FROM EOQ TO JIT WITH STORAGE CONSIDERATION: COORDINATING A TWO LEVEL SUPPLY CHAIN

by

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A thesis

Presented to Ryerson University

in partial fulfillment of the requirements for
the degree of

Master of Applied Science in the Program of
Mechanical Engineering

Toronto, Ontario, Canada, 2004

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ABSTRACT

FROM EOQ TO JIT WITH STORAGE
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A TWO LEVEL SUPPLY CHAIN

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Many organizations are faced with a decision to choose between two inventory systems namely JIT (Just in Time) and EOQ (Economic Order Quantity). This thesis models the cost drivers into the EOQ model and extends it to the JIT scenario. They include cost savings like space, synergy of coordination, and other cost factors like rework and penalty costs. It looks at the total cost of the supply chain with two players and calculates space in terms of storage spaces of equal capacity.

Results showed that considering space in EOQ brought savings to the chain. It has brought down the order quantity closer to, and many times equal to JIT ordering quantities. Coordination in the chain has brought further savings. Moving to JIT (ordering daily supply of demand) from the point, where space is accounted and there is coordination between the two levels, did not require much reduction in ordering costs.

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ACKNOWLEDGMENTS

I would like to thank Dr. M. Y. Jaber for his consistent and valuable input to this thesis. His patience, encouragement and guidance throughout the duration of the thesis were substantial. His comments and suggestions were of great importance and have helped me in the completion of the thesis. I would further like to sincerely appreciate Dr. Jaber for adjusting his schedule to meet with me out of the way in order to help me complete this thesis.

I would also like to extend my gratitude to Dr. Greg Kawall, the Chair of the examining committee and its members, Dr. M. Y. Jaber, Dr. Ahmed El-Bouri and Dr. Saeed Zolfaghari for their valuable suggestion to improve the thesis.

I would also thank my parents and my in-laws for the moral support, encouragement and valuable prayers for me during this time. The members of my church, "Zion Gospel Assembly", were particularly helpful when I needed their support and prayers. Many helped by taking care of my son Jonathan when my wife and I could not juggle our schedules. Most of all I would like to thank my Lord and Savior Jesus Christ who helped me in every situation and worked it for my good.

Finally, this thesis would not have come to completion but for the consistent and constant encouragement and prayers of my wife and dear friend Serene John and the delight that my son Jonathan brings me. Thank you.

Chapter 1

INTRODUCTION AND LITERATURE REVIEW

1.1 Research Background

In a competitive environment, corporations face many challenges to survive. They are constantly looking for ways to bring value to their share holders. Analysts judge the success of an organization through various measures like earnings per share, net earnings before interest and tax, and a host of ratios like the acid test or quick ratio, debt to equity, current ratio and the likes. Many of these ratios pertain to inventory like inventory turnover, receivables ratio, days in inventory and so on. Managing inventory and the associated costs properly can produce significant and positive impact on these ratios. When it comes to inventory, a few important decisions need to be made like, how much needs to be ordered and how often it needs to be ordered. A delicate balance needs to be struck between the cost of carrying this inventory and the cost of ordering or setup (in case of manufacturing). This quantity is referred to as the Economic Order Quantity (EOQ) or the Economic Manufacturing Quantity (EMQ). For the sake of consistency in this thesis, the term Economic Order Quantity will used.

1.2 Overview of Economic Order Quantity

The EOQ model in its basic form has essentially two components, namely, the order costs and the holding costs. The order cost is often referred to as ordering cost when the context is that of retailer or buyer. This component of the total cost is the cost incurred for a particular order or lot (in case of manufacturing). This is fixed and is incurred for every order, irrespective of the quantity ordered. Thus, if the quantity ordered is less, more orders need to be placed to fill the demand. This causes the order cost to increase. In order to reduce the impact of setup on the cost, large quantities are ordered on each order and thus the number of times it is ordered per year is reduced. However, this causes another problem, since large quantities increase the holding cost.

The holding cost is cost associated with keeping inventory. This has many components to it like insurance on inventory, property tax, obsolescence, spoilage, shrinkage, utilities, interest, handling, etc. This component increases in proportion with the quantity stored. This component works against the drive to reduce the ordering frequency and order large quantities.

Numerous research papers have been published ever since the concept of Economic Order Quantity (EOQ) had been introduced by Ford Harris in 1915. The problem, however, is that EOQ in its simplest form leads to a lot of miscalculations on the optimum lot size problem (Jones 1991). Jones (1991) argued that if EOQ has properly accounted for the costs it would lead to Just In Time (JIT) lot sizes.

1.3 Overview of Just In Time (JIT)

The recurring theme for Just In Time is to eliminate waste and improve flow of materials (Fuller 1995). This means that the adoption of JIT requires cutting down or eliminating inventory that is not needed. Orders are placed on an immediate need basis and enough for a day or till next delivery. In a manufacturing environment this means that orders are processed on a pull basis instead of push.

To make it worth while to order or manufacture in small quantities or batches, the setup costs have to be brought down or holding costs have to be higher. The following two sections discuss these in detail.

1.2.1 Setup/ Ordering Costs

One of the key components to moving the basic EOQ model towards JIT is to reduce the setup cost. This can drive the quantity down as well as bring the total cost down besides bringing the benefits of operating in JIT. The JIT literature identifies a variety of ways to bring down the setup cost, namely, the need for inspection or setup of inspection station for incoming stock is eliminated, no annual re-bidding or re-tendering, paper work reduced and a more informal ordering process, long term contracts established, etc.

One of the suggested ways in the literature (Pan & Liao 1989) to reduce the order cost component of the cost is to increase deliveries for a particular order. Thus, the cost is split over a number of deliveries. Pan & Liao (1989) argue that the purchase cost does not necessarily increase when going to JIT, since the total order is the same but it is just that the frequency of deliveries increases. One of the issues that are ignored in this assumption is that costs for such deliveries have to be accounted somewhere. If the vendor bears this cost it would be passed on to the retailer as cost increase. The other problem is based on the reaction of the vendor to the decision of the retailer. There are two scenarios here: one, the vendor also decides to move to

JIT and two, the vendor decides to continue to produce in EOQ. In the former scenario, costs are incurred from the purchase of raw materials (Fazel et al 1998), increased inspection to deliver zero defects to the retailer and so on. In the latter scenario, the vendor has to hold the inventory till it is shipped. This situation does not help in zero defect purchases. To ensure zero defects, the vendor has to increase the costs associated with inspection. In either case scenario, there would be a cost increase that would be transmitted to the retailer in terms of price increase.

Ramasesh (1990) also approaches the problem in terms of blanket orders with multiple shipments like Pan & Liao (1989). However, Ramasesh (1990) includes the cost of multiple shipments in the model, namely, the freight cost associated. In a way this is a cost that needs to be considered. But it is often the case that the purchase cost includes the cost of landing the product at the premises. This supports the argument that the price increases when moving towards JIT. Another factor that needs to be considered is that many organizations that purchase from a variety of sources optimize on their own transportation network by combining different loads from vendors in the same locality. This helps in bringing the shipment cost considerably lower. In such circumstances, the cost associated would be calculated in terms of the volume of the product rather than the number of shipments.

Thus it is possible to bring order cost down by increasing the number of shipments per order, but this comes at a cost in terms of increased price for the product.

1.2.2 Holding Cost

The other aspect that needs to be considered in moving towards JIT is the holding cost. Billington (2003) considered the total cost reduction in lot sizing by reducing the holding cost. The author argued that it is possible to reduce the holding cost by investing in automating the factory thereby reducing handling costs. Further, capital investments can reduce the cost of obsolescence and spoilage. Billington (2003) found that reducing holding cost does not necessarily bring the total cost down. The investment needed to reduce the holding cost should be considered.

However, Billington (2003) he did not consider the impact of cost of storage for these additional units. Reducing the holding cost leads to increase in lot size which in turn would impact the storage cost. It can be argued that cost saved in reducing the holding cost is offset by the cost incurred in increased storage space. This, however, is very dependant on the size of product. Small but expensive products can bring savings if holding cost is reduced. It is also worth mentioning that the handling costs of small products would be relatively small compared to larger ones so also the savings that can be obtained.

Further, in this thesis, the cost associated with obsolescence, spoilage or returns has been considered separately. This is due to the fact that these items, although they constitute a small percentage, have other costs like penalty costs, opportunity costs and storage costs associated with them. Thus it is essential for them to be considered separately.

On the other hand, some authors (e.g. Jones 1991) have argued that the holding costs used in the standard EOQ equation do not reflect the actual costs incurred for holding the inventory. Costs like facilities leasing costs, depreciation, interest, taxes, insurance, utilities, handling costs, inspection costs, rework, scrap, and administration costs are not considered in the holding cost function. Considering these factors as well as reducing the set up costs can reconcile JIT with EOQ.

While it can be accepted that these costs are not included in the holding costs, it can also be argued that these are often not a direct linear function of the lot size quantity and therefore need to be separated from the holding cost function. Functions like facilities costs and rework have been accounted for separately in this thesis. Including these factors in the lot sizing equation has brought the lot size closer to JIT.

The other factors that are not a linear function are space cost and rework cost. Rework is often part of the EOQ process. When JIT is implemented, the approach is to eliminate rework. This cost is often neglected in the analysis when considering the cost of inventory. The model proposed considers the impact of this quantity. The following section looks in to the literature specifically pertaining to space.

1.4 EOQ/JIT Model Consideration of Inventory

Many authors have discussed and modeled space in the classical EOQ equation. In this section, a critical analysis on these papers is done followed by a detailed look into a few articles.

Among the authors that worked on space is Aghazadeh (2001), who illustrated that the best course for a retailer is to opt for a quantity discount model that will result in the lowest cost. The author argues that JIT philosophy is not applicable in a retailer situation. The example of Wal Mart is quoted where they use distribution centers to store inventory rather than at the individual stores. The holding cost of the distribution stores is presumably lower than that of the individual stores. It is also argued that holding inventory on a JIT basis could result in stock outs which would prove very expensive for a retailer. Through the illustration, the author concludes that taking advantage of the price quantity discount is much cheaper that ordering in JIT quantities.

The model that was used, however, has one major drawback. It does not consider space as a cost factor. The holding cost only includes the carrying cost. It does not consider the cost of warehousing these items. The author considers coffee filter as an example which admittedly is an item with low volume, but retailers do not deal with just coffee filters. Many of the products are of high volume and sometimes low profit margin. A typical example of this will be ice salt or water conditioning salts that cost very low but has a high volume. Space needs to be considered in determining the EOQ for these situations.

Further, storing these filters for a long period of time (one year in the example quoted) could lead to spoilage or shrinkage or even obsolescence. This need not be the case with coffee filters if packaged properly but many products cannot be stored for this long.

A closer look into the work of some of the authors is done in the following sections.

1.4.1 Joshi

In his article Joshi (1990) discusses the case where bulky, inexpensive and low risk items (BIL) are incorporated in the EOQ equation. The author contends that the holding costs look at all items in the same manner irrespective of the volume the item occupies. It is mentioned that the bulky items considered do not require bins or can be stacked in single or multi-story storage rack systems. They are stored in pallets and sometimes can be double stacked.

If A is the fixed component of the order cost, D is the annual demand, Q is the order quantity, h is the holding cost and SC is the annual space cost, the total cost is given by

$$TC = D\frac{A}{Q} + \frac{Qh}{2} + SC \qquad (1.1)$$

The storage cost component in this situation is given as

$$SC = p \times k \times N$$

Where $N = \dfrac{Q}{m \times n}$

n is the number of units that fit on a pallet, m is the number of pallets that can be stacked one over the other, p is the area occupied by one pallet and k is the storage cost per square foot.

The author considers two scenarios. One is where the space is fixed assignment per product and the other is when the assignment is dynamic. Fixed assignment means that space for Q units is allocated for this product and not utilized for any other products. For the dynamic assignment the space utilization is far better. The space that is available is allocated for other products. This is what is done in many warehouses where the space allocation is computerized. The author argues that due to the efficient utilization of space the annual requirement would be that for $\frac{Q}{2}$ units. The space cost therefore in this scenario would be denoted as

$$SC = p \times k \times N$$

$$SC = (p \times k) \times \left(\frac{Q}{2 \times m \times n} \right)$$

Thus the EOQ for the two scenarios are given as

$$\begin{matrix} EOQ\ Space \\ (Fixed) \end{matrix} = \sqrt{\frac{2AD}{h + \frac{2pk}{mn}}} \qquad (1.2)$$

$$\begin{matrix} EOQ\ Space \\ (Dynamic) \end{matrix} = \sqrt{\frac{2AD}{h + \frac{pk}{mn}}} \qquad (1.3)$$

The article is limited in its application that the author considers only floor space where pallets are placed. This poses a problem since most large warehouses rely on bins and multi-story storage rack system for most of their products. Further space is a very valuable commodity that irrespective of the kind of the product space should be considered in the calculation of the EOQ.

The other question that this article raises is the consideration of space as a continuous function. It is a common practice that inventory is not mixed in bins or locations. If the EOQ is calculated as a little over a bin, the space actually occupied would be that of two bins. Thus the term $N = \frac{Q}{m \times n}$ should be an integer and should in actuality read as $N = Int\left(\frac{Q}{m \times n} + 1\right)$. In the example quoted by the author, the maximum units that can be accommodated in a pallet (double stacked) is 40 units. But if the EOQ results in recommending 41 units then the space occupied would be that of 80 units. The error is higher for fixed space allocation than for dynamic. Because of the nature of the bins it would be beneficial to use equation 1.2 and 1.3 to get an approximation and then iteratively calculating the total cost for the range of $\pm m \times n$. In the calculation the author did not utilize the model but iteratively calculated all the multiples of the space capacity to obtain the optimum solution. Adopting this method further neglects the impact of holding cost.

1.4.2 Rao & Bahari-Kashani

Rao & Bahari-Kashani (1990) took a similar approach to accounting for space in their model. In their model they considered the space function to be given as

$$SC = \sum_{j=1}^{P} S_j I_j \qquad (1.4)$$

Where S_j is the fixed cost for storage of the j-th unit per year, P is the number of available storage units and

$$I_j = \begin{cases} 1 & \text{if } Q > \sum_{i=1}^{j} C_{i-1} \quad for \quad j = 1 \quad to \quad P, \\ 0 & Otherwise, \end{cases} \qquad (1.5)$$

Where C_i is the capacity of storage i. The consideration of space in this way would flexible to include multiple storage locations with various size and corresponding costs.

The total cost is given by

$$C(Q) = \frac{AD}{Q} + h\frac{Q}{2} + SC \qquad (1.6)$$

The authors consider storage space to be fixed and hence the first derivative eliminates *SC* (Storage Cost). Thus EOQ

$$Q = \sqrt{\frac{2AD}{h}} \qquad (1.7)$$

The author concludes that the optimum cost can be obtained by computing the maximum capacity of fewer than the number of storage areas needed for Q to obtain the optimum.

Considering space as a constant in calculating the EOQ brings a lot of inaccuracies. The problem being that storage cost is directly dependant on the value of Q although it is a stepped function. The author also suggests that the maximum capacity of the storage areas be considered and the total cost computed and compared for the optimum. Considering only the maximum capacity of the storage areas negates the influence of the holding cost and ordering cost if any on the EOQ. The optimum quantity could be less than a full storage capacity.

1.4.3 Fazel

Fazel (1997) and Fazel et al (1998) compared the cost of EOQ to that of JIT. Fazel (1997) considered EOQ with no price quantity discount and Fazel et al (1998) with price quantity discount. The price quantity discount considered is an all unit quantity discount. The attempt in both the papers was to model the costs to aid decision makers regarding the move to JIT form EOQ.

Since both the papers have a lot of similarities, the latter article will be the subject of study.

In this paper the total cost for EOQ is given as

$$TC_E = \frac{AD}{Q} + \frac{Qh}{2} + (c_E^0 - \pi_E Q)D \text{ for } \quad Q \le Q_{max} \qquad (1.8)$$

$$TC_E = \frac{AD}{Q} + \frac{Qh}{2} + (c_E^{min})D \text{ for } \quad Q \ge Q_{max} \qquad (1.9)$$

Where c_E is the cost of one unit of product when ordering in EOQ, Q_{max} is the order quantity beyond which there would not be any further price quantity discount and π_E is the quantity discount rate. The price quantity discount is an all units quantity discount that is considered, thus the price changes as follows with the increase in Q as follows:

$$c_E = c_E^0 - \pi_E Q \text{ for } \quad Q \le Q_{max}$$

$$c_E = c_E^{min} \text{ for } \quad Q \ge Q_{max}$$

Where, c_E^0 is the price of the product for $Q = 0$, and c_E^{min} is the fixed price when the quantity exceeds a certain level (Q_{max}).The optimum quantity for this would be the minimum total cost for either

$$Q^* = \sqrt{\frac{2AD}{h - 2\pi_E D}} \text{ for } \quad Q^* \le Q_{max} \qquad (1.10)$$

or

$$Q^{**} = \sqrt{\frac{2AD}{h}} \text{ for } \quad Q^{**} > Q_{max} \qquad (1.11)$$ or that of Q_{max}.

The author assumes in the article that the total cost for operating in JIT is a simple form of the product of JIT price and demand as follows:

$$TC_J = c_J D \qquad (1.12)$$

Here c_J is the JIT cost of the product. It is argued that the supplier would raise cost due to the frequent deliveries inspection and other costs involved in moving to JIT or JIT deliveries.

The paper then narrows down the application of the model to where $Q^* \leq Q_{max}$ and derives the cost difference for JIT and EOQ as

$$Z = \frac{AD}{Q} + \frac{Qh}{2} + \left(c_E^0 - \pi_E Q\right)D - c_J D \qquad (1.13)$$

Fazel et al (1998) concluded that for lower demands it would be cheaper to use JIT instead of EOQ and for higher demands it would be worthwhile to use EOQ. The indifference point in demand was also calculated. This is the demand at which it does not make any difference whether the EOQ or JIT is adopted. This means that the value of Z is zero at this point.

The model described in both the articles has some drawbacks. Like Schniederjans & Cao (2000) pointed out, it ignores the space saved by the fact that the batch size is very small. The space savings should be considered as a major cost savings. This is especially the case when the products involved are voluminous. This point is further elaborated on the upcoming sections.

The other assumption that the author considers is that there is no holding costs and ordering (setup) cost. It is argued that these costs are largely reduced and thus can be neglected. This is flawed since ordering in JIT is many a times characterized by long term contracts. To establish long term contracts more resources are spent and thus this cost is high, although it is spread over many deliveries. Thus to assume that there is no ordering cost would lead to erroneous decisions. The other aspect that is ignored is that of the holding cost. Here again since the quantities are small the holding cost is assumed as negligible. Although, there is some truth to this assumption, the error compounds when a huge retailers warehouse containing 40,000 different products is considered. The holding cost can also be high when the expensive items are considered like refrigerators, washing machines, dryers etc.

One of the points that the model neglects is that, the holding cost is constant, irrespective of the price of the product. Since the holding cost includes the cost of capital among other things, presumably the cost should vary with the increase in price. Granted the difference ($0.3/dollar discount/unit/year) is very minor in the example that the author illustrated. If the order quantity is 2500 units the holding cost would be change by negative $375. If the item is more expensive or the demand is higher this could make significant impact.

1.4.4 Schniederjans & Cao

Cao & Schniederjans (2004) introduced the concept of considering the savings in space in comparing the JIT model with EOQ and price discounts. This was the latest in a series of articles by the authors, where model by Fazel (1997) and Fazel et al. (1998) was improved upon with the consideration of space savings.

The nomenclature used here is in continuation to that of by Fazel (1997) and Fazel et al. (1998)'s model. Cao & Schniederjans (2004) extended to total cost of JIT to include

$$TC_J = c_J D + \frac{mAD}{Q} - CF \qquad (1.14)$$

Where $m = \frac{Q}{q}\ for \quad m \geq 1$

q is the order quantity under JIT which is less than Q. C is the annual cost of a square foot of facility and F is the square feet saved by initially adopting a JIT system.

Thus the difference in cost is given by:

$$Z = \frac{AD}{Q} - \frac{mAD}{Q} + \frac{Qh}{2} + (c_E - c_J)D + CF \qquad (1.15)$$

or by substituting the value of Q from $Q = \sqrt{\frac{2AD}{h}}$ see working in Appendix 1.1

$$Z = \sqrt{2ADh}\left(1 - \frac{m}{2}\right) + (c_E - c_J)D + CF \qquad (1.16)$$

The authors suggested that the TC_J could be presented without the holding costs. This is never true even when $q = u$ where u is the usage rate ($D = u$ * demand periods in a year). That is, if the production facility shifts away from EOQ to JIT, the recommended lot size is $q = u$, which is producing as demanded per demand period (say 1 day). Even though $q = u$, inventory may be negligible, but surely not zero. This is also the case with cross dock. There is a point where the products enter and /or leave the warehouse. There are handling charges associated with the volume and facilities costs while in the facility. Thus although the holding charges are small due to the low units it can never be zero.

Another aspect is that of the annual facility space reduction termed as *CxF*. The authors suggest the term as savings to be deducted from Fazel's equation (1.12) for the total cost for JIT. The space requirements for any given product in either an EOQ or JIT system would be the lot size quantity, which in turn determines the space requirements for finished items, work-in-progress, raw material, other components, etc. If *CxF* represents the space requirement when using EOQ, then savings when adopting JIT policy would be

$$C \times F = C \times S\left(1 - \frac{q}{Q}\right) \qquad (1.17)$$

$F = S\left(1 - \frac{q}{Q}\right)$ where S is the total space available to store Q. If $q = Q$, then annual facility space reduction is zero. The authors have not considered the magnitude of facility reduction which could directly impact the decision on the value of q.

The cost indifference function is another aspect that would be impractical concerning the papers by Cao & Schniederjans (2004). It suggests that for a particular demand point the EOQ is more cost effective than JIT. Going along with the author's argument, which suggests that if demand in the next period is such that JIT is more cost effective then shift to JIT now, otherwise retain EOQ system. This will also suggest that if you have a JIT system implemented, then shift to EOQ system if the demand in the coming period favors EOQ to JIT. In a dynamic environment, it would then suggest to follow a flip-flop policy! Firms can only move towards a JIT system if continuous improvement programs are implemented, an issue that authors ignored.

As a summary, it can be said that all the authors considered the JIT in isolation. Although it was mentioned that there is a need of cooperation with the vendors this was not modeled. In the next section, papers that address the issue of cooperation with the other members of the chain are discussed. This is important when modeling the move to JIT since it is very difficult to implement JIT without cooperation with the vendors.

1.5 Coordination in Supply chain

As mentioned earlier, while many papers discussed earlier considered JIT and various aspects therein, the problem was approached with the picture of only the organization concerned. It did not model partnership with suppliers, although it was discussed and agreed upon as important to the success of JIT. It has been found that larger firms could exercise more power in implementing JIT than smaller firms that found it hard to obtain cooperation from its suppliers (Munson et al 1999).

The impact of coordination within the supply chain was discussed at length by Munson & Rosenblatt (2001). The authors modeled a three level supply chain namely a single supplier, single manufacturer and single retailer and the impact of savings that can be obtained if there is coordination between the three parties.

Under circumstances where there is no coordination, the manufacturer sets their orders quantity based on the order (EOQ) of the retailer. This would be a lot size multiplier of the order of the retailer. The supplier would in turn base their order quantity based on the quantity ordered by the manufacturer. This would be a lot size multiplier of the manufacturers order quantity.

In a situation where there is coordination the leader in the supply chain would initiate the change by enticing the retailer to increase their order quantity by offering quantity discounts. This would prompt the retailer to increase the order quantity to a certain point that would bring savings to the manufacturer. A similar approach would be done for the supplier.

Coordination would not necessarily bring savings to every party concerned, however, Munson & Rosenblatt (2001) argues that when considering the chain as a whole there would be savings and these savings can be shared through quantity discounts, price increases and so on. The savings could be concentrated at the manufacturer's or the supplier's or the retailer's side. The challenge is to ensure that the savings obtained at one point in the chain is distributed evenly or fairly among the players concerned. This responsibility would depend on the leader of the chain. If the leader exercises a great influence in the chain it need not share the savings and can force the others in the chain to take the losses.

This issue was addressed by Munson et al (1999), where the authors examined the dynamics of the industry concerning the aspect of coordination. Their article was a practical outlook on the industry and the dynamics of coordination. The authors argued that there are definite advantages in exploiting coordination but if the power of the supply chain leader is used in an abusive way to maximize profits at the leader's end, it could lead to negative consequences like boycott or legal implications. In the short term, this approach could prove profitable to the chain leader but in the long run it weakens the chain and forces the abused to take retaliatory measures.

This leads to another question namely, if the members of the chain do decide to cooperate how will the resulting savings be distributed. The following section takes a brief survey of the literature in this scenario.

1.6 Profit Sharing

The cooperation between the members of the supply chain to improve efficiency and costs raises another issue namely, how the savings generated would be distributed among the players. The brash approach for a leader would be to maximize and hold on to all the savings obtained being the leader. This would lead to a weakened chain as discussed earlier and long term effects.

One of the approaches would be adopting the model developed by Abad & Jaggi (2003). In this model the dynamics of setting price in a price sensitive environment through the length of credit was highlighted. The influence of price affects the demand and thereby the order quantity. They examine the difference in cooperative and non-cooperative scenarios. The model balances the choice between offering a small unit price and no credit against high unit price and some trade credit.

Although this aspect is not discussed at length in this thesis, it is acknowledged that this issue is imperative to coordinating in a supply chain. Besides Abad & Jaggi's (2003) model other ways of profit sharing can be based on the investment ratio of the players or a simplistic 50% split through quantity discounts and trade credit and so on.

1.7 Objectives and Scope of Thesis

In an environment of continuous improvement and increased competitiveness many organizations are adopting JIT. The benefits of JIT are often understated in many models due to the fact that the synergy of 1)space savings, 2)coordination, 3)reduction in defects and 4)reduced setups are not modeled. Through the past sections in the literature review the object was to highlight the research done so far on the EOQ model. The model that is developed here, unlike others that studied the impact of these factors independently, illustrates the impact of each of these factors together on the total cost of a two level supply chain.

1.8 Thesis Layout

Chapter 2 of the thesis contains the problem description and the development of the mathematical model. The solution procedure is dealt with in a separate section. Chapter 3 illustrates the solution through an example. The behavior of the model is then studied through the generation of random scenarios and the results computed. The results are analyzed to see the overall impact coordination has on the total cost. This is followed, in chapter 4, by the conclusion of the analysis and suggestions for further research. The appendices that follow includes the results in a tabulated form, and the step in the working of various equations.

Chapter 2

PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

2.1 Problem Objectives

The problem that is being addressed in this thesis has five aspects that can be described in a figure 2.1. The matrix describes a two level supply chain and the relationship between the two levels. The two can be representative of a supplier and retailer or a manufacturer and retailer or a supplier and a manufacturer. In this thesis, it is assumed that there are only two parties involved, i.e, there is only one player in the upper level that supplies to the lower level that consists of only one player. This kind of relationship is often seen in the marketplace where the retailer requires from the manufacturer exclusive rights to a product.

Under the initial circumstances, the two parties make order quantity decisions independent of each other. The lower level (say retailer) decides independently what the optimum order quantity is based on their parameters. The upper level (say supplier) then bases their optimum manufacturing quantity based on a lot size multiplier λ of the order quantity of the retailer. This situation is described in the first box (No Space[1], No Coordination).

Moving towards the right would be when the two levels of the chain decide to cooperate and decide on an optimum order quantity. This could mean that the lower level moves away from the local optimum but results in a greater savings for the

[1] In this thesis, the term "No Space" is used to mean that the cost for space is not considered as a function of Q (order quantity). It is factored as a constant in the mathematical model.

chain as a whole. This situation is described in the second box (No Space, Coordination). The retailer can be compensated by the supplier for the savings foregone in terms of discounts or lump sum compensation. The distribution of this savings, however, is not dealt within detail in this thesis.

Next, the impact of space on these two scenarios is examined. Space is often an expensive commodity and the model integrates the impact of space and bin capacity to develop the optimum order quantity and lot size multiplier.

Moving to JIT involves reducing the order quantity and processing orders on a lot for lot basis. This means that the supplier and retailer do not have a lot of inventory. It would also be fair to say that the supplier has a lot size multiplier of one. It is also worth mentioning here that it is not possible to have JIT without proper coordination between the two players. Thus, this area of the matrix is indicated by the shade.

Another consideration that has been included in the model is that when adopting EOQ, there maybe a lot of defects in a shipment. This is resource lost in the chain and needs to be accounted for in the model. Along with the defects brings the need to inspect the products on arrival. This causes the retailer to incur the cost of screening the shipment on arrival. These costs are eliminated when considering JIT since the approach is zero defects.

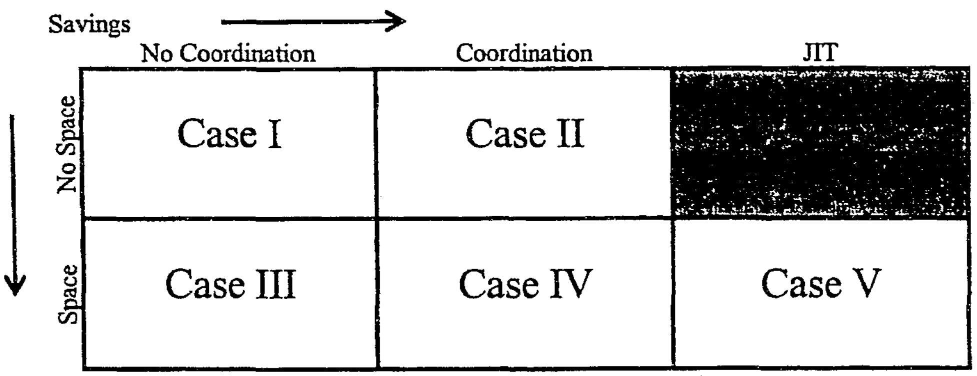

Figure 2.1: Two Level supply chain Matrix

The model developed below explores each scenario in detail and elucidates the impact that each relationship indicated in the matrix has on the overall cost of a two level supply chain. Thus players in a two-level supply chain can evaluate the options and corresponding savings they can have in their decision to move towards JIT.

In this chapter, the nomenclature used is first defined followed by the section on the mathematical model. The section on the mathematical model first establishes the assumptions and then models 1) no coordination no space 2) coordination no space 3) no coordination space 4) coordination space and finally 5) JIT. This is followed by the stepwise description of the optimal solution procedure.

2.2 Nomenclature

2.2.1 Input parameters:

i=r,s subscript for retailer or supplier

A_i = the fixed component of the order cost ($).

a = The variable component of the order cost. It is assumed that when operating according to the EOQ policy, lots received need to be screened for quality. The variable cost is the screening cost per unit ($/unit).

β= the fraction of units not conforming to quality that cannot be sold and thus needs to be returned. ($0 < \beta < 1$).

ω= the cost of reworking ($/unit)

c_i= the unit purchasing cost for EOQ purchasing ($/unit).

h_i = the unit holding cost when operating according to EOQ policy ($/unit/yr).

N_i = total number of storage areas, where N=1, 2, ... and each storage area has a maximum capacity of V.

R_i = the annual cost per a storage area ($/storage area).

v = opportunity cost per unit for not being able to sell the defective items ($/unit)

p = penalty charge per returned unit to the supplier ($/unit).

D = annual demand rate (Units/yr)

n=Number of working days per year

2.2.2 Decision Variables:

Q_E = order quantity when operating under EOQ policy (units)

Q_J = order quantity when operating under JIT policy (units)

λ = lot size multiplier (1,2,3,.....)

2.3 Mathematical Model

In this model, each scenario is formulated. The model considers the impact of defects, returns and penalties for returned goods. This is incorporated into the model to see the impact it would have when approaching JIT where it is assumed that defects are zero (i.e., total quality., Fuller 1995).

2.3.1 Assumptions

The assumptions associated with the classical Economic Order Quantity Model are held in this model, namely:

1) Demand is known, constant and independent.

2) Lead time is known and constant.

3) Receipt of inventory is instant and complete

4) There are no quantity discounts.

5) There is no shelf life for the product

6) Equipment capacity is infinite

7) Unlimited storage capacity is available

Following the assumptions mentioned above the first case scenario is considered.

2.3.2 No Coordination, No Space.

When a retailer is operating on EOQ, it is assumed that not all items conform to quality and a fraction (β) of quantity (Q) is assumed to be defective. These defective items are either sold at a discounted price or returned to the supplier at the end of each cycle. In this model, it is assumed that the retailer returns the defective products to the supplier and charges the supplier a penalty p per returned item. The penalty cost would include the holding cost of the defective items, the cost of screening and an opportunity cost for not having these items of good quality in the first place.

2.3.2.a Retailer's Cost

The figure 2.2 below illustrates the inventory cycle of the retailer following EOQ. The maximum and minimum inventory levels in a cycle are Q and βQ. It is assumed that the retailer holds on to the defective items till the next shipment arrives. Once the new shipment of Q units arrive, the defective items are loaded back on the trailer and returned to the supplier. Again, the Q units that are supplied by the supplier has βQ units that are defective and are held by the retailer till the next shipment. Thus, the cycle length T is given by:

$$T = \frac{Q(1-\beta)}{D}$$

With a fraction defective, the model can be illustrated as follows

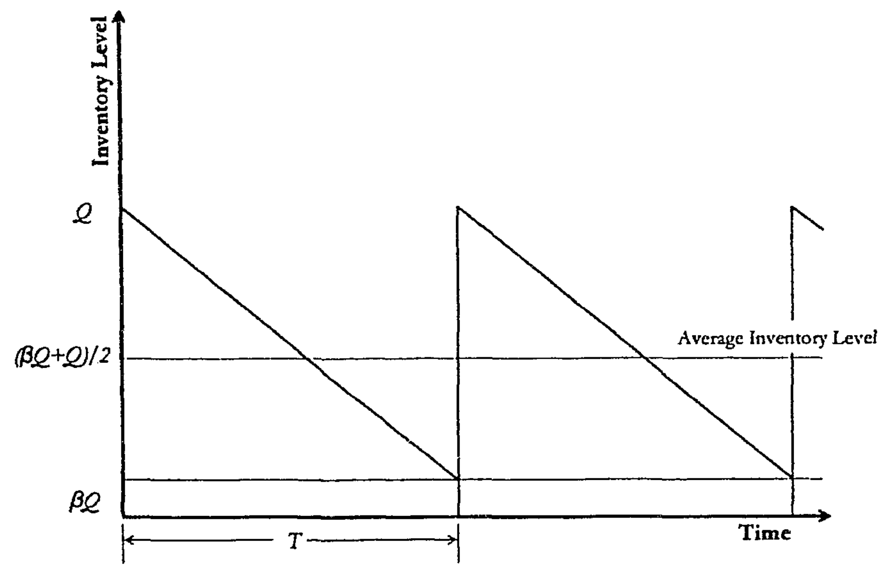

Figure 2.2 : Inventory cycle of the retailer

The maximum and minimum inventory levels in a cycle are Q and βQ.

The per lot cost for the retailer, $LC_r(Q)$, is the sum of all the costs in a given cycle, which are:

1. Procurement cost, $PC_r(Q)$

   $PC_r(Q)=A_r + aQ + c_rQ$ (2.1.1a)

2. Holding Cost, $HC_r(Q)$

   $HC_r(Q)$=Average Inventory Level x T

   $$HC_r(Q) = h_r \times \frac{Q(1+\beta)}{2} \times \frac{Q(1-\beta)}{D} = h_r \times \frac{Q^2(1-\beta^2)}{2D} \qquad (2.1.1b)$$

3. Storage Cost, SC_r

   $SC_r=K_r$ (2.1.1c)

   a constant such that it does not impact the decision variable. In this scenario, it is assumed that there is plenty of storage. However, for the sake of comparison of the costs of the different scenarios in a like manner, the storage cost has been given a value that is the greater, required for Q when there is coordination and no coordination. The assumption here is that space is not included in the decision variable, however that cost is valid and accounted for as overheads or otherwise. It is argued here that this function is significant and produces an impact on overall cost.

4. Penalty Refund, $PnC_r(Q)$

   $$PnC_r(Q) = h_r\beta\frac{Q^2(1-\beta)}{D} + (c_r + v + a)\beta Q \qquad (2.1.1d)$$

Therefore, the per lot cost for the retailer,

$LC_r(Q)$= (2.1.1a)+(2.1.1b)+(2.1.1c)-(2.1.1d)

$$LC_r(Q) = A_r + aQ + c_rQ + h_r \frac{Q^2(1-\beta^2)}{2D} + K_r - h_r\beta \frac{Q^2(1-\beta)}{D} - (c_r + v + a)\beta Q \quad (2.1.2)$$

The retailer's annual cost,

$$C_r(Q) = \frac{LC_r(Q)}{T}$$

$$C_r(Q) = \frac{A_rD}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + SC_r - h_r\beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)} \quad (2.1.3)$$

The first derivative of the above equation is given as:

$$\frac{dC_r(Q)}{dQ} = -\frac{A_rD}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta)$$

SC_r is considered a constant. It is assumed that space is available and not utilized for any other purposes.

The second derivative is as follows:

$$\frac{d^2C_r(Q)}{dQ^2} = \frac{2A_rD}{Q^3(1-\beta)} > 0, \forall Q \geq 1$$

Setting the first derivative equal to zero and solving for Q

$$Q_E = \sqrt{\frac{2A_rD}{h_r(1-\beta)^2}}$$ (2.1.4) working in Appendix 1.2

or

$$Q_E = \sqrt{\frac{2A_rD_{adj}}{h_r}} \quad (2.1.5)$$

where $D_{adj} = \frac{D}{(1-\beta)^2}$ is the adjusted demand.

2.3.2.b Supplier's Cost

The supplier delivers Q units every T units of time. The decision that the supplier has to make is on the lot size multiplier λ. The lot size multiplier in this thesis is considered as an integer, since the assumption is that the supplier provides this product only to this retailer. These can be exclusive products provided to a particular retailer like retail brands. This research adopts the assumption used in Munson & Rosenblatt (2001) in this matter.

Figure 2.3 illustrates the inventory cycle of the supplier. The supplier's inventory follows a stepped model. They manufacture or purchase in lot sizes of λQ and deplete inventory in steps of Q. The dotted line indicates the actual inventory that the supplier is holding. This includes the defects that are to be reworked or being reworked that are returned to the supplier. Each shipment returns with a load of βQ defective units. It is assumed here that these items can be turned around before the next shipment is delivered.

The following figure illustrates the Inventory level of the supplier

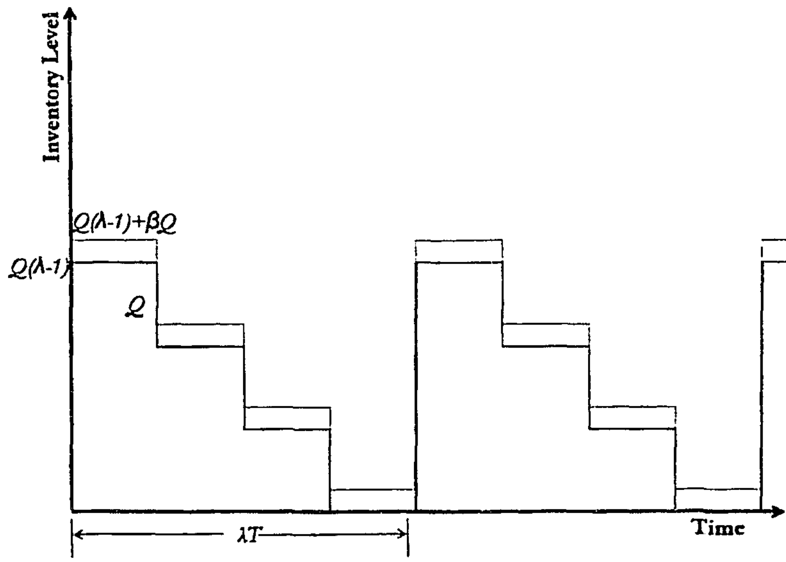

Figure 2.3 : Inventory cycle of the supplier

The per lot cost for the supplier, $LC_s(\lambda, Q)$, is the sum of the following costs in a given cycle, which are:

1. Procurement cost, $PC_s(\lambda, Q)$

   $PC_s(\lambda, Q) = A_s + c_s\lambda Q$ (2.1.6a)

2. Holding Cost, $HC_s(\lambda, Q)$

$$HC_s(\lambda,Q) = h_s\left\{\sum_{i=1}^{\lambda}(i-1)Q\right\}\frac{Q(1-\beta)}{D} + h'_s\lambda Q\beta$$
$$= \frac{h_s}{2D}\lambda(\lambda-1)(1-\beta)Q^2 + h'_s\lambda Q\beta \qquad (2.1.6b)$$

   The second term $h'_s\lambda Q\beta$ is representative of the carrying cost of the returned / defective items, where $h'_s = \frac{h_s}{c_s}(c_s + a + v + \omega)$.

3. Storage Cost, SC_s

   $SC_s = K_s$ (2.1.6c)

   a constant. The storage cost for the supplier in this scenario follows the same pattern as that of the retailer. It is assumed that there is plenty of storage available and this cost will not impact the decision variable. Again, for the sake of comparison of the costs of the different scenarios in a like manner, the storage cost has been given a value that is the greater, required for λQ, when there is coordination and no coordination.

4. Rework Cost, $RC_s(\lambda, Q)$

   $RC_s(\lambda,Q) = \omega\lambda\beta Q$ (2.1.6d)

5. Penalty Cost $PnC_s(Q)$ charged by the retailer. This would be a cost to the supplier for supplying defective items to the retailer.

$$PnC_s(Q) = PnC_r(Q) = h_r \beta \frac{Q^2(1-\beta)}{D} + (c_r + v + a)\beta Q \qquad (2.1.6e)$$

Therefore, the per lot cost for the supplier,

$LC_s(\lambda, Q)$= (2.1.6a)+(2.1.6b)+(2.1.6c)+(2.1.6d)+(2.1.6e)

The Supplier's annual cost,

$$C_s(\lambda, Q) = \frac{LC_s(\lambda, Q)}{\lambda T}$$

Where $\lambda T = \dfrac{\lambda Q(1-\beta)}{D}$

Thus

$$C_s(\lambda, Q) = \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + SC_s + \frac{\omega \beta D}{(1-\beta)} + h_r \beta Q + (c_r + v + a)\frac{\beta D}{(1-\beta)} \qquad (2.1.7)$$

The value of Q_E is substituted in the above equation and the optimal value of λ is computed such that

$$C_s(\lambda-1, Q_E) > C_s(\lambda, Q_E) < C_s(\lambda+1, Q_E) \ \forall \ \lambda \geq 2 \qquad (2.1.8)$$

and

$$C_s(\lambda, Q_E) < C_s(\lambda+1, Q_E) \text{ for } \lambda=1 \qquad (2.1.9)$$

Thus, in this scenario the supplier and the retailer independently evaluate the optimum cost that each can obtain. There is no attempt on the supplier's or the retailer's side to influence the other sides order quantity. In the next section the scenario of coordination between the supplier and retailer is examined.

2.3.3 Coordination, No Space.

In this scenario the two levels of the chain cooperate with each other to bring the total chain cost down to the optimum level. This means that the supplier or the retailer departs from their optimum order quantity in the interest of the total chain cost. Thus the total chain cost is optimized instead of the retailer's and the supplier's costs individually. The total chain cost is therefore given as

$$C_c(\lambda,Q)=C_s(\lambda,Q)+C_r(Q) \qquad (2.2.1)$$

$$\begin{aligned}C_c(\lambda,Q)=&\frac{A_rD}{Q(1-\beta)}+\frac{(a+c_r)D}{(1-\beta)}+h_r\frac{Q(1+\beta)}{2}+SC_r-h_r\beta Q-\\&(c_r+v+a)\frac{\beta D}{(1-\beta)}+\frac{A_sD}{Q(1-\beta)\lambda}+\frac{c_sD}{(1-\beta)}+h_s\frac{Q(\lambda-1)}{2}+\frac{h_s'D\beta}{(1-\beta)}+\\&SC_s+\frac{\omega\beta D}{(1-\beta)}+h_r\beta Q+(c_r+v+a)\frac{\beta D}{(1-\beta)} \qquad (2.2.2)\end{aligned}$$

The partial first derivative is:

$$\frac{\partial C_c(\lambda,Q)}{\partial Q}=-\frac{A_rD}{Q^2(1-\beta)}+\frac{h_r}{2}(1+\beta)-\frac{A_sD}{Q^2(1-\beta)\lambda}+h_s\frac{(\lambda-1)}{2} \qquad (2.2.3)$$

Setting the above equation to zero the following is obtained (working in Appendix 1.3)

$$Q_E(\lambda)=\sqrt{\frac{2D(\lambda A_r+A_s)}{\lambda(1-\beta)\{h_r(1+\beta)+h_s(\lambda-1)\}}} \qquad (2.2.4)$$

The partial second derivative is:

$$\frac{\partial^2 C_c(\lambda,Q)}{\partial Q^2}=\frac{2A_rD}{Q^3(1-\beta)}+\frac{2A_sD}{Q^3(1-\beta)\lambda}>0;\forall Q\geq 1 \qquad (2.2.5)$$

For every value of λ, a unique value for $Q_E(\lambda)$ is obtained. The value of λ and $Q_E(\lambda)$ is substituted in $C_c(\lambda, Q)$ and the optimal value is computed such that

$$C_c\{(\lambda-1), Q_E(\lambda-1)\} > C_c\{\lambda, Q_E(\lambda)\} < C_c\{(\lambda+1), Q_E(\lambda+1)\} \;\forall\; \lambda \geq 2 \qquad (2.2.6)$$

And

$$C_c\{\lambda, Q_E(\lambda)\} < C_c\{(\lambda+1), Q_E(\lambda+1)\} \text{ for } \lambda = 1 \qquad (2.2.7)$$

Thus the optimal cost of the total chain is computed. This cost can be less than or equal to that of no coordination. It may be noted that when the cost of the chain is optimized the retailer could be loosing money since they have gone away from their optimum and the supplier saves over and above what the retailer has lost. Thus the savings get clustered at the suppliers end. Based on the dynamics of the supply chain relationship, as discussed in the literature review, this savings can be distributed.

In the next two scenarios, space is considered as an input variable and the impact on the cost of the chain examined.

2.3.4 No Coordination, Space.

In this scenario Space is considered as a function of Q. It can be argued, however, that this function is a stepped function for incremental increases in Q. Most retailers/ suppliers store items in pallets, bins, kanbans or some kind of storage area. The storage area used each have a capacity V. The quantity that needs to be stored is given by Q and the number of storage areas required is given as

$$N = \left\lceil \frac{Q}{V} \right\rceil + 1 \qquad (2.3.1)$$

where $\lceil x \rceil$ is a round-down function. $\lceil x \rceil$ is the smallest integer less than or equal to x.

Thus the Storage Cost for the retailer per cycle denoted by $SC_r(Q)$ is

$$SC_r(Q) = \frac{R_r N_r}{T} = \frac{R_r}{T}\left(\frac{Q}{V_r} + 1\right) \qquad (2.3.2)$$

It is assumed that $V = V_r = V_s$ since the volume of the product does not usually change from the supplier to the retailer. It is accepted that there are situations where $V_r \neq V_s$. These are examples where a supplier processes the products and the manufacturer further processes it before it reaches the end consumer. A typical example of this can be that of a printed circuit board making its way through the supplier to the manufacturer who assembles components on them and assembles them in a box that is passed to the end consumer. The space required to accommodate the printed circuit board is a lot smaller that the space required for the fully assembled product. However, in a supplier and retailer context there is rarely a case where the volume of the product changes. The former can be very easily accommodated in to the model with minor modification.

Thus, when space is considered the retailers annual cost equation (2.1.3) now becomes:

$$\begin{aligned} C_r(Q) &= \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + R_r N_r - h_r \beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)} \\ &= \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + R_r\left(\frac{Q}{V} + 1\right) - h_r \beta Q - \\ &\quad (c_r + v + a)\frac{\beta D}{(1-\beta)} \end{aligned} \qquad (2.3.3)$$

The first derivative of the above equation is given as:

$$\frac{dC_r(Q)}{dQ} = -\frac{A_r D}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta) + \frac{R_r}{V} \qquad (2.3.4)$$

The second derivative is as follows:

$$\frac{d^2C_r(Q)}{dQ^2} = \frac{2A_r D}{Q^3(1-\beta)} > 0, \forall Q \geq 1$$

Setting the first derivative equation (2.3.4) equal to zero and solving for Q

$$Q_E = \sqrt{\frac{2A_r DC}{(1-\beta)\{h_r V(1-\beta) + 2R_r\}}} \qquad (2.3.5)$$ working is in Appendix 1.4

Since the storage has been taken as a continuous function, the value $C_r(Q)$ can be iteratively computed for all values between $(Q-\gamma V)$ to $(Q+\gamma V)$ to obtain Q_E . γ is a sufficiently large number that is enough to accommodate the error due to the non-continuous function.

The storage cost for the supplier is given as

$$SC_s(\lambda, Q) = R_s N_s = R_s\left(\frac{Q(\lambda-1)+\beta Q}{V}+1\right) = R_s\left(\frac{Q(\beta+\lambda-1)}{V}+1\right) \qquad (2.3.6)$$

The above equation ignores the space required for the total shipment of the supplier. This is because Q units get shipped out the moment it arrives. Thus the space required to accommodate that is not considered.

Substituting (2.3.6) in (2.2.2) the following can be obtained

$$\begin{aligned} C_s(\lambda, Q) &= \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + SC_s(\lambda, Q) + \frac{\omega\beta D}{(1-\beta)} + PnC_s(Q) \\ &= \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + R_s\left(\frac{Q(\beta+\lambda-1)}{V}+1\right) + \\ &\quad \frac{\omega\beta D}{(1-\beta)} + h_r \beta Q + (c_r + v + a)\frac{\beta D}{(1-\beta)} \end{aligned} \qquad (2.3.7)$$

Again, the supplier would try to optimize their cost by adjusting the lot size multiplier by iteratively substituting the value of $\lambda = 1, 2, 3, 4, \ldots$ etc.

The value of Q_E from equation (2.3.5) is substituted in the equation (2.3.7) and the optimal value of λ is computed such that

$$C_s(\lambda-1,Q_E) > C_s(\lambda,Q_E) < C_s(\lambda+1,Q_E)\ \forall\ \lambda \geq 2 \qquad (2.3.8)$$

and

$$C_s(\lambda,Q_E) < C_s(\lambda+1,Q_E) \text{ for } \lambda=1 \qquad (2.3.9)$$

The total chain cost would be the sum of the supplier's and retailer's cost function. This would represent the total cost of the chain if both the supplier and the retailer consider space as an important cost factor, but do not influence each other's order quantity.

2.3.5 Coordination, Space.

In this scenario, the two levels of the chain cooperate with each other to bring the total chain cost down to the optimum level. The difference here is that space cost is considered. Thus substituting the space cost equation (2.3.2) and (2.3.6) in (2.2.2) the total chain cost can be given as:

$$C_c(\lambda,Q) = C_s(\lambda,Q) + C_r(Q)$$

$$\begin{aligned} C_c(\lambda,Q) = {} & \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + R_r\left(\frac{Q}{V}+1\right) - h_r \beta Q - \\ & (c_r+v+a)\frac{\beta D}{(1-\beta)} + \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + \\ & R_s\left(\frac{Q(\beta+\lambda-1)}{V}+1\right) + \frac{\omega\beta D}{(1-\beta)} + h_r \beta Q + (c_r+v+a)\frac{\beta D}{(1-\beta)} \end{aligned} \qquad (2.4.1)$$

The Supplier cost and the retailer cost include storage cost as a function of Q.

The partial first derivative is:

$$\frac{\partial C_c(\lambda,Q)}{\partial Q}=-\frac{A_rD}{Q^2(1-\beta)}+\frac{h_r}{2}(1+\beta)+\frac{R_r}{V}-\frac{A_sD}{Q^2(1-\beta)\lambda}+h_s\frac{(\lambda-1)}{2}+\frac{R_s(\beta+\lambda-1)}{V} \tag{2.4.2}$$

Setting the above equation to zero the following is obtained

$$Q_E(\lambda)=\sqrt{\frac{2VD(\lambda A_r+A_s)}{\lambda(1-\beta)\{Vh_r(1+\beta)+Vh_s(\lambda-1)+2R_r+2R_s(\beta+\lambda-1)\}}} \tag{2.4.3}$$

The partial second derivative is:

$$\frac{\partial^2 C_c(\lambda,Q)}{\partial Q^2}=\frac{2A_rD}{Q^3(1-\beta)}+\frac{2A_sD}{Q^3(1-\beta)\lambda}>0;\forall Q\geq 1 \tag{2.4.4}$$

Again for every value of λ, a unique value for $Q_E(\lambda)$ is obtained. Since the storage has been taken as a continuous function the value $C_c(\lambda\ Q)$can be iteratively computed for all values between $(Q-\gamma V)$ to $(Q+\gamma V)$ to obtain Q_E for each value of λ. γ is a sufficiently large arbitrary number that is enough to accommodate the error due to the non-continuous function. The value of λ and $Q_E(\lambda)$ from equation (2.4.3) is substituted in equation (2.4.1) and $C_c(\lambda,Q)$ computed such that

$$C_c\{(\lambda-1),Q_E(\lambda-1)\}>C_c\{\lambda,Q_E(\lambda)\}<C_c\{(\lambda+1),Q_E(\lambda+1)\}\ \forall\ \lambda\geq 2 \tag{2.4.5}$$

And

$$C_c\{\lambda,Q_E(\lambda)\}<C_c\{(\lambda+1),Q_E(\lambda+1)\}\text{ For }\lambda=1 \tag{2.4.6}$$

Thus, the cost of coordination with space can be calculated. In each scenario, the cost of the chain either remains the same or reduces, due to various factors modeled. When space is considered, it influences the cost and the quantity ordered.

This in itself is an approach to JIT. Reducing the order quantity to a day to day basis can be considered as JIT. This means that there is no wastage of space and products do not wait in inventory bins to be processed. The following subsection looks at how far does the chain need to reduce costs from Coordination Space to JIT.

2.3.6 Operating Under JIT

The approach in this section is different from the other sections. Here the values are known and what needs to be obtained is the amount of reduction in setup needed to reach that level.

When operating under JIT, the lot size multiplier λ is equal to one since the supplier would be building inventory on a lot for lot basis. Further, owing to continuous improvement and immediate identification and corresponding rectification of problems it can be assumed that β approaches Zero.

Substituting these values in (2.4.3),

$$Q_J = \sqrt{\frac{2VD(A_r + A_s)}{(Vh_r + 2R_r)}} \qquad (2.5.1)$$

Ideally in a JIT world $Q_J = \frac{D}{n}$. Where it is assumed here that $n = 365$ working days per year. This would be the scenario where products are ordered on a day to day basis. However, bringing Q_J down would cause the total cost to increase. For this scenario to be profitable A_r and A_s should be brought down to a feasible level. Having computed the total cost in each scenario, the optimum cost is considered as a benchmark. The cost for operating under JIT is computed and the reduction needed in setup costs is computed.

2.4 Optimal Solution Procedure

The following sections will illustrate the solution procedure to arrive at the optimal order quantity in each of the abovementioned scenarios. It is acknowledged that the model assumes the order quantity and number of storage bins to be continuous for calculation purposes but in reality it is not possible to order 566.6667 units of a product. It should be either 566 or 567 units. This is also the case where number of bins is concerned. If 100 units fit into one bin the number of bins required to accommodate 566 units would be 6 bins. It cannot be 5.66 bin or 5. This necessitates the search for the optimum solution by the iterative search surrounding solution generated by the model.

The following two flowcharts illustrates the solution procedure for No Coordination and Coordination

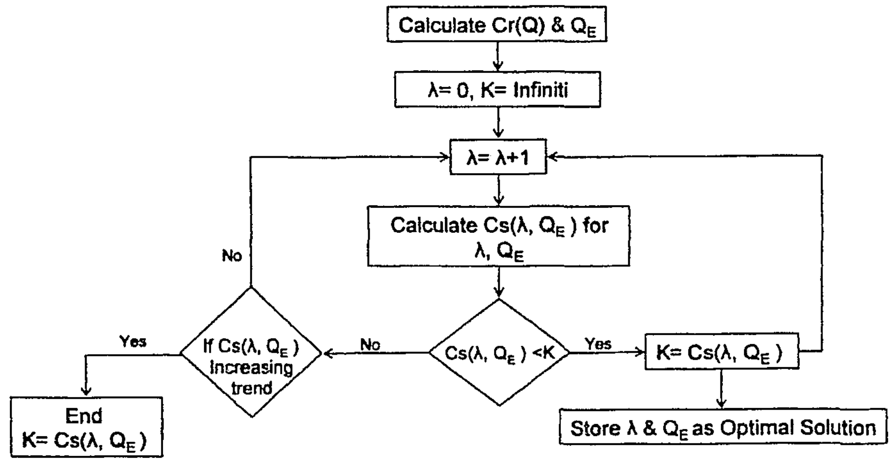

Figure 2.4: Optimal solution procedure for no coordination (case I and case III)

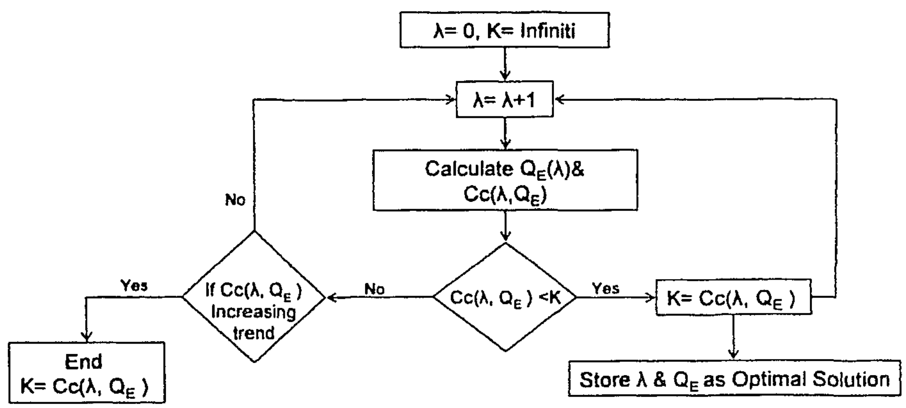

Figure 2.5: Optimal solution procedure for coordination (case II and case IV)

The following is the solution algorithm adopted:

2.4.1 No Coordination, No Space

Step 1: Set $Q_E = Int\left(\sqrt{\frac{2A_rD}{h_r(1-\beta)^2}}\right)$

Step 2 : For $Q = Q_E - \gamma V$ to $Q_E + \gamma V$

$$C_r(Q) = \frac{A_rD}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r\frac{Q(1+\beta)}{2} + SC_r - h_r\beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

The value of Q for which $C_r(Q)$ is minimum is the retailer's EOQ.

γ is a sufficiently large arbitrary number that is enough to accommodate the error due to the non-continuous function.

Step 3 : For $\lambda = 1$ to sufficiently large integer (say λ_{max}). λ_{max} is usually a value beyond which the cost function for the supplier is constantly increasing.

$$C_s(\lambda,Q) = \frac{A_sD}{Q(1-\beta)\lambda} + \frac{c_sD}{(1-\beta)} + h_s\frac{Q(\lambda-1)}{2} + \frac{h_s'D\beta}{(1-\beta)} + SC_s + \frac{\omega\beta D}{(1-\beta)} + h_r\beta Q + (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

The value of λQ for which $C_s(\lambda,Q)$ is minimum is the supplier's EOQ and the total chain cost is $C_c(\lambda,Q) = C_s(\lambda,Q) + C_r(Q)$

2.4.2 Coordination, No Space.

Step 1 : For $\lambda = 1$ to sufficiently large number (say λ_{max}) generate the values of

$$Q_E(\lambda) = \sqrt{\frac{2D(\lambda A_r + A_s)}{\lambda(1-\beta)\{h_r(1+\beta) + h_s(\lambda-1)\}}}$$

λ_{max} is a sufficiently large arbitrary integer beyond which the minimum total cost $C_c(\lambda,Q_E)$ is constantly increasing.

Step 2 : For $Q = Q_E - \gamma V$ to $Q_E + \gamma V$ for each λ

$$C_c(\lambda,Q) = \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + SC_r - h_r \beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)} + \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + SC_s + \frac{\omega\beta D}{(1-\beta)} + h_r \beta Q + (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

γ is again a sufficiently large arbitrary number that is enough to accommodate the error due to a non-continuous function. The value of Q and λ for which $C_c(\lambda,Q)$ is minimum is the retailer's EOQ and the suppliers lot size multiplier respectively.

2.4.3 No Coordination, Space

Step 1: Set $Q_E = \sqrt{\frac{2A_r DV}{(1-\beta)\{h_r V(1-\beta) + 2R_r\}}}$

Step 2 : For $Q = Q_E - \gamma V$ to $Q_E + \gamma V$

$$C_r(Q) = \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + R_r\left(\frac{Q}{V}+1\right) - h_r \beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

The value of Q for which $C_r(Q)$ is minimum is the retailer's EOQ.

γ is a sufficiently large number that is enough to accommodate error due to a non-continuous function.

Step 3 : For $\lambda = 1$ to sufficiently large integer (say λ_{max})

$$C_s(\lambda,Q) = \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + R_s\left(\frac{Q(\beta+\lambda-1)}{V}+1\right) + \frac{\omega\beta D}{(1-\beta)} + h_r \beta Q + (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

The value of λQ for which $C_s(\lambda,Q)$ is minimum is the supplier's EOQ and the total chain cost is $C_c(\lambda,Q) = C_s(\lambda,Q) + C_r(Q)$

λ_{max} is a sufficiently large integer beyond which the supplier cost is constantly increasing.

2.4.4 Coordination, Space.

Step 1 : For $\lambda = 1$ to sufficiently large number (say λ_{max}) generate the values of

$$Q_E(\lambda) = \sqrt{\frac{2VD(\lambda A_r + A_s)}{\lambda(1-\beta)\{Vh_r(1+\beta)+Vh_s(\lambda-1)+2R_r+2R_s(\beta+\lambda-1)\}}}$$

λ_{max} is a sufficiently large integer beyond which the minimum total cost $C_c(\lambda, Q_E)$ is constantly increasing.

Step 2 : For $Q = Q_E - \gamma V$ to $Q_E + \gamma V$ for each λ

$$\begin{aligned} C_c(\lambda, Q) = {} & \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + R_r\left(\frac{Q}{V}+1\right) - h_r \beta Q - \\ & (c_r+v+a)\frac{\beta D}{(1-\beta)} + \frac{A_s D}{Q(1-\beta)\lambda} + \frac{c_s D}{(1-\beta)} + h_s \frac{Q(\lambda-1)}{2} + \frac{h_s' D\beta}{(1-\beta)} + \\ & R_s\left(\frac{Q(\beta+\lambda-1)}{V}+1\right) + \frac{\omega\beta D}{(1-\beta)} + h_r\beta Q + (c_r+v+a)\frac{\beta D}{(1-\beta)} \end{aligned}$$

γ is a sufficiently large arbitrary number that is enough to accommodate error due to a non-continuous function. The value of Q and λ for which $C_c(\lambda, Q)$ is minimum is the retailer's EOQ and the suppliers lot size multiplier.

2.4.5 Operating under JIT

Step 1: Compute the value of Q such that $Q_J = \frac{D}{n}$ where n=365 working days per year

Step 2: Substitute the value of Q_J in equation (2.4.1) with λ =1and β=0 and compute the total cost.

Step 3: If the total cost obtained is less than that computed in each of the above scenarios, no further action required. If the total cost obtained is greater than that of any of the above scenarios, then A_r and A_s has to be reduced together uniformly to a level such that the total cost is less.

In the following chapter, the procedure detailed in section 2.4 is illustrated through an example and the results of the statistical analysis performed using a number of sets on parameters described.

Chapter 3

NUMERICAL RESULTS AND STATISTICAL ANALYSIS

This chapter presents numerical examples to illustrate the solution procedure to the mathematical models developed in Chapter 2. It also investigates the behavior of the model tested under varying sets of parameters.

3.1 Numerical Example

This section presents numerical examples for the models developed in Chapter 2 to illustrate the savings that could be obtained when the inventory system shifts from tradition EOQ inventory policy to contemporary JIT policy. These models represent the cases when there is no-coordination between the supplier and the retailer and no space considerations (Case I: NC-NS), coordination between the supplier and the retailer and no space considerations (Case II: C-NS), no-coordination between the supplier and the retailer with space considerations (Case III: NC-S), coordination between the supplier and the retailer with space considerations (Case IV: C-S), and the case where the supplier and the retailer operate under just-in-time policy (Case V: JIT). Each case, i.e., NC-NS, C-NS, NC-S, C-S, and JIT, is examined to see the progress in savings as each step towards the implementation of the ideal JIT as described by Cao and Schniederjans (2004) is considered. The following is a solved example to illustrate the savings that could be obtained.

In the example that is illustrated let A_r =420 (\$), A_s = 480 (\$) , a =0.105 (\$/unit), β=0.046 ($0 < \beta < 1$), c_r= 21 (\$/unit), c_s = 10 (\$/unit), h_r = 2.73 (\$/unit/yr), h_s=1.8

(\$/unit/yr), $R_r = 1800$ (\$/storage area), $R_s = 1500$ (\$/storage area), $v = 3.675$ (\$/unit), $D = 120{,}000$ (units/yr), V=10 units

Case I: No Coordination, No Space

From equation (1.3):

$$Q_E = Int\left(\sqrt{\frac{2\times 420\times 120{,}000}{2.73\times(1-0.046)^2}}\right) = Int(6{,}369.43) = 6{,}369$$

For $Q = Q_E\text{-}\gamma V$ to $Q_E+\gamma V$

Here γ is taken as 2. This is a sufficiently large arbitrary number from the results generated that cover the minimum value of $C_r(Q)$ and the error due to a non-continuous function.

For $Q = 6350$ to 6389 substituting the values in equation (2.1.3)

$$C_r(Q) = \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + SC_r - h_r \beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

Note that SC_r is kept constant and the value assigned is the cost of space needed to accommodate the greater of the two EOQ's obtained in case I and case II, namely "no coordination no space" and "coordination no space".

In the sub-section that follows, it can be seen that when the space requirement for "Coordination No space" was calculated, it was found to be 282 storage areas, since this scenario requires 637 storage areas the greater of the two is taken as SC_r for both the scenarios. This is done to bring consistency for the space cost to the two scenarios so that the two can be compared. Thus SC_r = \$1,146,600.

Appendix 1.5 shows the values for $C_r(Q)$ and their corresponding Q and graphically shows the variation.

It is obtained that :

$$C_r(Q_{min}) = C_r(6369) = \$3,674,525$$

It is interesting to note that the change in total cost is in decimals as the optimum is approached. For practical purposes, this gives the decision maker a lot of flexibility to adjust the order quantity to satisfy other decision parameters with limited impact on the cost.

To calculate the supplier's side of the cost, equation (2.1.7) is used for λ from 1 to λ_{max} for the optimal cost. The computational logic used here is that λ is tested from 1 to a very large number. As the values of cost for each λ, $C_s(\lambda, Q)$, are computed they are stored and if this displays an increasing trend the program exits. Here λ is computed to a λ_{max} of 9. The software is set this way since the increase in λ, $C_s(\lambda, Q)$ takes a parabolic shape. This means that if two or three consecutive values show an increasing trend then the minimum has been reached. Appendix 1.6 shows the values for $C_s(\lambda, Q)$ and the corresponding value of λ and depicted graphically.

The suppliers optimum cost is at:

$$C_s(\lambda_{min}, Q_E) = C_s(5, 6369) = \$5,218,309.37$$

Thus the total chain cost is

$$C_c(\lambda, Q) = C_s(\lambda, Q) + C_r(Q) = \$8,892,834$$

Case II: Coordination, No Space

The approach here is combined where for each λ, look for the optimum Q. To save computing time, the range of λ for which the total cost is minimum, is first established. This is done by iteratively substituting the value of λ in equation (2.2.4) and generating the value of Q. This in turn is substituted in equation (2.2.2) to get the total cost of the chain. The values of λ for which the total chain cost is minimum is identified and a search for the optimum value of λ and Q is done. Thus the value of λ for which the total cost was minimum was 8 and optimum value was searched for from λ= 6 to 10 is substituted in equation (2.2.2). The values of Q obtained are tabulated in table 3.1.

λ	Q	$C_c(\lambda,Q)$
6	3257	\$8,884,758
7	3000	\$8,883,713
8	2795	\$8,883,401
9	2626	\$8,883,552
10	2485	\$8,884,008

Table 3.1: Value of Q obtained for each λ from equation (2.2.4)

For each λ and Q, iteratively the surrounding values of Q were searched for the best solution. i.e. if Q(8)=2795, the best solution is searched for in the range of ±30 units. This range is selected to be sufficiently high arbitrary number such that the minimum lies within this range. The results are tabulated in Appendix 1.7. Thus, from this tabulated results, the optimal value is

$$C_c(8,2815) = C_s(8,2815) + C_r(2815) = \$3,680,369 + \$5,203,031 = \$8,883,400$$

Here again, it is interesting to note that the change in total cost is in decimals as the optimum is approached. However, this is not the case for the supplier cost and the retailer cost taken individually, that in turn varies by a few dollars. Also note the difference in the value generated from table 3.1 and the results obtained from appendix 1.7. This is due to the error of a non-continuous function.

It is worthwhile also to note that the supplier's cost reduced ($5,218,309-$5,203,031= $15,278), while the retailer cost increased ($3,674,525-$3,680,369=-$5,844), however, the total chain cost has reduced ($8,892,834-$8,883,400=$9,434).

The retailer can be compensated by the supplier for the change through quantity discounts or can be forced to order in lot sizes of 2,815 units by the supplier and the savings kept by the supplier, if the supplier is the chain leader. Alternately, if cooperation is the aim then the net savings can be shared based on investments or total purchases or so.

Case III: No Coordination, Space

When No Coordination with space is considered the EOQ is represented by equation (2.3.5) and the total cost of the retailer is represented by (2.3.3).

Thus substituting the input variables in (2.3.5).

Q_E=539 units

$C_r(539) = \$2,707,253$

The best solution is searched for from Q=520 to 559 and tabulated in Appendix 1.8. The graph of the variation in the total costs takes the form of a saw tooth pattern. This is the impact of the storage areas that are taken into account.

Now searching for λ (lot size multiplier) that is optimal to the supplier, consider λ from 1 to λ_{max} where λ_{max} is 6. The value of λ_{max} is different for each set of parameters since the software stops searching for the optimal solution when the result of increasing λ starts to generate increasing values of supplier cost. Substituting these values and all the input parameters in equation (2.3.7) $\lambda = 2$

$$C_s(2{,}539) = \$1{,}512{,}867$$

Again the supplier cost for $\lambda = 1$ to 6 is tabulated in Appendix 1.9

The total cost is therefore

$$C_c(2{,}539) = C_s(2{,}539) + C_r(539) = \$2{,}707{,}253 + \$1{,}512{,}867 = \$4{,}220{,}120$$

Thus, the optimal cost for No Coordination, Space is found.

Case IV: Coordination, Space

In this scenario the approach is combined where for each λ the optimum Q is obtained. From equation (2.4.3) $\lambda = 1$ to λ_{max} where λ_{max} is a large number beyond which the values are increasing. The values of Q obtained for each value of λ are tabulated in table 3.2.

λ	Q
1	775
2	494
3	385
4	325
5	286

Table 3.2: Value of Q obtained for each λ from equation (2.4.3)

For each λ and Q, iteratively the surrounding values of Q were searched for the best solution. The best solution is searched for in the range of ±30 units. The results are not included but follows the pattern of that of Coordination No Space. Only that this has a saw tooth pattern to the graph due to the cost of storage space that is included.

Thus, from the table the optimal result is

$$C_c(\lambda,Q)=C_s(\lambda,Q)+C_r(Q)$$

$$C_c(2,478)=C_s(2,478)+C_r(478)$$

$$C_c(2,478)=\$2,708,882+\$1,509,456$$

$$C_c(2,179)==\$4,218,338$$

Note that the similar situation has occurred with regard to the supplier cost where it is reduced (\$1,512,867-\$1,509,456= \$3,411), the retailers cost increased (\$2,707,253-\$2,708,882= -\$1,629), and the total chain has cost has a net reduction (\$4,220,120-\$4,218,338=\$1,782). The sharing of the savings can be done based on the dynamics of the supply chain relationship and the exchange of funds through quantity discounts or methods suggested by Abed & Jaggi (2003).

Case (V): Operating under JIT

As the supplier and retailer move towards JIT, it is assumed that the retailer ships product on a need to basis. That is the retailer buys only what is required for that day and the supplier buys or manufactures only what would be ordered by the retailer.

This means that λ=1 and $Q=\frac{D}{365}=\frac{120,000}{365}=329$ units

Moving to JIT means that the supplier and the retailer move towards total quality. This would increase the cost of c_r and c_s, thereby increasing the holding cost parameters h_r and h_s. It would be safe to assume that c_r and c_s increased by a minimum of one times the screening cost. It is also assumed that β is zero and the retailer does not need to do any screening since it is total quality that is being received.

Thus c_r and c_s would be \$21.11 and \$10.11per unit and h_r and h_s would be \$2.74 and \$1.82 per unit per year. Substituting these values in the equations:

$$C_r(Q) = \$2,745,751$$

$$C_s(\lambda, Q) = \$1,389,300$$

$$C_c(\lambda, Q) = \$4,135,051$$

Thus in this scenario adopting JIT has already proved cheaper than all the other alternatives. The major factor that contributed to the lower cost is the supply of total quality products (ie β=0). It is worthwhile to note that more savings can be obtained by reducing the set up cost or ordering costs although no reduction is required. Thus, it can be said that for this set of parameters it would be very valuable for the retailer and the supplier to move towards JIT.

The summary results on these five scenarios are tabulated in table 3.3.

Scenarios	Retailer's Cost	Supplier's Cost	Total Cost	Q	λ
NC-NS	$3,674,525	$5,218,309	$8,892,834	6,369	5
C-NS	$3,680,369	$5,203,031	$8,883,400	2,815	8
NC-S	$2,707,253	$1,512,867	$4,220,120	539	2
C-S	$2,708,882	$1,509,456	$4,218,338	478	2
JIT	$2,745,751	$1,389,300	$4,135,051	329	1

NC- No Coordination, C- Coordination, NS – No Space, S – Space, JIT- Just in Time

Table 3.3: Summary table of the total cost for all scenarios

3.2 Statistical Analysis

In the following section the model is tested through varying sets of input parameters to understand the behavior of the model. The parameters selected are adopted from the Munson & Rosenblatt (2001) example.

The demand (D) is varied randomly (following a uniform distribution) from 10,000 to 500,000 units in increments of 10,000. The setup costs (A_r & A_s) for the supplier and the retailer is also varied randomly (following a uniform distribution) from $50 to $500 in increments of $10. The cost for the storage area (R_r & R_s) for the supplier and the retailer follows the similar randomness pattern from $1000 to $2000 in increments of $100. The fraction of defects (β) ranges uniformly from 0.01 to 0.05 in increments of 0.001.

The cost for the supplier (c_s) varies uniformly from $10 to $100 in increments of $1/-. The cost for the retailer (c_r) is based on the supplier cost such that:

$c_r = ac_s + v$ where α is the markup that varies between 1.3 and 1.8 uniformly with intervals of 0.1 and v is the value added by the supplier to the product. In the analysis the value of v has been taken as 5. Thus the value of c_r is dependant on c_s but the randomness is introduced in the markup using the above-mentioned relationship.

The holding cost (h_r & h_s) is dependant on the value of c_r and c_s. This can be expressed as:

$h_i = \varphi_i \times c_i$ where φ_i is the annual holding cost percentage for firm i. φ_i varies from 12 to 25% of the cost for the supplier or the retailer in increments of 1%. This again is generated randomly following a uniform distribution.

The randomly generated values were tabulated and the total cost in each scenario calculated. The base reference cost is the total cost for No Coordination No Space. The summary of the averages of output measures from the computational study of 8410 trials are tabulated in Table 3.4.

	[illegible] Measure	Mean	Standard Deviation
1	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}NS)$	\$48,337	\$44,903
2	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}NS)$ %	0.11%	0.05%
3	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(NC\text{-}S)$	\$2,944,919	\$1,930,626
4	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(NC\text{-}S)$ %	10.95%	9.49%
5	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}S)$	\$3,005,102	\$1,957,834
6	$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}S)$ %	11.08%	9.47%
7	% Setup reduction on JIT	1.99%	9.22%
8	% Reduction in Q (NC-NS to C-NS)	60.39%	10.82%
9	% Reduction in λQ (NC-NS to C-NS)	35.60%	9.54%
10	% Reduction in Q (NC-NS to NC-S)	78.03%	6.23%
11	% Reduction in λQ (NC-NS to NC-S)	90.77%	3.56%
12	% Reduction in Q (NC-NS to C-S)	86.63%	4.55%
13	% Reduction in λQ (NC-NS to C-S)	92.73%	2.58%

Table 3.4: Summary averages from computational trials

The output parameters are defined below

$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}NS)$=> This is the difference in the total cost of the chain between two scenarios. (NC-NS (No Coordination No Space), C-NS (Coordination No Space), NC-S (No Coordination Space), C-S (Coordination Space).)

$C_c(\lambda,Q)(NC\text{-}NS) - C_c(\lambda,Q)(C\text{-}NS)$%=> This is the percentage difference between the total cost of the two scenarios.

% Setup reduction on JIT=> the cost to operate on JIT is compared with the minimum cost of operating in the other scenarios. This parameter is the percentage reduction in setup needed to bring the total cost of operating in JIT to the minimum.

It was observed that in all the experiments that coordinating in the chain brought some cost savings. When space was considered there has always been considerable savings that can be seen from the summary table

It can be said that it is definitely advantageous for organizations to move to JIT but computing the cost involved is often conducted in an erroneous manner since the actual costs and savings are hidden. Following the model developed above can lead to the conclusion that the cost involved in reducing setup costs to move to JIT is far lower than scenario of no coordination no space. This is illustrated by the point that the percentage reduction in setup required is around 1.99 % on average.

Among the other observations from the trials are the maximum savings computed when moving from no coordination, no space to coordination no space is 0.33% and the minimum is close to zero. Similarly, maximum savings from no coordination no space to no coordination space is 71.42% and the minimum is 1%. The numbers are slightly higher when comparing coordination space with no coordination no space namely 71.55% and 1.07%. This is due to the savings obtained due to coordination. This supports the fact that coordination between the supplier and retailer can bring a lot more savings.

Considering the percentage reduction in setup to make JIT feasible, the maximum reduction in setup required is 75.24% and the minimum is 0%. Another surprising observation is that 93.76% of the trials did not require any reduction in setup cost for JIT to be feasible. This means that for these case scenarios, moving to JIT lot sizes require little if no investments to reduce setup cost.

Other observations include, that if the supplier and retailer setup costs are equal the average percentage of improvement reduces slightly to below average from 10.95% to 10.10% for No Coordination Space and from 11.08% to 10.22% for Coordination Space.

This chapter details the solution of an example set of parameters and the overall behavior of the results of a number of trials conducted. The next chapter concludes this thesis and contains suggestions for further research.

Chapter 4

CONCLUSION AND FUTURE RESEARCH

In this research, the classical EOQ model was extended to reflect and therein compare the actual costs associated with adopting the classical EOQ approach and the JIT approach for inventory. The model brings out the advantages of coordination within the supply chain which is often an integral part of an organization that successfully adopts JIT.

The results from the random experiments illustrated that if an organization considered coordination (Case II) with its supplier or retailer on its own to begin with, can reduce the total chain cost by an average of $48,337. In a competitive market this saving can be used to the advantage of the organizations concerned. It must however be noted that the percentage reduction in order quantity for the retailer was down by an average of 60% and that of the supplier by 35.60%. The huge reduction in order quantity did not reflect in total cost since space occupied by the inventory was not properly accounted for in this scenario.

Considering space alone as a cost factor without coordination (Case III) brings significant impact to the cost. The results indicate that a total chain cost savings of 10.95%, on average, can be obtained in this scenario. The reduction in order quantity and there in the average inventory on hand reduced by 78.03% for the retailer and 90.77% for the supplier. This indicates considerable savings in the chain. One can argue that these quantities approach JIT lot sizes.

However, the best case is combining both the scenarios, namely considering space and coordination (Case IV), which brings us closest to JIT. In this scenario the total chain cost savings that was obtained was an average of 11.08%. The reduction in order quantity was the best with an average reduction of 86.63% for the retailer and 92.73% for the supplier.

In case III and case IV, it can be observed that order quantities of the supplier are reduced to an average of 10%. This can be an indication that the supplier is working on a lot for lot basis that would be in accordance with the principles of JIT. Ordering in JIT is characterized by total quality shipment that are manufactured or delivered on a need-to basis. This requires establishing long term contracts (Waters-Fuller 1995) and coordination between the supplier and the retailer. It can be observed that case IV characterized by coordination and consideration of space in itself is the closest to JIT ordering. Moving form case IV to JIT ordering (Case V), it can be observed that the reduction in setup needed to make JIT profitable is around 1.99%. It was also observed that in 93% cases no reduction in setup was ever required.

This result concurs with Jones (1991) argument that in EOQ, if all costs are appropriately accounted for would approximate JIT lot sizes. This is the conclusion that the model alludes to. The example illustrated in chapter 3 (table 3.3) shows the reduction in order quantity to JIT levels. Thus reducing setup costs, although needed, does not need to be of that much magnitude than estimated with no coordination and no space. This opens scope for lot more improvement in costs as setup is further improved.

There is however a lot more scope for further research. In the analysis that was done in this thesis the space allocation considered was fixed (Joshi 1990). Considering the space to be dynamic can improve the savings further more. Although this would be of benefit to organizations that have numerous products, it is not the scenario when organizations deal with a few products only.

Abed & Jaggi's (2003) extensive research could be extended into the model to illustrate the dynamics of the players and the mode in which the profit sharing can be done. This can be further investigated in future research.

The model assumes that the production rate to be infinite. Further research can be done to study the impact of finite production rates on the model. The economics of Price sensitivity to demand and demand sensitivity to price are other interesting areas that can be incorporated into the model.

It is also suggested that further research can be done by extending this model to three members of the supply chain. This would be in line with Munson & Rosenblatt (2001) model extended to JIT with the consideration of space and defects. The model can also be extended to consider single retailer with multiple vendors as well as multiple retailers and single vendor.

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Appendix 1.1

Working of equation from Scheniederjans & Cao

$Z = \frac{AD}{Q} - \frac{mAD}{Q} + \frac{Qh}{2} + (c_E - c_J)D + CF$ or by substituting the value of Q from

$$Q = \sqrt{\frac{2AD}{h}}$$

$$Z = \frac{AD}{\sqrt{\frac{2AD}{h}}} - \frac{mAD}{\sqrt{\frac{2AD}{h}}} + \frac{\sqrt{\frac{2AD}{h}}h}{2} + (c_E - c_J)D + CF$$

$$Z = \frac{\sqrt{AD}}{\sqrt{\frac{2}{h}}} - \frac{m\sqrt{AD}}{\sqrt{\frac{2}{h}}} + \frac{\sqrt{2AD}\sqrt{h}}{2} + (c_E - c_J)D + CF$$

$$Z = \sqrt{\frac{ADh}{2}} - m\sqrt{\frac{ADh}{2}} + \sqrt{\frac{ADh}{2}} + (c_E - c_J)D + CF$$

$$Z = 2\sqrt{\frac{ADh}{2}} - \frac{2m}{2}\sqrt{\frac{ADh}{2}} + (c_E - c_J)D + CF$$

$$Z = \sqrt{2ADh} - \frac{m}{2}\sqrt{2ADh} + (c_E - c_J)D + CF$$

$$Z = \sqrt{2ADh}\left(1 - \frac{m}{2}\right) + (c_E - c_J)D + CF$$

Appendix 1.2

Working of Retailer's Order quantity for no coordination no space

From Equation (2.1.3)

$$C_r(Q) = \frac{A_r D}{Q(1-\beta)} + \frac{(a+c_r)D}{(1-\beta)} + h_r \frac{Q(1+\beta)}{2} + SC_r - h_r \beta Q - (c_r + v + a)\frac{\beta D}{(1-\beta)}$$

(2.1.3)

The first derivative of the above equation is given as:

$$\frac{dC_r(Q)}{dQ} = -\frac{A_r D}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta)$$

Setting the above equal to zero

$$0 = -\frac{A_r D}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta)$$

$$\frac{A_r D}{Q^2(1-\beta)} = \frac{h_r}{2}(1-\beta)$$

$$\frac{2A_r D}{h_r(1-\beta)^2} = Q^2$$

$$Q_E = \sqrt{\frac{2A_r D}{h_r(1-\beta)^2}} \quad (2.1.4)$$

Appendix 1.3

The following is the working of the retailer's order quantity for Coordination, No Space.

From Equation (2.2.4)

$$\frac{\partial C_c(\lambda,Q)}{\partial Q}=-\frac{A_rD}{Q^2(1-\beta)}+\frac{h_r}{2}(1-\beta)-\frac{A_sD}{Q^2(1-\beta)\lambda}+h_s\frac{(\lambda-1)}{2} \qquad (2.2.4)$$

Setting equation (2.2.4) equal to zero

$$0=-\frac{A_rD}{Q^2(1-\beta)}+\frac{h_r}{2}(1-\beta)-\frac{A_sD}{Q^2(1-\beta)\lambda}+h_s\frac{(\lambda-1)}{2}$$

$$\frac{A_rD}{Q^2(1-\beta)}+\frac{A_sD}{Q^2(1-\beta)\lambda}=\frac{h_r}{2}(1-\beta)+\frac{h_s}{2}(\lambda-1)$$

$$\frac{\lambda A_rD+A_sD}{Q^2(1-\beta)\lambda}=\frac{1}{2}\left(h_r(1-\beta)+h_s(\lambda-1)\right)$$

$$\frac{2D(\lambda A_r+A_s)}{(1-\beta)\lambda\left(h_r(1-\beta)+h_s(\lambda-1)\right)}=Q^2$$

$$Q_E(\lambda)=\sqrt{\frac{2D(\lambda A_r+A_s)}{\lambda(1-\beta)\{h_r(1-\beta)+h_s(\lambda-1)\}}} \qquad (2.2.4)$$

Appendix 1.4

Working of the Retailer's order quantity for No Coordination, Space

From Equation (2.3.4)

$$\frac{dC_r(Q)}{dQ} = -\frac{A_r D}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta) + \frac{R_r}{V} \qquad (2.3.4)$$

Equating to zero and solving for Q

$$0 = -\frac{A_r D}{Q^2(1-\beta)} + \frac{h_r}{2}(1-\beta) + \frac{R_r}{V}$$

$$\frac{A_r D}{Q^2(1-\beta)} = \frac{h_r}{2}(1-\beta) + \frac{R_r}{V}$$

$$\frac{A_r D}{Q^2(1-\beta)} = \left(\frac{h_r V(1-\beta) + 2R_r}{2V}\right)$$

$$\frac{2A_r DV}{(1-\beta)(h_r V(1-\beta) + 2R_r)} = Q^2$$

$$Q_E = \sqrt{\frac{2A_r DV}{(1-\beta)\{h_r V(1-\beta) + 2R_r\}}} \qquad (2.3.5)$$

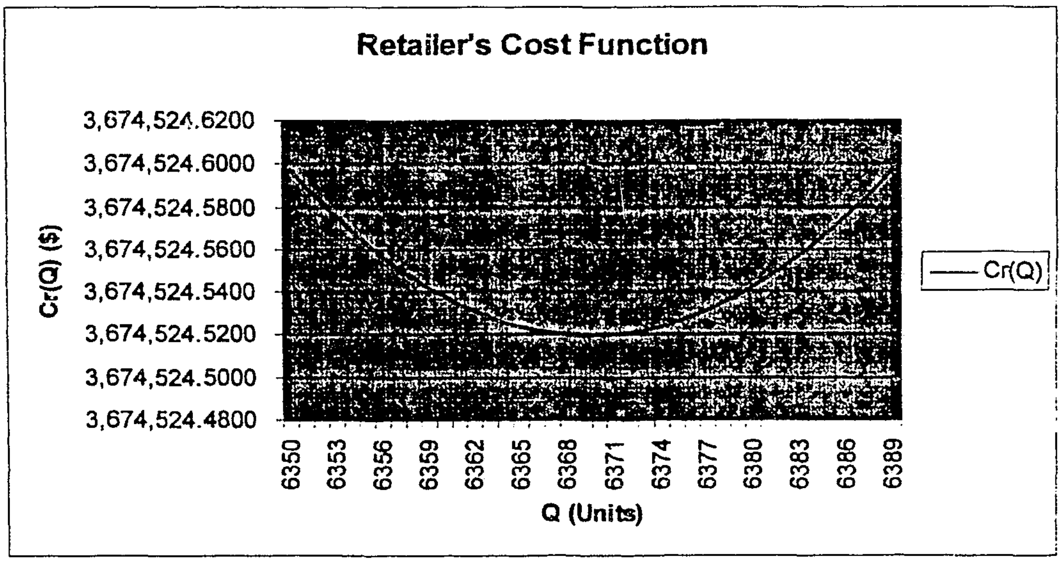
Appendix 1.5

Case I No Coordination No Space: The table below shows the total retailers cost ($C_T(Q)$) tabulated against the value of each order quantity (Q). Highlighted in black is the minimum cost for Q=6369

Q	$C_T(Q)$
6350	3,674,524.5973
6351	3,674,524.5895
6352	3,674,524.5822
6353	3,674,524.5752
6354	3,674,524.5687
6355	3,674,524.5626
6356	3,674,524.5568
6357	3,674,524.5515
6358	3,674,524.5466
6359	3,674,524.5422
6360	3,674,524.5381
6361	3,674,524.5344
6362	3,674,524.5312
6363	3,674,524.5284
6364	3,674,524.5259
6365	3,674,524.5239
6366	3,674,524.5223
6367	3,674,524.5211
6368	3,674,524.5203
6369	3,674,524.5199
6370	3,674,524.5200
6371	3,674,524.5204
6372	3,674,524.5212
6373	3,674,524.5225
6374	3,674,524.5242
6375	3,674,524.5262
6376	3,674,524.5287
6377	3,674,524.5316
6378	3,674,524.5349
6379	3,674,524.5386
6380	3,674,524.5427
6381	3,674,524.5472
6382	3,674,524.5521

Q	Cr(Q)
6383	3,674,524.5575
6384	3,674,524.5632
6385	3,674,524.5693
6386	3,674,524.5759
6387	3,674,524.5828
6388	3,674,524.5902
6389	3,674,524.5979

Case I No Coordination No Space: Figure below illustrates the change in the retailers cost with the change in Q

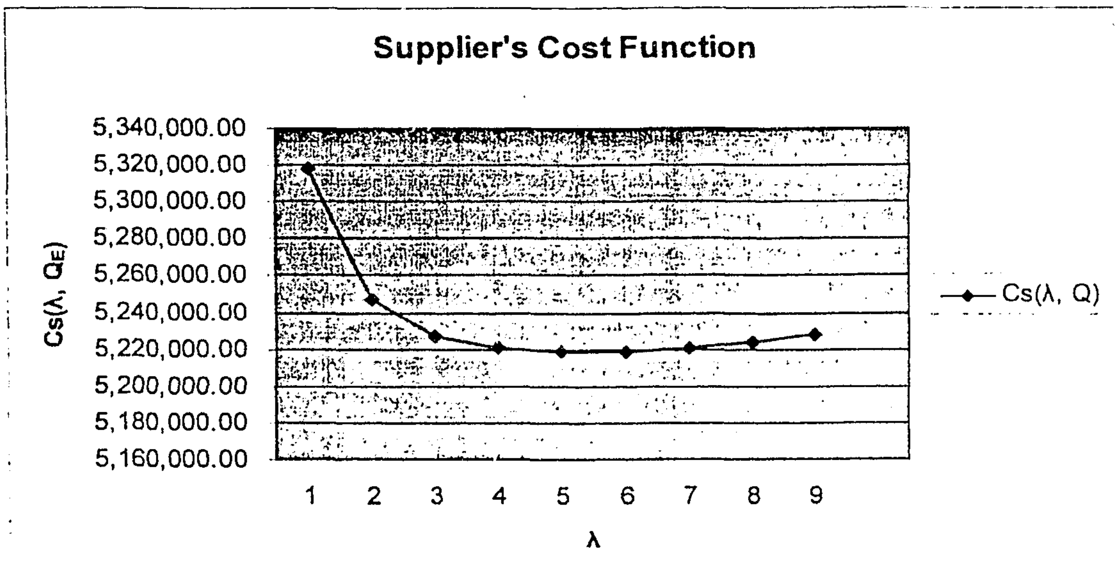


Appendix 1.6

Case I No Cordination No Space: Table below shows the variation the supplier cost with the change in the lot size multiplier (λ)

[illegible]	$C_s(\lambda,Q)$
1	5,318,309.64
2	5,247,211.32
3	5,227,333.28
4	5,220,260.31
5	5,218,309.37
6	5,218,919.44
7	5,220,992.95
8	5,223,981.11
9	5,227,579.03

The figure below illustrates the variation the suppliers cost ($C_s(\lambda,Q_E)$) with the lot size multiplier (λ)



Appendix 1.7

Case II: Coordination, No Space

The following are the values obtained for each value of λ for Q=Q-3V *to* Q+3V

$\lambda=6$			
Q	**Cr(Q)**	**Cs(λ,Q)**	**Cc(λ,Q)**
3,228	3,678,505.6124	5,206,256.5995	8,884,762.2118
3,229	3,678,501.8461	5,206,260.1550	8,884,762.0010
3,230	3,678,498.0829	5,206,263.7111	8,884,761.7940
3,231	3,678,494.3229	5,206,267.2677	8,884,761.5906
3,232	3,678,490.5660	5,206,270.8250	8,884,761.3910
3,233	3,678,486.8122	5,206,274.3829	8,884,761.1952
3,234	3,678,483.0616	5,206,277.9414	8,884,761.0030
3,235	3,678,479.3141	5,206,281.5005	8,884,760.8146
3,236	3,678,475.5697	5,206,285.0602	8,884,760.6298
3,237	3,678,471.8284	5,206,288.6204	8,884,760.4488
3,238	3,678,468.0902	5,206,292.1813	8,884,760.2715
3,239	3,678,464.3552	5,206,295.7427	8,884,760.0979
3,240	3,678,460.6232	5,206,299.3048	8,884,759.9280
3,241	3,678,456.8944	5,206,302.8674	8,884,759.7618
3,242	3,678,453.1687	5,206,306.4306	8,884,759.5993
3,243	3,678,449.4461	5,206,309.9945	8,884,759.4405
3,244	3,678,445.7265	5,206,313.5589	8,884,759.2854
3,245	3,678,442.0101	5,206,317.1239	8,884,759.1339
3,246	3,678,438.2967	5,206,320.6894	8,884,758.9862
3,247	3,678,434.5865	5,206,324.2556	8,884,758.8421
3,248	3,678,430.8793	5,206,327.8224	8,884,758.7017
3,249	3,678,427.1752	5,206,331.3897	8,884,758.5650
3,250	3,678,423.4742	5,206,334.9577	8,884,758.4319
3,251	3,678,419.7763	5,206,338.5262	8,884,758.3025
3,252	3,678,416.0814	5,206,342.0953	8,884,758.1768
3,253	3,678,412.3897	5,206,345.6650	8,884,758.0547
3,254	3,678,408.7010	5,206,349.2353	8,884,757.9362
3,255	3,678,405.0153	5,206,352.8061	8,884,757.8215
3,256	3,678,401.3327	5,206,356.3776	8,884,757.7103
3,257	3,678,397.6532	5,206,359.9496	8,884,757.6028
3,258	3,678,393.9768	5,206,363.5222	8,884,757.4990
3,259	3,678,390.3034	5,206,367.0954	8,884,757.3988
3,260	3,678,386.6330	5,206,370.6692	8,884,757.3022
3,261	3,678,382.9657	5,206,374.2436	8,884,757.2093
3,262	3,678,379.3014	5,206,377.8185	8,884,757.1199
3,263	3,678,375.6402	5,206,381.3940	8,884,757.0342
3,264	3,678,371.9821	5,206,384.9701	8,884,756.9522

λ=6			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
3,265	3,678,368.3269	5,206,388.5468	8,884,756.8737
3,266	3,678,364.6748	5,206,392.1240	8,884,756.7989
3,267	3,678,361.0258	5,206,395.7019	8,884,756.7276
3,268	3,678,357.3797	5,206,399.2803	8,884,756.6600
3,269	3,678,353.7367	5,206,402.8593	8,884,756.5960
3,270	3,678,350.0968	5,206,406.4388	8,884,756.5356
3,271	3,678,346.4598	5,206,410.0189	8,884,756.4787
3,272	3,678,342.8259	5,206,413.5997	8,884,756.4255
3,273	3,678,339.1949	5,206,417.1809	8,884,756.3759
3,274	3,678,335.5670	5,206,420.7628	8,884,756.3298
3,275	3,678,331.9421	5,206,424.3452	8,884,756.2874
3,276	3,678,328.3202	5,206,427.9282	8,884,756.2485
3,277	3,678,324.7014	5,206,431.5118	8,884,756.2132
3,278	3,678,321.0855	5,206,435.0960	8,884,756.1814
3,279	3,678,317.4726	5,206,438.6807	8,884,756.1533
3,280	3,678,313.8627	5,206,442.2660	8,884,756.1287
3,281	3,678,310.2558	5,206,445.8518	8,884,756.1077
3,282	3,678,306.6519	5,206,449.4383	8,884,756.0902
3,283	3,678,303.0510	5,206,453.0253	8,884,756.0763
3,284	3,678,299.4531	5,206,456.6129	8,884,756.0659
3,285	3,678,295.8581	5,206,460.2010	8,884,756.0591
3,286	3,678,292.2662	5,206,463.7897	8,884,756.0559
3,287	3,678,288.6772	5,206,467.3790	8,884,756.0562

λ=7			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,971	3,679,586.6701	5,204,131.7000	8,883,718.3701
2,972	3,679,581.9891	5,204,136.1411	8,883,718.1302
2,973	3,679,577.3122	5,204,140.5828	8,883,717.8951
2,974	3,679,572.6393	5,204,145.0252	8,883,717.6646
2,975	3,679,567.9704	5,204,149.4683	8,883,717.4387
2,976	3,679,563.3055	5,204,153.9120	8,883,717.2176
2,977	3,679,558.6447	5,204,158.3564	8,883,717.0011
2,978	3,679,553.9878	5,204,162.8014	8,883,716.7893
2,979	3,679,549.3350	5,204,167.2471	8,883,716.5821
2,980	3,679,544.6861	5,204,171.6935	8,883,716.3796
2,981	3,679,540.0412	5,204,176.1405	8,883,716.1817
2,982	3,679,535.4003	5,204,180.5881	8,883,715.9884
2,983	3,679,530.7634	5,204,185.0364	8,883,715.7998
2,984	3,679,526.1305	5,204,189.4853	8,883,715.6158
2,985	3,679,521.5016	5,204,193.9349	8,883,715.4365
2,986	3,679,516.8766	5,204,198.3851	8,883,715.2618
2,987	3,679,512.2556	5,204,202.8360	8,883,715.0916

λ=7			
Q	Cr(Q)	Cs(λ,Q)	Cc(λ,Q)
2,988	3,679,507.6386	5,204,207.2876	8,883,714.9261
2,989	3,679,503.0255	5,204,211.7397	8,883,714.7652
2,990	3,679,498.4164	5,204,216.1926	8,883,714.6089
2,991	3,679,493.8112	5,204,220.6460	8,883,714.4573
2,992	3,679,489.2100	5,204,225.1002	8,883,714.3102
2,993	3,679,484.6127	5,204,229.5549	8,883,714.1676
2,994	3,679,480.0194	5,204,234.0103	8,883,714.0297
2,995	3,679,475.4300	5,204,238.4664	8,883,713.8964
2,996	3,679,470.8445	5,204,242.9230	8,883,713.7676
2,997	3,679,466.2630	5,204,247.3804	8,883,713.6434
2,998	3,679,461.6854	5,204,251.8383	8,883,713.5238
2,999	3,679,457.1117	5,204,256.2970	8,883,713.4087
3,000	3,679,452.5419	5,204,260.7562	8,883,713.2982
3,001	3,679,447.9761	5,204,265.2161	8,883,713.1922
3,002	3,679,443.4141	5,204,269.6766	8,883,713.0908
3,003	3,679,438.8561	5,204,274.1378	8,883,712.9939
3,004	3,679,434.3020	5,204,278.5996	8,883,712.9016
3,005	3,679,429.7517	5,204,283.0620	8,883,712.8138
3,006	3,679,425.2054	5,204,287.5251	8,883,712.7305
3,007	3,679,420.6629	5,204,291.9888	8,883,712.6517
3,008	3,679,416.1244	5,204,296.4532	8,883,712.5775
3,009	3,679,411.5897	5,204,300.9181	8,883,712.5078
3,010	3,679,407.0589	5,204,305.3837	8,883,712.4426
3,011	3,679,402.5319	5,204,309.8500	8,883,712.3819
3,012	3,679,398.0089	5,204,314.3169	8,883,712.3257
3,013	3,679,393.4897	5,204,318.7844	8,883,712.2740
3,014	3,679,388.9743	5,204,323.2525	8,883,712.2268
3,015	3,679,384.4628	5,204,327.7213	8,883,712.1841
3,016	3,679,379.9552	5,204,332.1907	8,883,712.1459
3,017	3,679,375.4515	5,204,336.6607	8,883,712.1121
3,018	3,679,370.9515	5,204,341.1313	8,883,712.0829
3,019	3,679,366.4555	5,204,345.6026	8,883,712.0581
3,020	3,679,361.9632	5,204,350.0745	8,883,712.0377
3,021	3,679,357.4748	5,204,354.5471	8,883,712.0219
3,022	3,679,352.9902	5,204,359.0202	8,883,712.0105
3,023	3,679,348.5095	5,204,363.4940	8,883,712.0035
3,024	3,679,344.0326	5,204,367.9684	8,883,712.0010
3,025	3,679,339.5595	5,204,372.4434	8,883,712.0029
3,026	3,679,335.0902	5,204,376.9191	8,883,712.0093
3,027	3,679,330.6247	5,204,381.3954	8,883,712.0201
3,028	3,679,326.1631	5,204,385.8723	8,883,712.0354
3,029	3,679,321.7052	5,204,390.3498	8,883,712.0550
3,030	3,679,317.2512	5,204,394.8279	8,883,712.0791

[illegible] $\lambda=8$			
Q	$C_r(Q)$	$C_s(\lambda, Q)$	$C_c(\lambda, Q)$
2,766	3,680,637.6132	5,202,769.2160	8,883,406.8292
2,767	3,680,632.0127	5,202,774.5456	8,883,406.5583
2,768	3,680,626.4172	5,202,779.8759	8,883,406.2931
2,769	3,680,620.8266	5,202,785.2069	8,883,406.0335
2,770	3,680,615.2410	5,202,790.5387	8,883,405.7797
2,771	3,680,609.6604	5,202,795.8711	8,883,405.5315
2,772	3,680,604.0848	5,202,801.2042	8,883,405.2890
2,773	3,680,598.5141	5,202,806.5381	8,883,405.0522
2,774	3,680,592.9484	5,202,811.8727	8,883,404.8211
2,775	3,680,587.3876	5,202,817.2079	8,883,404.5956
2,776	3,680,581.8318	5,202,822.5439	8,883,404.3757
2,777	3,680,576.2809	5,202,827.8806	8,883,404.1615
2,778	3,680,570.7350	5,202,833.2180	8,883,403.9530
2,779	3,680,565.1939	5,202,838.5561	8,883,403.7500
2,780	3,680,559.6578	5,202,843.8949	8,883,403.5527
2,781	3,680,554.1267	5,202,849.2344	8,883,403.3610
2,782	3,680,548.6004	5,202,854.5746	8,883,403.1750
2,783	3,680,543.0790	5,202,859.9155	8,883,402.9945
2,784	3,680,537.5626	5,202,865.2571	8,883,402.8197
2,785	3,680,532.0510	5,202,870.5994	8,883,402.6504
2,786	3,680,526.5443	5,202,875.9424	8,883,402.4867
2,787	3,680,521.0425	5,202,881.2861	8,883,402.3286
2,788	3,680,515.5456	5,202,886.6305	8,883,402.1761
2,789	3,680,510.0536	5,202,891.9756	8,883,402.0292
2,790	3,680,504.5665	5,202,897.3214	8,883,401.8878
2,791	3,680,499.0842	5,202,902.6678	8,883,401.7520
2,792	3,680,493.6067	5,202,908.0150	8,883,401.6217
2,793	3,680,488.1341	5,202,913.3629	8,883,401.4970
2,794	3,680,482.6664	5,202,918.7114	8,883,401.3779
2,795	3,680,477.2035	5,202,924.0607	8,883,401.2642
2,796	3,680,471.7455	5,202,929.4106	8,883,401.1561
2,797	3,680,466.2923	5,202,934.7613	8,883,401.0536
2,798	3,680,460.8439	5,202,940.1126	8,883,400.9565
2,799	3,680,455.4003	5,202,945.4646	8,883,400.8650
2,800	3,680,449.9616	5,202,950.8173	8,883,400.7789
2,801	3,680,444.5277	5,202,956.1707	8,883,400.6984
2,802	3,680,439.0985	5,202,961.5248	8,883,400.6233
2,803	3,680,433.6742	5,202,966.8796	8,883,400.5538
2,804	3,680,428.2547	5,202,972.2350	8,883,400.4897
2,805	3,680,422.8400	5,202,977.5912	8,883,400.4311
2,806	3,680,417.4300	5,202,982.9480	8,883,400.3780
2,807	3,680,412.0249	5,202,988.3055	8,883,400.3303
2,808	3,680,406.6245	5,202,993.6637	8,883,400.2882

[illegible]			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,809	3,680,401.2289	5,202,999.0225	8,883,400.2514
2,810	3,680,395.8380	5,203,004.3821	8,883,400.2201
2,811	3,680,390.4520	5,203,009.7423	8,883,400.1943
2,812	3,680,385.0706	5,203,015.1032	8,883,400.1739
2,813	3,680,379.6941	5,203,020.4648	8,883,400.1589
2,814	3,680,374.3223	5,203,025.8271	8,883,400.1493
2,815	3,680,368.9552	5,203,031.1900	8,883,400.1452
2,816	3,680,363.5928	5,203,036.5536	8,883,400.1464
2,817	3,680,358.2352	5,203,041.9179	8,883,400.1531
2,818	3,680,352.8823	5,203,047.2829	8,883,400.1652
2,819	3,680,347.5342	5,203,052.6485	8,883,400.1827
2,820	3,680,342.1907	5,203,058.0148	8,883,400.2056
2,821	3,680,336.8520	5,203,063.3818	8,883,400.2338
2,822	3,680,331.5180	5,203,068.7495	8,883,400.2674
2,823	3,680,326.1886	5,203,074.1178	8,883,400.3065
2,824	3,680,320.8640	5,203,079.4868	8,883,400.3508
2,825	3,680,315.5441	5,203,084.8565	8,883,400.4006

[illegible]			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,597	3,681,660.4642	5,201,897.8884	8,883,558.3526
2,598	3,681,653.9362	5,201,904.1080	8,883,558.0442
2,599	3,681,647.4143	5,201,910.3284	8,883,557.7427
2,600	3,681,640.8984	5,201,916.5496	8,883,557.4480
2,601	3,681,634.3885	5,201,922.7715	8,883,557.1600
2,602	3,681,627.8846	5,201,928.9942	8,883,556.8788
2,603	3,681,621.3867	5,201,935.2177	8,883,556.6044
2,604	3,681,614.8948	5,201,941.4419	8,883,556.3367
2,605	3,681,608.4089	5,201,947.6669	8,883,556.0758
2,606	3,681,601.9289	5,201,953.8926	8,883,555.8216
2,607	3,681,595.4549	5,201,960.1191	8,883,555.5741
2,608	3,681,588.9869	5,201,966.3464	8,883,555.3333
2,609	3,681,582.5249	5,201,972.5744	8,883,555.0993
2,610	3,681,576.0688	5,201,978.8032	8,883,554.8719
2,611	3,681,569.6186	5,201,985.0327	8,883,554.6513
2,612	3,681,563.1744	5,201,991.2630	8,883,554.4374
2,613	3,681,556.7361	5,201,997.4940	8,883,554.2301
2,614	3,681,550.3037	5,202,003.7258	8,883,554.0295
2,615	3,681,543.8772	5,202,009.9583	8,883,553.8356
2,616	3,681,537.4567	5,202,016.1916	8,883,553.6483
2,617	3,681,531.0420	5,202,022.4257	8,883,553.4677
2,618	3,681,524.6333	5,202,028.6604	8,883,553.2937
2,619	3,681,518.2304	5,202,034.8960	8,883,553.1264

λ=9			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,620	3,681,511.8334	5,202,041.1323	8,883,552.9657
2,621	3,681,505.4423	5,202,047.3693	8,883,552.8116
2,622	3,681,499.0571	5,202,053.6070	8,883,552.6641
2,623	3,681,492.6777	5,202,059.8456	8,883,552.5232
2,624	3,681,486.3041	5,202,066.0848	8,883,552.3890
2,625	3,681,479.9365	5,202,072.3248	8,883,552.2613
2,626	3,681,473.5746	5,202,078.5656	8,883,552.1402
2,627	3,681,467.2186	5,202,084.8070	8,883,552.0257
2,628	3,681,460.8685	5,202,091.0493	8,883,551.9177
2,629	3,681,454.5241	5,202,097.2922	8,883,551.8163
2,630	3,681,448.1856	5,202,103.5359	8,883,551.7215
2,631	3,681,441.8528	5,202,109.7804	8,883,551.6332
2,632	3,681,435.5259	5,202,116.0255	8,883,551.5514
2,633	3,681,429.2048	5,202,122.2714	8,883,551.4762
2,634	3,681,422.8894	5,202,128.5181	8,883,551.4075
2,635	3,681,416.5799	5,202,134.7655	8,883,551.3453
2,636	3,681,410.2761	5,202,141.0136	8,883,551.2897
2,637	3,681,403.9781	5,202,147.2624	8,883,551.2405
2,638	3,681,397.6858	5,202,153.5120	8,883,551.1978
2,639	3,681,391.3993	5,202,159.7623	8,883,551.1616
2,640	3,681,385.1186	5,202,166.0133	8,883,551.1319
2,641	3,681,378.8436	5,202,172.2651	8,883,551.1087
2,642	3,681,372.5743	5,202,178.5176	8,883,551.0919
2,643	3,681,366.3107	5,202,184.7708	8,883,551.0816
2,644	3,681,360.0529	5,202,191.0248	8,883,551.0777
2,645	3,681,353.8008	5,202,197.2795	8,883,551.0803
2,646	3,681,347.5544	5,202,203.5349	8,883,551.0893
2,647	3,681,341.3137	5,202,209.7910	8,883,551.1047
2,648	3,681,335.0787	5,202,216.0478	8,883,551.1266
2,649	3,681,328.8494	5,202,222.3054	8,883,551.1548
2,650	3,681,322.6258	5,202,228.5637	8,883,551.1895
2,651	3,681,316.4079	5,202,234.8227	8,883,551.2306
2,652	3,681,310.1956	5,202,241.0824	8,883,551.2780
2,653	3,681,303.9890	5,202,247.3429	8,883,551.3319
2,654	3,681,297.7881	5,202,253.6041	8,883,551.3921
2,655	3,681,291.5928	5,202,259.8660	8,883,551.4587
2,656	3,681,285.4031	5,202,266.1286	8,883,551.5317

λ=10			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,456	3,682,644.7400	5,201,369.7222	8,884,014.4621
2,457	3,682,637.2873	5,201,376.8342	8,884,014.1215

[illegible] 1-10 [illegible]			
Q	Cr(Q)	Cs(C,Q)	Ce(C,Q)
2,458	3,682,629.8418	5,201,383.9470	8,884,013.7888
2,459	3,682,622.4034	5,201,391.0606	8,884,013.4640
2,460	3,682,614.9721	5,201,398.1751	8,884,013.1472
2,461	3,682,607.5479	5,201,405.2903	8,884,012.8383
2,462	3,682,600.1308	5,201,412.4064	8,884,012.5372
2,463	3,682,592.7208	5,201,419.5233	8,884,012.2441
2,464	3,682,585.3178	5,201,426.6410	8,884,011.9588
2,465	3,682,577.9219	5,201,433.7594	8,884,011.6814
2,466	3,682,570.5331	5,201,440.8787	8,884,011.4118
2,467	3,682,563.1513	5,201,447.9988	8,884,011.1501
2,468	3,682,555.7765	5,201,455.1197	8,884,010.8963
2,469	3,682,548.4088	5,201,462.2415	8,884,010.6503
2,470	3,682,541.0481	5,201,469.3640	8,884,010.4121
2,471	3,682,533.6944	5,201,476.4873	8,884,010.1817
2,472	3,682,526.3477	5,201,483.6114	8,884,009.9591
2,473	3,682,519.0080	5,201,490.7363	8,884,009.7443
2,474	3,682,511.6753	5,201,497.8620	8,884,009.5373
2,475	3,682,504.3496	5,201,504.9885	8,884,009.3381
2,476	3,682,497.0308	5,201,512.1158	8,884,009.1466
2,477	3,682,489.7190	5,201,519.2439	8,884,008.9629
2,478	3,682,482.4142	5,201,526.3728	8,884,008.7870
2,479	3,682,475.1163	5,201,533.5025	8,884,008.6188
2,480	3,682,467.8253	5,201,540.6330	8,884,008.4583
2,481	3,682,460.5412	5,201,547.7643	8,884,008.3055
2,482	3,682,453.2641	5,201,554.8963	8,884,008.1604
2,483	3,682,445.9939	5,201,562.0292	8,884,008.0231
2,484	3,682,438.7306	5,201,569.1628	8,884,007.8934
2,485	3,682,431.4742	5,201,576.2972	8,884,007.7714
2,486	3,682,424.2246	5,201,583.4325	8,884,007.6571
2,487	3,682,416.9820	5,201,590.5685	8,884,007.5504
2,488	3,682,409.7462	5,201,597.7052	8,884,007.4514
2,489	3,682,402.5173	5,201,604.8428	8,884,007.3601
2,490	3,682,395.2952	5,201,611.9812	8,884,007.2764
2,491	3,682,388.0800	5,201,619.1203	8,884,007.2003
2,492	3,682,380.8716	5,201,626.2602	8,884,007.1318
2,493	3,682,373.6700	5,201,633.4009	8,884,007.0709
2,494	3,682,366.4753	5,201,640.5424	8,884,007.0177
2,495	3,682,359.2873	5,201,647.6847	8,884,006.9720
2,496	3,682,352.1062	5,201,654.8277	8,884,006.9339
2,497	3,682,344.9319	5,201,661.9715	8,884,006.9034
2,498	3,682,337.7643	5,201,669.1161	8,884,006.8804
2,499	3,682,330.6035	5,201,676.2614	8,884,006.8650
2,500	3,682,323.4495	5,201,683.4076	8,884,006.8571

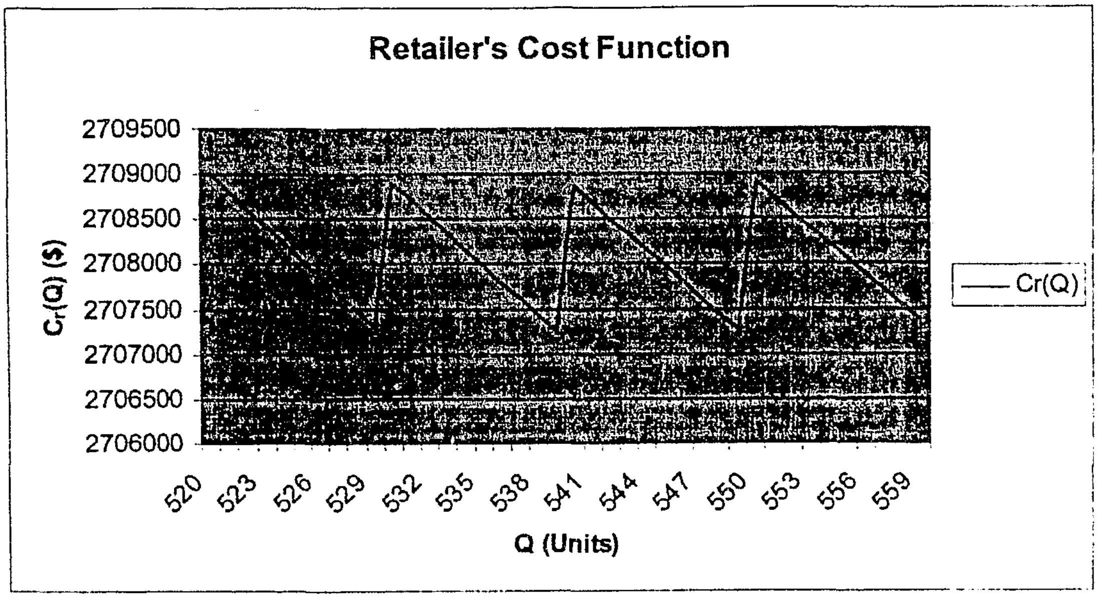
$\lambda=10$			
Q	Cr(Q)	Cs(λ, Q)	Cc(λ, Q)
2,501	3,682,316.3023	5,201,690.5545	8,884,006.8568
2,502	3,682,309.1618	5,201,697.7022	8,884,006.8640
2,503	3,682,302.0281	5,201,704.8506	8,884,006.8787
2,504	3,682,294.9011	5,201,711.9998	8,884,006.9009
2,505	3,682,287.7808	5,201,719.1498	8,884,006.9306
2,506	3,682,280.6672	5,201,726.3006	8,884,006.9678
2,507	3,682,273.5604	5,201,733.4521	8,884,007.0125
2,508	3,682,266.4603	5,201,740.6044	8,884,007.0647
2,509	3,682,259.3668	5,201,747.7574	8,884,007.1243
2,510	3,682,252.2801	5,201,754.9113	8,884,007.1914
2,511	3,682,245.2000	5,201,762.0659	8,884,007.2659
2,512	3,682,238.1266	5,201,769.2212	8,884,007.3478
2,513	3,682,231.0599	5,201,776.3773	8,884,007.4372
2,514	3,682,223.9999	5,201,783.5342	8,884,007.5340
2,515	3,682,216.9464	5,201,790.6918	8,884,007.6383

Appendix 1.8

Case III No Cordination Space: The table below shows the total retailers cost ($C_T(Q)$) tabulated against the value of each order quantity (Q).

Q	$C_T(Q)$	Q	$C_T(Q)$
520	2709010	540	2708873
521	2708816	541	2708693
522	2708623	542	2708514
523	2708431	543	2708336
524	2708239	544	2708159
525	2708048	545	2707982
526	2707858	546	2707805
527	2707669	547	2707630
528	2707481	548	2707455
529	2707293	549	2707281
530	2708906	550	2708907
531	2708719	551	2708734
532	2708533	552	2708562
533	2708348	553	2708390
534	2708164	554	2708219
535	2707981	555	2708048
536	2707798	556	2707878
537	2707615	557	2707709
538	2707434	558	2707540
539	2707253	559	2707372

Case III No Coordination Space: Figure below illustrates the change in the retailers cost with the change in Q



Appendix 1.9

Case III No Coordination Space: Table below shows the variation the supplier cost with the change in the lot size multiplier (λ)

λ	$Cs(\lambda,Q_E)$
1	1,559,115
2	1,512,867
3	1,551,774
4	1,611,971
5	1,680,682
6	1,752,152

The figure below illustrates the variation the suppliers cost ($C_s(\lambda,Q_E)$) with the lot size multiplier (λ)