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A Probabilistic Approach for Optimal Capacitor Planning in Distribution Systems with Wind Generators

by

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ABSTRACT

This thesis proposes a probabilistic approach based on the Cumulant method for optimal capacitor planning in distribution systems with high penetration of wind generators. To account for the problem uncertainties, the probabilistic behaviour of load forecasts and wind generators are modeled using Probability Density Functions. Once the probabilistic framework is defined, an optimization problem can be formulated to minimize the total costs of the capacitors and of the annual energy losses. The optimization problem is then solved by using the Logarithm Barrier Interior Point Method, which provides a linear relationship between the cumulants of load and wind variables and the cumulants of the system parameters and solution cost.

The Cumulant method offers a generous advantage in speed, while maintaining acceptable accuracy, as compared to the traditional Monte Carlo Simulation method. The proposed method is tested on a 7-bus and on a 33-bus systems, and the results are reported and discussed.

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LIST OF SYMBOLS

$f_x(x)$	PDF of random variable x
$F_x(x)$	CDF of random variable x
m_n	n^{th}-order moment of random variable x
k_n	n^{th}-order cumulant of random variable x
$\Phi_x(s)$	Moment Generating Function
$\Psi_x(s)$	Cumulant Generating Function
σ^2	Variance of random variable x
$\hat{f}_x(t)$	Characteristic function of random variable x
μ	mean of random variable x Shape Factor of Weibull Distribution
γ	Shape Factor of Weibull Distribution
β	Scale Factor of Weibull Distribution
Z_l	impedance of Line l
T_l	The Tap Setting of Transformer
V_i	voltage magnitude at i^{th} bus
SD_i	Complex Power of Loads at i^{th} bus
SB_i	The difference between generation and load at bus i
P_i, Q_i	Real and Reactive Power Receiving at i^{th} bus
S_i	Complex Power Receiving at i^{th} bus
PW_i	Real Power Output of i^{th}WG
QS_i	Reactive Power Injected into the i^{th} bus
nb, NB	Number of Buses in the RDS
PL_l	Real Power Loss on Line l
TPL	Total Real Power Loss of RDS
TC	Total cost
KL	Cost of Energy
KC	Annual Cost of per unit Capacitor Bank Installed in a RDS
QC	Total Cost of Capacitor Banks Installed in a RDS
ELC	Total Cost of Annual Energy Loss
T_D	Loading Time Interval
LF_D	Load Factor at D^{th} time interval
H	Hessian of the Lagrangian
NQS	Number of capacitors
He_i	i^{th} Hermite polynomial
$L(u)$	Lagrangian function
λ	Lagrangian multiplier
s_i	Slack variable
μ_j	Barrier parameter
$FV(X)$	Voltage drop equation
$FS(X)$	power balance equation
J_g	Jacobian of the equality constraints, g(x),
$\nabla L(u)$	Gradient of the Lagrangian
$\nabla_x^2 L_u$	Second derivative of the Lagrangian with respect to x

LIST OF ABBREVIATIONS

PDF	Probability Density Function
CDF	Cumulative distribution function
RDS	Radial Distribution Systems
WG	Wind Generator
SOE	Set of Equation
MCS	Monte Carlo Simulations
LBIPM	Logarithmic-Barrier Interior Point Method
KKT	Karush–Kuhn–Tucker Conditions
ND	Number of the load samples
NW	Number of the wind generation samples

CHAPTER 1

INTRODUCTION

Large scale integration of wind generators has been promoted across the globe in a drive to harness renewable energy. Most of these generators are sources connected to distribution systems. Due to the intermittent nature of wind, these generators introduce uncertainties on power system planning, which is in addition to difficulties associated with the steady state solution of distribution systems. Furthermore, loads are probabilistic in nature as well.

Probabilistic and stochastic programming introduces random variables and uncertainty into conventional linear and nonlinear programming [1]. The randomness and uncertainty are generally represented by using probability density functions (PDFs) [2]. Many of the power system components are often modeled as a known constant parameter, such as a wind generator or a customer load. However, these components do not have a constant behaviour, which can be treated as a random variable. As a consequence, the power system model becomes a probabilistic framework and as such, requires a probabilistic approach to deal with the load and generator uncertainties. Due to the probabilistic nature of loads and availability of wind, to achieve a precise and optimum planning, it is necessary to propose an efficient solution method that includes the load and generator probabilistic models. The ultimate goal is to determine the probabilistic density function of typical variables.

Distribution system optimization is a multidimensional optimization problem [3]. Finding the optimum capacitor size, conductor size, location of substations and the best network configuration are some of such objectives. Capacitor planning in a distribution system must account for forecasted load and planned Wind Generators [4]. In distribution systems with large penetration of wind generators, these sources supply a significant part of the total load on an average.

With forecasts of loads and generations being probabilistic in nature with differing PDFs, their proper representation in a capacitor planning exercise is imperative. The objective is to minimize the total cost of the capacitors and the annual energy loss in the system.

1.1 Integration of Wind Energy into the Power Systems

Wind energy is beginning to play an important role in the energy supply in many regions around the world by growing at the rate of 30% annually, with a world total installed capacity of 196 Gigawatts (GW) in 2010 as shown in Figure 1 [5].

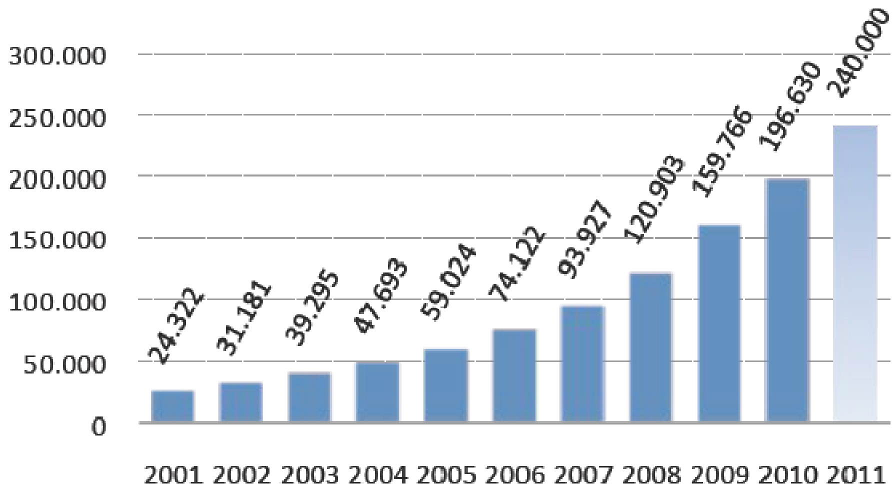

Figure 1 World Total Installed Capacity (MW)

The integration of wind energy into the electricity grids raises challenges due to its availability and geographical distribution. Wind Energy Generators (WEGs) can be integrated in either the transmission or distribution systems. However, in their traditional configuration, WEGs are mostly connecting to transmission lines and appear much less commonly as embedded distributed generation connecting to distribution feeders. Due to some challenges, such distribution scenarios are not well researched yet. One of the major challenges in using wind energy is the uncertainty in wind speed. Commonly, the probability of wind speed is presented by its PDF. Currently, there are other tools to forecast wind speed but wind speed forecasts errors are inevitable. This thesis deals with these limitations by using a probabilistic approach, as it will be shown in the later chapters.

Next section introduces the problem of distribution system capacitor planning and provides a review of existing capacitor planning methods.

1.2 Capacitor Planning in Distribution Systems

In power systems, keeping the voltage, frequency, and the amount of power supplied to the loads and their power factor in line with expectations are some of the challenges of distribution system planning. Loads with low power factor draw more current than loads with high power factor for the same amount of active power consumed, increasing the losses in the distribution system. In addition to increased power consumption, the increased current drawn by low power factor loads can reach the limit of the conductors, deteriorating their performances and reducing their useful life.

To avoid problems related to the low power factor of loads, utilities typically install reactive power compensators at several locations of a distribution feeder. For example, reactive power can be compensated by using synchronous condensers or capacitor banks. Synchronous condensers have the advantage of being dynamic devices, but they are more complex and require greater amount of electrical components. A more cost-effective and prevalent way to compensate for reactive power is through the use of capacitors banks. In addition to compensating for reactive power and correcting power factor, shunt capacitor banks are also used in distribution systems to improve stability margin, manage voltage profile, and reduce losses.

1.3 Capacitor Planning Methods

Substantial research has been carried out on the solution of optimal capacitor planning in the distribution systems for the tasks of power factor correction, voltage profile improvement and loss reduction. In some of the pioneer research works (first category of solutions) the algorithms were formulated using a voltage-independent reactive current model. Using these models, the problem was solved by analytical methods [6-8].

Researchers later proposed a general formulation for the cost function comprising the cost of energy losses with respect to constraints such as the number of fixed or switchable capacitors and the limits on system variables such as the voltage. This set of algorithms can be classified as the second category of planning. In [9-12], iterative methods were applied to select a sequence of candidate nodes among which the optimal node can be determined such that by compensating the bus results in the highest loss reduction. Once the sequence of candidate buses are evaluated, the size of the installed capacitors and the corresponding loss saving are determined by solving the related equations.

There is a third category of distribution system capacitor planning that refers to the use of evolutionary and swarm optimization techniques [13-17]. In [13], the author formulated the optimization problem as a mixed-integer nonlinear programming problem. The problem was decomposed into two phases and an evolution search technique was developed for the first phase called master problem. Firstly the location of capacitors was determined and secondly the type and size of capacitor was found. The system formulation is solved using a mixed integer nonlinear programming method. Souza et al. [16] applied a microgenetic algorithm combined with a fuzzy logic technique in two steps:

a) By using fuzzy logic the optimal locations for the capacitor banks installation was determined, and b) the formulated optimal placement of capacitor banks was minimised using a microgenetic algorithm. In addition to the other conventional algorithms described earlier, Venkatesh et al. [17] proposed a single dynamic data structure for an evolutionary programming (EP) algorithm that handles the problems of siting and sizing of new shunt capacitors simultaneously. This proposed method is very efficient since it considers other system control parameters such as transformer taps, reconfiguration options and existing reactive power sources.

However, one may note that these three categories of algorithms discussed above do not consider the integration of wind generators into distribution systems. Furthermore, the probabilistic behaviour of the annual load is not regarded into the modeling. In conclusion, previous problem formulations as well as the obtained solutions are deterministic in nature and therefore unable to represent the true behaviour of the system in the presence of stochastic wind generators and loads.

It can be said that [4] is the pioneer of the fourth and last category of algorithms to take probabilistic behaviour of the wind generation and load into the account. In that work, a Fuzzy stochastic capacitor planning method was formulated to minimize the total costs of newly sited and sized capacitors, as well as the annual energy loss in a distribution system by using probabilistic models of load and wind generation as Normal and Weibull PDFs, respectively. The load and wind generation PDFs are divided into load and wind generation segments (ND and NW), respectively, to provide ND random load and NW random generation variables. Then the proposed fuzzy stochastic programming method is linearized to be successively solved using a robust mixed integer linear programming solver while updating the solution.

The probabilistic capacitor planning method proposed in this thesis can be classified under the fourth category as it presents a completely probabilistic formulation. It is intended for distribution systems with high penetration of wind generators and uses probabilistic density functions for both wind speed and load consumption.

1.4 Thesis Outline

This thesis is structured in following manner:

Chapter two presents a detailed explanation provided to identify the probabilistic distributions adapted in this thesis for the load consumption and wind speed. These are the Normal and the Weibull distribution, respectively. In addition, it defines moments and cumulants, as well as their generating functions, which are used to develop the probabilistic models of load a generation variables used in this thesis.

Chapter three formulates the capacitor planning problem to be solved in this thesis. In the first part of the chapter, the distribution system model is presented. In the second part of the chapter, the probabilistic formulation of the capacitor planning in distribution systems is given.

Chapter four presents the results of the proposed probabilistic approach tested on a 7-bus and on a 33-bus distribution system with some wind generators connected to the buses. The results are compared with those from Monte Carlo Simulations (MCS). Errors are evaluated and the probabilistic distributions of the variables are re-generated.

Chapter 5 presents the conclusion of this research.

CHAPTER 2

PROBABILISTIC MODELING OF LOAD AND WIND GENERATORS

This chapter presents the probabilistic background used in this thesis to handle the load behaviour and wind generator speeds. Firstly, the concept of moments and cumulants are introduced. Secondly, the random characteristics of load and wind generators are embodied in terms of Probabilistic Density Functions.

2.1 Random Variables and Probability Density Functions

In probability and statistics, a random variable or stochastic variable is a variable which is obtained from some type of random process and can be defined as a function which maps events or outcomes.

The probability of the value of random variable is determined by the probability density function. There are various probability distributions that are used in different applications. Two of the most important ones related to this work are the Normal distribution and the Weibull distribution. The Normal and Weibull distributions have been commonly used to describe the load consumption and wind speed behaviours, respectively. While it is widely known that the Normal distribution can be described by its mean and variance (first and second moments), the Weibull distribution can be described by higher order of moments or cumulants. Next sections define moments and cumulants and their relationships with probabilistic distributions.

2.2 Definition of Moments

In probability and statistic theory, moments and cumulants are two sets quantities of a random variable x which are mathematically equivalent. The definition of the moments is explained in this section and the definition of cumulants is presented in the next one.

Let $f_x(x)$ be a probability density function of a random variable x, then the expected value of x is defined as [2]

$$E[x] = \int_{-\infty}^{\infty} x\, f_x(x) dx. \tag{1}$$

Accordingly the n^{th}-order moment m_n is calculated using the following definition:

$$m_n = E[x^n] = \int_{-\infty}^{\infty} x^n\, f_x(x) dx. \tag{2}$$

Alternatively, the moments of random variable x can be determined directly by the use of the moment generating function, which is explained in the next subsection.

2.2.1 Moment Generating Function

Mathematically, the moment generating function $\Phi_x(s)$ of a random variable x is

$$\Phi_x(s) = E[e^{sx}]. \tag{3}$$

The n^{th}order moment of random variable x can be computed by taking the n^{th}derivative of moment generating function with respect to s and setting $s = 0$.

For example the second moment is computed as follows [2]:

$$m_2 = \frac{d^2\Phi_x(s)}{ds^2}|_{s=0} = \frac{d^2E[e^{sx}]}{ds^2}|_{s=0} = E\left[\frac{d^2e^{sx}}{ds^2}|_{s=0}\right] = E[x^2e^{sx}|_{s=0}] = E[x^2] \tag{4}$$

which is the second order moment of random variable x.

2.3 Definition of Cumulants

Moments are widely used to describe the distribution of random variables. However, in some cases it is preferable to use cumulants due to their simplicity over using moments. Furthermore, the several orders of cumulants of a random variable x can be calculated by using its several orders of moments. The definition of the cumulant $k_n$is explained as follows:

Given a scalar random variable x, its characteristic function $\hat{f}_x(t)$ is defined as:

$$\hat{f}_x(t) = E\left[e^{itx}\right] \tag{5}$$

The Cumulants k_n are defined by:

$$\ln\left(\hat{f}_x(t)\right) = \sum_{n=1}^{\infty} k_n \frac{(it)^n}{n!}, \tag{6}$$

where k_n is the cumulant of x. In particular, the cumulants can be related to the moments by the following formula:

$$k_n = m_n - \sum_{i=1}^{n-1} \binom{n-1}{i-1} k_i m_{n-i} \tag{7}$$

where m_n is the n^{th}-order moment. In this work, we use the above formula to calculate the first six cumulants of the known random variable from its moments as follows:

$$k_1 = m_1, \tag{8}$$

$$k_2 = m_2 - m_1^2,$$

$$k_3 = m_3 - 3 \times m_2 \times m_1 + 2 \times m_1^3,$$

$$k_4 = m_4 - 4 \times m_3 \times m_1 - 3 \times m_2^2 + 12 \times m_2 \times m_1^2 - 6 \times m_1^4$$

$$k_5 = m_5 - 5 \times m_4 \times m_1 - 10 \times m_3 \times m_2 + 20 \times m_3 \times m_1^2 + 30 \times m_2^2 \times m_1 - 60 \times m_2 \times m_1^3 + 24 \times m_1^5$$

$$k_6 = m_6 - 6 \times m_5 \times m_1 - 15 \times m_4 \times m_2 + 30 \times m_4 \times m_1^2 - 10 \times m_3^2 + 120 \times m_3 \times m_2 \times m_1 - 120 \times m_3 \times m_1^3 + 30 \times m_2^3 - 270 \times m_2^2 \times m_1^2 + 360 \times m_2 \times m_1^4 - 120 \times m_1^6.$$

Alternatively, the cumulants of random variable x can be determined directly by the use of the cumulant generating function, which is explained in the next subsection.

2.3.1 Cumulant Generating Function

The cumulant generating function can be written in terms of the moment generating function $\Phi_x(s)$ in the following manner:

$$\Psi_x(s) = \ln\left(\Phi_x(s)\right) \tag{9}$$

In a similar manner as done for the moment generating function, by taking the successive derivatives from the cumulant generating function with respect to s and evaluating $s = 0$, the different order of cumulant of random variable x can be obtained. The n^{th} order cumulant is usually denoted as k_n. [2].

The advantage of the cumulants over moments are evident when the aim is to find different orders of cumulant of the random variable x comprised by a linear combination of some other independent random variables with known cumulants. From the above explanation, it can be shown that the cumulants of the random variable x can be computed easily by using the given cumulants of the other random variables in the linear combination.

In the next chapter it will be shown that the proposed solution method takes advantage of this concept to provide a linear combination of the system variables to obtain the cumulants of the unknown variables from the cumulants of the known variables.

2.4 Probabilistic Characteristics of Annual Load Demand

The behaviours of the load buses uncertainties including active and reactive loads have been modeled in different research works. In [18], the authors formulated a probabilistic load flow for power system expansion while the loads were modeled using normal probability distribution. However, in [19] the proposed method was examined while the loads were modeled as the gamma distribution in addition to the normal distribution model for the loads. Furthermore, Billinton et al. [20, 21] pointed out that it is difficult to obtain sufficient historical data to determine the distribution type and therefore, it is appropriate to represent uncertainties in the load forecasts with a normal distribution and a given value of standard deviation. [4]

Following the aforementioned ideas, the annual load demand will be modeled in this thesis as a normal distribution. The graph of the associated PDF is depicted in Figure 2.

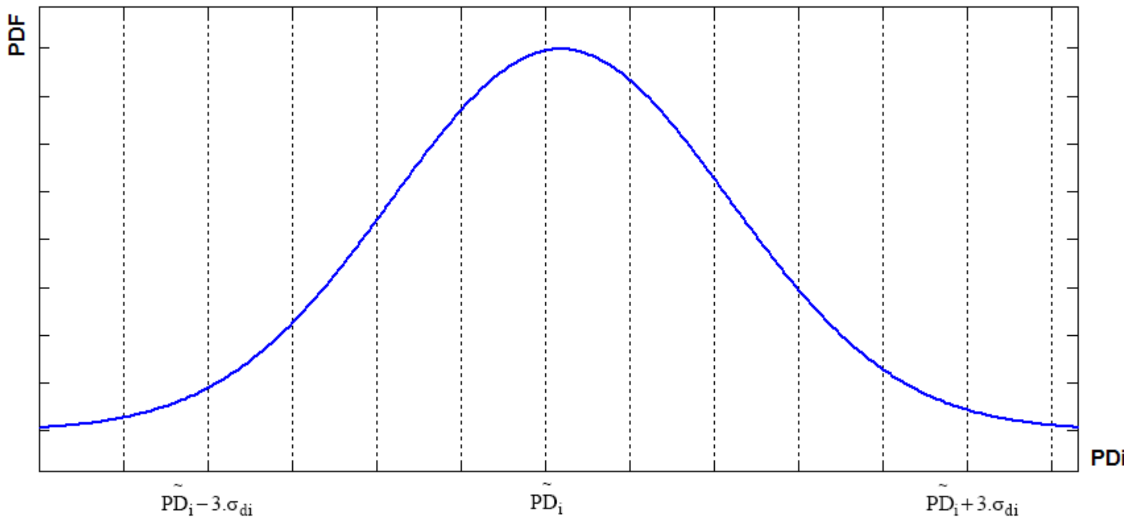

Figure 2   Probability Density Function of Load Modeling

The Gaussian function of the system loading is defined as follow:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(PD-\widetilde{PD})^2}{2\sigma^2}} \tag{10}$$

where $\widetilde{PD}$ is the mean of the load and σ^2 is the variance of the load. The distribution with $\widetilde{PD} = 0$ and $\sigma^2 = 1$ is called the standard normal of the system loading.

2.5   Probabilistic Characteristics of Wind Energy Systems

As explained earlier in this thesis, a useful way to describe the wind characteristics is by using a probabilistic distribution. Extensive research has been published in the last two decades to describe wind speed distribution. However, the proposed methods should be able to analyze the wind speed statistically since the speed of wind is continuously changing. The annual wind speed forecasts can be modeled using Weibull distributions as shown in Figure 3 [22]. The probability density function of the wind forecast for a generator at the i^{th} bus is given as [4]

$$\rho w_i = \left(\gamma / \beta_i^{\gamma} \right) w_i^{\gamma-1} . e^{-\left(w_i / \beta_i \right)^{\gamma}}, \tag{11}$$

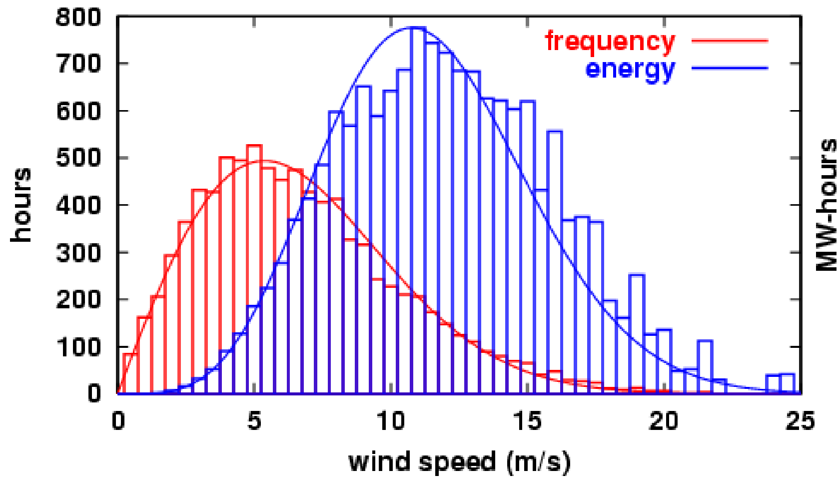

Figure 3 Weibull distribution function of the wind speed

where W_i is the wind speed in m/s, γ and β_i are the shape and scale parameters respectively. Due to the fact it is reasonable to assume that all WGs installed in a distribution feeder are in the same geographical area, this thesis also assumes that γ is the same for all WGs. The scale parameter β_i can be determined for the WG_i, given the mean forecast $\widetilde{W}_i$ and the shape parameter γ.

This concludes the load and wind generator probabilistic behaviours as applicable to the capacitor sizing and allocation problem. Chapter 3 presents the distribution system model and the probabilistic approach to deal with the capacitor placement problem.

CHAPTER 3
PROBLEM FORMULATION

The probabilistic model for the capacitor planning problem formulation in distribution systems is presented in this chapter. The distribution system model used in this thesis is provided in the first part of the chapter, and the complete nonlinear problem, along with the proposed solution, is formulated thereafter.

3.1 The Load Flow Solution Method Adapted for RDS [23]

The accurate voltage solution method for radial distribution systems is developed by formulating a set of equations (SOE) to describe a radial distribution system with high R/X ratio. This SOE comprises $3(N-1)$ nonlinear equations for an N-bus distribution system. Then the SOE is solved by using the first-order Newton-Raphson technique. These equations are devoid of bus phase angles and can be subsequently solved using the Newton-Raphson method [23].

The SOE is formed by a voltage equation and real and reactive power balance equations at every bus, and is described as follows. A single-line representation of a tree-like distribution systems structure is shown in Figure 4.

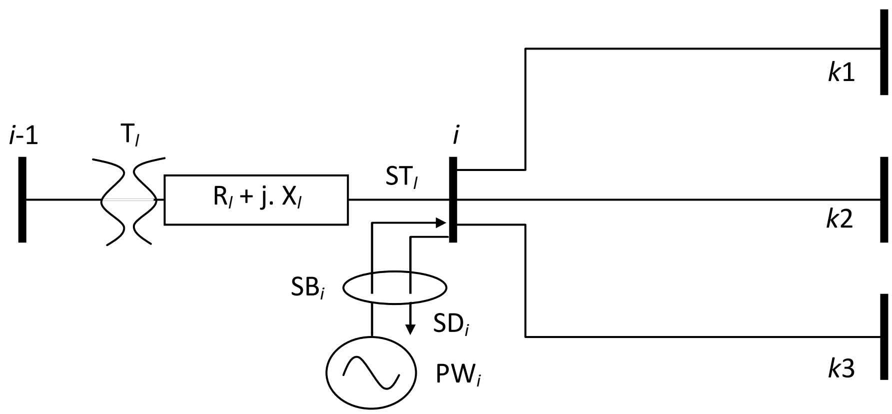

Figure 4 A tree-like generic distribution system

Consider the i^{th} bus in Figure 4. It has a wind turbine connected to it that injects only real power equal to $\mathrm{PW_i}$. Its bus load is represented by $\mathrm{SD_i = PD_i + jQD_i}$. The total power injected into this bus is $\mathrm{SB_i = PB_i + jQB_i}$, and it is the difference between generation and load at that bus. Consider the $\mathrm{l^{th}}$ line/transformer between buses i-1 and i. The tap setting of this transformer/line is represented by $\mathrm{T_l}$ and it has an impedance $\mathrm{Z_l = R_l + j.X_l}$. The total apparent power reaching the downstream end of this line equals $\mathrm{ST_l}$. The real power loss on this line equals

$$\mathrm{PL_l = R_l.|ST_l|^2.V_i^{-2}}. \tag{12}$$

The total real power loss in all feeders of the system equals

$$\mathrm{TPL} = \sum_{\mathrm{l=1}}^{\mathrm{nl}} \mathrm{R_l.|ST_l|^2.V_i^{-2}}, \tag{13}$$

where Vi is the bus voltage magnitude, nb is the number of buses in the system and nl is the number of lines/transformers.

In Figure 4, the power balance at the i^{th} bus can be expressed as

$$\mathrm{P_i} = \mathrm{PD_i} + \sum_{l=kl1,k=k1}^{kl3,k3} \mathrm{R_l} \cdot (\mathrm{P_k}^2 + \mathrm{Q_k}^2) \cdot \mathrm{V_k}^{-2} + \sum_{k=k1}^{k3} \mathrm{PT_k} - \mathrm{PW_i}, \tag{14}$$

$$\mathrm{Q_i} = \mathrm{QD_i} + \sum_{l=kl1,k=k1}^{kl3,k3} \mathrm{X_l} \cdot (\mathrm{P_k}^2 + \mathrm{Q_k}^2) \cdot \mathrm{V_k}^{-2} + \sum_{k=k1}^{k3} \mathrm{QT_k} - \mathrm{QS_i} \tag{15}$$

where $\mathrm{QS_i}$ is the reactive power injected into the $\mathrm{i^{th}}$ bus. Writing the voltage drop equation across line/transformer l gives

$$2 \cdot \mathrm{V_i^2} \cdot \left[\mathrm{PT_l} \cdot \mathrm{R_l} + \mathrm{QT_l} \cdot \mathrm{X_l} - \frac{1}{2} \cdot \mathrm{V_{i-1}^2} \cdot \mathrm{T_l}^{-2}\right] + \mathrm{V_i^4} = -|\mathrm{Z_l}|^2 \cdot |\mathrm{ST_l}|^2, \tag{16}$$

$\mathrm{i} = 2{:}\,\mathrm{nb} - 1.$

For a system with N-buses, the nonlinear SOE will have 3(N-1) equations and 3(N-1) unknowns. Once the SOE is formulated for each bus, a first-order Newton-Raphson Method can be applied to obtain a solution.

3.2 Capacitor Planning Problem Definition using the Proposed Probabilistic Approach

In this section the probabilistic formulation of the capacitor planning problem for distribution systems is presented.

3.2.1 Objective Functions

The goal of a capacitor planning study is to select the optimal size and location for a capacitor banks in a distribution system in such a way that the sum of the total cost of the newly sited and sized capacitor, as well as the annual energy losses, are minimized while the voltage profile remains within a prescribed limit. By siting a capacitor unit at a bus along the feeder, the nodal power and voltages are changed, leading to a reduction in real power loss in the system.

This reduction in real power losses is economical and can be obtained by multiplying the annual energy loss by the average cost of energy (KL). However, purchasing new unit of capacitor sustains a cost which is in conflict with the financial benefits accrued by the reduction of real power loss. Consequently, the problem comprises multi-objective functions which conflict with each other.

The objective functions are:

a) minimize the total cost of capacitors ($)

$$\mathrm{QC} = \mathrm{KC}.\sum_{\mathrm{i}=1}^{\mathrm{NB}} \mathrm{QS}_{\mathrm{i}}, \tag{17}$$

where KC is the annual cost of per unit capacitor bank installed in a RDS ($/kVAR) and $\mathrm{QS_i}$ is the size of capacitor located at the bus $\mathrm{i^{th}}$.

b) minimize the total cost of annual energy losses ($).

$$\mathrm{ELC} = \mathrm{KL}.\sum_{\mathrm{D}=1}^{\mathrm{nt}} \sum_{\mathrm{i}=1}^{\mathrm{NB}} \mathrm{T_D} \,.\, \mathrm{PL}_{\mathrm{i,D}}, \tag{18}$$

where KL is the annual cost of per unit energy loss ($/kWh), $\mathrm{T_D}$ is loading time interval and $\mathrm{PL_{i,D}}$ is the real power loss at the bus $\mathrm{i^{th}}$and on load level D.

3.2.2 The Complete Nonlinear Optimization Formulation

The aim is to minimize sum of the total capacitor cost and the cost of the annual energy losses in the system. Since the unit of both objective functions is the dollar value ($), the multi-objective problem can be reduced to a single objective function by summing up two functions. Hence, the complete model including the constraints can be written as a nonlinear programming problem as below:

$$\text{Minimize } TC = QC + ELC = KC.\sum_{i=1}^{NB} QS_i + KL.\sum_{D=1}^{nt}\sum_{i=1}^{NB} T_D \,.\, PL_{i,D} \tag{19}$$

Subject to

$$P_i = PD_i + \sum_{l=kl1,k=k1}^{kl3,k3} R_l \,.\,({P_k}^2 + {Q_k}^2).{V_k}^{-2} + \sum_{k=k1}^{k3} PT_k - PW_i \tag{19-1}$$

$$Q_i = QD_i + \sum_{l=kl1,k=k1}^{kl3,k3} X_l \,.\,({P_k}^2 + {Q_k}^2).{V_k}^{-2} + \sum_{k=k1}^{k3} QT_k - QS_i \tag{19-2}$$

$$2.V_i^2.\left[PT_l.R_l + QT_l.X_l - \frac{1}{2}.V_{i-1}^2.T_l^{-2}\right] + V_i^4 = -|Z_l|^2.|ST_l|^2 \tag{19-3}$$

$i = 2{:}\,nb - 1$

$$V_{min} \le V_D \le V_{max} \qquad \forall D = 1{:}\,NH \tag{19-4}$$

$$QS_{min} \le QS_i \le QS_{max} \tag{19-5}$$

$$S_{i,j}^{min} \le S_{i,j} \le S_{i,j}^{max} \tag{19-6}$$

Equations (19-1) - (19-3) are equality constraints which correspond to the power balance equation and the voltage drop equation across line l, respectively. Equation (19-4) refers to the limitation of the bus voltage magnitude. Equation (19-5) corresponds to the limits on output of capacitors at the bus i^{th} and (19-6) describes the branches apparent power limits.

The optimization problem described as in (19) is solved using the Logarithmic-Barrier interior point method (LBIPM). The LBIPM is an interior point solution method that is able to effectively solve the aforementioned constrained nonlinear optimization problem. It was chosen in this thesis because of its desirable convergence speed and characteristics. It is explained in the next section.

3.3 The Cumulant Method for Optimizing Power Systems

The solution strategy for the problem is detailed in this section. The goal of this solution method is to verify the probabilistic behaviour of the problem variables which can be observed by their PDF. Therefore, the first step is to formulate the optimization problem. In order to solve the problem, the LBIPM method is used. After convergence, the cumulants are obtained using some characteristics of the LBIPM. These cumulants are the key parameters to describe the probabilistic distributions. Once obtained, they are used to re-construct the distribution curves by using the Gram Charlier expansion.

In summary, the main steps in the cumulant-based optimal capacitor planning problem are:

1) Do the sensitivity analysis to find the best locations for capacitor siting
2) Solve the optimization problem assuming the mean value for the loads and wind generations using a LBIPM
3) Find a linear relationship between problem variables and system loads and wind generators using the Hessian of the Lagrangian function
4) Find the cumulants of the problem variables using the linear relationship obtained in step3
5) Reconstruct the PDFs of the problem variables using the Gram-Charlier expansion theory

The first step to solve the problem is to determine the candidate buses. This step is presented in the next subsection.

3.3.1 Sensitivity Analysis to Determine the Candidate Buses (Step 1)

The solution procedure for minimizing (19) can become computationally difficult and complicated if all variables corresponding to the capacitor, i.e., size, location and switching time of capacitors, need to be determined simultaneously. Consequently, one efficient way to reduce the complexity of the problem is to find the optimal locations (candidate buses) to be compensated which lead to the greatest loss reduction prior to the optimization problem.

For locating the candidate buses, a sensitivity analysis is used to reduce the search space for the optimization procedure. The sensitivity analysis chooses the buses with maximum effect on the real power losses in the system with respect to the nodal reactive power. Such technique has been widely adapted in capacitor allocation problems. For instance, a novel approach for sensitivity calculations in the radial distribution system was proposed in [24]. In this thesis, we borrow the idea for formulating the sensitivity index from [24] and adapt it to the new power balance equations and the voltage relations described in section 3.1.

Consider the model of a branch in radial distribution systems given in Figure 5.

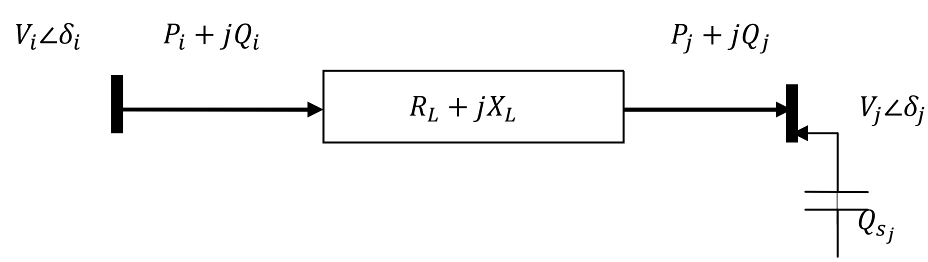

Figure 5 A branch model representation in distribution systems

Accordingly, the total real power loss is given by:

$$\mathrm{TPL} = \sum_{l=1}^{nl} R_l.(P_j^2 + Q_j^2).V_j^{-2}, \tag{20}$$

where $P_j + Q_j$ is the complex power reaching bus j. For finding the sensitivity of the real power losses in the system with respect to reactive power injection, one can get the derivative of the real power loss with respect to reactive power injected at the bus by using the following relationship:

$$\left(\frac{\partial TPL}{\partial Q_s}\right) = \left(\frac{\partial TPL}{\partial P_j}\right).\left(\frac{\partial P_j}{\partial Q_s}\right) + \left(\frac{\partial TPL}{\partial Q_j}\right).\left(\frac{\partial Q_j}{\partial Q_s}\right) + \left(\frac{\partial TPL}{\partial V_j}\right).\left(\frac{\partial V_j}{\partial Q_s}\right). \tag{21}$$

Expanding equation (21) yields:

$$\frac{\partial TPL}{\partial Q_s} = \sum_{l=1}^{nl} \left(\frac{2.R_l.P_j}{{V_j}^2}\right)\left(\frac{\partial P_j}{\partial Q_s}\right) + \left(\frac{2.R_l.Q_j}{{V_j}^2}\right)\left(\frac{\partial Q_j}{\partial Q_s}\right) - \left(\frac{2.R_l.(P_j^2+Q_j^2)}{{V_j}^3}\right).\left(\frac{\partial V_j}{\partial Q_s}\right). \tag{22}$$

Equation (22) can be written in vector format as follows:

$$\frac{\partial TPL}{\partial Q_s} = [A]^T[X_j], \tag{23}$$

where the vector $[A]^T$ is of size $1 \times 3.(NB-1)$ and includes the coefficient of (22). Once a load flow is solved and the value of the nodal power and voltage magnitude are obtained, the vector $[A]$ can be calculated easily. The vector $[X_j]$ consists of partial derivative terms of (22) of size $3.(NB-1) \times 1$, and can be expanded as follows:

$$[X_j] = \left[\ \left[\frac{\partial P_1}{\partial Q_s}\ \frac{\partial P_2}{\partial Q_s}\ .\ .\ .\ \frac{\partial P_{(NB-1)}}{\partial Q_s}\right], \left[\frac{\partial Q_1}{\partial Q_s}\ \frac{\partial Q_2}{\partial Q_s}\ .\ .\ .\ \frac{\partial Q_{(NB-1)}}{\partial Q_s}\right], \left[\frac{\partial V_1}{\partial Q_s}\ \frac{\partial V_2}{\partial Q_s}\ .\ .\ .\ \frac{\partial V_{(NB-1)}}{\partial Q_s}\right]\ \right]^T. \tag{24}$$

The next step is to calculate the vector $[X_j]$. As the expansion of the vector $[X_j]$ is given in (24), the vector $[X_j]$ contains the partial derivative of the active and reactive power flow as well as the voltage relation with respect to Q_s. Referring to Figure 5, to calculate these partial derivatives, the power balance equations and voltage relationship for this branch model are written as

$$P_i = R_l.\left(P_j^2 + Q_j^2\right).V_j^{-2} + P_j + PD_j, \tag{25}$$

$$Q_i = X_l.\left(P_j^2 + Q_j^2\right).V_j^{-2} + Q_j - Q_s + QD_j, \tag{26}$$

$$V_i^2 = V_j^2 + 2.\left(P_j.R_l + Q_j.X_l\right) + \frac{\left(R_l^2+X_l^2\right).\left(P_j^2+Q_j^2\right)}{V_j^2}. \tag{27}$$

By taking the derivative of (24)-(26) with respect to Q_s, we obtain:

$$\left(\frac{2.R_l.P_j}{V_j^2} + 1\right).\left(\frac{\partial P_j}{\partial Q_s}\right) + \left(\frac{2.R_l.Q_j}{V_j^2}\right).\left(\frac{\partial Q_j}{\partial Q_s}\right) - \left(\frac{2.R_l.\left(P_j^2+Q_j^2\right)}{V_j^3}\right).\left(\frac{\partial V_j}{\partial Q_s}\right) - \left(\frac{\partial P_i}{\partial Q_s}\right) = 0, \tag{28}$$

$$\left(\frac{2.X_l.P_j}{V_j^2}\right).\left(\frac{\partial P_j}{\partial Q_s}\right) + \left(\frac{2.X_l.Q_j}{V_j^2} + 1\right).\left(\frac{\partial Q_j}{\partial Q_s}\right) - \left(\frac{2.X_l.\left(P_j^2+Q_j^2\right)}{V_j^3}\right)\left(\frac{\partial V_j}{\partial Q_s}\right) - \left(\frac{\partial Q_i}{\partial Q_s}\right) = 1, \tag{29}$$

$$\left(V_j - \frac{\left(R_l^2+X_l^2\right).\left(P_j^2+Q_j^2\right)}{V_j^3}\right).\left(\frac{\partial V_j}{\partial Q_s}\right) + \left(R_l + \frac{\left(R_l^2+X_l^2\right).P_j}{V_j^2}\right).\left(\frac{\partial P_j}{\partial Q_s}\right) + \left(X_l + \frac{\left(R_l^2+X_l^2\right).Q_j}{V_j^2}\right).\left(\frac{\partial Q_j}{\partial Q_s}\right) -$$

$$V_i.\left(\frac{\partial V_i}{\partial Q_s}\right) = 0, \tag{30}$$

where

$$\left(\frac{\partial P_i}{\partial Q_s}\right) = \textstyle\sum_j \left(\frac{\partial P_j}{\partial Q_s}\right), \tag{31}$$

$$\left(\frac{\partial Q_i}{\partial Q_s}\right) = \textstyle\sum_j \left(\frac{\partial Q_j}{\partial Q_s}\right).$$

Writing equations (25)-(27) for all the lines in the systems, i.e., $3.(\mathrm{NB}-1)$ lines, the following system in a compact format is achieved:

$$[\mathrm{B}]_{3.(\mathrm{NB}-1)\times 3.(\mathrm{NB}-1)}.\left[\mathrm{X_j}\right]_{3.(\mathrm{NB}-1)\times 1} = \left[\mathrm{C_j}\right]_{3.(\mathrm{NB}-1)\times 1}, \quad (32)$$

where $\left[\mathrm{C_j}\right]$ is a vector of constant value at the right hand side of the equations (27)-(29). By solving (32) for $\left[\mathrm{X_j}\right]$ and substituting that in (23), $\frac{\partial \mathrm{TPL}}{\partial \mathrm{Q_s}}$ of the system can be determined. The buses with the highest real power loss sensitivity with respect to the reactive power injected into the system are ranked in descending order and can be chosen in the selection procedure. In summary the algorithm of the selection procedure can be adapted to the capacitor planning problem using the following steps:

Step 1- Set i=1, NQS=1 (number of capacitor) and find TC (i) from solving the deterministic capacitor planning problem

Step 2- Set i= i+1 and find the active power loss sensitivity for reactive power injection using equation (32) and (23)

Step 3- Rank the candidate buses with the highest sensitivity found in Step 2

Step 5 - Connect a capacitor in the first ranked bus

Step 6- Solve deterministic capacitor planning problem

Step 7- if TC (i) < TC (i-1) and min(V) < $\mathrm{V_{min}}$, set NQS = NQS+1 and go to Step 2; otherwise exit the loop

By performing the above steps, the best candidate buses for capacitor sitting are selected.

3.3.2 The LBIPM Solution (Step 2)

The Logarithmic Barrier Interior Point Method (LBIPM) is one of the applications of the interior-point methods that have been widely used to efficiently solve large power system constrained optimization problems for many years. However, the application of the LBIPM is not only restricted to the use in nonlinear programming problems, but it can be also efficiently applied to Quadratic programming as well as to linear programming. The method is explained briefly as follows [25]:

Consider the following NLP problem model:

$$\text{minimize } f(x) \tag{33}$$

subject to:

$$g(x) = D$$

$$\hat{x}^{\min} \leq \hat{I}x \leq \hat{x}^{\max}$$

where, for the cost function minimization problem in capacitor planning, this system can be defined as:

- x consisting of the system variables $x = [\,V\ Q\ P\ \ QS]^T$.
- $g(x)$ comprises the equality constraints separated into two parts: the voltage relation $\left(FV(X) = [0]_{(nb-1)\times 1}\right)$ and the power balance equation $\left(FS(X) = [D]_{2\times(nb-1)\times 1}\right)$. [D] is the a vector of generation and loading (see equations (19-1) and (19-2)).

To apply the LBIPM to the above system, firstly all inequality constraints in the NLP problem should be converted to the equalities by adding slack variables as follows:

$$\text{minimize } f(x) \tag{34}$$

subject to:

$$g(x) - D = 0$$

$$-s_1 - s_2 - \hat{x}^{min} + \hat{x}^{max} = 0$$

$$-\hat{I}x - s_2 + \hat{x}^{max} = 0$$

$$s_1, s_2 \geq 0$$

By incorporating the slack variables $s_i \geq 0$ into the problem using logarithmic barrier terms, we get:

$$\text{minimize } f(x) - \mu \sum_{j=1}^{3.(NB-1)} \left(\ln s_{1_j} + \ln s_{2_j} \right) \tag{35}$$

subject to:

$$g(x) - D = 0$$

$$-s_1 - s_2 - \hat{x}^{min} + \hat{x}^{max} = 0$$

$$-\hat{I}x - s_2 + \hat{x}^{max} = 0$$

where $\mu_j > 0$ is a barrier parameter which tends to zero as iterations progress. Using the method of Lagrange multipliers, the next step is to write the Lagrangian function of the above systems:

$$L(u) = f(x) - \mu \sum_{j=1}^{3.(NB-1)} \left(\ln s_{1_j} + \ln s_{2_j}\right) - \lambda^T \sum_{j=1}^{3.(NB-1)} (g(x) - D) - \tag{36}$$

$$z_1^T\left(-s_1 - s_2 - \hat{x}^{min} + \hat{x}^{max}\right) - z_2^T(-\hat{I}x - s_2 + \hat{x}^{max})$$

where λ is the Lagrange multiplier, and u is a vector of primal and dual variables in the optimization problem.

Using the system variables, (35) becomes:

$$L(u) = f(X) - \lambda^T(g(X) - D), \tag{37}$$

where $u = [X\,\lambda\,]^T$ is the vector of primal and dual variables.

3.3.2.1 Gradient, Hessian and Newton Step

In optimization theory, the Karush–Kuhn–Tucker conditions (also known as the Kuhn–Tucker or KKT conditions) are necessary for a solution in nonlinear programming to be optimal [26, 27]. Accordingly, an optimal solution of (36) must satisfy the gradient of the Lagrangian function which is known as the first order Karush-Kuhn-Tucker (KKT) conditions:

$$\nabla_x L(u) = \nabla f(x) - J_g(x)^T\lambda + \hat{I}^T z_2 = 0, \tag{38}$$

$$\nabla_\lambda L(u) = -g(x) + D = 0,$$

$$\nabla_{s1} L(u) = -\mu s_1^{-1} + z_1 = 0,$$

$\nabla_{s2}L(u) = -\mu s_2^{-1} + z_1 + z_2 = 0,$

$\nabla_{z1}L(u) = -s_1 - s_2 - \hat{x}^{min} + \hat{x}^{max} = 0,$

$\nabla_{z2}L(u) = -\hat{I}x - s_2 + \hat{x}^{max} = 0,$

where $\nabla f(x)$ is the gradient of the f(x), and J_g is the Jacobian of the g(x). To solve the nonlinear system of (38) one can apply a Newton-Raphson-based solver which gives the following relationship:

$$H(u)\Delta u = -\nabla L(u), \tag{39}$$

where $H(u)$and $\nabla L(u)$ are the Hessian and the gradient of the Lagrangian, respectively and Δu stands for the Newton step. By expanding (39) the following symmetric system is obtained [25]:

$$\begin{bmatrix} \mu s_1^{-2} & 0 & I & 0 & 0 & 0 \\ 0 & \mu s_1^{-2} & I & I & 0 & 0 \\ I & I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & \hat{I} & 0 \\ 0 & 0 & 0 & \hat{I}^T & \nabla_x^2 L_u & -J_g^T \\ 0 & 0 & 0 & 0 & -J_g & 0 \end{bmatrix} \times \begin{bmatrix} \Delta s_1 \\ \Delta s_2 \\ \Delta z_1 \\ \Delta z_2 \\ \Delta x \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} \mu s_1^{-1} - z_1 \\ \mu s_2^{-1} - z_1 - z_2 \\ -s_1 - s_2 - \hat{x}^{min} + \hat{x}^{max} \\ -\hat{I}x - s_2 + \hat{x}^{max} \\ \nabla f(x) - J_g(x)^T\lambda + \hat{I}^T z_2 \\ g(x) - D \end{bmatrix}, \tag{40}$$

where $\nabla_x^2 L_u$ is the second derivative of the Lagrangian with respect to x. One idea to obtain the Newton step from the large system above is to solve for the Δx and $\Delta\lambda$ first from the reduced system represented below:

$$\begin{bmatrix} M & -J_g^T \\ -J_g & 0 \end{bmatrix} \times \begin{bmatrix} \Delta x \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} -N \\ g(x) - D \end{bmatrix}, \tag{41}$$

where

$$M = \nabla_x^2 L_u + \mu \hat{I}^T (s_1^{-2} + s_2^{-2}) \hat{I}, \tag{42}$$

$$N = \nabla f(x) - J_g(x)^T \lambda + \hat{I}^T z_2. \tag{43}$$

Once the primal and dual variables of the system are obtained, $\Delta s_1, \Delta s_2, \Delta z_1, \Delta z_2$ can be computed. Comparing equation (39) with equation (41), one can conclude that the Hessian of the Lagrangian in this system is the matrix terms of (41), i.e.

$$H = \begin{bmatrix} M & -J_g^T \\ -J_g & 0 \end{bmatrix}. \tag{44}$$

3.3.3 The Cumulant Method (Steps 3 and 4)

The cumulant method is adapted to the probabilistic capacitor planning problem to obtain the statistic parameters of unknown variables, which are formulated making use of the statistic parameters of known variables, such as the system loading and WG power output. This relationship is linear, as it will be explained in this subsection.

Consider a linear combination of n independent random variable x used to create a new random variable Y as follows [28]:

$$Y = c_1 x_1 + c_2 x_2 + c_3 x_3 + \ldots + c_n x_n , \tag{45}$$

where c_i is the i^{th} coefficient in the linear combination. From the probabilistic definitions discussed in chapter 2, the moment generation function of random variable Y ,$\Phi_Y(s)$, can be written as:

$$\Phi_Y(s) = E[e^{sY}] = E\left[e^{s(c_1x_1+c_2x_2+...+c_nx_n}\right] = E[e^{s(c_1x_1)}e^{s(c_2x_2)}...e^{s(c_nx_n)}] \quad (46)$$

Assuming that x_1 , x_2 , x_3 , . . . , x_n are independent, the above relationship can be written as

$$\Phi_Y(s) = E\left[e^{s(c_1x_1)}\right]E\left[e^{s(c_2x_2)}\right]...E\left[e^{s(c_nx_n)}\right] = \Phi_{x1}(c_1s)\Phi_{x2}(c_2s)...\Phi_{xn}(c_ns). \quad (47)$$

For simplification purposes, the multiplication of the moment generating function of x_i in the left-hand side of equation (47) can be converted into the summation of them. To achieve that, the natural logarithm (ln) function should be used. By taking the natural logarithm from the above relationship, we get:

$$\ln(\Phi_Y(s)) = \ln\left(\Phi_{x1}(c_1s)\Phi_{x2}(c_2s)...\Phi_{xn}(c_ns)\right) =$$

$$\ln\left(\Phi_{x1}(c_1s)\right) + \ln\left(\Phi_{x2}(c_2s)\right) + ... + \ln\left(\Phi_{xn}(c_ns)\right). \quad (48)$$

From chapter 2, the cumulant generating function, $\Psi_x(s)$, can be written in terms of the moment generating function $\Phi_x(s)$ as follows:

$$\Psi_x(s) = \ln(\Phi_x(s)). \quad (49)$$

Consequently, writing (48) in the terms of cumulant generating function gives:

$$\Psi_Z(s) = \Psi_{x1}(c_1 s) + \Psi_{x2}(c_2 s) + \ldots + \Psi_{xn}(c_n s). \tag{50}$$

To compute different orders of cumulant, different order derivatives of the cumulant generating function should be taken while setting $s = 0$. For example, for computing zero, first and second order cumulants, the zero, first and second order derivatives of the cumulant generating function are taken as:

$$\Psi_Y(s) = \Psi_{x1}(s)(c_1 s) + \Psi_{x2}(s)(c_2 s) + \ldots + \Psi_{xn}(s)(c_n s), \tag{51}$$

$$\Psi'_Y(s) = c_1 \Psi'_{x1}(s)(c_1 s) + c_2 \Psi'_{x2}(s)(c_2 s) + \ldots + c_n \Psi'_{xn}(s)(c_n s),$$

$$\Psi''_Y(s) = c_1{}^2\, \Psi''_{x1}(s)(c_1 s) + c_2{}^2\, \Psi''_{x2}(s)(c_2 s) + \ldots + c_n{}^2\, \Psi''_{xn}(s)(c_n s).$$

Setting $s = 0$ yields:

$$\Psi''_Y(s) = c_1{}^2\, \Psi''_{x1}(0) + c_2{}^2\, \Psi''_{x2}(s)(0) + \ldots + c_n{}^2\, \Psi''_{xn}(s)(0). \tag{52}$$

Cumulants of third and higher order can be computed following the same procedure. A general equation for the n^{th}order cumulant of Y is the following:

$$k_n = \Psi_{Y1}^{(n)}(0) = c_1^n \Psi_{x1}^{(n)}(0) + c_2^n \Psi_{x2}^{(n)}(0) + \ldots + c_n^n \Psi_{xn}^{(n)}(0). \tag{53}$$

In (53), it can be observed that the n^{th}order cumulant of Y can be obtained if the value of the n^{th}order cumulant of random variables x_i and the corresponding coefficients c_i are known. In conclusion, if a random variable Y denotes an unknown variable of the problem, and x_i denotes the known variable with known probabilistic information given as the different order of cumulants of x_i, the different order cumulants of the unknown random variables can be easily obtained from the same order cumulant of the x_i.

Using the aforementioned reasoning, (39) can be rearranged by replacing $\nabla L(u)$ with $F(u)$, giving

$$\Delta u = -H(u)^{-1}F(u), \tag{54}$$

where $F(u)$ consists power flow (FS) and voltage drop (FV) equations and N, (42).

With a vector of change in bus generation and loading, the vector of changes in the unknown variables can be linearly mapped with the vector of changes in bus generation and loading, i.e., ΔD, using the inverse of Hessian:

$$\Delta u = -H(u)^{-1}.F(u) = -H(u)^{-1}.\begin{bmatrix}[0]\\ \Delta D\end{bmatrix}. \tag{55}$$

While the vector $F(u)$ includes power flow and voltage drop equations, the corresponding equation in each row of the power flow equations is replaced by the cumulants of the random loads. Accordingly, the cumulants of the output variables are obtained by using the following relation:

$$k_{u,n} = (-H^{-1})^n \times k_{D,n}, \tag{56}$$

where $k_{u,n}$ is a vector of n^{th}-order cumulants for the unknown variables and $k_{D,n}$ is a vector of n^{th}-order cumulants for the random bus generation and loading. Consequently, the Hessian contains of the constant multipliers given by:

$$-H^{-1} = \begin{bmatrix} c_{1,1} & c_{1,2} & c_{1,3} & \cdots & c_{1,n} \\ c_{2,1} & c_{2,2} & c_{2,3} & \cdots & c_{2,n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{n,1} & c_{n,2} & c_{n,3} & \dots & c_{n,n} \end{bmatrix}.$$

Accordingly, the n^{th}-order cumulant for the i^{th} in u is computed using the following equation [19]:

$$k_{u_{i,n}} = c_{i,1}^{n} k_{D_{1,n}} + c_{i,2}^{n} k_{D_{2,n}} + \cdots + c_{i,1}^{n} k_{D_{n,n}}, \tag{57}$$

where u_i is the i^{th} element in u and $k_{D_{j,n}}$ is the n^{th} cumulant for the j^{th}component variable. Once the cumulants of the output variables are computed, their PDFs are reconstructed using Gram-Charlier/Edgeworth Expansion theory.

3.3.4 Gram-Charlier Expansion Theory (Step 5)

According to the Gram-Charlier Expansion, the PDFs of the many distributions can be formed as a series comprised of a standard normal distribution and its derivatives. The proposed solution method makes use of this characteristic to reconstruct the PDFs of the problem variables after their cumulants are defined in terms of the known cumulants (of the known variables).

Consider a random variable x with the cumulants of k_n. The PDF of x can be written in terms of the known PDF of φ. This known PDF is usually chosen as the standard normal distribution, i.e., with $\mu = 0$ and $\sigma = 1$. Thus the PDF of x is written as:

$$f(x) = \varphi(x) + \frac{c_1}{1!}\varphi^{(1)}(x) + \frac{c_2}{2!}\varphi^{(2)}(x) + \frac{c_3}{3!}\varphi^{(3)}(x) + \cdots \quad (58)$$

where $\varphi^{(n)}$ represents the n^{th}derivative of the standard normal distribution and c_i is the is the i^{th} Edgeworth series coefficient. The Edgeworth form of the series uses the cumulants of x to compute the series coefficient. Rearranging the above summation in terms of φ, a new parameter called Tchebycheff-Hermite or Hermite polynomial is introduced:

$$f(x) = c_0 He_0 \varphi(x) + c_1 He_1 \varphi(x) + c_2 He_2 \varphi(x) + c_3 He_3 \varphi(x) + \cdots \quad (59)$$

where $He_i$is the i^{th}Hermite polynomial. The above series form is called A series [19]. In the next sections the Hermite Polynomials and the computation of the Edgeworth series coefficient are discussed.

3.3.4.1 Hermite Polynomials

In the previous section, it was shown that the Gram-Charlier Expansion series allows to the PDF of random variable x to be constructed in a series form called A series. The series introduces two variables that can be defined in terms of the standard distribution functions and the cumulants of the random variable x. In this subsection one of these two variables, called the Hermite polynomial, is introduced.

Comparing the equation (58) and (59), it is showed that i^{th} Hermite polynomials are defined in terms of the i^{th} order derivative of the standard normal distribution as follows [29, 30]:

$$\mathrm{He}_i(x) = (-1)^i . e^{\frac{x^2}{2}} . \frac{d^i}{dx^i} \varphi(x) = (-1)^i e^{\frac{x^2}{2}} . \frac{d^i}{dx^i} e^{\frac{-x^2}{2}} \tag{60}$$

Accordingly, the first five Hermite polynomials are:

$$\mathrm{He}_0(x) = 1, \tag{61}$$

$$\mathrm{He}_1(x) = x,$$

$$\mathrm{He}_2(x) = x^2 - 1,$$

$$\mathrm{He}_3(x) = x^3 - 3x,$$

$$\mathrm{He}_4(x) = x^4 - 6x^2 + 3$$

$$\mathrm{He}_5(x) = x^5 - 10x^3 + 15x$$

3.3.4.2 Edgeworth A-Series Coefficients

Recalling equation (58), the probability density function of x can be rewritten in an exponential form as follow when the cumulant of the f(x) is given.

$$f(x) = \left[e^{\sum_{i=3}^{\infty} K_i \frac{(-D)^i}{i!}} \right] \varphi(x) \tag{62}$$

where k_i is the i^{th} cumulant of the f(x) and D is the differential operator with respect to x. Expanding (62) gives [29]:

$$f(x) = [1 + \frac{\left(-\frac{k_3}{3!}D^3 + \frac{k_4}{4!}D^4 - \frac{k_5}{5!}D^5 + \cdots\right)}{1!} + \frac{\left(-\frac{k_3}{3!}D^3 + \frac{k_4}{4!}D^4 - \frac{k_5}{5!}D^5 + \cdots\right)^2}{2!} + \frac{\left(-\frac{k_3}{3!}D^3 + \frac{k_4}{4!}D^4 - \frac{k_5}{5!}D^5 + \cdots\right)^3}{3!} + \cdots]\varphi(x). \tag{63}$$

By rearranging (63) the following is obtained:

$$f(x) = \left[1 - \frac{k_3}{3!}D^3 + \frac{k_4}{4!}D^4 - \frac{k_5}{5!}D^5 + \left(\frac{k_6}{6!} + \frac{{k_3}^2}{2!3!^2}\right)D^6 + \cdots\right]\varphi(x) \tag{64}$$

The differential operator in (64) can be substituted with the Hermite polynomials obtained in the previous section by the following relationship:

$$He_i(x).\varphi(x) = (-D)^n\varphi(x) \tag{65}$$

Substituting (65) in (64) yields:

$$f(x) = 1 + \frac{k_3}{3!}He_3\alpha(x) + \frac{k_4}{4!}He_4\alpha(x) + \frac{k_5}{5!}He_5\alpha(x) + \frac{(k_6 + 10{k_3}^2)}{6!}He_6\alpha(x) + \cdots \tag{66}$$

Recalling equation (59), the values of the Edgeworth series coefficients can be obtained. Table 1 presents the first six values of these coefficients.

Table 1 Edgeworth Coefficients

Coefficient (c_n)	value
0	1
1	0
2	0
3	$\frac{1}{6}k_3$
4	$\frac{1}{24}k_4$
5	$\frac{1}{120}k_5$
6	$\frac{1}{720}(k_6 + 10k_3^2)$

CHAPTER 4
RESULTS AND DISCUSSIONS

The solution method described in the preceding chapter is applied to a 7-bus and 33-bus RDS with one and two wind generators connected to them respectively. In order to assess the accuracy of the solution method, comparison studies have been carried out with Monte Carlo simulations (MCS) consisting of 2500 samples. The absolute percentage errors of the probabilistic information of the system variables are presented.

This section presents and discusses the results from the solution method.

4.1 7-Bus System

The methodology described in Chapter 3 is tested on the 7-bus test system with one wind generator shown in Figure 6. The system load and line data are given in the Appendix A.

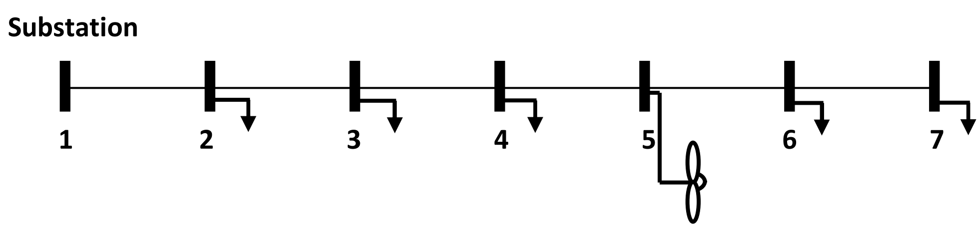

Figure 6 Single line diagram of the 7-bus distribution test system

4.1.1 Comparison of the results of the Cumulant method with those of MCS

In this section the results of the cumulant method are compared with those of MCS using 2500 samples. The loads are modeled using Gaussian distribution with the standard deviation equal to the 10% of the nominal bus loading value.

Table 2 gives the statistical parameters assumed for the Weibull distribution representing the wind generator. The data of the system including the mean values of the loads are given in the Appendix A.

Table 2 Parameters of the Weibull PDF for the wind generator

Bus No.	Mean Generation(kW)	Shape Factor	Standard Deviation [%]
5	400	2	55% of the Mean

In the present case, three different load levels, namely light, medium and heavy loads, have been considered with the corresponding time duration given in table 3, which also presents the system cost details. That makes the structure of the Hessian matrix consist of three submatrices such that the inverse of it contains the coefficients for the linear relation between system variables and the input random variables (loads and WGs) in the corresponding load level. Similarly, the cumulant vector of the loads and WGs consists of three subvectors. Equation (67) shows these segments clearly.

In the inverse of the Hessian matrix and vectors below, the subscripts of the coefficients (C) and Cumulant (k) can be L, M, and H, standing for Low, Medium and High, respectively.

$$\begin{bmatrix} k_{u_{L,n}} \\ k_{u_{M,n}} \\ k_{u_{H,n}} \end{bmatrix} = \begin{bmatrix} C_L & C_M & C_H \\ C_M & C_M & C_H \\ C_H & C_H & C_H \end{bmatrix}^{-n} \times \begin{bmatrix} [0] \\ k_{d_{L,n}} + k_{WG_{L,n}} \\ k_{d_{M,n}} + k_{WG_{M,n}} \\ k_{d_{H,n}} + k_{WG_{H,n}} \end{bmatrix} \tag{67}$$

Consequently, $k_{u_{L,n}}$, $k_{u_{M,n}}$ and $k_{u_{H,n}}$ are the Cumulants of the system variables in the light, medium and heavy load level and $(k_{d_{L,n}} + k_{WG_{L,n}})$,$(k_{d_{M,n}} + k_{WG_{M,n}})$ and $(k_{d_{H,n}} + k_{WG_{H,n}})$ are the Cumulants of the system loads and WGs in the prescribed load levels.

Table 3 Load Duration Data

	Load Level			Variable Capacitor Cost KS ($/kVAr)	Cost of Energy KL ($/kWh)
	Low	Medium	High		
Amount(p.u.)	0.5	0.7	1.5	90	0.06
Duration(Hrs)	1000	1000	6760		

The sensitivity index calculation corresponding to the procedure detailed in section 3.3.1 is carried out to find the best locations for siting the capacitor banks. These locations are known as the candidate buses. It is assumed that a capacitor switch is connected to all buses except the first bus. The bus bars are sorted in descending order of their active power loss sensitivity, which was calculated with respect to the reactive power injection in the corresponding bus. Once the bus with the highest sensitivity is determined, the capacitor switch is closed and a new sensitivity calculation is performed. It can be seen from the step procedure given in section 3.3.1 that a candidate bus can be selected more than once to allow more than one capacitor can to be connected to a bus.

Figure 7 shows the active power loss sensitivity value of each bus at the third time interval, before capacitor installation. According to this figure, the best candidate bus for capacitor sitting is bus 7. The sensitivity step procedure is carried out again until all conditions are satisfied.

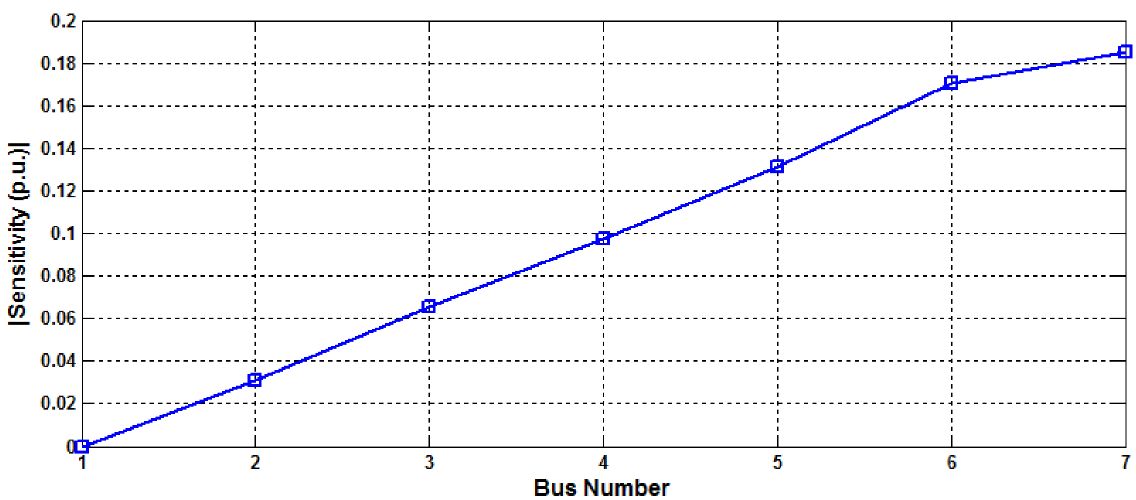

Figure 7 Sensitivity Index Solutions

Table 4 presents in detail the probabilistic optimal solutions for the real power loss and the cost of energy loss at each time duration, with and without capacitors as per cumulant method and MCS solutions. The table also gives the size of capacitors at each candidate bus. A significant decrease in real power loss and consequently in the annual cost of energy loss after capacitor installation can be seen from the table. In addition, the results of cumulant method are in close agreement with those of MCS.

Table 4 Optimal solutions and comparison of the two methods

Method	Time Duration	Capacitors		Real Power Loss (kW)	Annual Cost of Energy Losses ($)
		Bus No.	Size (kVAr)		
Base Case Cumulant	1	0		23.757	4,123,100
	2			52.359	
	3			327.59	
Base Case MCS	1	0		22.847	4,101,800
	2			51.588	
	3			326.09	
Cumulant using OPF	1	7	1738.7	9.9915	2,137,500
	2			25.068	
	3			170.48	
MCS using OPF	1	7	1738.6	10.785	2,139,000
	2			25.615	
	3			170.40	

Table 5 presents the probabilistic optimal solutions for the total cost. From this table it can be seen that the mean value of the total cost yields a cost of 2.2940 million dollars using the cumulant method and 2.2964 million dollars using the MCS. It can be seen that the mean values of the optimal solutions of both methods closely match each other with a percentage error of 0.10%.

The percentage error of the Standard deviation is 57.23%. This large percentage error is due to the power loss formulation, which adds the errors of the power and voltage values at all buses. Consequently, the existing variance errors at all buses are combined, yielding a total large percentage error. Moreover, there is a two stage procedure to obtain the cumulants of the total costs from the known cumulants of loads and wind generation. Firstly, the cumulants of the system power flow and nodal voltage should be obtained using the method as explained earlier, and secondly the total cost equation should be linearized so that the cumulant of the objective function can be calculated from the cumulant of the system variables. It can be concluded that the cumulant of the total cost is indirectly related to the cumulant of the known variables. This results in a further increased error as well.

Note that the total cost of employing capacitors clearly offset the annual energy loss for the uncompensated cases. The savings will be more pronounced after the first year.

Table 5 Total Cost Values

Cumulant		MCS		Error of Mean [%]	Error of Standard Deviation [%]
Mean value of Total Cost (millions of $)	Standard Deviation of Total Cost (millions of $)	Mean value of Total Cost (millions of $)	Standard Deviation of Total Cost (millions of $)		
2.2940	0.188	2.2964	0.441	0.10	57.23

Figure 8 shows the voltage improvement before and after the capacitor installation at the third time interval. From this figure it can be seen that the minimum voltage before installing any capacitor, i.e., at the base case, is 0.86 p.u. which is improved to 0.927 p.u. after capacitor installation.

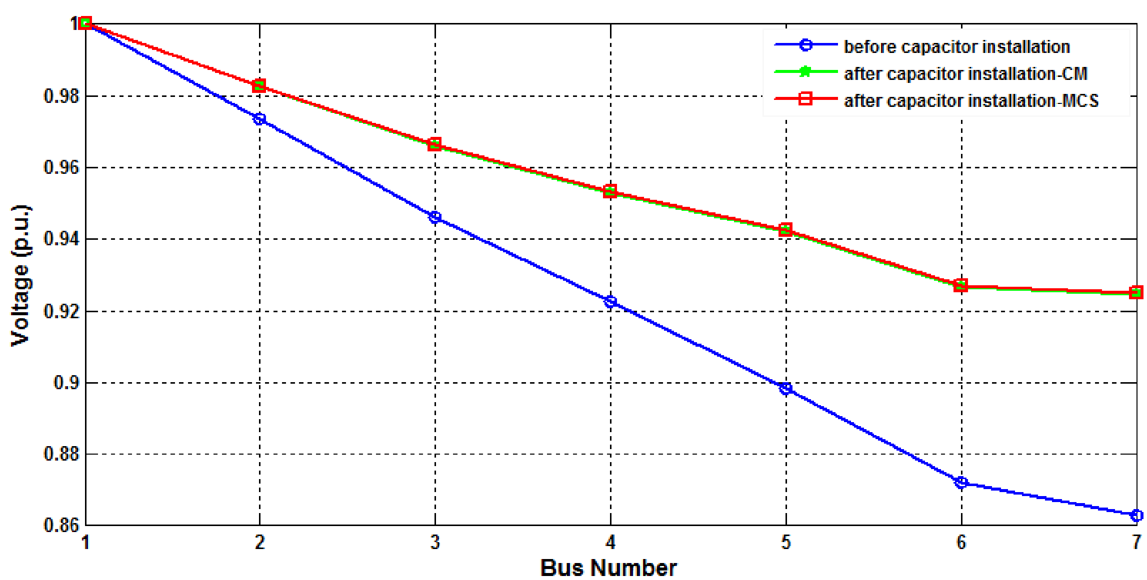

Figure 8 Voltage improvement before and after capacitor installation

The probabilistic optimal results for the system variables, voltage magnitude, active and reactive power flow solutions, and capacitor size in all time durations are detailed in table 6 and table 7. These results are divided into two portions. The first portion includes the results for the mean value and the discussion about the percentage error between both cumulant and MCS. The second portion of the results presents the standard deviation values and the discussion about the percent error between both cumulant and MCS. The mean values for the loads and WGs were taken at the nominal value from the 7-bus system. The results of both mean and standard deviation values are discussed as follows:

1) *Mean Values:* Table 6 contains the results related to the mean values for all system variables in per unit as well as a comparison between the cumulant method and MCS presented as an absolute percentage error in all TDs. The results for the percentage error of the voltage mean value is very small, with the maximum of 0.06% which occurs at TD3 at buses 5 and 6. The percentage errors of the mean value for the other system variables are not as small, but well below 10%.
2) *Variance Value:* Table 7 contains the results of the standard deviation values for all system variables in per unit as well as a percent error comparison between the cumulant method and MCS in all TDs. From the table it can be seen that the percentage error of the standard deviation of all variables are below 10% .The maximum percentage error for standard deviation of voltage, active and reactive

power and capacitor size are 2.25% at TD1, 3.01% at TD3, 9.45%at TD3 and 1.46% at TD2, respectively.

Table 6 Mean values of the system variables

	Bus No.	Cumulant Method Mean value			MCS-2500 samples Mean value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	1	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
	2	0.996	0.993	0.983	0.996	0.993	0.983	0.011	0.012	0.013
	3	0.992	0.987	0.966	0.992	0.988	0.966	0.025	0.025	0.027
	4	0.990	0.983	0.953	0.990	0.983	0.953	0.038	0.039	0.041
	5	0.988	0.980	0.942	0.989	0.980	0.943	0.055	0.056	0.060
	6	0.983	0.973	0.927	0.984	0.974	0.927	0.055	0.056	0.060
	7	0.983	0.972	0.925	0.983	0.973	0.925	0.055	0.056	0.059
Active Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.633	1.055	2.804	0.608	1.029	2.779	4.097	2.432	0.906
	3	0.508	0.877	2.400	0.483	0.853	2.376	5.121	2.894	1.013
	4	0.397	0.720	2.044	0.372	0.695	2.020	6.743	3.594	1.207
	5	0.280	0.555	1.675	0.255	0.530	1.650	9.951	4.757	1.482
	6	0.677	0.948	2.039	0.677	0.949	2.040	0.045	0.046	0.049
	7	0.325	0.455	0.975	0.326	0.456	0.977	0.167	0.167	0.167
Reactive Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.283	0.399	0.974	0.283	0.398	0.972	0.105	0.130	0.138
	3	0.183	0.257	0.644	0.182	0.256	0.643	0.329	0.315	0.253
	4	0.079	0.110	0.311	0.079	0.109	0.311	0.489	0.338	0.114
	5	-0.064	-0.093	-0.139	-0.065	-0.093	-0.139	1.010	0.620	0.275
	6	-0.067	-0.098	-0.163	-0.068	-0.098	-0.164	0.993	0.618	0.334
	7	-0.339	-0.479	-0.984	-0.339	-0.478	-0.983	0.007	0.076	0.152
Capacitor Size(PU)	7	0.590	0.831	1.739	0.591	0.831	1.739	0.079	0.039	0.001

Table 7 Standard deviation values of the system variables

	Bus No.	Cumulant Method Mean value			MCS-2500 samples Mean value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.001	0.001	0.001	0.001	0.001	0.001	1.644	1.596	0.239
	3	0.002	0.002	0.003	0.002	0.002	0.003	1.832	1.842	0.624
	4	0.004	0.004	0.005	0.004	0.004	0.005	1.841	1.839	0.550
	5	0.005	0.005	0.007	0.005	0.005	0.007	1.948	1.964	0.676
	6	0.005	0.005	0.007	0.005	0.006	0.007	2.157	2.145	0.481
	7	0.005	0.006	0.008	0.005	0.006	0.008	2.255	2.227	0.356
Active Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.232	0.241	0.297	0.237	0.246	0.305	2.152	2.312	2.409
	3	0.230	0.238	0.287	0.236	0.244	0.296	2.418	2.656	3.020
	4	0.229	0.235	0.278	0.235	0.242	0.286	2.394	2.592	2.863
	5	0.228	0.233	0.269	0.234	0.239	0.277	2.465	2.652	2.913
	6	0.048	0.067	0.144	0.048	0.067	0.144	0.154	0.152	0.124
	7	0.033	0.046	0.098	0.033	0.046	0.098	0.286	0.286	0.286
Reactive Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.015	0.021	0.047	0.015	0.021	0.046	0.552	0.883	3.233
	3	0.012	0.017	0.038	0.012	0.017	0.037	1.124	0.951	0.183
	4	0.010	0.014	0.030	0.010	0.014	0.031	0.613	0.865	3.143
	5	0.008	0.012	0.026	0.008	0.012	0.028	1.171	1.267	8.697
	6	0.008	0.012	0.027	0.008	0.012	0.030	1.209	1.413	9.451
	7	0.024	0.034	0.072	0.024	0.034	0.074	1.549	1.466	2.690
Capacitor Size(PU)	7	0.035	0.049	0.105	0.034	0.048	0.103	1.329	1.464	1.073

4.2 33-Bus System

The methodology described in Chapter 3 is tested on the 33-bus, 32-branch test system described in [31]. The system, however, is modified to accommodate two wind generators with a mean capacity equal to 500kW each. The single line diagram of the system is depicted on Figure 9. The system load and line data are given in the Appendix A.

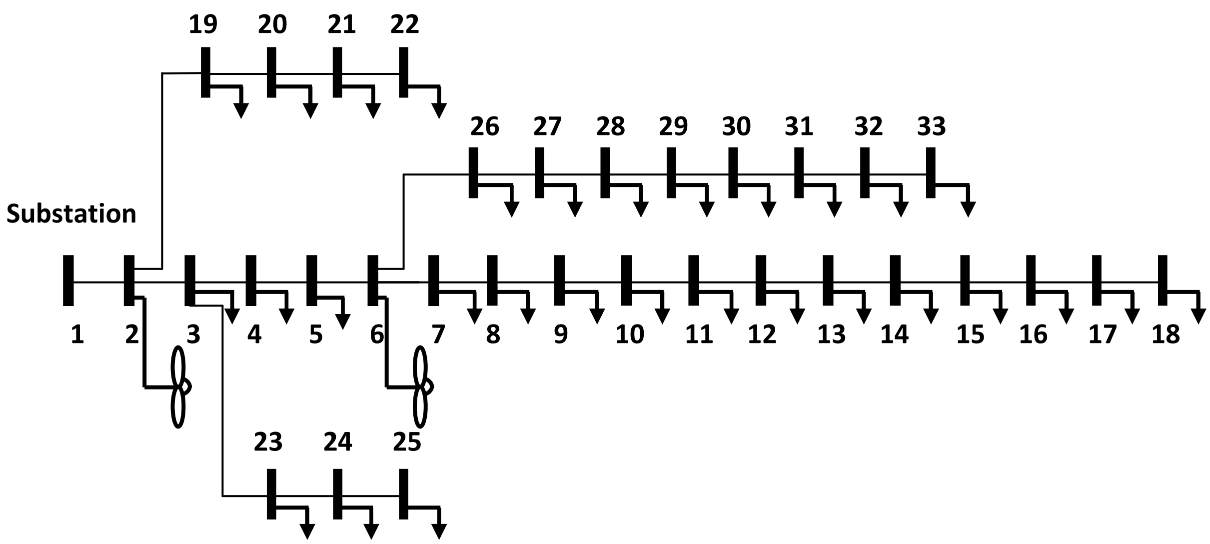

Figure 9 Single line diagram of the 33-bus distribution test system

4.2.1 Comparison of the results of the cumulant method with those of MCS

The cumulant solution method is tested on a 33-bus system with two wind generators connected to it. The standard deviations of the loads are equal to 10% of the nominal bus loading value. The load duration data of the system is same as table 3.

Table 8 includes the statistical parameters of the Weibull distribution used to represent the wind generators. Note both wind generators are assumed to be identical.

Table 8 Parameters of the Weibull PDF for the wind generator

Bus No.	Mean Generation (kW)	Shape Factor	Standard Deviation [%]
2 and 6	500	2	55% of the Mean

Figure 10 shows the active power loss sensitivity value of each bus before capacitor allocation at the third time interval. According to this figure, the best candidate bus for capacitor sitting is bus 33. The sensitivity step procedure is carried out again until all conditions are satisfied.

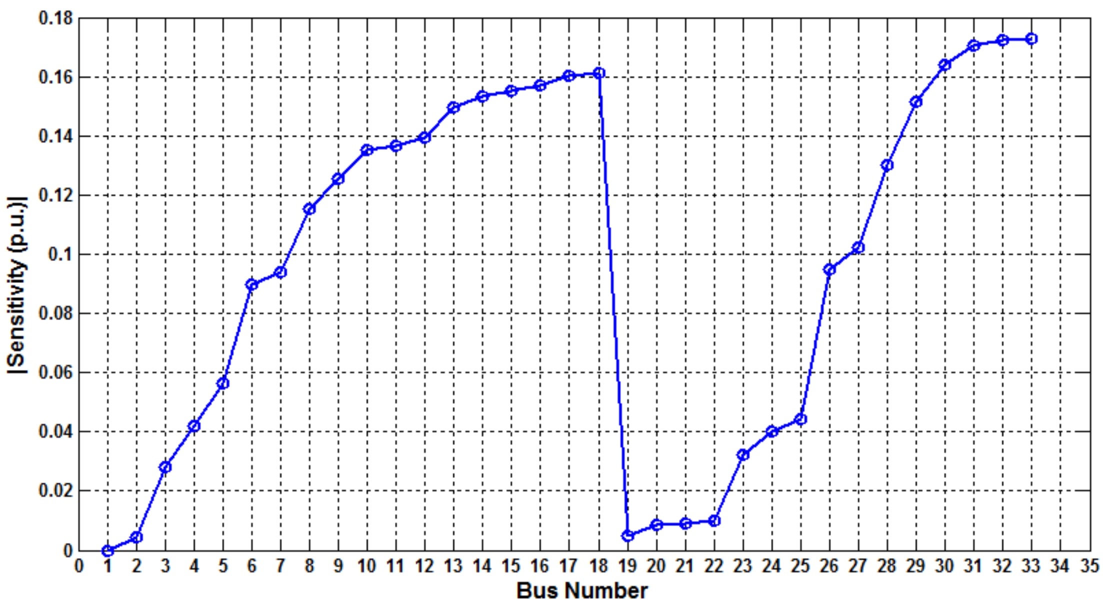

Figure 10 Sensitivity Index Solutions

The results obtained using the Cumulant method have been compared with those of MCS consisting of 2500 samples. Table 9 presents the probabilistic optimal solutions for the real power loss and the cost of energy loss at each time duration, with and without capacitors as per cumulant method and MCS solutions.

The table also gives the size of capacitors at each candidate bus. A significant decrease in real power loss and consequently in the annual cost of energy loss after capacitor installation can be seen from the table. In addition, the results of the cumulant method are in close agreement with those of MCS.

Table 9 Optimal solutions and comparison of the two methods

Method	Time Duration	Capacitors Bus No.	Capacitors Size (kVAr)	Real Power Loss (kW)	Annual Cost of Energy Losses($)
Base Case Cumulant	1	0		32.56	5,473,900
	2			72.11	
	3			434.38	
Base Case MCS	1	0		33.36	5,474,900
	2			72.69	
	3			434.26	
Cumulant using OPF	1	8	722.04	18.39	3,373,300
	2	18	263.30	42.97	
	3	25	483.27	268.15	
		33	1062.4		
MCS using OPF	1	8	721.25	18.87	3,351,500
	2	18	263.14	43.02	
	3	25	481.94	266.28	
		33	1061.6		

Table 10 represents the probabilistic optimal solutions for the total cost. From this table the mean value of the total cost is 3.6011 million dollars using the cumulant method and 3.5803 million dollars using the MCS. The corresponding percentage error is only 0.57%. The percentage error of Standard deviation, however, is 55.02%. This large error can is due to the same reason to that of the 7-bus system.

Table 10 Total Cost Values

Cumulant		MCS		Error of Mean [%]	Error of Standard Deviation [%]
Mean value of Total Cost (millions of $)	Standard Deviation of Total Cost (millions of $)	Mean value of Total Cost (millions of $)	Standard Deviation of Total Cost (millions of $)		
3.6011	0.173	3.5803	0.386	0.57	55.02

Figure 11 shows the voltage improvement before and after capacitor planning at TD3. The figure shows that the minimum of voltage before installing any capacitor, i.e. base case, is 0.86 p.u., which is improved to 0.91 p.u. after capacitor planning.

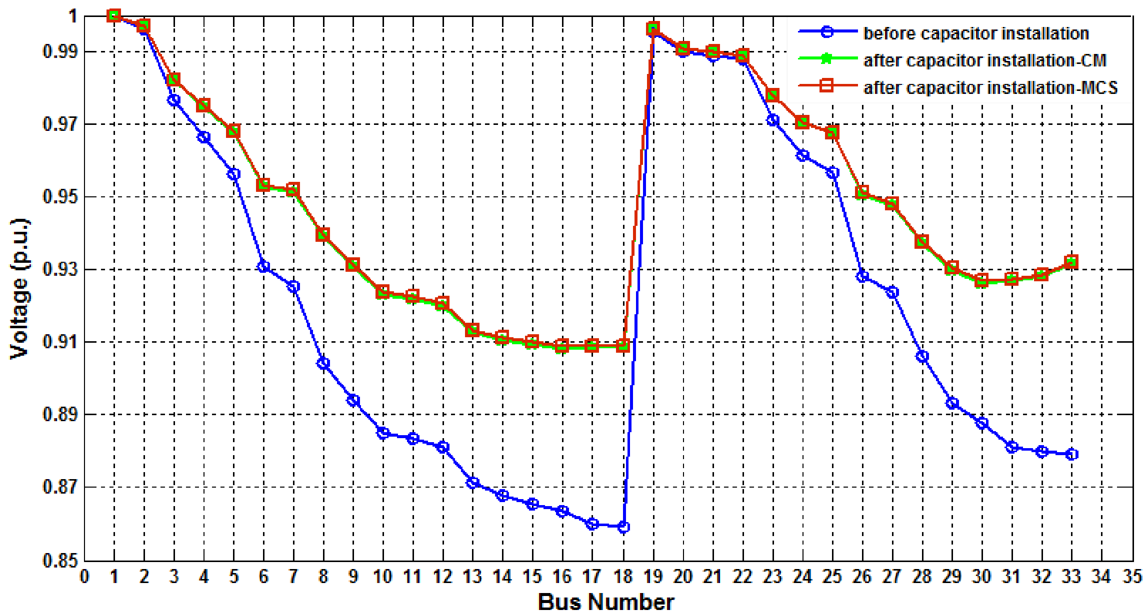

Figure 11 Voltage improvement before and after capacitor installation

Figure 12 and 13 show the reconstructed PDF and CDF of the objective function using Gram-Charlier/Edgeworth Series, respectively.

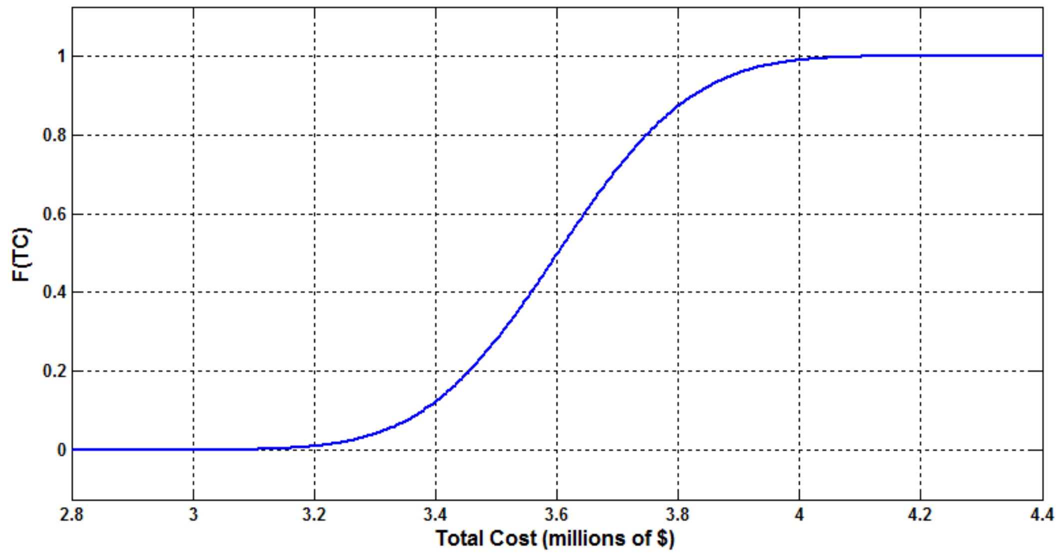

Figure 12 Reconstructed PDF of Total Cost

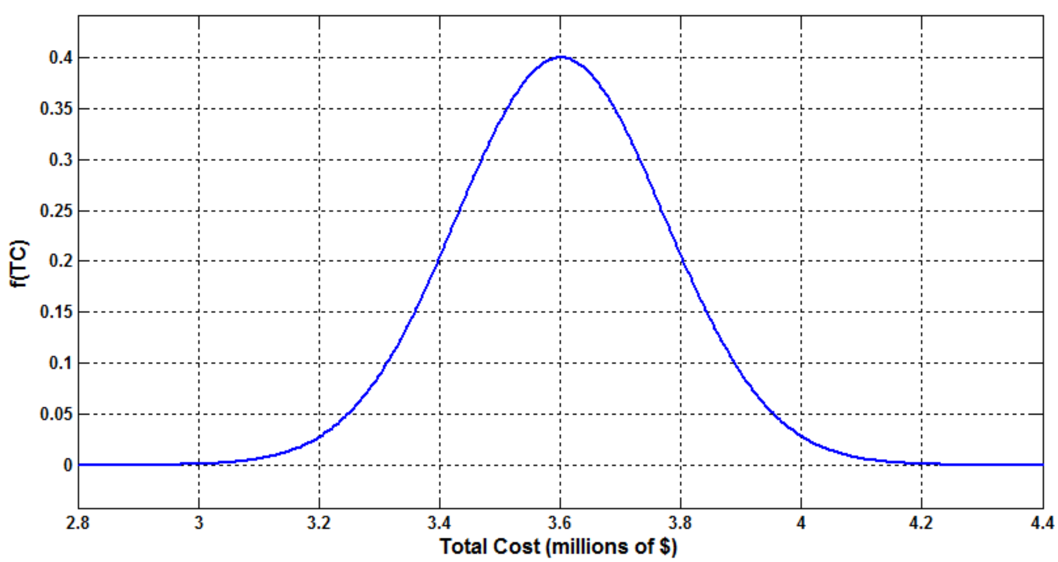

Figure 13 Reconstructed CDF of Total Cost

The probabilistic optimal results for the system variables, voltage magnitude, active and reactive power flow solutions and the capacitor size in all time durations are detailed in table 23 and table 24 in Appendix B. However, in table 11 and table 12 the maximum corresponding error are given. From table 11 can be seen that the percentage errors of the mean value for the other system variables are all below 9%.

The results presented in table 12 and 24 show that the percentage error of the standard deviation of all variables are below 10% except the percentage error of the standard deviation of the reactive power at bus 2. It should be noted that one of the wind generators is connected at this bus without any reactive loads. Consequently, the reactive power injected to this bus has a small variation, which causes a larger error in absolute percentage error in comparison with other buses.

The maximum percentage error for standard deviation of voltage, active and reactive power and capacitor size are 5.37%, 13.87%, 7.96% and 4.67%, respectively, which occur at the first time interval for active power and third time interval for other variables.

Table 11 maximum error of mean values of the system variables

	Bus No.	Cumulant Method Mean value			MCS-2500 samples Mean value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	16,17, 18	0.976	0.964	0.908	0.977	0.964	0.909	0.05	0.052	0.062
Active Power(PU)	2	0.776	1.502	4.528	0.713	1.439	4.462	8.827	4.421	1.49
Reactive Power(PU)	14	-0.001	-0.002	-0.006	-0.001	-0.002	-0.006	4.415	4.05	3.06
Capacitor Size(PU)	25	0.209	0.294	0.483	0.209	0.294	0.482	0.14	0.164	0.276

Table 12 maximum error of standard deviation values of the system variables

	Bus No.	Cumulant Method SD value			MCS-2500 samples SD value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	18	0.0039	0.0041	0.0051	0.0038	0.0039	0.0048	2.6264	2.8657	5.3733
Active Power(PU)	31	0.0132	0.0185	0.0398	0.0129	0.0181	0.0389	2.2412	2.2385	2.2184
Reactive Power(PU)	2	0.0071	0.0101	0.0242	0.0069	0.0097	0.0212	2.6173	4.0918	13.877
Capacitor Size(PU)	8	0.0079	0.0111	0.0243	0.0078	0.0111	0.0255	0.3101	0.1816	4.6717

The PDFs of all system variables are reconstructed from their cumulants using Gram Charlier/ Edgeworth series. Figures 14, 15 and 16 show the reconstructed PDFs of the voltage, nodal real power and capacitor size at bus 8 in the first time interval and compare it with those of MCS. In order to illustrate the PDF of capacitor size, bus 8 is selected randomly from other candidate buses. From these figures it can be seen that the PDFs have a non-Gaussian behaviour. That can be seen more clearly in Figure 15, which illustrates the nodal real power at this bus.

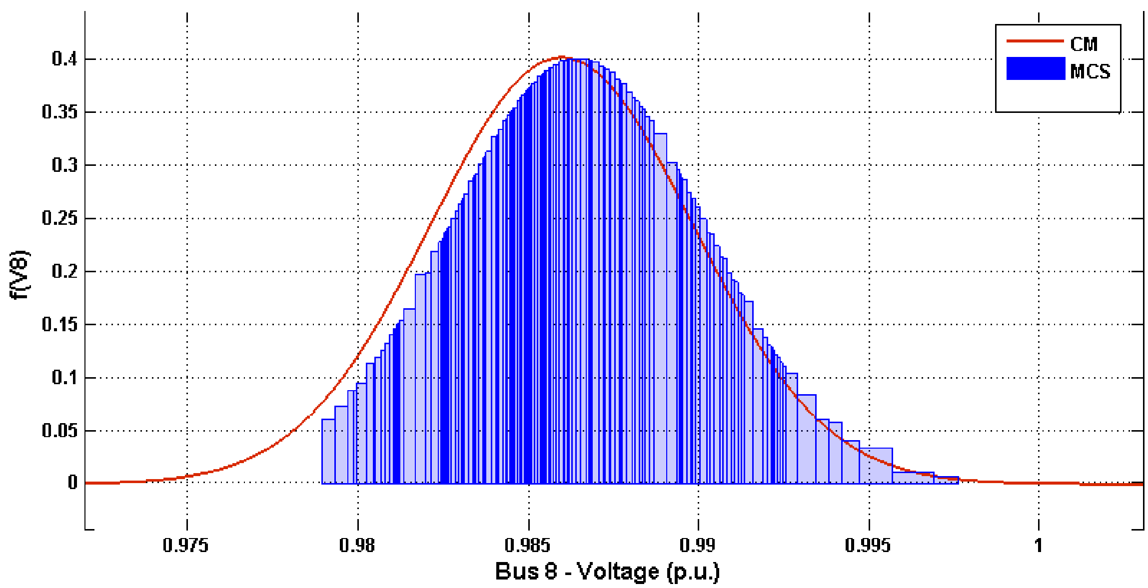

Figure 14 PDF of Voltage at Bus 8 reconstructed by Gram-Charlier/Edgeworth

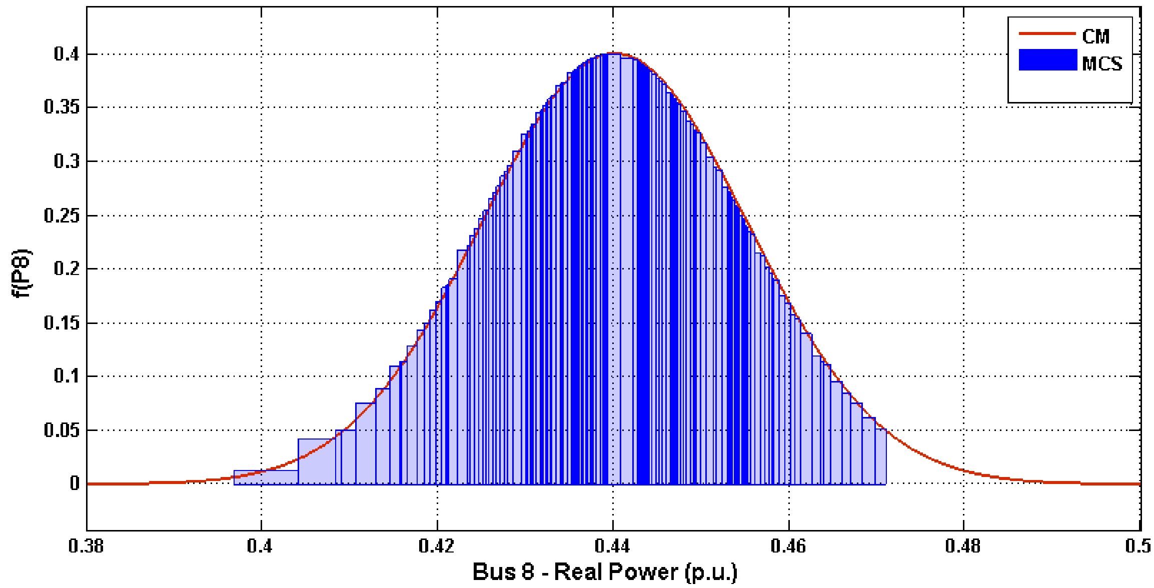

Figure 15 PDF of real Power at Bus 8 reconstructed by Gram-Charlier/Edgeworth

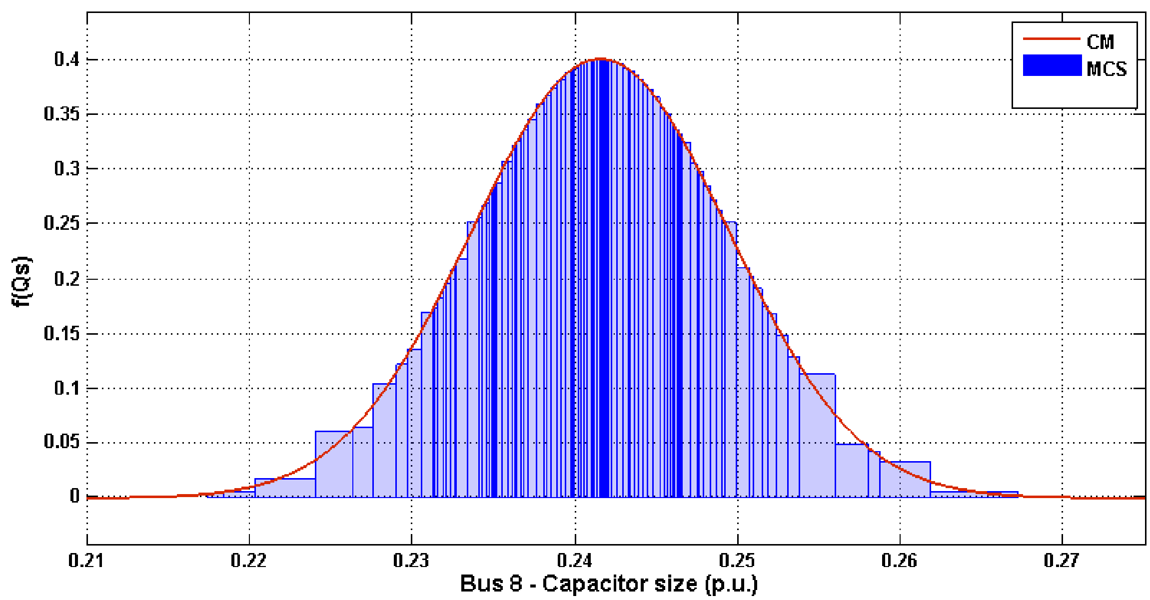

Figure 16 PDF of capacitor size at Bus 8 reconstructed by Gram-Charlier/Edgeworth

To explain the non-Guassian behaviour of the system random variables at this bus, one may note that one of the WGs is connected at bus 6 and since the WGs were modeled as Weibull distributions, the probabilistic behaviour of the systems variables in the buses nearby have a tendency to behave more like a Weibull distribution. The small difference between PDFs of cumulant and MCS in Figure 14 is due to the small error between the standard deviation values obtained from both methods.

These results clearly imply that the cumulant method provides results acceptably close to those obtained from MCS. Therefore, the method has been validated. The cumulant method brings the advantage of computational speed and analytical soundness, as opposed to the exhaustive nature of MCS.

Next chapter presents the main conclusions of this research work.

CHAPTER 5

CONCLUSIONS

This thesis studied a new probabilistic formulation for the optimal capacitor planning problem in distribution systems with high penetration of wind generators. The probabilistic components of the systems are loads and wind generators which were modeled as Gaussian and Weibull distributions, respectively. The different orders of their cumulants were obtained from their PDFs. To find the best locations to site the capacitor banks, a selection procedure was formulated to identify the buses with highest active power loss sensitivity with respect to the reactive power injection. By selecting the candidate buses beforehand, this task greatly reduces the complexity and variable size of the optimization problem. Once the best locations are determined, the power system is formulated as an optimization problem and solved by using the LBIPM. This solution delivers the Hessian matrix, which is inverted to provide a linear mapping from the cumulants of the system loadings and WGs to those of the unknown system variables (voltage, reactive and active powers).

The method was implemented and tested on a 7-bus and on a 33-bus system. Once the cumulants of all unknown system variables are obtained, their PDFs as well as the objective function were reconstructed using the Gram Charlier/ Edgeworth series.

In order to illustrate the efficiency and accuracy of the cumulant method, the results obtained using the cumulant method were compared with those of MCS with 2500 samples. The errors were found to be acceptable. The proposed probabilistic algorithm has much less computational burden and complexity, making it very practical and advantageous. Moreover, the PDFs provided by the proposed solution offer great insight to power system planners when finding the most cost effective solutions for capacitor allocation in distribution systems.

APPENDIX A

TEST SYSTEMS DATA

7-Bus System Data

Table 13 Voltage data: 7-bus RDS

Slack Bus Voltage (p.u.)	Maximum Voltage Limit (p.u.)	Minimum Voltage Limit (p.u.)
1.00	1.10	0.9

Table 14 Feeder data: 7-bus RDS

PG (MW)	QG (Mvar)	QGMAX (Mvar)	QGMIN (Mvar)
1.00	1.10	1000	-1000

Table 15 Shunt capacitor limits: 7-bus RDS

QSMAX (Mvar)	QSMIN (Mvar)	QSSTP (Mvar)
2	0	1.00

Table 16 Mean values and standard deviation of the loads

Bus Number	Mean		Standard Deviation [%]
	Real Power (kW)	Reactive Power (kW)	
1	0	0	10
2	0.2465	0.197	10
3	0.2209	0.206	10
4	0.232	0.286	10
5	0	0	10
6	0.701	0.542	10
7	0.65	0.503	10

Table 17 Line data 7-bus RDS

From Bus	To Bus	R (Ω)	X (Ω)	Rating (MVA)	System Voltage (kV)
1	2	0.7314	0.7158	100	12.66
2	3	0.8411	0.8231	100	12.66
3	4	0.8411	0.8231	100	12.66
4	5	1.0605	1.0379	100	12.66
5	6	1.197	0.808	100	12.66
6	7	0.8542	0.5766	100	12.66

33-Bus System Data

Table 18 Voltage data: 33-bus RDS

Slack Bus Voltage (p.u.)	Maximum Voltage Limit (p.u.)	Maximum Voltage Limit (p.u.)
1.00	1.10	0.9

Table 19 Feeder data: 33-bus RDS

PG (MW)	QG (Mvar)	QGMAX (Mvar)	QGMIN (Mvar)
1.00	1.10	1000	-1000

Table 20 Shunt capacitor limits: 33-bus RDS

QSMAX (Mvar)	QSMIN (Mvar)	QSSTP (Mvar)
100	0	1.00

Table 21 Mean values and standard deviation of the loads

Bus Number	Mean		Standard Deviation [%]
	Real Power (kW)	Reactive Power (kW)	
1	0	0	10
2	0	0	10
3	0.09	0.04	10
4	0.12	0.08	10
5	0.06	0.03	10
6	0	0	10
7	0.2	0.1	10
8	0.2	0.1	10
9	0.06	0.02	10
10	0.06	0.02	10
11	0.045	0.03	10
12	0.06	0.035	10
13	0.06	0.035	10
14	0.12	0.08	10
15	0.06	0.01	10
16	0.06	0.02	10
17	0.06	0.02	10
18	0.09	0.04	10
19	0.09	0.04	10
20	0.09	0.04	10
21	0.09	0.04	10
22	0.09	0.04	10
23	0.09	0.05	10
24	0.4	0.2	10

25	0.4	0.2	10
26	0.06	0.025	10
27	0.06	0.025	10
28	0.06	0.02	10
29	0.12	0.07	10
30	0.2	0.6	10
31	0.15	0.07	10
32	0.21	0.1	10
33	0.06	0.04	10

Table 22 Line data 33-bus RDS

From Bus	To Bus	R (Ω)	X (Ω)	Rating (MVA)	System Voltage (kV)
1	2	0.0922	0.047	100	12.66
2	3	0.493	0.2511	100	12.66
3	4	0.3662	0.1864	100	12.66
4	5	0.3811	0.1941	100	12.66
5	6	0.819	0.707	100	12.66
6	7	0.1872	0.6188	100	12.66
7	8	1.7114	1.2351	100	12.66
8	9	1.03	0.74	100	12.66
9	10	1.044	0.74	100	12.66
10	11	0.1966	0.065	100	12.66
11	12	0.3744	0.1238	100	12.66
12	13	1.468	1.155	100	12.66
13	14	0.5416	0.7129	100	12.66
14	15	0.591	0.526	100	12.66
15	16	0.7463	0.545	100	12.66
16	17	1.289	1.721	100	12.66
17	18	0.732	0.574	100	12.66
2	19	0.164	0.1565	100	12.66
19	20	1.5042	1.3554	100	12.66

20	**21**	**0.4095**	**0.4784**	**100**	**12.66**
21	**22**	**0.7089**	**0.9373**	**100**	**12.66**
3	**23**	**0.4512**	**0.3083**	**100**	**12.66**
23	**24**	**0.898**	**0.7091**	**100**	**12.66**
24	**25**	**0.896**	**0.7011**	**100**	**12.66**
6	**26**	**0.203**	**0.1034**	**100**	**12.66**
26	**27**	**0.2842**	**0.1447**	**100**	**12.66**
27	**28**	**1.059**	**0.9337**	**100**	**12.66**
28	**29**	**0.8042**	**0.7006**	**100**	**12.66**
29	**30**	**0.5075**	**0.2585**	**100**	**12.66**
30	**31**	**0.9744**	**0.963**	**100**	**12.66**
31	**32**	**0.3105**	**0.3619**	**100**	**12.66**
32	**33**	**0.341**	**0.5302**	**100**	**12.66**

APPENDIX B

RESULT COMPARISON OF THE 33-BUS SYSTEM

Table 23 Mean values of the system variables

	Bus No.	Cumulant Method Mean value			MCS-2500 samples Mean value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	1	1.000	1.000	1.000	1.000	1.000	1.000	0.000	0.000	0.000
	2	0.999	0.999	0.997	1.000	0.999	0.997	0.004	0.004	0.004
	3	0.996	0.993	0.982	0.996	0.993	0.982	0.014	0.014	0.016
	4	0.994	0.991	0.975	0.995	0.991	0.975	0.022	0.022	0.024
	5	0.993	0.988	0.968	0.993	0.989	0.968	0.030	0.030	0.033
	6	0.990	0.983	0.953	0.991	0.984	0.953	0.047	0.048	0.053
	7	0.990	0.983	0.951	0.990	0.983	0.952	0.047	0.048	0.053
	8	0.986	0.977	0.939	0.986	0.978	0.939	0.048	0.049	0.055
	9	0.983	0.974	0.931	0.984	0.974	0.931	0.048	0.050	0.057
	10	0.981	0.970	0.923	0.982	0.971	0.924	0.049	0.050	0.058
	11	0.981	0.970	0.922	0.981	0.970	0.923	0.049	0.050	0.058
	12	0.980	0.969	0.920	0.981	0.969	0.921	0.049	0.051	0.059
	13	0.978	0.966	0.913	0.978	0.966	0.913	0.049	0.051	0.060
	14	0.977	0.965	0.911	0.978	0.965	0.911	0.049	0.051	0.061
	15	0.977	0.964	0.909	0.977	0.965	0.910	0.049	0.052	0.061
	16	0.976	0.964	0.908	0.977	0.964	0.909	0.050	0.052	0.062
	17	0.976	0.964	0.908	0.977	0.964	0.909	0.050	0.052	0.062
	18	0.977	0.964	0.908	0.977	0.964	0.909	0.050	0.052	0.062
	19	0.999	0.999	0.996	0.999	0.999	0.996	0.004	0.004	0.004

	20	0.997	0.996	0.991	0.997	0.996	0.991	0.004	0.004	0.004
	21	0.997	0.996	0.990	0.997	0.996	0.990	0.004	0.004	0.004
	22	0.997	0.995	0.989	0.997	0.995	0.989	0.004	0.004	0.004
	23	0.995	0.992	0.978	0.995	0.992	0.978	0.014	0.014	0.016
	24	0.992	0.988	0.970	0.993	0.989	0.971	0.014	0.015	0.016
	25	0.992	0.987	0.968	0.992	0.988	0.968	0.014	0.015	0.017
	26	0.990	0.982	0.951	0.990	0.983	0.951	0.047	0.048	0.053
	27	0.989	0.981	0.948	0.989	0.982	0.948	0.047	0.048	0.053
	28	0.985	0.976	0.937	0.986	0.977	0.938	0.047	0.048	0.054
	29	0.983	0.973	0.930	0.984	0.974	0.930	0.047	0.049	0.054
	30	0.982	0.972	0.926	0.982	0.972	0.927	0.047	0.049	0.054
	31	0.982	0.972	0.927	0.983	0.973	0.927	0.047	0.048	0.053
	32	0.983	0.973	0.928	0.983	0.973	0.929	0.047	0.048	0.052
	33	0.984	0.974	0.931	0.984	0.975	0.932	0.047	0.048	0.052
Active Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.776	1.502	4.528	0.713	1.439	4.462	8.827	4.421	1.490
	3	1.092	1.740	4.422	1.058	1.706	4.386	3.170	1.995	0.822
	4	0.599	1.048	2.914	0.566	1.014	2.880	5.877	3.305	1.205
	5	0.538	0.962	2.715	0.505	0.928	2.681	6.636	3.639	1.302
	6	0.507	0.915	2.587	0.473	0.881	2.553	7.126	3.841	1.347
	7	0.542	0.762	1.663	0.542	0.761	1.661	0.119	0.119	0.118
	8	0.440	0.618	1.340	0.440	0.617	1.339	0.069	0.069	0.069
	9	0.339	0.476	1.031	0.339	0.476	1.031	0.038	0.038	0.039
	10	0.309	0.433	0.934	0.309	0.433	0.934	0.037	0.037	0.037
	11	0.279	0.391	0.843	0.278	0.390	0.843	0.071	0.071	0.071
	12	0.256	0.359	0.774	0.256	0.358	0.773	0.084	0.084	0.084
	13	0.225	0.316	0.679	0.225	0.316	0.678	0.083	0.083	0.083
	14	0.195	0.274	0.588	0.195	0.273	0.587	0.060	0.060	0.060
	15	0.135	0.189	0.407	0.135	0.189	0.406	0.208	0.208	0.208

	16	0.105	0.147	0.316	0.105	0.147	0.315	0.268	0.268	0.267
	17	0.075	0.105	0.225	0.075	0.105	0.225	0.206	0.205	0.205
	18	0.045	0.063	0.135	0.045	0.063	0.135	0.349	0.349	0.349
	19	0.180	0.252	0.542	0.180	0.252	0.542	0.098	0.098	0.097
	20	0.135	0.189	0.405	0.135	0.189	0.405	0.074	0.074	0.074
	21	0.090	0.126	0.270	0.090	0.126	0.270	0.177	0.177	0.176
	22	0.045	0.063	0.135	0.045	0.063	0.135	0.314	0.314	0.314
	23	0.446	0.625	1.346	0.446	0.625	1.345	0.057	0.056	0.054
	24	0.400	0.561	1.202	0.400	0.560	1.202	0.068	0.068	0.067
	25	0.200	0.280	0.600	0.200	0.280	0.599	0.092	0.092	0.092
	26	0.464	0.652	1.418	0.464	0.652	1.419	0.058	0.059	0.064
	27	0.433	0.609	1.324	0.434	0.609	1.325	0.066	0.067	0.072
	28	0.402	0.564	1.222	0.403	0.565	1.223	0.085	0.086	0.090
	29	0.371	0.521	1.125	0.372	0.522	1.126	0.111	0.111	0.114
	30	0.311	0.436	0.941	0.311	0.437	0.942	0.079	0.079	0.082
	31	0.210	0.295	0.634	0.211	0.295	0.635	0.159	0.159	0.160
	32	0.135	0.190	0.407	0.135	0.190	0.408	0.059	0.059	0.059
	33	0.030	0.042	0.090	0.030	0.042	0.090	0.123	0.123	0.123
Reactive Power (PU)	1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	2	0.228	0.320	0.984	0.228	0.320	0.983	0.022	0.045	0.035
	3	0.146	0.203	0.709	0.145	0.202	0.708	0.120	0.123	0.049
	4	0.108	0.150	0.434	0.107	0.150	0.433	0.174	0.196	0.140
	5	0.067	0.093	0.304	0.067	0.093	0.304	0.216	0.196	0.066
	6	0.051	0.068	0.226	0.051	0.068	0.227	0.681	0.398	0.095
	7	-0.069	-0.102	-0.183	-0.070	-0.102	-0.183	0.228	0.069	0.227
	8	-0.121	-0.175	-0.350	-0.121	-0.175	-0.350	0.062	0.027	0.194
	9	0.070	0.098	0.216	0.070	0.098	0.216	0.070	0.070	0.073
	10	0.059	0.083	0.181	0.059	0.083	0.181	0.046	0.047	0.051
	11	0.049	0.069	0.151	0.049	0.069	0.151	0.062	0.063	0.067

	12	0.034	0.048	0.105	0.034	0.048	0.105	0.160	0.161	0.166
	13	0.016	0.023	0.049	0.016	0.023	0.048	0.202	0.206	0.220
	14	-0.001	-0.002	-0.006	-0.001	-0.002	-0.006	4.415	4.050	3.060
	15	-0.041	-0.058	-0.126	-0.041	-0.058	-0.126	0.132	0.131	0.124
	16	-0.046	-0.065	-0.142	-0.046	-0.065	-0.142	0.108	0.107	0.101
	17	-0.056	-0.079	-0.173	-0.056	-0.079	-0.173	0.085	0.084	0.079
	18	-0.066	-0.093	-0.203	-0.066	-0.093	-0.203	0.095	0.094	0.090
	19	0.080	0.112	0.242	0.080	0.113	0.242	0.143	0.143	0.144
	20	0.060	0.084	0.180	0.060	0.084	0.181	0.129	0.129	0.129
	21	0.040	0.056	0.120	0.040	0.056	0.120	0.301	0.301	0.301
	22	0.020	0.028	0.060	0.020	0.028	0.060	0.392	0.392	0.392
	23	0.017	0.023	0.200	0.017	0.023	0.201	0.109	0.224	0.212
	24	-0.009	-0.014	0.119	-0.009	-0.014	0.119	0.834	0.183	0.209
	25	-0.109	-0.154	-0.183	-0.109	-0.154	-0.183	0.006	0.042	0.237
	26	0.119	0.167	0.396	0.119	0.167	0.396	0.155	0.117	0.045
	27	0.107	0.149	0.357	0.106	0.149	0.357	0.165	0.122	0.043
	28	0.093	0.130	0.309	0.093	0.130	0.309	0.188	0.141	0.057
	29	0.082	0.114	0.272	0.082	0.114	0.272	0.195	0.144	0.057
	30	0.047	0.065	0.165	0.047	0.065	0.166	0.094	0.005	0.112
	31	-0.253	-0.356	-0.741	-0.253	-0.356	-0.740	0.056	0.071	0.097
	32	-0.289	-0.406	-0.849	-0.289	-0.406	-0.848	0.027	0.041	0.062
	33	-0.339	-0.476	-1.002	-0.339	-0.476	-1.002	0.059	0.071	0.088
Capacitor Size (PU)	8	0.242	0.345	0.722	0.242	0.345	0.721	0.018	0.027	0.110
	18	0.086	0.121	0.263	0.086	0.121	0.263	0.064	0.063	0.060
	25	0.209	0.294	0.483	0.209	0.294	0.482	0.140	0.164	0.276
	33	0.359	0.504	1.062	0.359	0.504	1.062	0.047	0.058	0.074

Table 24 Standard deviation values of the system variables

	Bus No.	Cumulant Method S.D value			MCS-2500 samples S.D value			Error (%)		
		TD1	TD2	TD3	TD1	TD2	TD3	TD1	TD2	TD3
Voltage (PU)	1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	2	0.00023	0.00023	0.00025	0.00023	0.00023	0.00025	1.54070	1.51180	1.77500
	3	0.00106	0.00109	0.00125	0.00104	0.00106	0.00121	2.46080	2.40900	2.75780
	4	0.00171	0.00174	0.00196	0.00166	0.00170	0.00191	2.47990	2.47120	3.00830
	5	0.00238	0.00243	0.00272	0.00232	0.00237	0.00264	2.47100	2.48120	3.10640
	6	0.00383	0.00391	0.00435	0.00374	0.00381	0.00420	2.46760	2.52840	3.56120
	7	0.00383	0.00391	0.00436	0.00374	0.00381	0.00419	2.49550	2.60770	4.15170
	8	0.00386	0.00395	0.00453	0.00376	0.00385	0.00431	2.53710	2.73910	5.19250
	9	0.00387	0.00398	0.00464	0.00378	0.00387	0.00441	2.53510	2.73560	5.16410
	10	0.00389	0.00401	0.00475	0.00379	0.00390	0.00451	2.54510	2.74810	5.15680
	11	0.00389	0.00401	0.00476	0.00380	0.00390	0.00453	2.54880	2.75240	5.15380
	12	0.00390	0.00402	0.00480	0.00380	0.00391	0.00456	2.55220	2.75610	5.14260
	13	0.00392	0.00406	0.00494	0.00382	0.00395	0.00470	2.54800	2.74900	5.08730
	14	0.00393	0.00407	0.00498	0.00383	0.00396	0.00474	2.54030	2.73850	5.06360
	15	0.00393	0.00408	0.00501	0.00383	0.00397	0.00477	2.55290	2.75620	5.09730
	16	0.00393	0.00408	0.00505	0.00384	0.00397	0.00480	2.56850	2.77830	5.14060
	17	0.00394	0.00409	0.00508	0.00384	0.00398	0.00483	2.60980	2.84090	5.31170
	18	0.00394	0.00410	0.00510	0.00384	0.00398	0.00484	2.62640	2.86570	5.37330
	19	0.00023	0.00023	0.00026	0.00023	0.00023	0.00025	1.63790	1.65020	2.07070
	20	0.00025	0.00027	0.00038	0.00024	0.00026	0.00037	2.13240	2.23930	2.47400
	21	0.00025	0.00028	0.00042	0.00025	0.00027	0.00041	2.23180	2.32400	2.41770
	22	0.00026	0.00029	0.00045	0.00026	0.00028	0.00044	2.45180	2.52680	2.43630
	23	0.00107	0.00111	0.00133	0.00105	0.00109	0.00131	2.23660	2.09210	2.10110
	24	0.00111	0.00118	0.00161	0.00109	0.00116	0.00159	1.77310	1.46630	1.09330
	25	0.00114	0.00123	0.00179	0.00112	0.00122	0.00177	1.53580	1.19030	0.86691

	26	0.00383	0.00391	0.00437	0.00374	0.00382	0.00422	2.46410	2.52370	3.55390
	27	0.00384	0.00392	0.00441	0.00375	0.00383	0.00426	2.45720	2.51420	3.53530
	28	0.00387	0.00397	0.00458	0.00377	0.00387	0.00443	2.42270	2.47330	3.51880
	29	0.00389	0.00400	0.00473	0.00380	0.00391	0.00457	2.38700	2.42650	3.45310
	30	0.00390	0.00402	0.00481	0.00381	0.00393	0.00465	2.36170	2.39260	3.39210
	31	0.00390	0.00403	0.00486	0.00381	0.00393	0.00467	2.48250	2.57680	3.89320
	32	0.00390	0.00403	0.00486	0.00381	0.00393	0.00467	2.53060	2.65010	4.09010
	33	0.00390	0.00403	0.00485	0.00380	0.00392	0.00464	2.60700	2.76910	4.42950
Active Power (PU)	1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	2	0.39913	0.40391	0.43318	0.39343	0.39852	0.42910	1.44900	1.35440	0.94900
	3	0.28410	0.28937	0.32156	0.27832	0.28387	0.31749	2.07890	1.94030	1.28240
	4	0.28181	0.28504	0.30450	0.27576	0.27908	0.29916	2.19250	2.13290	1.78740
	5	0.28101	0.28357	0.29972	0.27513	0.27780	0.29457	2.13720	2.07960	1.74920
	6	0.27951	0.28079	0.29077	0.27391	0.27530	0.28592	2.04320	1.99470	1.69540
	7	0.01824	0.02569	0.05662	0.01847	0.02600	0.05730	1.20450	1.19940	1.18740
	8	0.01511	0.02123	0.04639	0.01509	0.02121	0.04633	0.12500	0.12594	0.13388
	9	0.01127	0.01586	0.03478	0.01106	0.01555	0.03411	1.95880	1.95930	1.96140
	10	0.01082	0.01520	0.03310	0.01061	0.01490	0.03246	1.98190	1.98090	1.97470
	11	0.01039	0.01459	0.03177	0.01024	0.01439	0.03132	1.41640	1.41770	1.42160
	12	0.01013	0.01422	0.03090	0.00994	0.01395	0.03032	1.90850	1.90870	1.90750
	13	0.00963	0.01350	0.02911	0.00947	0.01328	0.02862	1.70090	1.70020	1.69420
	14	0.00914	0.01281	0.02754	0.00898	0.01258	0.02706	1.78490	1.78320	1.77210
	15	0.00689	0.00965	0.02076	0.00690	0.00967	0.02080	0.20162	0.20134	0.20226
	16	0.00619	0.00867	0.01864	0.00622	0.00872	0.01873	0.48037	0.48017	0.48087
	17	0.00541	0.00758	0.01624	0.00544	0.00761	0.01632	0.47712	0.47722	0.47835
	18	0.00450	0.00630	0.01350	0.00452	0.00632	0.01355	0.33396	0.33396	0.33396
	19	0.00902	0.01264	0.02719	0.00905	0.01269	0.02729	0.37612	0.37525	0.37166
	20	0.00780	0.01092	0.02341	0.00782	0.01095	0.02348	0.26404	0.26349	0.26126
	21	0.00637	0.00891	0.01910	0.00632	0.00885	0.01897	0.70676	0.70683	0.70713

	22	0.00450	0.00630	0.01350	0.00447	0.00626	0.01342	0.58930	0.58930	0.58930
	23	0.02880	0.04041	0.08745	0.02881	0.04042	0.08747	0.02571	0.02568	0.02504
	24	0.02832	0.03966	0.08517	0.02838	0.03975	0.08535	0.21706	0.21675	0.21557
	25	0.02000	0.02800	0.06000	0.02000	0.02800	0.06000	0.00159	0.00159	0.00159
	26	0.01865	0.02627	0.05794	0.01872	0.02637	0.05816	0.35383	0.35832	0.38292
	27	0.01838	0.02588	0.05693	0.01850	0.02604	0.05729	0.62161	0.62309	0.63510
	28	0.01804	0.02533	0.05515	0.01808	0.02539	0.05530	0.23163	0.23552	0.26232
	29	0.01771	0.02484	0.05368	0.01785	0.02503	0.05409	0.74425	0.74496	0.75997
	30	0.01663	0.02331	0.05021	0.01648	0.02309	0.04974	0.94228	0.94394	0.93517
	31	0.01325	0.01856	0.03982	0.01296	0.01815	0.03896	2.24120	2.23850	2.21840
	32	0.01092	0.01529	0.03278	0.01090	0.01526	0.03271	0.22709	0.22481	0.21101
	33	0.00300	0.00420	0.00900	0.00295	0.00413	0.00884	1.78200	1.78200	1.78200
Reactive Power (PU)	1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	2	0.00710	0.01019	0.02422	0.00692	0.00979	0.02127	2.61730	4.09180	13.87700
	3	0.00577	0.00821	0.01917	0.00566	0.00798	0.01768	1.84870	2.86300	8.45630
	4	0.00593	0.00847	0.02021	0.00588	0.00830	0.01823	0.86272	2.12050	10.83200
	5	0.00512	0.00728	0.01719	0.00503	0.00708	0.01572	1.86130	2.84640	9.35410
	6	0.00501	0.00709	0.01567	0.00503	0.00719	0.01751	0.32531	1.36680	10.55300
	7	0.00554	0.00780	0.01708	0.00554	0.00788	0.01864	0.02108	1.03730	8.34110
	8	0.00545	0.00768	0.01687	0.00549	0.00781	0.01869	0.61933	1.67490	9.76940
	9	0.00276	0.00387	0.00848	0.00276	0.00387	0.00845	0.02501	0.00706	0.34299
	10	0.00262	0.00367	0.00803	0.00261	0.00367	0.00799	0.17236	0.19003	0.47858
	11	0.00252	0.00354	0.00773	0.00256	0.00360	0.00784	1.65490	1.63080	1.30930
	12	0.00231	0.00325	0.00710	0.00238	0.00334	0.00726	2.64790	2.62280	2.28350
	13	0.00207	0.00291	0.00632	0.00209	0.00293	0.00636	0.89540	0.86487	0.57730
	14	0.00208	0.00292	0.00632	0.00207	0.00290	0.00626	0.66206	0.68795	0.87798
	15	0.00261	0.00364	0.00771	0.00265	0.00369	0.00782	1.33200	1.35390	1.48640
	16	0.00263	0.00367	0.00776	0.00267	0.00373	0.00791	1.78050	1.80200	1.94050
	17	0.00272	0.00380	0.00807	0.00272	0.00381	0.00810	0.11529	0.13631	0.29095

	18	0.00287	0.00401	0.00853	0.00285	0.00399	0.00850	0.50109	0.47854	0.31770
	19	0.00400	0.00561	0.01204	0.00399	0.00559	0.01200	0.32482	0.32842	0.34299
	20	0.00346	0.00485	0.01040	0.00350	0.00490	0.01052	1.09200	1.09190	1.09170
	21	0.00283	0.00396	0.00849	0.00284	0.00398	0.00852	0.38503	0.38471	0.38339
	22	0.00200	0.00280	0.00600	0.00203	0.00284	0.00609	1.50750	1.50750	1.50750
	23	0.00396	0.00560	0.01256	0.00403	0.00571	0.01294	1.84330	1.90490	2.93760
	24	0.00370	0.00524	0.01186	0.00373	0.00529	0.01219	0.74990	0.95576	2.71470
	25	0.00691	0.00969	0.02097	0.00698	0.00979	0.02129	0.94828	0.97037	1.53640
	26	0.00928	0.01305	0.02861	0.00922	0.01296	0.02832	0.63009	0.70143	1.01530
	27	0.00923	0.01297	0.02843	0.00912	0.01281	0.02799	1.23910	1.30780	1.60050
	28	0.00918	0.01289	0.02815	0.00907	0.01273	0.02772	1.21740	1.29130	1.54590
	29	0.00917	0.01287	0.02804	0.00908	0.01275	0.02770	0.92747	1.00270	1.22130
	30	0.00931	0.01307	0.02840	0.00931	0.01306	0.02833	0.02883	0.05311	0.26575
	31	0.02123	0.02973	0.06363	0.02109	0.02954	0.06344	0.68335	0.62796	0.30000
	32	0.02148	0.03009	0.06457	0.02134	0.02992	0.06441	0.63312	0.57781	0.24650
	33	0.02203	0.03090	0.06658	0.02199	0.03085	0.06669	0.20842	0.15612	0.16392
Capacitor Size (PU)	8	0.00790	0.01111	0.02433	0.00788	0.01113	0.02552	0.31018	0.18169	4.67170
	18	0.00350	0.00489	0.01043	0.00349	0.00489	0.01043	0.08274	0.07395	0.00278
	25	0.01216	0.01703	0.03660	0.01222	0.01713	0.03688	0.54787	0.56426	0.76619
	33	0.02212	0.03102	0.06685	0.02207	0.03097	0.06694	0.23159	0.18023	0.13348

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