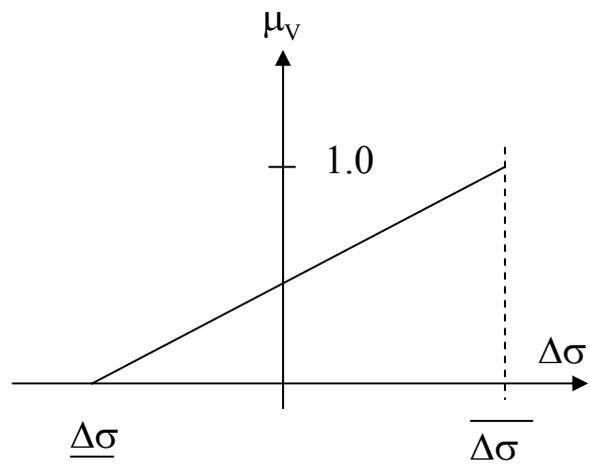


Figure 8 Satisfaction function  $\mu_V$  for the second objective of Voltage Stability Margin Maximization

Then the fuzzy optimization process changes to maximization of  $\lambda$ . The complete fuzzy optimization problem may be stated as:

$$\text{Maximize } \lambda \quad (67)$$

Subject to the constraints (65) & (66)

$$\lambda \leq \mu_{TC} = \frac{\overline{\Delta TC} - \Delta TC(\Delta X)}{\overline{\Delta TC} - \underline{\Delta TC}} \quad \text{or} \quad (\overline{\Delta TC} - \underline{\Delta TC})\lambda + \Delta TC(\Delta X) \leq \overline{\Delta TC} \quad (68)$$

$$\lambda \leq \mu_S = \frac{\Delta \sigma(\Delta X) - \underline{\Delta \sigma}}{\overline{\Delta \sigma} - \underline{\Delta \sigma}} \quad \text{or} \quad \overline{\Delta \sigma} \leq \Delta \sigma(\Delta X) - (\overline{\Delta \sigma} - \underline{\Delta \sigma})\lambda \quad (69)$$

Subject to the constraints (47) – (60)

a) Power Balance Equations:

$$\Delta PG_i - [J1 \ J2] \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = 0 \quad (70)$$

$$\Delta QG_i - [J3 \ J4] \begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} + \sum_{j \in i} \Delta QS_j = 0 \quad (71)$$

b) Limits on real power outputs

$$(UG_i + \Delta UG_i) \underline{PG}_i \leq \sum_{n=1}^{NK} (PG_{in} + \Delta PG_{in}) \leq (UG_i + \Delta UG_i) \overline{PG}_i + \sum_{n=1}^{NM} d_{in} \cdot (QG_{in} + \Delta QG_{in}) \quad (72)$$

$$- PG_{in} \leq \Delta PG_{in} \leq \overline{PG}_{in} - PG_{in} \quad (73)$$

$$-\overline{\Delta PG_i} \leq \sum_{n=1}^{NK} \Delta PG_{in} \leq +\overline{\Delta PG_i} \quad (\text{Limits on } \Delta PG \text{ step}) \quad (74)$$

c) Limits on reactive power outputs

$$(\overline{UG_i} + \Delta UG_i) \cdot \underline{QG_i} \leq \sum_{n=1}^{NM} (QG_{in} + \Delta QG_{in}) \leq (\overline{UG_i} + \Delta UG_i) \cdot \overline{QG_i} \quad (75)$$

$$\underline{QG_{in}} - QG_{in} \leq \Delta QG_{in} \leq \overline{QG_{in}} - QG_{in} \quad (76)$$

d) Limits on capacitor output

$$(\overline{US_i} + \Delta US_i) \cdot \underline{QS_i} \leq QS_i + \Delta QS_i \leq (\overline{US_i} + \Delta US_i) \cdot \overline{QS_i} \quad (77)$$

$$-\overline{\Delta QS_i} \leq \Delta QS_i \leq +\overline{\Delta QS_i} \quad (\text{Limits on } \Delta QS \text{ step}) \quad (78)$$

e) Limits on generator voltage (control) and load bus voltages (dependent)

$$\underline{V_G} - V_G \leq +\Delta V_G \leq \overline{V_G} - V_G \quad (79)$$

$$\underline{V_L} - V_L \leq +\Delta V_L \leq \overline{V_L} - V_L \quad (80)$$

$$-\overline{\Delta V_{Gi}} \leq \Delta V_{Gi} \leq +\overline{\Delta V_{Gi}} \quad (\text{Limits on } \Delta VG \text{ step}) \quad (81)$$

f) Limits on UG and US

$$UG \leq +\Delta UG \leq 1 - UG \quad (82)$$

$$US \leq +\Delta US \leq 1 - US \quad (83)$$

Additionally, the optimization must ensure that vectors  $[UG + \Delta UG]$  and  $[US + \Delta US]$  remain binary integers.

### 3.5 Solution Algorithm

The formulation (67)-(83) is written and programmed in MATLAB® using the optimization function “mosekopt”, an optimization toolbox. The study results are discussed in the next chapter. The proposed solution algorithm has the following steps:

- **Step 1:** Load Flow analysis for the System is done with the initial values of X.
- **Step 2:** The constraint equations and limits are set using the values of X.
- **Step 3:** The optimization formulation (67)-(83) is solved to maximize  $\lambda$  to determine the optimal increments  $\Delta X$ .
- **Step 4:** Update the values of the control variables of X with their optimal increments in  $\Delta X$  ( $\Delta PG$ ,  $\Delta QS$ ,  $\Delta UG$ ,  $\Delta US$  and  $\Delta V$ ).  $X = X + \Delta X$ .
- **Step 5:** Load Flow Analysis is done using the new values of X achieved from Step 4.
- **Step 6:** The values of Total Cost, MW Margin to Voltage Collapse and the marginal price are calculated and recorded.
- **Step 7:** Steps 2-6 are called an LPMOVE. They are repeated for a number of times to get the optimal solution.

Note: In case of the single objective solution (Only minimizing the cost), the constraint equation (69) (for maximizing the stability) is not included in Step 2.

## CHAPTER 4

# RESULTS AND DISCUSSIONS

In order to study the effect of the voltage stability margin on the market efficiency, the optimization problem is solved in two steps.

1. First, the fuzzy optimization problem is solved without considering the second objective (eq. 69) of maximizing the min singular value of the system Jacobian. This solution, which is called the single objective solution, yields the least cost generation dispatch solution.

The results are recorded and the bidding curve is drawn. The *CMC* (Competitive Market Clearing) point, the net seller surplus and the market efficiency values are calculated accordingly. The net seller surplus in this point is maximum and the market efficiency is 100% (maximum efficiency).

2. Then, the formulation is solved considering both objectives of minimizing costs (68) and maximizing the min singular value of the Jacobian (voltage stability margin maximization) (69).

The Net Seller Surplus and the Market Efficiency values for this solution, which is called the double objective solution, are then calculated and compared to the single objective solution.

The above procedure was applied to two cases of the 5-bus test system (Figure 9) and a modified version of the IEEE 118-bus test system.

### 4.1 5-bus System

For the 5-bus test system, two cases are set up. In case #1, expensive generators are set close to bus 4 (system load centre), while in case #2 the inexpensive generators are set close to bus 4. Data for the system is given in the Appendices (Appendix B).

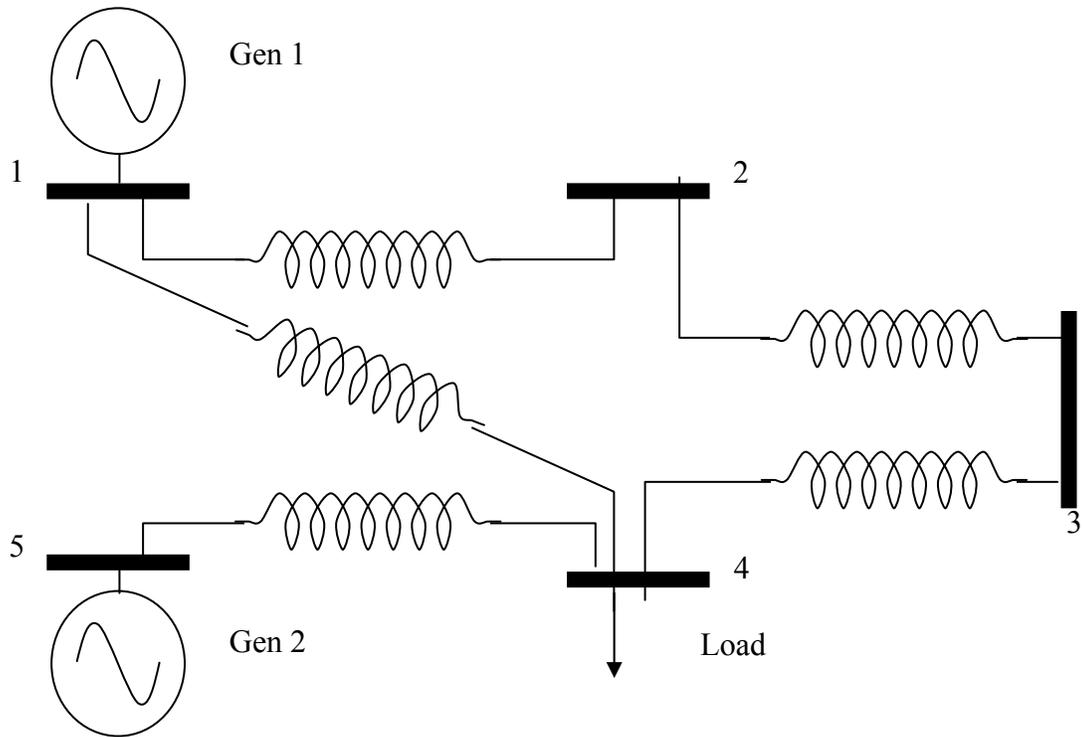


Figure 9 5-bus System

#### 4.1.1 Case #1

Generator 1, which is connected to bus 1 (away from the load at bus 4), sells energy at a lesser price whereas Generator 2 connected to bus 5 (closer to the load at bus 4) sells energy at a higher price.

Before optimizing, the two generators share the system load equally by supplying 200 MW each. This is the *starting state*. Following Step 1, the single objective solution is obtained minimizing the cost only. The cost decreases from \$8,000 to \$7,000 as the generation is optimally moved to the inexpensive generator 1 so that it now supplies 300 MW of the 400 MW load. The minimum singular value decreases from 4.1080 to 3.9135 as generator 1 (bus 1) is far away from load at bus 4 as compared to generator 2 (bus 5). In addition, the MW Margin to Voltage Collapse drops from 212 MW to 168 MW. Thereafter, following step 2, the double objective solution moves generation to the expensive generator 2 (bus 5) so that it now supplies 300 MW and generator 1 at bus 1 supplies only 100 MW. It increases the cost to \$10,000 and the minimum singular value increases from 4.1080 (Starting State) to 4.2162. The MW Margin to Voltage Collapse increased from 212 MW to 228 MW. These results can be seen in Figure 10.

As a result of the configuration of the system, the objective of increasing the system voltage stability shifts most of the generation to the more expensive generator which is closer to the load in this case. These results are reflected on the bidding curves developed for the generators and shown in Figure 11. It is found that the market efficiency drops from 100% in single objective case to 50% ( $= A1 / [A1+A2]$ ) in the double objective case.

Summarizing, in Table 4, the optimal results of single and double objective optimization are presented. In this case, it is evident that as the voltage stability margin is maximized, the market efficiency reduces.

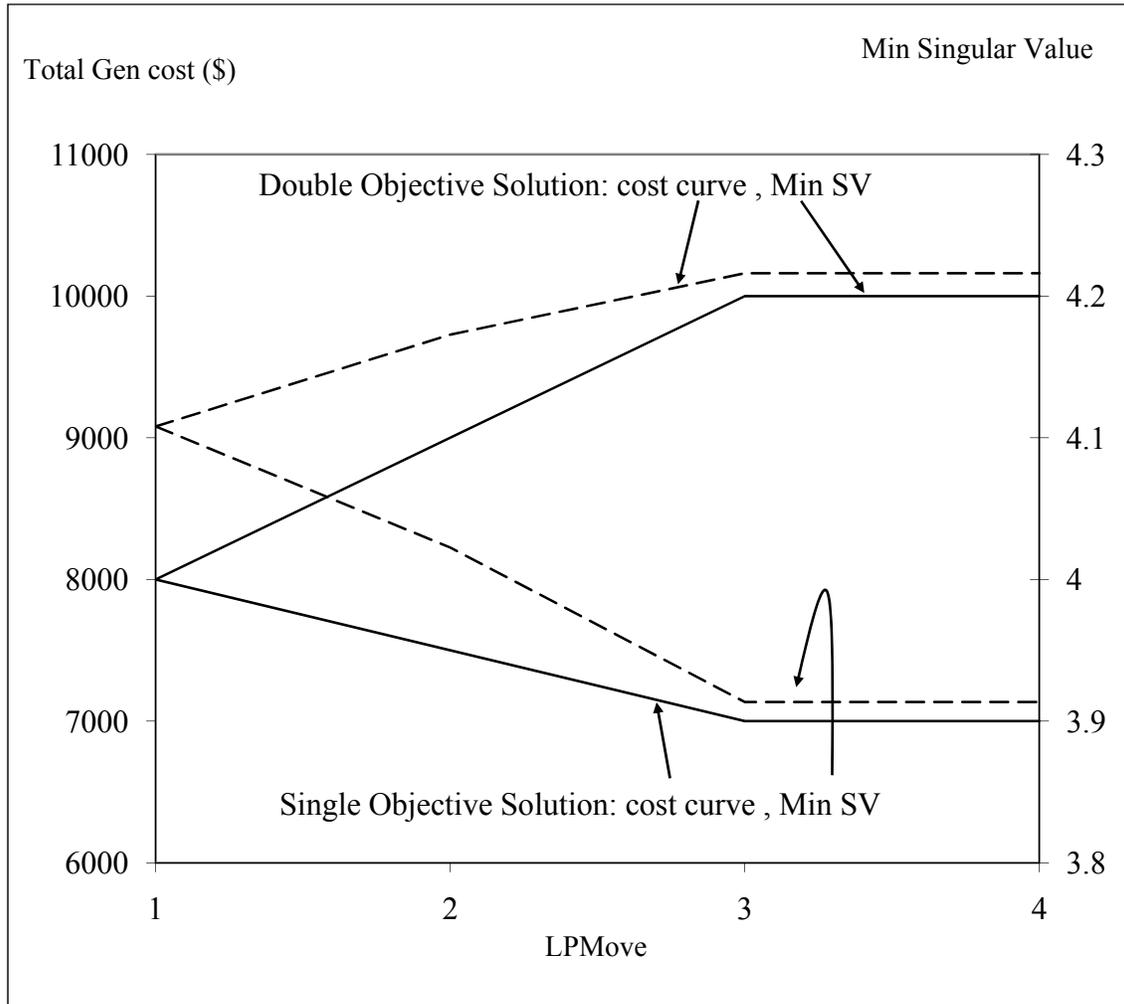


Figure 10 5-bus system case #1

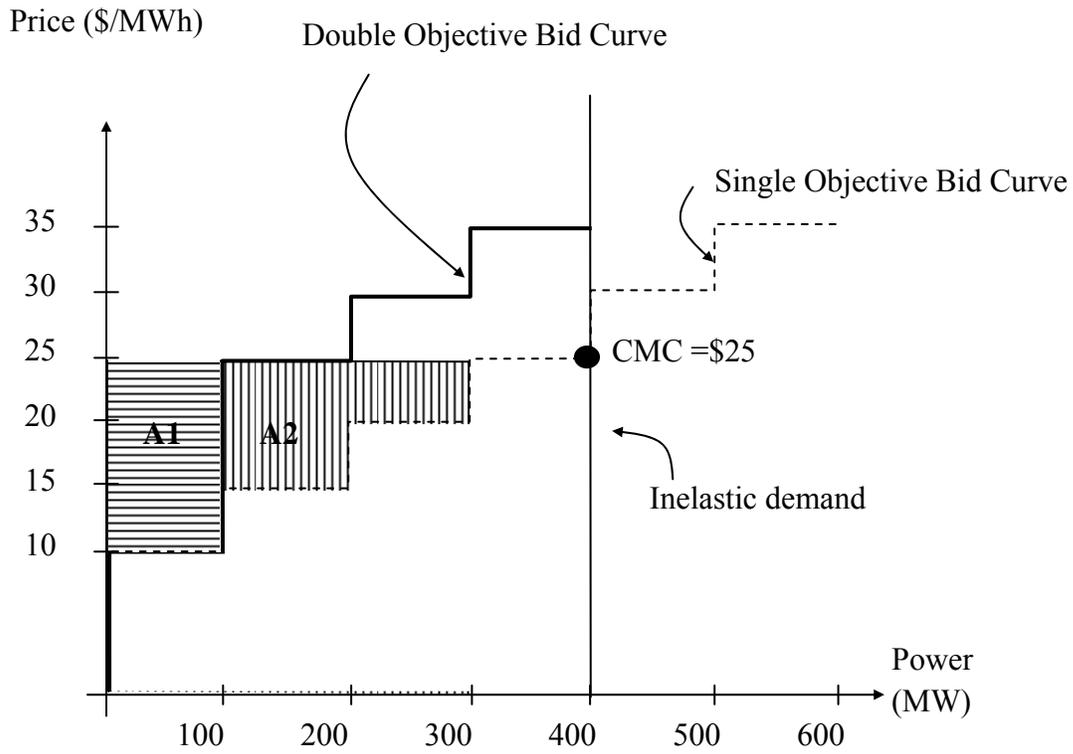


Figure 11 5-bus system case #1(Bidding Curve)

A1 = \$1500; A2 = \$1500

TABLE 4 FINAL RESULTS OF 5-BUS CASE I

Optimization	Real Power Cost (\$)	Min Singular Value ( $\sigma_{\min}$ )	Load Margin to Collapse (MW)	System Marginal Price (\$/MWh)	Market Efficiency %
Single Objective	7000	3.9135	168	25	100
Double Objective	10,000	4.2162	228	35	50

### 4.1.2 Case #2

In case #2, we reverse the price data of generators 1 and 2 so that generator 1 (away from load) is expensive and generator 2 (closer to the load) is inexpensive. The data for this system is given in the Appendix.

After getting the single objective solution (minimizing the cost only) in Step 1, the cost decreases from \$8,000 to \$7,000 and the minimum singular value increases from 4.1080 to 4.2144. The MW Margin to Voltage Collapse increased from 212 MW to 252 MW. Following step 2, the double objective solution also increases the min singular value from the same point to 4.2144 and the MW Margin to Voltage Collapse from the same point to 252 MW while the cost decreases to \$7,000. Hence, in this case #2, the two optimization solutions yield the same result. The results are shown in Figure 12.

In case #2, the generator pricing aids the system's voltage stability. Hence, optimizing for the single objective (step 1) shifts generation to the inexpensive generator 2 (bus 5) which is closer to the load. Consequently, the voltage stability increases automatically. In step 2 of case #2, optimizing for double objectives yields the same result. Hence, the market efficiency remains at 100%. The bid curves for the single objective and double objective solutions are identical (they are identical to the single objective solution bid curve of case #1).

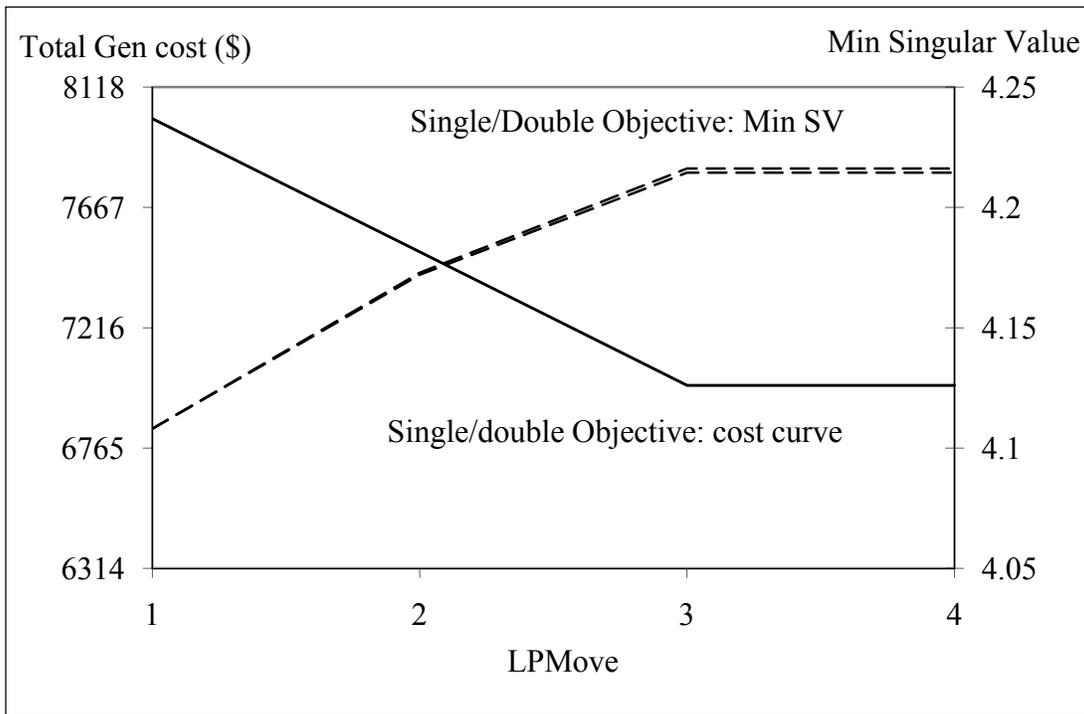


Figure 12 5- bus system case #2

On surveying real power systems, one sees that usually, large coal fired plants and hydro generators are placed far away from large urban load centers. These power plants supply inexpensive electric energy. Small peaking plants tend to supply expensive electric energy and are situated closer to the loads. Hence, not as a rule, in the usual scenario case #1 is more likely in a real power system. Hence, while optimizing both objectives, where the second objective tries to maximize voltage stability, the market efficiency reduces.

#### **4.2 118-bus System**

We now consider the standard 118-bus IEEE system. It is modified by changing line resistance values to zero such that it becomes a lossless system. Data for the system is given in the Appendices Section (Appendix C). The results of optimization are given in Figure 13. On optimizing the system considering only the single objective of cost minimization, the optimum cost equals \$79,451 with a min singular value of 0.1975 and a MW Margin to Voltage Collapse of 186.65 MW. In contrast, when the system is optimized for double objectives of cost minimization and voltage stability maximization, it has the optimum costs of \$115,573, a min singular value of 0.2007 and a MW Margin to Voltage Collapse of 306.65 MW.

From the results, one may clearly see that by optimizing the system in the double objective case, the generation moves towards the loads to increase min singular value and it costs more. Now, by looking at the bid curves (Figure 14), it can be seen that the marginal price is driven much higher as lower priced generator segments (away from loads) are switched off and higher priced generator segments (closer to the load) are turned on.

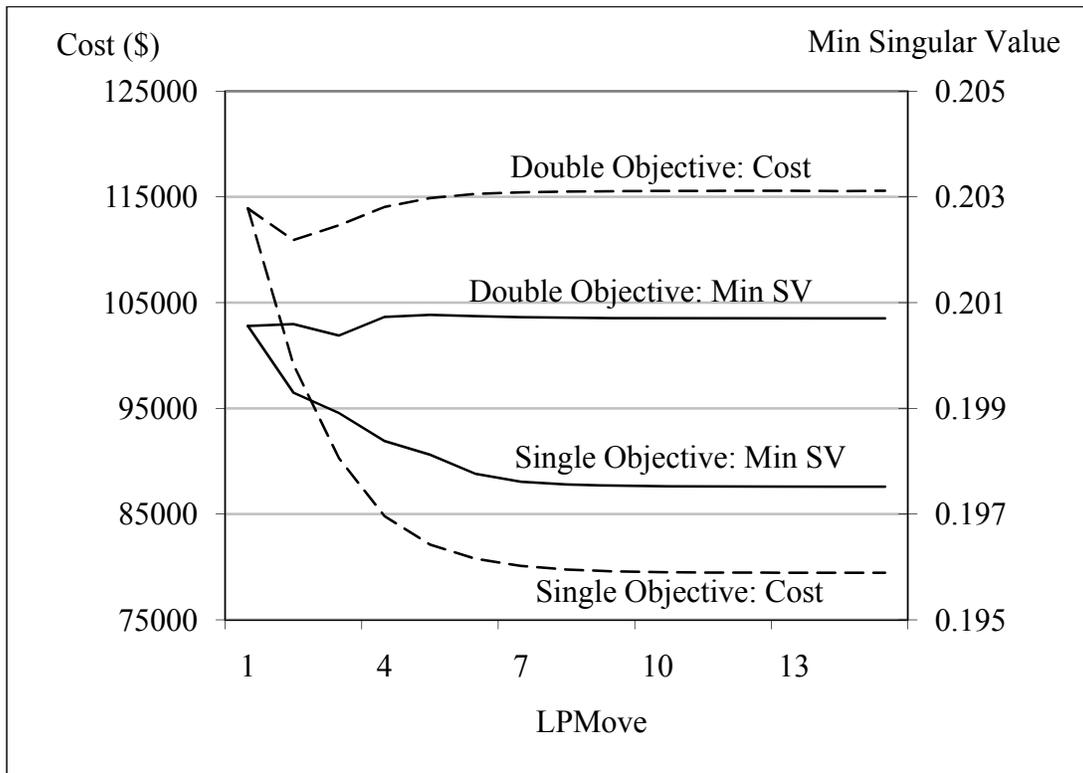


Figure 13 118-bus system

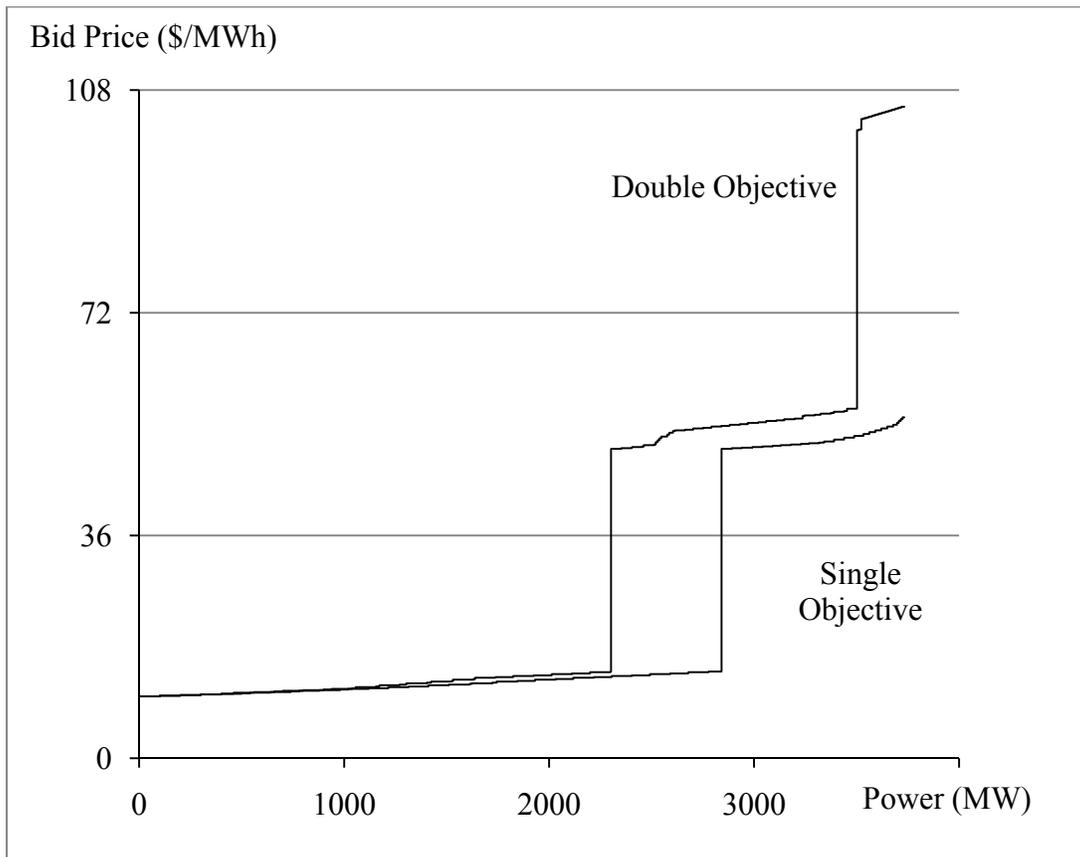


Figure 14 118-bus system (Bidding Curve)

TABLE 5 FINAL RESULTS OF 118-BUS SYSTEM

Optimization	Real Power Cost (\$)	Min Singular Value ( $\sigma_{\min}$ )	Load Margin to Collapse (MW)	System Marginal Price (\$/MWh)	Market Efficiency %
Single Objective	79,451	0.1975	186.65	55.1	100
Double Objective	115,573	0.2007	306.65	105.3	80.59

From Figure 14, it is evident that the system marginal price has increased to \$105.30 /MWh in the double objective case from \$55.10/MWh in the single objective case. The market efficiency is lowered to 80.59% in the double objective case from 100% in the single objective case. Table 5 summarizes the relationship between voltage stability margin maximization and market efficiency in the 118-bus system.

From the preceding analysis, the following are clear:

- When load centers are away from inexpensive generators, rescheduling to maximize voltage stability moves generation to expensive generators closer to loads so that voltage stability improves.
- A higher voltage stability therefore means, a higher system marginal price.
- A higher voltage stability also yields a lower market efficiency.

## CHAPTER 5

# CONCLUSIONS

This thesis studies the effect of maximizing voltage stability margin on market efficiency. To this end, this thesis proposes a formulation and its solution that works on two stages. The first stage minimizes the generation cost and settles the market to get the maximum seller surplus that can be extracted from the system and accordingly the maximum efficiency of the electricity market for the system. The second stage tries to maximize the voltage stability of the system in order to see how the market efficiency changes from the initial value when the system moves towards a more stable configuration.

Thereafter, the thesis studies two systems, a 5-bus test system and the IEEE 118-bus test system. The 5-bus system is created with two cases. The results of the three study cases show the negative effect that a voltage stability maximization objective has on the market efficiency of a power system.

a) In the first case (case #1), the load is located far away from inexpensive generation. In this case, optimizing only for the cost moves the generation to inexpensive generators (inframarginal units) leading to a low voltage stability margin, a low MW Margin to Voltage Collapse and high market efficiency. When both the objectives are optimized, it assigns generation to expensive units (extramarginal units) located closer to loads where by the solution cost increases and voltage stability represented in the min singular value and the MW Margin to Voltage Collapse improves while the market efficiency lowers.

b) In the second case (case #2), loads are located closer to inexpensive generators and it shows that optimizing for one or both objectives moves generation to the inexpensive generator (inframarginal units) that minimizes costs and maximizes voltage stability margin whilst having the best market efficiency.

c) As a final example, the 118-bus system with modifications is studied. This study shows once more that as we optimize both objectives, the solution moves generation to expensive units (extramarginal units) closer to the loads. This achieves a higher voltage stability margin but lower market efficiency.

While one cannot make a definite case that inexpensive generators are always located away from the loads, it is most likely the case in a real power system. Hence, optimizing for voltage stability margin would lead to a reduced market efficiency.

In future, the research could be extended to consider a multi-commodity market. A more general configuration for the electricity market in which supply offers are made for both real and reactive powers can be designed.

## APPENDICES

### Appendix A: Minimum Singular Value Decomposition of the Jacobian [16]

The incremental change in any singular value of the load flow Jacobian is represented in terms of the incremental change in the state of the power system. The details are as below. Using Singular Value Decomposition, the load flow Jacobian,  $[J]$  is represented as:

$$[J(\delta, V)] = [SL][\sigma(\delta, V)][SR]^t \quad (A1)$$

where  $[SL]$  and  $[SR]$  are orthogonal singular vector matrices and  $[\Sigma]$  is a diagonal matrix comprising the singular values. Considering a small perturbation in the state,  $\Delta\delta$  and  $\Delta V$ , (A1) is written as:

$$[J(\delta + \Delta\delta, V + \Delta V)] = [SL + \Delta SL][\Sigma + \Delta\Sigma][SR + \Delta SR]^t \quad (A2)$$

Left Hand Side of (A2) is expanded using Taylor's series and only the first order term of the series containing the load flow hessian  $[H]$  is retained while neglecting the higher order terms to get:

$$[\Delta J] = J(\delta + \Delta\delta, V + \Delta V) - J(\delta, V) = [H] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (A2.1)$$

Using above to represent Left Hand Side of (A2) and expanding Right Hand Side of (A2) retaining only the first order terms, (A2) is re-written as:

$$[H] \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = [\Delta SL][\Sigma][SR]^t + [SL][\Delta\Sigma][SR]^t + [SL][\Sigma][\Delta SR]^t \quad (A3)$$

Imposing orthogonality constraints on the updated left and right singular vector matrices one gets:

## Appendices

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$$[\text{SL} + \Delta\text{SL}] [\text{SL} + \Delta\text{SL}] = \text{I} \quad \text{and} \quad [\text{SR} + \Delta\text{SR}] [\text{SR} + \Delta\text{SR}] = \text{I} \quad (\text{A4})$$

Expanding (A4) neglecting the second order terms and using the orthogonality property of [SL] and [SR], the matrices [SM] and [SN] are written as:

$$[\text{SN}] = [\text{SL}]^t [\Delta\text{SL}] = -[\Delta\text{SL}]^t [\text{SL}] \quad \text{and}$$

$$[\text{SM}] = [\Delta\text{SR}]^t [\text{SR}] = -[\text{SR}]^t [\Delta\text{SR}] \quad (\text{A5})$$

From (A5), it can be seen that the diagonal elements of [SN] and [SR] are zeros. Premultiplying and postmultiplying (A3) with [SL]<sup>t</sup> and [SR] respectively, (A3) is written as:

$$[\text{SL}]^t [\text{H}] \begin{bmatrix} \Delta\delta \\ \Delta\text{V} \end{bmatrix} [\text{SR}] = [\text{SN}][\Sigma] + [\Delta\Sigma] + [\Sigma][\text{SM}] \quad (\text{A6})$$

Since [SN][Σ] and [Σ][SM] have zeros as their diagonal elements, the diagonal elements of (A6) are equated as:

$$\Delta\sigma_i = [\Delta\Sigma]_{ii} = \left[ [\text{SL}]^t [\text{H}] \begin{bmatrix} \Delta\delta \\ \Delta\text{V} \end{bmatrix} [\text{SR}] \right]_{ii} \quad (\text{A7})$$

**Appendix B: 5-Bus System Data**

NUMBER OF BUSES	5
SLACK BUS NUMBER	1001
NUMBER OF GENERATORS	2
NUMBER OF LOAD BUSES	3
NUMBER OF TRANSFORMERS	2
NUMBER OF TRANSMISSION LINES	3
NUMBER OF SHUNT CAPACITORS	0
NUMBER OF SWITCHABLE CAPACITORS	2
NUMBER OF SHUNT REACTORS	0
SLACK BUS VOLTAGE	1.00
TOLERANCE (MW)	0.0100
BASE MVA	100.00
MINIMUM LOAD BUS VOLTAGE	0.9500
MAXIMUM LOAD BUS VOLTAGE	1.0500
MAXIMUM NUMBER OF ITERATIONS	200

## Appendices

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### Generator Buses

#	Bus Number	QGMax (Mvar)	QGMin (Mvar)	V (pu)	PGMin (MW)	PGMax (MW)
1	1001	200	-40	1.0	0	300
2	2005	200	-40	1.0	0	300

#	Fixed costs (\$)	Segment 1		Segment 2		Segment 3	
		Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW
<b>Case #1</b>							
1	0	10	100	15	100	20	100
2	0	25	100	30	100	35	100
<b>Case #2</b>							
1	0	25	100	30	100	35	100
2	0	10	100	15	100	20	100

Notes: 1) The reactive power costs (ep, fp) are set to zero.

2) The linear relation values of relation dp are also assumed to be 0.

### Load Buses

#	Bus Number	PD (MW)	QD (Mvar)
3	2	0	0
4	3	0	0
5	4	400	50

## Appendices

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### Transformer Data

#	From Bus	To Bus	Resistance (pu)	Reactance (pu)	Off-nominal Tap Ratio	Rating (MVA)
1	1	4	0	0.2	1	500
2	3	4	0	0.2	1	500

### Transmission Line Data

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
1	1	2	0	0.02	0.001	100
2	2	3	0	0.02	0.001	100
3	4	5	0	0.02	0.001	100

### Switchable Capacitor Data

#	Bus Number	MVAR MAX	MVAR MIN	MVAR STEP	MVAR ACTUAL
1	1003	0.5	0	0.01	0
2	2004	0.5	0	0.01	0

**Appendix C: 118-Bus System Data**

NUMBER OF BUSES	118
SLACK BUS NUMBER	69
NUMBER OF GENERATORS	54
NUMBER OF LOAD BUSES	64
NUMBER OF TRANSFORMERS	9
NUMBER OF TRANSMISSION LINES	177
NUMBER OF SHUNT CAPACITORS	0
NUMBER OF SWITCHABLE CAPACITORS	14
NUMBER OF SHUNT REACTORS	0
SLACK BUS VOLTAGE	1.0350
TOLERANCE (MW)	0.100
BASE MVA	100.00
MINIMUM LOAD BUS VOLTAGE	0.9500
MAXIMUM LOAD BUS VOLTAGE	1.0500
MAXIMUM NUMBER OF ITERATIONS	20

Appendices

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Generator Buses

#	Bus Number	QGMax (Mvar)	QGMin (Mvar)	V (pu)	PGMin (MW)	PGMax (MW)
1	87	1000	-100	1.0	0	300
2	10	300	-300	1.0	0	300
3	80	280	-165	1.0	0	300
4	89	300	-210	1.0	0	300
5	65	200	-67	1.0	0	300
6	66	250	-250	1.0	0	300
7	26	1000	-1000	1.0	0	300
8	69	99999	-99999	1.0	0	300
9	12	120	-35	1.0	0	300
10	25	200	-200	1.0	0	300
11	92	10	-10	1.0	0	300
12	99	100	-100	1.0	0	300
13	100	155	-50	1.0	0	300
14	49	210	-85	1.0	0	300
15	54	300	-300	1.0	0	150
16	59	180	-100	1.0	0	150
17	61	300	-100	1.0	0	150
18	18	50	-30	1.0	0	150
19	32	42	-14	1.0	0	150

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#	Bus Number	QGMax (Mvar)	QGMin (Mvar)	V (pu)	PGMin (MW)	PGMax (MW)
20	36	24	-20	1.0	0	150
21	46	100	-100	1.0	0	150
22	55	23	-8	1.0	0	150
23	56	15	-8	1.0	0	150
24	62	20	-20	1.0	0	150
25	76	23	-8	1.0	0	150
26	77	70	-20	1.0	0	150
27	82	9900	-9900	1.0	0	150
28	104	23	-15	1.0	0	150
29	105	23	-8	1.0	0	150
30	111	1000	-100	1.0	0	150
31	112	1000	-100	1.0	0	150
32	113	200	-100	1.0	0	150
33	70	32	-25	1.0	0	150
34	91	100	-100	1.0	0	150
35	110	23	-8	1.0	0	150
36	116	1000	-1000	1.0	0	150
37	4	300	-300	1.0	0	30
38	6	50	-13	1.0	0	30

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#	Bus Number	QGMax (Mvar)	QGMin (Mvar)	V (pu)	PGMin (MW)	PGMax (MW)
39	8	300	-300	1.0	0	30
40	15	30	-25	1.0	0	30
41	19	24	-8	1.0	0	30
42	24	300	-300	1.0	0	30
43	27	300	-300	1.0	0	30
44	31	300	-300	1.0	0	30
45	34	150	-150	1.0	0	30
46	40	300	-300	1.0	0	30
47	42	300	-300	1.0	0	30
48	72	100	-100	1.0	0	30
49	73	100	-100	1.0	0	30
50	85	200	-200	1.0	0	30
51	74	30	-25	1.0	0	30
52	90	300	-300	1.0	0	30
53	103	60	-20	1.0	0	30
54	107	200	-200	1.0	0	30

Appendices

#	Fixed costs (\$)	Segment 1		Segment 2		Segment 3	
		Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW
1	87	10	100	10.1	100	10.2	100
2	10	10.3	100	10.4	100	10.5	100
3	80	10.6	100	10.7	100	10.8	100
4	89	10.9	100	11	100	11.1	100
5	65	11.2	100	11.3	100	11.4	100
6	66	11.5	100	11.6	100	11.7	100
7	26	11.8	100	11.9	100	12	100
8	69	12.1	100	12.2	100	12.3	100
9	12	12.4	100	12.5	100	12.6	100
10	25	12.7	100	12.8	100	12.9	100
11	92	13	100	13.1	100	13.2	100
12	99	13.3	100	13.4	100	13.5	100
13	100	13.6	100	13.7	100	13.8	100
14	49	13.9	100	14	100	14.1	100
15	54	50	50	50.1	50	50.2	50
16	59	50.3	50	50.4	50	50.5	50
17	61	50.6	50	50.7	50	50.8	50
18	18	50.9	50	51	50	51.1	50
19	32	51.2	50	51.3	50	51.4	50

Appendices

#	Fixed costs (\$)	Segment 1		Segment 2		Segment 3	
		Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW
20	36	51.5	50	51.6	50	51.7	50
21	46	51.8	50	51.9	50	52	50
22	55	52.1	50	52.2	50	52.3	50
23	56	52.4	50	52.5	50	52.6	50
24	62	52.7	50	52.8	50	52.9	50
25	76	53	50	53.1	50	53.2	50
26	77	53.3	50	53.4	50	53.5	50
27	82	53.6	50	53.7	50	53.8	50
28	104	53.9	50	54	50	54.1	50
29	105	54.2	50	54.3	50	54.4	50
30	111	54.5	50	54.6	50	54.7	50
31	112	54.8	50	54.9	50	55	50
32	113	55.1	50	55.2	50	55.3	50
33	70	55.4	50	55.5	50	55.6	50
34	91	55.7	50	55.8	50	55.9	50
35	110	56	50	56.1	50	56.2	50
36	116	56.3	50	56.4	50	56.5	50
37	4	100	10	100.1	10	100.2	10
38	6	100.3	10	100.4	10	100.5	10

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#	Fixed costs (\$)	Segment 1		Segment 2		Segment 3	
		Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW	Linear Cost \$/MWh	Capacity MW
39	8	100.6	10	100.7	10	100.8	10
40	15	100.9	10	101	10	101.1	10
41	19	101.2	10	101.3	10	101.4	10
42	24	101.5	10	101.6	10	101.7	10
43	27	101.8	10	101.9	10	102	10
44	31	102.1	10	102.2	10	102.3	10
45	34	102.4	10	102.5	10	102.6	10
46	40	102.7	10	102.8	10	102.9	10
47	42	103	10	103.1	10	103.2	10
48	72	103.3	10	103.4	10	103.5	10
49	73	103.6	10	103.7	10	103.8	10
50	85	103.9	10	104	10	104.1	10
51	74	104.2	10	104.3	10	104.4	10
52	90	104.5	10	104.6	10	104.7	10
53	103	104.8	10	104.9	10	105	10
54	107	105.1	10	105.2	10	105.3	10

Notes: 1) The reactive power costs (ep, fp) are set to zero.

2) The linear relation values of relation dp are also assumed to be 0.

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### Load Buses

#	Bus Number	PD (MW)	QD (Mvar)
1	1	54.14	28.66
2	2	21.23	9.55
3	3	41.4	10.62
4	5	0	0
5	7	20.17	2.12
6	9	0	0
7	11	74.31	24.42
8	13	36.09	16.99
9	14	14.86	1.06
10	16	26.54	10.62
11	17	11.68	3.18
12	20	19.11	3.18
13	21	14.86	8.49
14	22	10.62	5.31
15	23	7.43	3.18
16	28	18.05	7.43
17	29	25.48	4.25
18	30	0	0
19	33	24.42	9.55
20	35	35.03	9.55

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#	Bus Number	PD (MW)	QD (Mvar)
21	37	0	0
22	38	0	0
23	39	27	11
24	41	37	10
25	43	18	7
26	44	16	8
27	45	53	22
28	47	34	0
29	48	20	11
30	50	17	4
31	51	17	8
32	52	18	5
33	53	23	11
34	57	12	3
35	58	12	3
36	60	78	3
37	63	0	0
38	64	0	0
39	67	28	7
40	68	0	0

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#	Bus Number	PD (MW)	QD (Mvar)
41	71	0	0
42	75	47	11
43	78	71	26
44	79	39	32
45	81	0	0
46	83	20	10
47	84	11	7
48	86	21	2
49	88	48	10
50	93	12	7
51	94	30	16
52	95	42	31
53	96	38	15
54	97	15	9
55	98	34	8
56	101	22	15
57	102	5	3
58	106	43	16
59	108	2	1

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#	Bus Number	PD (MW)	QD (Mvar)
60	109	8	3
61	114	8.49	3.18
62	115	23.35	7.43
63	117	21.23	8.49
64	118	33	15

### Transformer Data

#	From Bus	To Bus	Resistance (pu)	Reactance (pu)	Off-nominal Tap Ratio	Rating (MVA)
1	8	5	0	0.0267	0.985	500
2	26	25	0	0.0382	0.96	500
3	30	17	0	0.0388	0.96	500
4	38	37	0	0.0375	0.935	500
5	63	59	0	0.0386	0.96	500
6	64	61	0	0.0268	0.985	500
7	65	66	0	0.037	0.935	500
8	68	69	0	0.037	0.935	500
9	81	80	0	0.037	0.935	500

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Transmission Line Data

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
1	1	2	0	0.0999	0.0254	400
2	1	3	0	0.0424	0.0108	400
3	4	5	0	0.008	0.0021	400
4	3	5	0	0.108	0.0284	400
5	5	6	0	0.054	0.0143	400
6	6	7	0	0.0208	0.0055	400
7	8	9	0	0.0305	1.162	400
8	9	10	0	0.0322	1.23	400
9	4	11	0	0.0688	0.0175	400
10	5	11	0	0.0682	0.0174	400
11	11	12	0	0.0196	0.005	400
12	2	12	0	0.0616	0.0157	400
13	3	12	0	0.16	0.0406	400
14	7	12	0	0.034	0.0087	400
15	11	13	0	0.0731	0.0188	400
16	12	14	0	0.0707	0.0182	400
17	13	15	0	0.2444	0.0627	400
18	14	15	0	0.195	0.0502	400
19	12	16	0	0.0834	0.0214	400

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#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
20	15	17	0	0.0437	0.0444	400
21	16	17	0	0.1801	0.0466	400
22	17	18	0	0.0505	0.013	400
23	18	19	0	0.0493	0.0114	400
24	19	20	0	0.117	0.0298	400
25	15	19	0	0.0394	0.0101	400
26	20	21	0	0.0849	0.0216	400
27	21	22	0	0.097	0.0246	400
28	22	23	0	0.159	0.0404	400
29	23	24	0	0.0492	0.0498	400
30	23	25	0	0.08	0.0864	400
31	25	27	0	0.163	0.1764	400
32	27	28	0	0.0855	0.0216	400
33	28	29	0	0.0943	0.0238	400
34	8	30	0	0.0504	0.514	400
35	26	30	0	0.086	0.908	400
36	17	31	0	0.1563	0.0399	400
37	29	31	0	0.0331	0.0083	400
38	23	32	0	0.1153	0.1173	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
39	31	32	0	0.0985	0.0251	400
40	27	32	0	0.0755	0.0193	400
41	15	33	0	0.1244	0.0319	400
42	19	34	0	0.247	0.0632	400
43	35	36	0	0.0102	0.0027	400
44	35	37	0	0.0497	0.0132	400
45	33	37	0	0.142	0.0366	400
46	34	36	0	0.0268	0.0057	400
47	34	37	0	0.0094	0.0098	400
48	37	39	0	0.106	0.027	400
49	37	40	0	0.168	0.042	400
50	30	38	0	0.054	0.422	400
51	39	40	0	0.0605	0.0155	400
52	40	41	0	0.0487	0.0122	400
53	40	42	0	0.183	0.0466	400
54	41	42	0	0.135	0.0344	400
55	43	44	0	0.2454	0.0607	400
56	34	43	0	0.1681	0.0423	400
57	44	45	0	0.0901	0.0224	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
58	45	46	0	0.1356	0.0332	400
59	46	47	0	0.127	0.0316	400
60	46	48	0	0.189	0.0472	400
61	47	49	0	0.0625	0.016	400
62	42	49	0	0.323	0.086	400
63	42	49	0	0.323	0.086	400
64	45	49	0	0.186	0.0444	400
65	48	49	0	0.0505	0.0126	400
66	49	50	0	0.0752	0.0187	400
67	49	51	0	0.137	0.0342	400
68	51	52	0	0.0588	0.014	400
69	52	53	0	0.1635	0.0406	400
70	53	54	0	0.122	0.031	400
71	49	54	0	0.289	0.0738	400
72	49	54	0	0.291	0.073	400
73	54	55	0	0.0707	0.0202	400
74	54	56	0	0.0095	0.0073	400
75	55	56	0	0.0151	0.0037	400
76	56	57	0	0.0966	0.0242	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
77	50	57	0	0.134	0.0332	400
78	56	58	0	0.0966	0.0242	400
79	51	58	0	0.0719	0.0179	400
80	54	59	0	0.2293	0.0598	400
81	56	59	0	0.251	0.0569	400
82	56	59	0	0.239	0.0536	400
83	55	59	0	0.2158	0.0565	400
84	59	60	0	0.145	0.0376	400
85	59	61	0	0.15	0.0388	400
86	60	61	0	0.0135	0.0146	400
87	60	62	0	0.0561	0.0147	400
88	61	62	0	0.0376	0.0098	400
89	63	64	0	0.02	0.216	400
90	38	65	0	0.0986	1.046	400
91	64	65	0	0.0302	0.38	400
92	49	66	0	0.0919	0.0248	400
93	49	66	0	0.0919	0.0248	400
94	62	66	0	0.218	0.0578	400
95	62	67	0	0.117	0.031	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
96	66	67	0	0.1015	0.0268	400
97	65	68	0	0.016	0.638	400
98	47	69	0	0.2778	0.0709	400
99	49	69	0	0.324	0.0828	400
100	69	70	0	0.127	0.122	400
101	24	70	0	0.4115	0.102	400
102	70	71	0	0.0355	0.0088	400
103	24	72	0	0.196	0.0488	400
104	71	72	0	0.18	0.0444	400
105	71	73	0	0.0454	0.0118	400
106	70	74	0	0.1323	0.0337	400
107	70	75	0	0.141	0.036	400
108	69	75	0	0.122	0.124	400
109	74	75	0	0.0406	0.0103	400
110	76	77	0	0.148	0.0368	400
111	69	77	0	0.101	0.1038	400
112	75	77	0	0.1999	0.0498	400
113	77	78	0	0.0124	0.0126	400
114	78	79	0	0.0244	0.0065	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
115	77	80	0	0.0485	0.0472	400
116	77	80	0	0.105	0.0228	400
117	79	80	0	0.0704	0.0187	400
118	68	81	0	0.0202	0.808	400
119	77	82	0	0.0853	0.0817	400
120	82	83	0	0.0367	0.038	400
121	83	84	0	0.132	0.0258	400
122	83	85	0	0.148	0.0348	400
123	84	85	0	0.0641	0.0123	400
124	85	86	0	0.123	0.0276	400
125	86	87	0	0.2074	0.0445	400
126	85	88	0	0.102	0.0276	400
127	85	89	0	0.173	0.047	400
128	88	89	0	0.0712	0.0193	400
129	89	90	0	0.188	0.0528	400
130	89	90	0	0.0997	0.106	400
131	90	91	0	0.0836	0.0214	400
132	89	92	0	0.0505	0.0548	400
133	89	92	0	0.1581	0.0414	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
134	91	92	0	0.1272	0.0327	400
135	92	93	0	0.0848	0.0218	400
136	92	94	0	0.158	0.0406	400
137	93	94	0	0.0732	0.0188	400
138	94	95	0	0.0434	0.0111	400
139	80	96	0	0.182	0.0494	400
130	82	96	0	0.053	0.0544	400
141	94	96	0	0.0869	0.023	400
142	80	97	0	0.0934	0.0254	400
143	80	98	0	0.108	0.0286	400
144	80	99	0	0.206	0.0546	400
145	92	100	0	0.295	0.0472	400
146	94	100	0	0.058	0.0604	400
147	95	96	0	0.0547	0.0147	400
148	96	97	0	0.0885	0.024	400
149	98	100	0	0.179	0.0476	400
150	99	100	0	0.0813	0.0216	400
151	100	101	0	0.1262	0.0328	400
152	92	102	0	0.0559	0.0146	400

## Appendices

#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
153	101	102	0	0.112	0.0294	400
154	100	103	0	0.0525	0.0536	400
155	100	104	0	0.204	0.0541	400
156	103	104	0	0.1584	0.0407	400
157	103	105	0	0.1625	0.0408	400
158	100	106	0	0.229	0.062	400
159	104	105	0	0.0378	0.0099	400
160	105	106	0	0.0547	0.0143	400
161	105	107	0	0.183	0.0472	400
162	105	108	0	0.0703	0.0184	400
163	106	107	0	0.183	0.0472	400
164	108	109	0	0.0288	0.0076	400
165	103	110	0	0.1813	0.0461	400
166	109	110	0	0.0762	0.0202	400
167	110	111	0	0.0755	0.02	400
168	110	112	0	0.064	0.062	400
169	17	113	0	0.0301	0.0077	400
170	32	113	0	0.203	0.0518	400
171	32	114	0	0.0612	0.0163	400

## Appendices

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#	From Number	To Number	Resistance (pu)	Reactance (pu)	Half Line Charging (pu)	Rating (MVA)
172	27	115	0	0.0741	0.0197	400
173	114	115	0	0.0104	0.0028	400
174	68	116	0	0.004	0.164	400
175	12	117	0	0.14	0.0358	400
176	75	118	0	0.0481	0.012	400
177	76	118	0	0.0544	0.0136	400

### Switchable Capacitor Data

#	Bus Number	MVAR MAX	MVAR MIN	MVAR STEP	MVAR ACTUAL
1	34	14	13.9	0	14
2	86	100	99.9	0	100
3	88	0	-0.1	0	0
4	44	10	9.9	0	10
5	45	10	9.9	0	10
6	46	10	9.9	0	10
7	48	15	14.9	0	15
8	74	12	11.9	0	12
9	79	20	19.9	0	20

## REFERENCES

- [1] T. Van Cutsem, T. and C. Vournas, “Voltage Stability of Electric Power Systems,” Kluwer Academic Publishers, 1998.
- [2] F. Milano, C.A. Canizares, M. Invernizzi, “Voltage stability constrained OPF market models considering N–1 contingency criteria,” *Elect. Power Syst. Res.* 74 (1): 27–36, 2005.
- [3] X. Lin, A. David, and C. Yu, “Reactive power optimization with voltage stability consideration in power market systems,” *Proc. Inst. Elect. Eng. Generation, Transmission, Distrib.*, 150(3): 305–310, May 2003.
- [4] K. N. Srivastava, S. C. Srivastava, “Effect of generation rescheduling on voltage stability margin,” *International Journal of Electrical Power & Energy Systems*, 19(1), January 1997.
- [5] G. M. Huang and N. C. Nair, “Voltage stability constrained load curtailment procedure to evaluate power system reliability measures,” in *Proc. IEEE PES. Winter Meeting*, 2: 761–765, New York, Jan. 2002.
- [6] A. J. Conejo, F. Milano, R. García-Bertrand, “Congestion Management Ensuring Voltage Stability, *IEEE Trans. on Power Systems*,” 21(1): 357-364, February 2006.
- [7] B. Venkatesh, G. Sadasivam and M. A. Khan, “A new optimal reactive power scheduling method for loss minimization and voltage stability margin maximization using successive multi-objective fuzzy LP technique,” *IEEE Transactions on Power Systems*, 15(2):844–851, May 2000.
- [8] F. L. Alvarado, J. Meng, C. L. DeMarco, and W. S. Mota. “Stability analysis of interconnected power systems coupled with market dynamics,” *IEEE Transactions on Power Systems*, 16(4):695–701, Nov 2001.

## References

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- [9] J. H. Yan, G. A. Stern, P. B. Luh, and F. Zhao, "Payment Versus Bid Cost Minimization in ISO Markets," *IEEE Power & Energy Magazine*, March/April 2008, pp. 24–36.
- [10] H. Li, J. Sun, and L. Tesfatsion, "Separation and volatility of locational marginal prices in restructured wholesale power markets," *ISU Economics Working Paper #09009*, June 2009.
- [11] Florin Capitanescu and Thierry Van Cutsem, "A unified management of congestions due to voltage instability and thermal overload," *Electric Power Syst. Res.*, 77:1274-1283, 2007.
- [12] Leigh Tesfatsion, "Auction Basics for Wholesale Power Markets: Objectives and Pricing Rules," *IEEE Power and Energy Society General Meeting*, August 2009.
- [13] B. Venkatesh, A. Arunagiri and H. B. Gooi, "Unified OPF method for maximizing voltage stability margin using successive fuzzy LP," *Electric Power Systems Research*, 64: 119-128, 2003.
- [14] B. Venkatesh, G. Sadasivam and M. A. Khan, "Optimal reactive power planning against voltage collapse using successive multi-objective fuzzy LP technique," *Proc. Inst. Elect. Eng. Generation, Transmission, Distrib.*, 146(4): 343-348, Jul 1999.
- [15] William D. Stevenson Jr., "Elements of Power System Analysis," Fourth edition, McGraw-Hill, New York (1962).
- [16] A. Tiranuchit and R. J. Thomas, "A posturing, strategy against voltage instabilities in electric power systems," *IEEE Transactions on Power System*, 3 (1): 87–93, Feb 1988.
- [17] L. A. Zadeh, *Fuzzy Sets. Inf. Control* 8 (1965), pp. 338–353.

## References

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- [18] I. Dobson, H. Glavitsch, C.C. Liu, Y. Tamura, K. Vu, "Voltage Collapse in Power Systems," IEEE Circuits and Devices Magazine, vol. 8, no. 3, May 1992, pp. 40-45.