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A comparison of some heuristics to improve shift generation in the labour scheduling problem

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A COMPARISON OF SOME HEURISTICS TO IMPROVE SHIFT GENERATION IN THE LABOUR SCHEDULING PROBLEM

By

Banafsheh Namiranian

(B.Sc, Isfahan University of Technology, Iran, 1997)

A thesis

presented to Ryerson University

in partial fulfillment of the

requirements for the degree of

Master of Applied Science

in

Mechanical Engineering

Toronto, Ontario, Canada

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BORROWER'S PAGE

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ABSTRACT

A COMPARISON OF SOME HEURISTICS TO IMPROVE SHIFT GENERATION IN THE LABOUR SCHEDULING PROBLEM

Banafsheh Namiranian
Master of Applied Science in Mechanical Engineering
Ryerson University, 2005

The use of part-time personnel with different skills and time-availability has risen dramatically over the last few decades. This fact, especially in service-oriented organizations with great demand fluctuations, makes the labour scheduling problem complicated. The Integer Programming (IP) technique is a common tool for assigning employees to right shifts but, due to its large size in this particular type of problem, the IP model cannot be solved optimally within a reasonable time. In order to reduce the problem size and, subsequently, the computational time, one approach is to reduce the number of decision variables which are defined as shifts in the labour scheduling problem. This thesis presents eight different heuristics to generate a smaller set of shifts instead of exhaustively generating all possible shifts. With the aid of statistical analysis, the proposed algorithms are compared, and the best heuristic in terms of efficiency and effectiveness is introduced.

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NOMENCLATURE

LP	Linear Programming
NP	Non-linear Programming
IP	Integer Programming
i	index on employees
j	index on shifts
k	index on days
m	index on skills
n	index on intervals
X_{ij}	employee-shift combination, 1 if employee i works shift j ; 0 otherwise
L_j	length of shift j
S_{mnk}	subset of shifts that contain skill m , interval n and day k
R_{mnk}	minimum staffing requirement for interval n of day k for skill m
D_{jk}	1 if shift j occurs in day k ; 0 otherwise
X_j	number of employees required to perform shift j
S_n	subset of shifts that contain interval n
R_n	minimum staffing requirement for interval n

CHAPTER 1: INTRODUCTION & LITERATURE REVIEW

1.1 Labour Scheduling

Service production systems are quite different from goods production or manufacturing systems. Aggarwal (1982) presented the following six major differences and special characteristics of services that distinguish them from manufacturing.

- 1) The output of service systems cannot be inventoried.
- 2) The demand fluctuation for services is considerably greater than manufacturing over time. It varies from month-to-month, day-to-day and even hour-to-hour. Also, seasonal variations of demand exist.
- 3) The demand for services cannot be backlogged as it gets lost.
- 4) Most service systems have a large amount of labour as customers or users receive the service directly from servers.
- 5) In order to meet the peak or near-peak demand, service systems need to be designed with an excessive equipment and space capacity.
- 6) Emergency services and electric power must be kept supplied around the clock.

As the subsequent of the above characteristics, Mabert and Watts (1982) enumerated two other differences between service industries and manufacturing. First, manufacturing firms have a few specific work schedules, for example, 7am-3pm, 3pm-11pm and 11pm-7am. This is in contrast with service operations that have numerous shifts, which sometimes happen to be overlapping in order to meet the requirements at peak periods, due to the fact that service cannot be inventoried. These shifts vary in the start time and the length, for example, 9am-5pm, 10am-6pm, 1pm-9pm, and 3pm-9pm. Second, in manufacturing environments, employees work full-time on a five-day basis but, in service systems, in order to handle the demand variation, many part-time workers who differ in their times of availability and skills are present.

Attributes of service-oriented organizations show that human resources are generally the most expensive resources for the service sector. Therefore, managers of these organizations are seeking efficient labour scheduling. In service industries, overstaffing results in excess labour cost, due to payroll expenditure, while understaffing is a greater concern that leads to poor customer service and a potential loss of future sales. It can also result in a lower quality of life for employees who work fewer hours than they desire. Labour scheduling is the act of balancing customer demand, employee work requests and profitability (Thompson, 1998). Thompson (1995) divided the labour scheduling problem into four steps: (1) forecast customer demand; (2) translate the forecasts into requirements for employees; (3) develop the labour schedule; and (4) control the delivery of the service in real time. In the literature, most researchers have focused upon step (3), or the problem of daily or weekly labour scheduling. In order to clarify the first two steps, an example from the literature is reviewed here. Loucks and Jacobs (1991) chose a fast food restaurant with eight tasks and a 128-hour, seven-day workweek. In the first step, by using one of the forecasting methods, they forecast the sales volume expected in each operating hour of the week. Suppose the forecast for Monday of a given week is as shown in Table 1.

Table 1: Forecast of Hourly Sales for Monday

Hour (starting time)	Forecast of Hourly Sales (\$)	Hour (starting time)	Forecast of Hourly Sales (\$)
6 a.m.	70	3 p.m.	170
7 a.m.	85	4 p.m.	130
8 a.m.	100	5 p.m.	250
9 a.m.	100	6 p.m.	265
10 a.m.	100	7 p.m.	180
11 a.m.	255	8 p.m.	100
12 p.m.	440	9 p.m.	100
1 p.m.	270	10 p.m.	50
2 p.m.	155	11 p.m.	40

In order to translate the forecast into requirements for employees (second step), Loucks and Jacobs (1991) used the conversion table shown in Table 2. This table expresses how the hourly sales forecast from step (1) can be converted to the number of required employees for every planning period (every hour). As an example, if at any given hour, the sales forecast is between \$246 and \$270, according to the conversion table, a total of nine people are required. Past experiences have shown that these people should be distributed over the working area as follows.

Three employees in Grill area, two in Drive-thru window, as well as two for Customer counter. Bin caller and French fry vats need one employee each.

Table 2: Sales-to-staffing Conversion Table

Forecast of Hourly Sales (\$)	Total Workers Required	Workers Required by Tasks*							
		A	B	C	D	E	F	G	H
0 to 80	4	2	1	1					
81 to 125	5	2	1	1	1				
126 to 160	6	2	1	1	1	1			
161 to 195	7	2	2	1	1	1			
196 to 245	8	2	2	2	1	1			
246 to 270	9	3	2	2	1	1			
271 to 320	10	3	3	2	1	1			
321 to 395	11	3	3	3	1	1			
396 to 445	12	4	3	3	1	1			
446 to 470	13	4	3	3	1	1	1		
471 to 520	14	4	4	3	1	1	1		
521 to 540	15	4	4	3	1	1	1	1	
541 to 640	16	4	4	3	1	1	1	1	1
641 to 700	17	5	4	3	1	1	1	1	1
701 to 790	18	5	4	4	1	1	1	1	1
791 to 985	19	5	4	4	2	1	1	1	1

*A= Grill Area, B= Drive-thru window, C= Customer counter, D= Bin caller, E= French fry vats, F= Lobby cleaning, G= Special cashier, H= Drink drawer

With the aid of the conversion table and the forecast from step (1), the total hourly requirements (for all tasks) for Monday range from a low of four workers to a high of twelve. For more details, refer to Table 3. To explain how the numbers in this table have been reached, take 3 p.m. as an instance. According to Table 1, the sale for that period has been forecasted as \$170. With a look at Table 2, \$170 would be translated into seven employees as it takes place in the fourth ranges of forecast (161-195). Notice that this

number (seven) has been reflected in Table 3 for 3 p.m. The right part of Table 2 also defines how to assign these employees to different working areas; two in Grill area, two in Drive-thru window and one in each of Customer counter, Bin caller and French fry vats.

Table 3: Workers Required (by Hours and Day) for Monday

Hour (starting time)	Total Workers Required	Hour (starting time)	Total Workers Required
6 a.m.	4	3 p.m.	7
7 a.m.	5	4 p.m.	6
8 a.m.	5	5 p.m.	9
9 a.m.	5	6 p.m.	9
10 a.m.	5	7 p.m.	7
11 a.m.	9	8 p.m.	5
12 p.m.	12	9 p.m.	5
1 p.m.	9	10 p.m.	4
2 p.m.	6	11 p.m.	4

Step (3), developing a “schedule” or a “roster”, has been defined by Litchfield et al. (2003) as “the assignment of specific employees to specific work patterns”. This phase, which is the focus of this study, will be discussed in more details in the following section.

Step (4) modifies the labour scheduling in real time (Thompson, 1993). There is a high possibility that the decisions made in the earlier stages are not right. The sales-dollar forecast can be far from the real sales. Employees might call in sick and not attend the workplace. In order to keep the service at a satisfactory level with a low cost, an effective real time control is needed.

1.2 Tour Scheduling

As mentioned earlier, step (3), known as the labour scheduling or rostering problem, has been the main focus of researchers. Baker (1976) classified the labour scheduling

problem into three main categories: (1) *shift scheduling*, (2) *days-off scheduling*, and (3) *tour scheduling*. Shift scheduling seeks a minimum number of employees together with their assigned shift schedules to satisfy fluctuating demand requirements over a single day (Morris and Showalter, 1983). The objective of the days-off scheduling problem is to minimize the work force size necessary to meet all predetermined daily staffing requirements for a week and allotting a specified number of consecutive idle days to each employee (Loucks and Jacobs, 1991). Tour scheduling is the combination of the first two models and involves the determination of both work hours of the day and workdays of the week for each employee (Alfares, 2004). Therefore, tour scheduling is applicable for organizations with a weekly planning horizon. Service industries such as fast food restaurants, retail stores, police stations and airlines which operate seven days a week with more than one shift a day, utilize this model.

According to Alfares' survey (2004), tour scheduling problem has proven difficult to solve optimally due to its large size and pure integer nature. Over the last decade, many studies have concentrated on modeling and solving this problem. Alfares (2004) has classified the solution techniques into ten categories. One of these techniques is decomposition.

In decomposition technique, the large tour scheduling problem is broken into some sub-problems which are smaller and easier. Breaking the tour scheduling problem into "shift scheduling" and "days-off scheduling" sub-problems is one of the most popular approaches. Melachrinoudis and Olafsson (1992) applied this method to schedule supermarket cashiers. They broke down the tour scheduling problem into seven daily shift scheduling problems, which were solved separately. The solutions then were combined to create feasible days-off schedules.

Khoong (1993) reversed the order of the two phases presented by Melachrinoudis and Olafsson (1992). After determination of shift labour requirements, he generated days-off schedules first and then in the last phase assigned the work shifts.

Lauer et al. (1994) implemented a decomposition method to schedule students who were working as part-time employees in some college computer labs. In phase one, an LP model helps to find the optimal daily shift schedules and in phase two, students are assigned to their schedules.

Mason, Ryan, and Panton (1998) used a decomposition procedure to schedule an airport customs staff. They combined heuristics, optimization, and simulation methods. First, a simulation model is used to determine the staffing requirements. Then, with the aid of an IP model, personnel are assigned to shifts for each day of the week. Finally, by integrating the seven daily shift schedules from the previous phase, a days-off schedule is created.

For the fast food industry, Love and Hoey (1990) developed an integer linear program and decomposed it into two sub-problems. The first sub-problem determines the shift requirements for employees to satisfy staffing needs and to minimize overstaffing. The second sub-problem determines the assignment of employees to the shift requirements from the first sub-problem. Other people who worked in the fast food area are Loucks and Jacobs (1991), and Litchfield et al. (2003) who have developed some heuristics.

Loucks and Jacobs (1991) considered a dual problem of tour scheduling involving workers with different time-availability and skills. They applied goal programming as a mathematical technique to prioritize and optimize a set of multiple objectives. The primary goal (the manager's concern) is to minimize overstaffing while meeting the labour demand for each planning period. The secondary objective (the workers' concern) is to minimize deviations from the target work hours for each employee. Since the proposed binary goal programming model is too large to be solved optimally, a heuristic approach consisting of two phases, a construction phase followed by an improvement phase, was suggested.

Litchfield et al. (2003) described a heuristic algorithm consisting of an initial construction phase followed by a tabu search. In the first phase, employees are assigned

to planning periods sequentially until the number of assignments reaches the sum of the staffing requirements over all days and all planning periods. The heuristic ensures that employees are not assigned to planning periods in which they are not available. It also prevents an employee from being assigned to more than one planning period in the same day. Another constraint that is considered in the construction phase is to prevent an employee from being assigned to a night shift one day and a morning shift the following day. The solution obtained from the first phase is used as a starting point for the tabu search heuristic (second phase).

A review of the literature shows that decomposition is used through a linear programming or integer programming approach that searches for an optimal solution. In this study, decomposition is applied to tour scheduling problems. However, unlike other studies, here a heuristic is used. The advantage of heuristics is that they may obtain a near optimal solution in a shorter time. Therefore, they may be preferable when computational efficiency is more important than optimality.

1.3 Optimization

Optimization can simply be defined as finding the best way to do something and clearly it is an inseparable part of everyday life. People try to find the best way of using their resources while considering some limitations. This is exactly what businesses face on a larger scale. They are seeking a solution to maximize their revenue, minimize their cost and, in short, reach an operational efficiency without violating the imposed constraints. All these goals may be attained by developing an appropriate optimization model.

Optimization models consist of three main elements:

- 1) An *objective function* that is going to be maximized or minimized. Besides maximizing the profit or minimizing the cost, sometimes the total deviation of observed data from predictions is minimized or, in an automobile industry, the strength of a panel is desired to be maximized while, in a service industry, minimizing the total hours worked by employees may be the goal.

- 2) A set of *variables* which affect the value of the objective function. Amount of money gained daily or time spent on each activity are examples of decision variables. In the panel problem, the variables can be the shape and dimensions of the panel while, in the service sector, the number of people working each day can be defined as the decision variable.
- 3) A set of *constraints* that allow the decision variables to take on certain values but exclude others. For example, it does not make sense to spend a negative amount of time on any activity, so all the "time" variables are constrained to be non-negative. In the panel design problem, there might be a limit on the weight of the product while limited time availability is a constraint for service operations.

Therefore, the optimization problem can be summarized as: finding values of the decision variables that minimize or maximize the objective function while satisfying the constraints. One concern in developing optimization problems is the size of the problem. The size of an optimization problem is defined as the number of decision variables in addition to the number of constraints.

1.4 Linear and Integer Linear Programming

Optimization problems are classified into two major categories: Linear Programming (LP) and Non-linear Programming (NP). If an optimization problem has a linear objective function and also each of its constraints is a linear equation or linear inequality, it is called an LP problem. In a first approach, linear optimization may seem quite theoretical but it has a lot of practical applications in real problems.

The problem of integer programming (or integer linear programming) requires some or all of the variables to take on integer values. Integer programs (IPs) often have the advantage of being more realistic than LPs but the disadvantage of being much harder to solve.

An IP in which all variables are required to be integers is called a *pure integer* programming problem. An IP in which only some of the variables are required to be

integers is called a *mixed integer* programming problem. Finally, an integer programming problem in which all the variables must equal 0 or 1 is called a *0-1* or *binary* IP. The latter is the most common one, because many kinds of combinatorial and logical restrictions can be modeled through the use of 0-1 variables.

1.5 Tour Scheduling Problem as an IP Problem

The employee tour scheduling problem has been modeled as an integer programming problem. According to Bechtold et al. (1991), in the literature, the following criteria have been suggested as objective functions:

- 1) Total labour hours scheduled
- 2) Total number of employees
- 3) Labour costs
- 4) Unscheduled labour costs
- 5) Customer service
- 6) Overstaffing
- 7) Understaffing
- 8) Number of schedules with consecutive days off
- 9) Number of different work schedules utilized
- 10) Some combination of the above

Basic workload assignment, government or union regulation, cost and budget considerations, as well as employee quality of life, form the constraints for this specific type of IP problem (Quan, 2004). Alfares (2004) identified various constraints involved in the tour scheduling problem as: (1) the allowable shift starting time, (2) the minimum and maximum length of each shift, (3) the frequency and duration of meal and rest breaks, (4) the minimum rest period between shift changes, (5) the operating hours per day, (6) the number of workdays per week, (7) the limits on the number of consecutive workdays, (8) shift rotation, (9) labour demand patterns, (10) employee preferences, (11) seniority rules, and (12) fairness in assigning employees to a schedule.

1.6 Research Scope and Objectives

In this research, the labour scheduling problem is broken into two parts. In the majority of past studies, both parts are considered at the same time, which leads to large size problems. The objective in these problems is to find the appropriate number of workers and assignment of shifts to them.

This thesis, unlike other studies, just focuses on the first part of the labour scheduling problem, which consequently reduces the size of the problem. The goal here is to show that, for improving the efficiency in solving the entire problem of labour scheduling, rather than the set of all possible shifts, a selected subset of shifts should be generated. To achieve this goal, various algorithms are developed and examined later to identify the best of them.

1.7 Thesis Organization

This thesis is organized as follows. In chapter 2, the problem is explained in more detail, along with the definition of terms, mathematical models and notation. To show the importance of the work, an example is also presented in this chapter. Chapter 3 introduces all heuristics, but first, based on the requirements trend, six patterns are differentiated. In order to evaluate the performance of the proposed methods, statistical analysis is conducted in chapter 4. Finally, chapter 5 provides conclusions and research extensions.

CHAPTER 2: PROBLEM DESCRIPTION

2.1 Overview

The objective function for the model studied in this research is to minimize the total labour hours scheduled subject to some of the basic constraints of employee scheduling problem such as:

- 1) Enforce the staffing requirement in every interval
- 2) Honour employee availability
- 3) Honour skill requirements
- 4) Do not schedule more than one shift per day
- 5) Do not assign staff more than maximum hours per day
- 6) Do not assign staff less than minimum hours per day

Since, in most practical cases, the large size of the problem makes IP inefficient for obtaining the optimal solution, this research suggests breaking down the problem into two parts: the *Pre-processor* and the *Main part*. The Pre-processor, which is the model construction part, develops the model. It prepares the necessary data for sending to the Main part, where the model is being solved. Pre-processor also helps to reduce the problem size and consequently computational time.

As a matter of fact, the Pre-processor is an assistant for the Main part that facilitates the process of problem solving (i.e., optimization). By considering some issues in advance, the Pre-processor does not let any unnecessary data proceed to the Main part. Another advantage of applying the Pre-processor is that what happens in this part can be simply coded into one of the programming languages in order to get the output (the desired data for entering the Main part) very quickly and effortlessly.

To have a better understanding of the Pre-processor part and the mathematical model which appears in the Main part, first the following terms should be defined:

- *Shift* – a work pattern specified by day, skill (job), start time and end time, for example, “9am-1pm on Monday as a cashier.”
- *Planning Interval* – the smallest segment of time that each day has been divided to for the purpose of scheduling, for example, in a store that demand is fluctuating every half an hour, the interval is considered 30 minutes.
- *Staffing/Interval Requirement* – number of employees required for a particular day, interval and skill, for example, 2 cashiers are needed for Monday 9am-9:30am.
- *Opening/Closing Time* – times that the service firms start/end working.
- *Min/Max Length* – minimum/maximum allowable shift length in the service firm, for example, Min/Max Length 4/8 means minimum length four hours and maximum length eight hours and it indicates that no shift shorter than four hours and longer than eight hours can exist.
- *Shift Start Time* – acceptable time to start a shift, for example, in some stores shifts must start on the hour like 9am, 10am, but not at 9:30am, or 10:15am.
- *Shift Increment* – portion of time that determines all possible shift lengths, for example, a 60-minute shift increment when Min/Max Length is 4/8 generates 4, 5, 6, 7 and 8 hours shifts, and thus shifts like 9am-1:30pm cannot exist.

2.2 Pre-processor

The Pre-processor contains two phases. In the first phase, by considering Opening/Closing Time, Min/Max Length, Shift Start Time and Shift Increment, all the potential shifts are generated. To clarify this, assume a store with the Opening/Closing Time of 9am-5pm every day, seven days a week. Min/Max Length for this location is 6/8 and Shift Start Time as well as Shift Increment is hourly. All the possible shifts generated in the first phase for cashiers on Monday would be as follows;

- Shift 1: (Monday, cashier, 9am, 3pm)
- Shift 2: (Monday, cashier, 9am, 4pm)
- Shift 3: (Monday, cashier, 9am, 5pm)
- Shift 4: (Monday, cashier, 10am, 4pm)

Shift 5: (Monday, cashier, 10am, 5pm)

Shift 6: (Monday, cashier, 11am, 5pm)

In the second phase, all choices for “who can do what job and when” are made. In other words, each employee is assigned to each possible shift generated in the first phase. Also, in this step, a few staffing rules are taken into consideration to eliminate some of the potential employee-shift combinations. These rules are:

- 1) Honour employee availability.
- 2) Honour skill requirements.
- 3) Do not assign staff more than maximum hours per day.
- 4) Do not assign staff less than minimum hours per day.

To explain what exactly happens at this stage, recall the example from the previous phase. If in the above store, Employee 1 is not available before 10am, then combinations (Employee 1, Shift 1), (Employee 1, Shift 2) and (Employee 1, Shift 3) cannot be created. Also, if Employee 2 does not have the right skill to work as a cashier, he/she cannot be paired up with any of the above shifts. As another scenario, if maximum hours that employee 3 can work every day is 7, then (Employee 3, Shift 3) will not exist.

Generally speaking, the resulting output of the Pre-processor is the set of potential valid employee-shift combinations that passes to the Main part. Figure 1 demonstrates different parts of the problem along with their inputs and outputs.

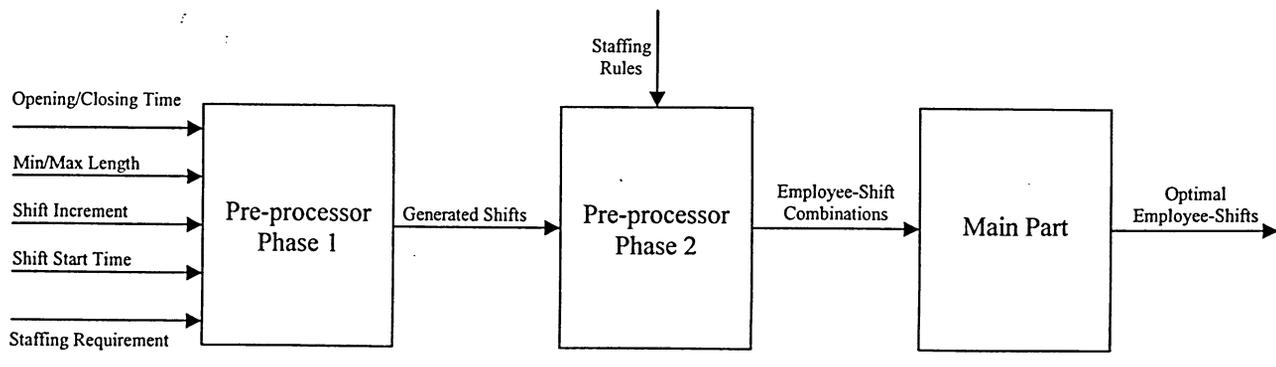


Figure 1- Flow of data through the problem

2.3 Main Part

The Main part is a 0-1 integer programming model with the following notations:

i = index on employees

j = index on shifts

k = index on days

m = index on skills

n = index on intervals

X_{ij} = 1 if employee i works shift j ; 0 otherwise

L_j = length of shift j

S_{mnk} = subset of shifts that contain skill m , interval n and day k

R_{mnk} = minimum staffing requirement for interval n of day k for skill m

D_{jk} = 1 if shift j occurs in day k ; 0 otherwise

The objective function is to minimize the total labour hours scheduled or

$$\text{Minimize} \quad \sum_i \sum_j X_{ij} * L_j$$

Subject to the constraint set

$$\sum_i \sum_{j \in S_{mnk}} X_{ij} \geq R_{mnk} \quad \text{For all } m, n, k \quad (1)$$

$$\sum_j X_{ij} * D_{jk} \leq 1 \quad \text{For all } i, k \quad (2)$$

$$X_{ij} \in \{0, 1\} \quad \text{For all } i, j \quad (3)$$

Constraint (1) enforces the staffing requirement in every interval. Constraint (2) ensures that employees are not assigned to more than one shift per day, and constraint (3) imposes binary assignment. There is no constraint on availability, skill, and the minimum

and maximum allowable working hours in a day, as they have already been considered in the Pre-processor part.

Note that the binary decision variables X_{ij} are the valid employee-shift combinations that were generated in the Pre-processor part. Therefore, by reducing the number of choices that have been paired up in the Pre-processor, the size of the problem and consequently the computational time will be cut. In this thesis, a few algorithms will be introduced that can be used in the first phase of the Pre-processor to cut the number of generated shifts. It means instead of exhaustively generating all possible shifts (master set), by utilizing one of these methods, a smaller subset of shifts (i) will be generated. Thus, the number of decision variables created in the second phase will be cut dramatically.

2.4 Example

In order to show that utilizing one of the proposed algorithms has a great impact on the problem size, a numerical example from a retail store is presented in this section. The problem involves 150 employees with 10 different skills such as cashiering, stocking, customer service, etc. Every employee has the knowledge of all 10 skills. This store runs seven days a week from 9 o'clock in the morning to 9 o'clock in the evening. Min/Max Length for this location is 4/8 and Shift Start Time as well as Shift Increment is quarter hourly or fifteen minute interval.

If there is no availability restriction (everybody is available any time any day), then the number of all possible shifts, just for one day and for one skill, is equal to 425 shifts. Start and end time of some of these shifts are as illustrated in Table 4.

As it is shown, the last possible shift starts at 5 o'clock in the evening. No shift can start after 5:00pm, because the Min length was set 4-hour long. If a shift starts later than 5:00pm, as it should end by Closing Time (9:00pm), the shift length would be shorter than 4 hours.

Table 4: Table of All Possible Shifts for One Day and One Skill

9:00am-1:00pm	9:15am-1:15pm		
9:00am-1:15pm	9:15am-1:30pm		
9:00am-1:30pm	9:15am-1:45pm		
.			
.			
.			
9:00am-4:45pm	9:15am-5:00pm		
9:00am-5:00pm	9:15am-5:15pm	.	5:00pm-9:00pm

For this store, the number of all possible shifts for a week considering all 10 skills is $425 * 7 * 10 = 29750$ shifts. This will be the number of shifts generated in the first phase of the Pre-processor if no reduction method is applied. In the second phase, these shifts are assigned to each employee to create the decision variables. As there are no skill and availability constraints in effect, the number of employee-shift combinations will be $29750 * 150 = 4462500$. The entire list of all combinations is the output of the Pre-processor that is fed into the Main part as binary decision variables.

Now, if using a method, one can reduce the number of the generated shifts for one day and one skill from 425 to 23 then, the size of the master set and subsequently employee-shift combinations will change to $23 * 7 * 10 = 1610$ and $1610 * 150 = 241500$, respectively.

According to the above example, it is obvious that the number of generated shifts in the first phase of the Pre-processor has a significant impact on the number of the decision variables in the Main part.

2.5 Assumptions

In this study the following assumptions have been made:

- 1) Max Length must be at least half as much as the difference between Opening Time and Closing Time or

$$\text{Closing Time} - \text{Opening Time} \leq 2 * \text{Max Length}$$

Preliminary studies show that without considering this assumption, in some particular cases, the problem might become infeasible due to the generated shifts which do not cover all intervals. In other words, there might be no shifts which include those intervals.

- 2) For each interval, number of available people who have the right skill is greater than the staffing requirement for that period. For example, if the staffing requirement for cashiering from 10am to 10:30am is 3 people, then there should be at least 3 cashiers available to perform the job starting at 10 o'clock.
- 3) For simplicity, the scheduling problem contains just one day and one skill. Therefore, from now on, shifts that are generated do not show the day and skill. This means shifts are only defined by start time and end time. This assumption helps to save time and test more cases, and it is not a limiting assumption at all.

In case of various skills and a full week scheduling, decomposition would be needed in order to break down the problem into individual one skill- one day problems and, therefore, there will be as many problems as the number of skill-day combinations that have to be solved. For example, if cashiering and stocking are the required skills through a week, then the following 14 individual problems appear:

- | | |
|---------------------------|--------------------------|
| 1. Cashiering – Monday | 8. Stocking – Monday |
| 2. Cashiering – Tuesday | 9. Stocking – Tuesday |
| 3. Cashiering – Wednesday | 10. Stocking – Wednesday |
| 4. Cashiering – Thursday | 11. Stocking – Thursday |
| 5. Cashiering – Friday | 12. Stocking – Friday |
| 6. Cashiering – Saturday | 13. Stocking – Saturday |
| 7. Cashiering – Sunday | 14. Stocking – Sunday |

Note that the second and third assumptions turn the previous mathematical model into the following smaller IP model;

$$\begin{array}{ll} \text{Minimize} & \sum_j X_j * L_j \\ \\ \text{Subject to} & \\ & \sum_{j \in S_n} X_j \geq R_n \quad \text{For all } n \\ & X_j \geq 0 \text{ and integer} \quad \text{For all } j \end{array}$$

where

X_j = number of employees required to perform shift j

L_j = length of shift j

S_n = subset of shifts that contain interval n

R_n = minimum staffing requirement for interval n

The recent model is almost the same as the basic tour scheduling problem developed by Dantzig (1954). As it can be seen, when availability is not a concern, the decision variable can just focus on the shift and X_{ij} changes to X_j .

According to the model, now the point is which shifts to select and determine the *number* of people needed for each shift.

CHAPTER 3: METHODOLOGY

3.1 Overview of Patterns

As indicated before, this study focuses on the first phase of the Pre-processor to reduce the number of generated shifts and consequently decision variables. Several algorithms that each of them is made up of some rules are suggested. One rule that appears in almost every method is demand-oriented and is working based on the fluctuation in staffing requirements. In this chapter, some popular patterns for staffing requirements have been introduced. These patterns are Flat, Increasing (Inc.), Decreasing (Dec.), One-Hump, Two-Hump, and Mixed patterns. They will all be explained through an example. Suppose there is a store that runs 9am to 9pm with the planning interval of an hour. Here are different patterns for one skill (e.g., cashiering) and one day (e.g., Monday).

3.1.1 Flat Pattern

When there is no change in the number of required people from one interval to another interval, it is said that the pattern is “Flat” as shown in Table 5.

Table 5: Flat Pattern

Interval	No. of required employees
9am-10am	3
10am-11am	3
11am-12pm	3
12pm-1pm	3
1pm-2pm	3
2pm-3pm	3
3pm-4pm	3
4pm-5pm	3
5pm-6pm	3
6pm-7pm	3
7pm-8pm	3
8pm-9pm	3

Table 5 displays the requirements for the above store. According to the table, in each interval, the constant number of 3 cashiers is required on Monday. It means that, during the operation hours, the requirements stay at the same level.

3.1.2 Increasing Pattern

When the staffing requirement for interval $i+1$ is greater than that of interval i , there is an “Increasing” pattern. Note that it is not necessary that the requirement monotonically increases from one interval to another, but it is important to follow an increasing trend even if in some intervals the requirement remains unchanged. Table 6 illustrates an example of increasing requirements.

Table 6: Increasing Pattern

Interval	No. of required employees
9am-10am	2
10am-11am	2
11am-12pm	4
12pm-1pm	4
1pm-2pm	7
2pm-3pm	7
3pm-4pm	7
4pm-5pm	8
5pm-6pm	8
6pm-7pm	8
7pm-8pm	8
8pm-9pm	10

3.1.3 Decreasing Pattern

When the staffing requirement for interval i is greater than that of interval $i+1$, there is a “Decreasing” pattern. As it is expected, decreasing pattern is behaving the opposite way of increasing one, so in decreasing, it is also allowed to have flat requirements in some points. An example is demonstrated in Table 7 with staffing requirement of seven people at 9am that declines to two people by the end of the day.

Table 7: Decreasing Pattern

Interval	No. of required employees
9am-10am	7
10am-11am	7
11am-12pm	7
12pm-1pm	6
1pm-2pm	6
2pm-3pm	5
3pm-4pm	4
4pm-5pm	3
5pm-6pm	3
6pm-7pm	3
7pm-8pm	2
8pm-9pm	2

3.1.4 One-Hump Pattern

When the staffing requirement is rising up to a point and then it declines, a “One-Hump” pattern forms. Thus, this pattern is partly increasing and partly decreasing as depicted in Table 8. The hump can happen in the beginning hours, in the middle of the working day, or toward the end of operation hours. In the following example, the hump is located in the last intervals. As it can be seen, until 5pm, the requirement is following an increasing pattern and after that it switches to a decreasing one.

Table 8: One-Hump Pattern

Interval	No. of required employees
9am-10am	2
10am-11am	2
11am-12pm	4
12pm-1pm	4
1pm-2pm	6
2pm-3pm	6
3pm-4pm	6
4pm-5pm	7
5pm-6pm	7
6pm-7pm	5
7pm-8pm	5
8pm-9pm	4

3.1.5 Two-Hump Pattern

When, within a day, an increasing pattern followed by a decreasing one happens twice, a “Two-Hump” pattern is created. Table 9 shows an example for this case.

Table 9: Two-Hump Pattern

Interval	No. of required employees
9am-10am	2
10am-11am	3
11am-12pm	4
12pm-1pm	4
1pm-2pm	3
2pm-3pm	3
3pm-4pm	2
4pm-5pm	2
5pm-6pm	4
6pm-7pm	4
7pm-8pm	3
8pm-9pm	3

3.1.6 Mixed Pattern

The Mixed pattern case contains several increasing, decreasing and flat patterns as shown in Table 10.

Table 10: Mixed Pattern

Interval	No. of required employees
9am-10am	4
10am-11am	4
11am-12pm	3
12pm-1pm	5
1pm-2pm	5
2pm-3pm	3
3pm-4pm	3
4pm-5pm	2
5pm-6pm	2
6pm-7pm	3
7pm-8pm	3
8pm-9pm	4

To have a better view of the different patterns, the above examples are presented graphically in Figure 2.

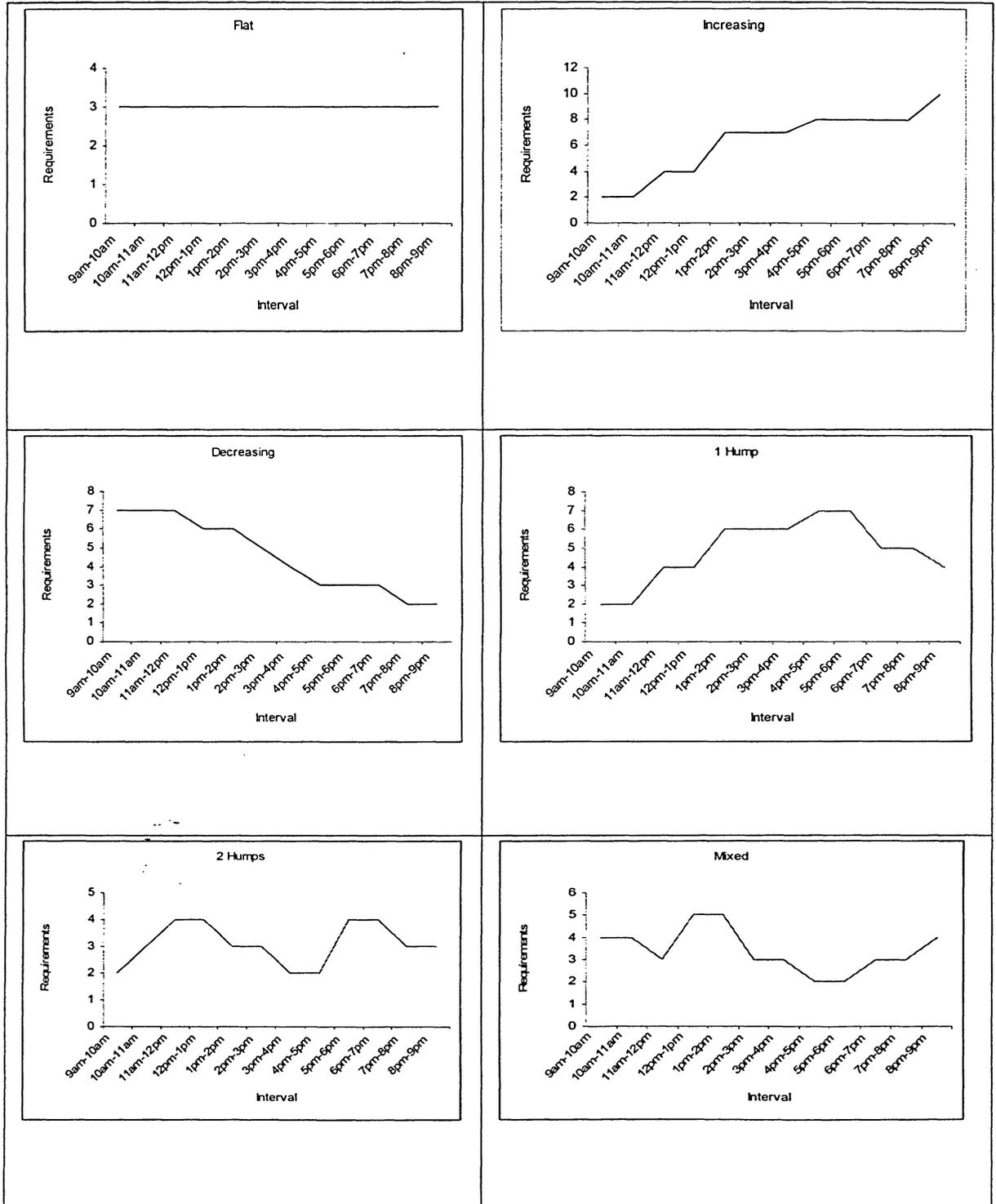


Figure 2: Graphic presentation of different patterns

3.2 Shift Generation Algorithms

The upcoming algorithms are the keys to generate a labour schedule within a reasonable time. The first method simply generates all possible shifts and is a basis for evaluating other methods in terms of number of variables (efficiency) and objective function value (effectiveness). The second and third methods are taken from the industry while other methods are developed by this research.

Methods 1 to 9 are represented by M1, M2, M3 ..., and M9 in this study. Description of each approach along with an example is given below. It should be noted that, in all examples, the same store mentioned above is used. The hours of operation for this location, called Store A, is 9am to 9pm while shifts vary between four and eight hours. Changes in start time and shift length for this store can only happen every sixty minutes. Also, in order to deal with a smaller problem, generated shifts are for one skill and one day. That is why they are presented just by the start time and end time.

3.2.1 Method 1 (M1)

According to Opening/Closing Time, Min/Max Length and Shift Start Time, as well as Shift Increment, this method simply creates all shifts, known as the master set of shifts. For Store A, the following 35 shifts will be generated by utilizing M1.

Table 11: Shifts for Store A Generated through M1

9am-1pm	9am-2pm	9am-3pm	9am-4pm	9am-5pm
10am-2pm	10am-3pm	10am-4pm	10am-5pm	10am-6pm
11am-3pm	11am-4pm	11am-5pm	11am-6pm	11am-7pm
12pm-4pm	12pm-5pm	12pm-6pm	12pm-7pm	12pm-8pm
1pm-5pm	1pm-6pm	1pm-7pm	1pm-8pm	1pm-9pm
2pm-6pm	2pm-7pm	2pm-8pm	2pm-9pm	
3pm-7pm	3pm-8pm	3pm-9pm		
4pm-8pm	4pm-9pm			
5pm-9pm				

The flowchart of shift generation for M1 is presented in Figure 3.

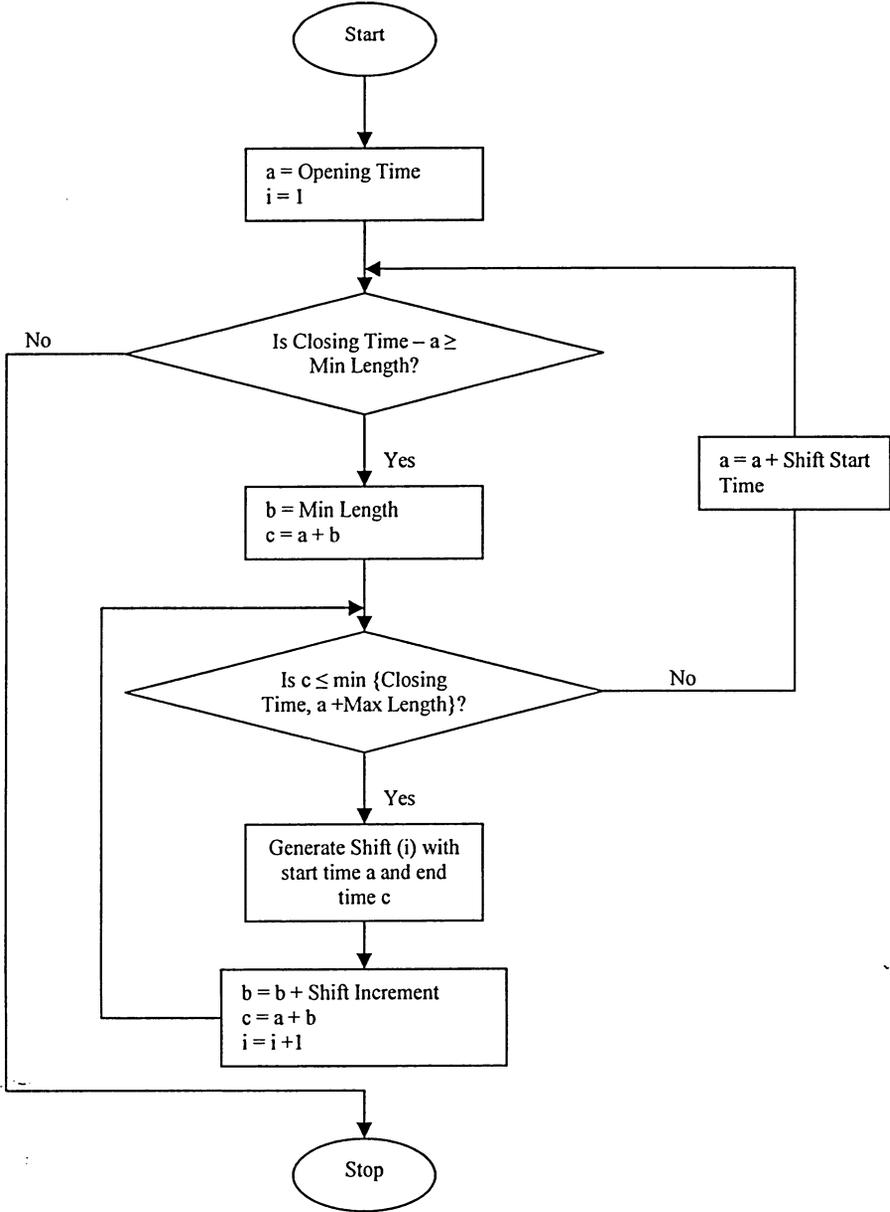


Figure 3: M1 Flowchart

The procedure of generating shifts based on Method 1 is coded in the Mosel¹ language and can be found in Appendix I. Since DASH’s optimizing engine (**Optimizer**) will be

¹ Mosel™ is a proprietary language developed by DASH Optimization (www.dashoptimization.com)

called later to solve the scheduling problem, all algorithms are coded in the Mosel™ language.

3.2.2 Method 2 (M2)

This method is demand-oriented. It means there is a rule in this algorithm that will be active if the staffing requirement varies from one interval to another. Based on this rule, a shift is generated whenever there is a jump in requirements. To be more specific, this rule (Rule 0) enforces to create all shifts starting in the interval that the jump in requirement happens. As an example, if the staffing requirements for Store A changes as indicated in Table 9 (Two-Hump pattern), shifts that are starting at 10am, 11am and 5pm will be generated in the first phase of Pre-processor as shown in Part I of Table 12.

In addition to Rule 0, there are two more rules in this method which are demand-free. Rule 1 makes sure that every shift whose start time is the same as Opening Time is created. Thus, for the above store, the shifts in Part II of Table 12 are added to the set of potential shifts that will enter the second phase of Pre-processor.

Rule 2 forces to have all shifts ending at Closing Time in the list, too. For Store A, this set includes the shifts in Table 12, Part III.

Table 12: Shifts for Store A Generated through M2

Part I	10am-2pm	10am-3pm	10am-4pm	10am-5pm	10am-6pm
	11am-3pm	11am-4pm	11am-5pm	11am-6pm	11am-7pm
	5pm-9pm				
Part II	9am-1pm	9am-2pm	9am-3pm	9am-4pm	9am-5pm
Part III	1pm-9pm	2pm-9pm	3pm-9pm	4pm-9pm	5pm-9pm

Comparing M1 and M2, it is obvious that the number of generated shifts have been reduced from 35 to 20. This means a 57% reduction in the number of decision variables.

3.2.3 Method 3 (M3)

M2 always outperforms M1 in terms of number of decision variables; however, it may not be as good as M1 with respect to effectiveness. A preliminary analysis shows that in some cases, M2 cannot attain the optimal objective function value (total labour hours scheduled) as M1 does. When there is no jump in requirements (number of required employees remains flat or decreases) over time, only few shifts are created. In this event, there is a high probability that one or some of the shifts that could potentially be part of the optimal solution are not generated. In order to resolve this problem M3 and M4, which generate a few more shifts compared to M2, are introduced.

Method 3 consists of all three rules from the previous method in addition to a new one (Rule 3). This new rule becomes effective when the staffing requirement does not increase over a period of longer than Min Length (This area is named P area). As an example, Store A with the requirements specified in Table 9 is recalled. If from 11am until 5pm there is no jump in requirements, the number of required employee remains either unchanged or decreases for six intervals (six hours). Since this period is longer than the Min length of four hours, then Rule 3 is triggered and generates the shifts displayed in Table 13.

Table 13: Shifts for Store A Generated by Rule 3

11am-3pm	11am-4pm	11am-5pm	11am-6pm	11am-7pm
3pm-7pm	4pm-9pm			

Here is how this rule works.

- Step 1: As soon as a P area (from 11am to 5pm) is recognized, with respect to the Min Length, the shortest possible shift starting at the beginning point of P area, is created (11am-3pm).

- Step 2: If the end time of the recently generated shift exceeds the final point of P area, go to Step 4, otherwise go to Step 3.
- Step 3: Generate another shift starting at the end time of the last shift and with the same length (3pm-7pm). Go back to Step 2.
- Step 4: Add the Shift Increment (hourly for this example) to the previous shift length. If the new shift length is greater than Max Length, stop, otherwise with that length, generate the shift which is starting at the beginning point of P area (11am-4pm) and then go back to Step 2.

Applying M3, only one shift (3pm-7pm) is added to the set of shifts generated by M2 for Store A. The other six shifts, which appear due to implementing Rule 3, have already been generated through Rules 0 to 2. Thus, by using M3, the number of decision variables will be 21 compared to 20 from M2 and 35 from M1.

3.2.4 Method 4 (M4)

Similar to M3, this method also contains the three main rules (Rules 0, 1, and 2), as well as a new rule which is called Rule 4. This rule does not take any of the requirements into considerations. It means Rule 4 is enabled under any circumstances, regardless of the requirement's profile. The four-step algorithm for M3 changes to the following for Method 4.

- Step 1: With respect to the Min Length, the shortest possible shift starting at the Opening Time is created (9am-1pm).
- Step 2: If the end time of the recently generated shift exceeds the Closing Time, go to Step 4; otherwise go to Step 3.
- Step 3: Generate another shift starting at the end time of the last shift and with the same length (1pm-5pm). Go back to Step 2.
- Step 4: Add the Shift Increment (hourly for this example) to the previous shift length. If the new shift length is greater than Max Length, stop; otherwise with that length, generate the shift which is starting at the Opening Time (9am-2pm) and then go to Step 2.

Implementing the above steps results in creating the shifts listed in Table 14.

Table 14: Shifts for Store A Generated by Rule 4

9am-1pm	9am-2pm	9am-3pm	9am-4pm	9am-5pm
1pm-5pm	2pm-7pm	3pm-9pm		
5pm-9pm				

Except “1pm-5pm” and “2pm-7pm”, other shifts have already been generated. The first row was in the set of generated shifts following Rule 1 and the rest were in the list due to Rule 2. Therefore, M4 ends up with 22 shifts or decision variables.

3.2.5 Method 5 (M5)

Method 5 has a major difference with other methods that have been introduced so far. M5 does not include Rule 0 and that means it is not sensitive to the change in requirements. However, it does obey Rule 1 and Rule 2 and its own specific rule (Rule 5). Rule 5 is summarized as below,

“With respect to the Min and Max Length, the shortest and longest shifts starting at each interval are created”.

To show how this rule works, Store A is recalled. Remember this algorithm does not depend on the requirement, so any pattern from Flat to Mixed can be chosen for Store A and still get the same shifts. Between all shifts that start at 9am, the shortest one (S), “9am-1pm”, and the longest shift (L), “9am-5pm”, are created. “10am-2pm” and “10am-6pm” are the “S” and “L” for the second interval. Outputs of applying Rule 5 to Store A are listed in Table 15.

The advantage of M5 is its consistency in generating the shifts. For a location like Store A, the number of decision variables always remains at 20, regardless of the staffing requirements.

Table 15: Shifts for Store A Generated by Rule 5

9am-1pm	9am-5pm
10am-2pm	10am-6pm
11am-3pm	11am-7pm
12pm-4pm	12pm-8pm
1pm-5pm	1pm-9pm
2pm-6pm	2pm-9pm
3pm-7pm	3pm-9pm
4pm-8pm	4pm-9pm
5pm-9pm	

3.2.6 Method 6 (M6)

Methods 6 to 9 have one thing in common. They are not only sensitive to a jump in requirements, but they are responsive to a fall in requirements.

There is a possibility that the previously presented methods (M2 to M5) sometimes lack in achieving the optimal objective value. In fact, they may not perform well in terms of effectiveness; however, they are good at efficiency. In the following methods, the new rule makes up for effectiveness. It generates a few shifts when a reduction in number of requirements is observed.

M6 consists of Rule 0, Rule 1 and Rule 2, in addition to the new rule, Rule 6. This rule forces to generate all shifts that are ending at the interval where requirements decline. Store A with Two-Hump pattern is recalled here. Referring to Table 9, the requirement decreases at 1pm, 3pm and 7pm. According to Rule 6, all shifts ending at these time intervals should be generated. They are as indicated in Table 16.

Table 16: Shifts for Store A Generated by Rule 6

9am-1pm				
9am-3pm	10am-3pm	11am-3pm		
11am-7pm	12pm-7pm	1pm-7pm	2pm-7pm	3pm-7pm

By adding the above shifts to the ones that have been created by Rules 0 to 2, the total number of shifts will be 24.

Depending on the requirements profile, M6 might end up with generating a relatively large number of shifts close to what derives from Method 1. For instance, if staffing requirements keep decreasing in each interval, the total number of shifts that M6 generates is equal to the one that is obtained by the first method. In this case, there is no improvement in reducing the number of decision variables. In order to overcome this problem, a combination of Method 5 and Method 6 is devised below.

3.2.7 Method 7 (M7)

Method 7 has the aforementioned three rules (Rules 0, 1, and 2) in addition to Rule 7. This new rule attempts to create fewer shifts when the requirement starts declining. Instead of exhaustively generating all shifts ending at the decreasing interval (what M6 does), Method 7 just creates the shortest and the longest one of those. As an example, for the last case when the requirement falls at 7pm, only “11am-7pm” and “3pm-7pm” are created.

Shift “11am-7pm” with eight hours length is the longest shift ending at 7 o’clock while “3pm-7pm” is the shortest possible shift that ends at that time. By implementing Method 7 for Store A with the requirements indicated in Table 9, 21 decision variables are produced. These shifts can be viewed in Table 17.

Table 17: Shifts for Store A Generated through M7

9am-1pm	9am-2pm	9am-3pm	9am-4pm	9am-5pm
10am-2pm	10am-3pm	10am-4pm	10am-5pm	10am-6pm
11am-3pm	11am-4pm	11am-5pm	11am-6pm	11am-7pm
1pm-9pm	2pm-9pm	3pm-9pm	4pm-9pm	5pm-9pm
3pm-7pm				

3.2.8 Method 8 (M8)

This method is very similar to the last method except in the way that Rule 2 is executed. In M8, instead of generating all possible shifts ending at Closing Time, just the shortest and the longest of them appear in the list of generated shifts. This modified Rule 2 is known as Rule 8.

Again, Store A with requirements matching Table 9 is recalled to implement all M8's rules. According to Rule 0, shifts in Table 18, Part I are generated at jumps.

Rule 1 as one of the active rules in Method 8 produces all the shifts that start at Opening Time. They are shown in the second part of Table 18.

Rule 8 also generates "5pm-9pm" shift as the shortest shift that ends at Closing Time and the eight-hour shift, "1pm-9pm", as the longest one ending at that time. Finally, based on Rule 7, which forces to generate the shortest and longest shifts when the requirement is declining, shifts displayed in Table 18, Part III are created.

Table 18: Shifts for Store A Generated by M8

Part I	10am-2pm	10am-3pm	10am-4pm	10am-5pm	10am-6pm
	11am-3pm	11am-4pm	11am-5pm	11am-6pm	11am-7pm
	5pm-9pm				
Part II	9am-1pm	9am-2pm	9am-3pm	9am-4pm	9am-5pm
Part III	9am-1pm				
	9am-3pm	11am-3pm			
	11am-7pm	3pm-7pm			

By eliminating the repetitive shifts from the above table, the total number of shifts which pass from the first phase of Pre-processor to the next phase is equal to 18.

3.2.9 Method 9 (M9)

The final method is M9 which is behaving opposite to M8 at some points. In Method 8, *all* possible shifts are generated when:

1. Start time of the shift is the same as Opening Time.
2. There is a jump in requirements.

M8 also creates the *shortest and longest* shifts of all when:

1. End time of the shift is the same as Closing Time.
2. There is a fall in requirement.

If the *shortest and longest* shifts are generated for the first two events and *all* shifts are generated for the last two, Method 9 is in effect. In fact M9 contains Rule 2, and Rule 6 plus two new rules, Rule 9 and Rule 10.

Rule 9 generates the shortest and the longest of the shifts starting at Opening Time, while Rule 10 creates the shortest and the longest shifts when the requirement starts going up. For clarity, assume Store A with the same requirements as before. M9 works as follows;

The shortest and longest shifts among the ones that start at Opening Time are “9am-1pm” and “9am-5pm”. When there is a jump in requirements, the shortest and longest shifts are as Table 19, Part I.

All shifts ending at Closing Time as well as those when requirements fall are the ones shown in Part II in Table 19.

In total, the number of shifts generated by executing Method 9 is 16 that is the lowest number of decision variables among all algorithms. However, it should be emphasized that in most methods number of variables or shifts depends on the requirement profile.

Table 19: Shifts for Store A Generated through M9

Part I	10am-2pm	10am-6pm			
	11am-3pm	11am-7pm			
	5pm-9pm				
Part II	1pm-9pm	2pm-9pm	3pm-9pm	4pm-9pm	5pm-9pm
	11am-7pm	12pm-7pm	1pm-7pm	2pm-7pm	3pm-7pm
	9am-3pm	10am-3pm	11am-3pm		
	9am-1pm				

3.3 Summary of Rules and Methods

In this section, a summary of the aforementioned rules followed by the table of methods and their including rules (Table 20) is presented.

Rule 0, enforces to create all shifts starting in the interval that the requirements jump.

Rule 1, makes sure that any shift whose start time is the same as Opening Time is created.

Rule 2, forces to generate all shifts ending at Closing Time.

Rule 3, becomes effective when the staffing requirement does not increase over a period of longer than Min Length.

Rule 4, is working similar to Rule 3, but starting at Opening Time regardless of the requirement's profile.

Rule 5, With respect to the Min and Max Length, the shortest and longest shifts starting at each interval are created.

Rule 6, forces to generate all shifts ending at the interval where requirements decline.

Rule 7, just creates the shortest and the longest shifts where requirements decline.

Rule 8, generates the shortest and the longest of the shifts ending at Closing Time.

Rule 9, generates the shortest and the longest of the shifts starting at Opening Time.

Rule 10, creates the shortest and the longest shifts when the requirement starts going up.

Table 20: Summary of Methods and Their Rules

Methods	Rules
Method 2	0, 1, 2
Method 3	0, 1, 2, 3
Method 4	0, 1, 2, 4
Method 5	1, 2, 5
Method 6	0, 1, 2, 6
Method 7	0, 1, 2, 7
Method 8	0, 1, 7, 8
Method 9	2, 6, 9, 10

CHAPTER 4: EXPERIMENTAL DESIGN & STATISTICAL ANALYSIS

4.1 Experimental Design - Overview

In an experiment, one or more process variables are deliberately changed in order to observe the effect of changes on one or more response variables. The objectives of the experiment may include:

1. Determining which factors are most influential on the response.
2. Determining how the levels of these factors should vary.
3. Determining in what way these levels should be combined.

Berger and Maurer (2002) introduced the main four steps of experimental design as follows.

- 1) Plan the experiment
- 2) Design the experiment
- 3) Perform the experiment
- 4) Analyze the data from the experiment

These four steps of experimental design process are explained in more detail in the following sections.

4.1.1 Planning the Experiment

The planning stage itself consists of different steps. Identifying the response variable(s) is the first step. The response (dependent) variable should be chosen carefully based on what the goal is. In the next step, the factors that potentially affect the response variable are determined. This is not as easy as it seems, because sometimes a large number of factors are involved and identifying all of them needs a precise study. A brainstorming

session is an effective tool to consider all candidate independent variables. In the last phase of planning, one should determine the number of levels for each factor and what those levels are.

It should be noted that sometimes variables are subjective and, in one step, they need to be translated to measurable quantities.

4.1.2 Designing the Experiment

This step is the core of an experimental design. At this stage, first, the design type should be chosen. *One-at-a-time* designs and *factorial* designs are two types of designs. In the first group, the level of each factor is varied, only one factor at a time. In factorial designs, the experiment contains various combinations of levels of factors. The latter design is significantly superior to the former one.

In factorial designs, when there are too many combinations of levels of factors, only a fraction of them can be run. Therefore, a critical decision is to determine which fraction (subset of combinations of levels of factors) is appropriate.

Another element of designing an experiment is the consideration of *replication*.

Replication, which is the number of data values obtained under the same experimental conditions, improves the significance of the experimental results.

4.1.3 Performing the Experiment

In order to obtain the data that are to be analyzed, the experiment should be performed. It is very important that the experiment that was designed is the experiment that is run.

Also, in order to prevent the occurrence of bias effects on the results, randomization is extremely important. In other words, the order of running the combinations of levels of factors should be random.

4.1.4 Analyzing the Data from the Experiment

The primary statistical method used for the analysis of the data in experimental design is called analysis of variance (ANOVA). This method, which was developed by Sir Ronald Fisher in 1925, determines if the level of a factor has impact on the value of the response variable.

Statistical analysis, which deals with testing different hypotheses, cannot prove that a factor makes a difference, but it can provide a measure of the consistency of the results. If the results from the previous step (running the experiment) are relatively inconsistent, it can be concluded that the factor has an effect on the response variable. In addition, the analysis can show how likely a given conclusion would be in error.

In the following sections, the details of the above steps for this research will be discussed.

4.2 Planning the Shift Generation Experiment (Step 1)

As discussed earlier, for the shift generation problem, there are two output variables: the objective function value and the number of decision variables. According to a preliminary study, the objective function value usually remains unchanged through the different methods, so it cannot be an appropriate response variable. Thus, in this thesis, the number of decision variables or shifts is set as the response variable. The input (independent) variables or factors involved in this experiment are:

1. Min Length
2. Max Length
3. Shift Increment
4. Shift Start Time
5. Method

The number of levels varies among factors. For this experiment, Min Length and Max Length have two levels each while the number of levels for Shift Increment and Shift Start Time is three. Method, with the most number of levels, contains nine levels.

The factors and their levels are presented below.

Min Length	(3 hrs, 4 hrs)
Max Length	(8 hrs, 10 hrs)
Shift Increment	(1/4 hr, 1/2 hr, 1 hr)
Shift Start Time	(1/4 hr, 1/2 hr, 1 hr)
Method	(M1, M2, M3, M4, M5, M6, M7, M8, M9)

Here, Min Length is introduced as factor A and Max length is referred to as factor B. Shift Increment, Shift Start Time, and Method are factors C, D, and E, respectively.

4.3 Designing the Shift Generation Experiment (Step 2)

Since in this experiment there are five factors with different number of levels, a full factorial design is chosen. This means all possible combinations of levels of factors are included and no fraction is involved.

Before revealing the number of runs or replications, it is worthwhile to mention that, instead of one experiment being conducted, five different experiments are conducted in this study. These experiments vary based on the requirements pattern that they follow.

The requirements pattern is not of primary interest here but, if not taken into consideration, it may add to the variability in the data and even obscure the true effect of the factors of real interest. Therefore, five different patterns namely: Increasing, Decreasing, One-Hump, Two-Hump, and Mixed patterns are considered individually, with three replications for each, as illustrated in Table 21.

In the case of Flat pattern (Table 21), the staffing requirement does not change over the time, so the number of generated shifts (the response variable in the experiment) always remains the same for any number of required employees. For example, if the staffing

requirement for Store A is three employees throughout the day, the total number of shifts for M1 through M9 is 35, 10, 12, 12, 20, 10, 10, 7, and 7, respectively. These numbers do not vary if the requirement changes to 1, 5, 100 or any other number as long as it remains unchanged from Opening Time to Closing Time.

Table 21: Replications for Different Patterns

Interval	Staffing Requirements for Pattern															
	FLAT	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED
9:00-9:15	3	3	3	2	7	4	12	4	7	2	4	4	6	7	2	7
9:15-9:30	3	3	3	2	7	4	12	4	7	2	4	4	6	7	2	7
9:30-9:45	3	3	3	2	7	4	12	4	7	2	4	4	6	7	2	7
9:45-10:00	3	3	3	2	7	4	12	4	7	2	4	4	6	7	2	7
10:00-10:15	3	3	3	2	6	4	12	4	7	2	6	6	6	7	2	5
10:15-10:30	3	3	3	2	6	4	12	4	7	2	6	6	6	7	2	5
10:30-10:45	3	3	3	2	6	4	12	4	7	2	6	6	6	7	2	5
10:45-11:00	3	3	3	2	6	4	12	4	7	2	6	6	6	7	2	5
11:00-11:15	3	3	4	4	5	4	9	4	7	4	6	4	6	6	1	3
11:15-11:30	3	3	4	4	5	4	9	4	7	4	6	4	6	6	1	3
11:30-11:45	3	3	4	4	5	4	9	4	7	4	6	4	6	6	1	3
11:45-12:00	3	3	4	4	5	4	9	4	7	4	6	4	6	6	1	3
12:00-12:15	3	3	4	4	4	3	9	4	7	4	7	4	10	7	5	4
12:15-12:30	3	3	4	4	4	3	9	4	7	4	7	4	10	7	5	4
12:30-12:45	3	3	4	4	4	3	9	4	7	4	7	4	10	7	5	4
12:45-13:00	3	3	4	4	4	3	9	4	7	4	7	4	10	7	5	4
13:00-13:15	3	4	4	6	4	3	9	4	7	4	7	4	5	9	2	5
13:15-13:30	3	4	4	6	4	3	9	4	7	4	7	4	5	9	2	5
13:30-13:45	3	4	4	6	4	3	9	4	7	4	7	4	5	9	2	5
13:45-14:00	3	4	4	6	4	3	9	4	7	4	7	4	5	9	2	5
14:00-14:15	3	4	5	6	3	3	5	4	7	4	8	4	8	8	2	6
14:15-14:30	3	4	5	6	3	3	5	4	7	4	8	4	8	8	2	6
14:30-14:45	3	4	5	6	3	3	5	4	7	4	8	4	8	8	2	6
14:45-15:00	3	4	5	6	3	3	5	4	7	4	8	4	8	8	2	6
15:00-15:15	3	4	5	8	3	3	5	4	7	5	8	4	9	6	7	3
15:15-15:30	3	4	5	8	3	3	5	4	7	5	8	4	9	6	7	3
15:30-15:45	3	4	5	8	3	3	5	4	7	5	8	4	9	6	7	3
15:45-16:00	3	4	5	8	3	3	5	4	7	5	8	4	9	6	7	3
16:00-16:15	3	4	5	8	3	3	5	4	7	5	4	4	12	5	2	2
16:15-16:30	3	4	5	8	3	3	5	4	7	5	4	4	12	5	2	2
16:30-16:45	3	4	5	8	3	3	5	4	7	5	4	4	12	5	2	2
16:45-17:00	3	4	5	8	3	3	5	4	7	5	4	4	12	5	2	2
17:00-17:15	3	5	5	9	2	3	1	4	7	5	4	4	5	5	7	1
17:15-17:30	3	5	5	9	2	3	1	4	7	5	4	4	5	5	7	1
17:30-17:45	3	5	5	9	2	3	1	4	7	5	4	4	5	5	7	1
17:45-18:00	3	5	5	9	2	3	1	4	7	5	4	4	5	5	7	1
18:00-18:15	3	5	6	9	2	2	1	5	7	9	7	4	5	4	7	1
18:15-18:30	3	5	6	9	2	2	1	5	7	9	7	4	5	4	7	1
18:30-18:45	3	5	6	9	2	2	1	5	7	9	7	4	5	4	7	1
18:45-19:00	3	5	6	9	2	2	1	5	7	9	7	4	5	4	7	1
19:00-19:15	3	5	6	10	1	2	1	5	12	9	7	6	4	7	2	1
19:15-19:30	3	5	6	10	1	2	1	5	12	9	7	6	4	7	2	1
19:30-19:45	3	5	6	10	1	2	1	5	12	9	7	6	4	7	2	1
19:45-20:00	3	5	6	10	1	2	1	5	12	9	7	6	4	7	2	1
20:00-20:15	3	5	7	10	1	1	1	2	4	3	4	4	4	9	1	7
20:15-20:30	3	5	7	10	1	1	1	2	4	3	4	4	4	9	1	7
20:30-20:45	3	5	7	10	1	1	1	2	4	3	4	4	4	9	1	7
20:45-21:00	3	5	7	10	1	1	1	2	4	3	4	4	4	9	1	7

As for Flat, no variation in the response variable (number of decision variables) exists; the experiment does not deal with this specific pattern.

4.4 Performing the Shift Generation Experiment (Step 3)

In order to perform the experiments, all algorithms (M1 to M9) are coded in Mosel™. Depending on the complexity of the method, one (i.e., M1) to three programs (i.e., M3) should be run sequentially to produce the desired shifts. In order to find out the objective function value (total labour hours scheduled) and the number of decision variables, the last program should be run.

As an example, all computer programs for M2 are presented in Appendix I. The first two programs for M2 generate the required shifts. These shifts, which are described with the start time and end time, are forwarded to the third program in which the mathematical model is built based on the decision variables and constraints. The constructed IP model from the third program can be run in DASH™ and solved by Optimizer (DASH's optimizing engine) as the last program. The total number of integer decision variables (shifts), as well as the optimal objective function value, appears in the "stat" page of the software.

To perform a full factorial experiment, all possible combinations of levels of factors were run for the three replications of each pattern. In total, $(2 \times 2 \times 3 \times 3 \times 9) \times 3 \times 5 = 4860$ IP models were solved. The results for the combination of Min Length = 3 hrs, Max Length = 8 hrs, Shift Increment = 1/4 hr, and Shift Start Time = 1/4 hr are displayed in Table 22. The outcomes of other combinations are summarized in Appendix II.

Table 22 consists of three parts. The first part shows the optimal objective function value derived from each method for each run corresponding to the requirements in Table 21. For example, the first column presents the results for all nine methods when the staffing requirement for the mentioned combination is following the first Increasing pattern in

Table 21. It means if a store, which operates 9am to 9pm, starts working with three employees and at 1pm the requirement changes to four people and then again at 5pm increases to five, the optimal “total labour hours scheduled” is 48 as shown in Table 22. This is true when Min and Max Length for this given location are three and eight hours while Shift Increment as well as Shift Start Time is quarterly (15 minutes).

It can be observed that under some staffing conditions, some of the methods do not produce the optimal objective function value. For instance, M2 for the first Decreasing pattern results in 46 hours instead of 44, which is the best answer.

Table 22: Results for Min Length=3, Max Length=8, Shift Increment=1/4, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE- HUMP	ONE- HUMP	ONE- HUMP	TWO- HUMP	TWO- HUMP	TWO- HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	567	567	567	567	567	567	567	567	567	567	567	567	567	567	567
	M2	66	79	99	42	42	42	42	42	75	100	63	99	83	79	99
	M3	74	81	99	58	58	58	58	58	79	102	76	99	87	79	101
	M4	81	94	113	58	58	58	58	58	90	114	79	111	97	93	112
	M5	92	92	92	92	92	92	92	92	92	92	92	92	92	92	92
	M6	66	79	99	91	84	70	63	63	95	133	84	138	135	135	135
	M7	66	79	99	46	46	44	44	44	77	102	65	101	86	83	99
	M8	48	61	82	27	27	24	25	25	59	84	46	85	67	66	81
	M9	24	26	28	73	65	52	44	44	47	65	46	72	84	89	74
REDUCTION IN NO. OF VARIABLES (%)	M1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	M2	88.36	86.07	82.54	92.59	92.59	92.59	92.59	92.59	86.77	82.36	88.89	82.54	85.36	86.07	82.54
	M3	86.95	85.71	82.54	89.77	89.77	89.77	89.77	89.77	86.07	82.01	86.60	82.54	84.66	86.07	82.19
	M4	85.71	83.42	80.07	89.77	89.77	89.77	89.77	89.77	84.13	79.89	86.07	80.42	82.89	83.60	80.25
	M5	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77	83.77
	M6	88.36	86.07	82.54	83.95	85.19	87.65	88.89	88.89	83.25	76.54	85.19	75.66	76.19	76.19	76.19
	M7	88.36	86.07	82.54	91.89	91.89	92.24	92.24	92.24	86.42	82.01	88.54	82.19	84.83	85.36	82.54
	M8	91.53	89.24	85.54	95.24	95.24	95.77	95.59	95.59	89.59	85.19	91.89	85.01	88.18	88.36	85.71
	M9	95.77	95.41	95.06	87.13	88.54	90.83	92.24	92.24	91.71	88.54	91.89	87.30	85.19	84.30	86.95

In order to be able to conclude that M2 to M9 are performing effectively, their objective function values should always be the same as that of M1.

The second part of the table reveals the number of decision variables, which are actually the number of shifts generated by implementing various methods. For example, if for solving the first Increasing case one chooses M1 (exhaustively generating all possible shifts), then 567 shifts will be generated. This number can be reduced to 66 by implementing the second method and even 24 by using M9 while the objective function value remains as good as M1's.

The third part in Table 22 determines how much each method is improving the result in terms of efficiency. In this section, figures are representing the percentage of reduction in the number of decision variables, or shifts. In order to find these numbers for Method i , the following calculations are done.

$$\frac{\text{Number of decision variables for Method 1} - \text{Number of decision variables for Method } i}{\text{Number of decision variable for Method 1}} \times 100$$

For example, according to Table 22, in the first replication of Decreasing, total number of generated shifts by Method 2 is 66 comparing to 567 shifts under M1. Therefore, $(567 - 66 / 567) \times 100 \approx 88.36\%$ is the percentage of improvement by M2. As it can be observed, for the same replication, M9 is making 95.77% improvement in terms of efficiency and referring to the first part of Table 22, it is also performing effectively.

4.5 Analyzing the Data from the Shift Generation Experiment (Step 4)

In this study, the statistical analysis is divided into two parts. The first part deals with the analysis of variance (ANOVA) in order to find out if the level of a factor has impact on the value of the response variable. In the second part, pairwise comparison is conducted

to determine which method is superior.

4.5.1 Analysis of Variance (ANOVA)

M1, as the basis of comparison, is not required to be considered in statistical analysis, because the main point is to study the efficiency of other methods and compare them with the existing one. Therefore, with the aid of “Design-Expert” software, ANOVA is conducted for the multifactor experiment consisting of the following factors and levels.

A: Min Length (3, 4)

B: Max Length (8, 10)

C: Shift Increment (1/4, 1/2, 1)

D: Shift Start Time (1/4, 1/2, 1)

E: Method (M2, M3, M4, M5, M6, M7, M8, M9)

Table 23 contains all 288 combinations of levels of the above factors with three replications for each. This table only presents the results for the response variable (number of decision variables or shifts) under Increasing Pattern. Values for other patterns can be derived from tables in Appendix II.

As discussed before, ANOVA is a tool to determine whether the level of a factor has impact on the value of the response variable. In other words, with the aid of ANOVA the equality of the level means for each factor can be tested. To do so, for the shift generation experiment, the appropriate hypotheses for main factors are:

$H_0^{(1)}$: Both Min Lengths have equal mean values.

$H_0^{(2)}$: Both Max Lengths have equal mean values.

$H_0^{(3)}$: All three Shift Increments have equal mean values.

$H_0^{(4)}$: All three Shift Start Times have equal mean values.

$H_0^{(5)}$: All eight methods have equal mean values.

When performing hypothesis testing (whether an F test, a t test, or any other test) a quantity called the P-value can be determined. This value is defined as the weight of evidence against a null hypothesis. Indeed, the P-value is the probability that the null hypothesis is true (there is no factor effect).

Tables 24 to 28 are ANOVA tables for Increasing, Decreasing, One-Hump, Two-Hump, and Mixed patterns, respectively. Small P-values for factors A, B, C, D, and E in all these tables signify that there is a very small probability (less than 0.01%) that $H_0^{(1)}$, $H_0^{(2)}$, $H_0^{(3)}$, $H_0^{(4)}$, and $H_0^{(5)}$ should be accepted. Therefore, these hypotheses are rejected, and it is concluded that all main factors have a significant impact on the response variable. In other words, regardless of the pattern, Min and Max Length, Shift Increment, Shift Start Time, and Method always have a significant impact on the number of decision variables.

In addition to single factor effects, interactions may exist. For the shift generation problem, potentially 10 two-factor interactions exist which increase the number of null hypotheses to 15.

$H_0^{(6)}$: No interactions between Min Length and Max Length exist.

$H_0^{(7)}$: No interactions between Min Length and Shift Increment exist.

$H_0^{(8)}$: No interactions between Min Length and Shift Start Time exist.

$H_0^{(9)}$: No interactions between Min Length and Method exist.

$H_0^{(10)}$: No interactions between Max Length and Shift Increment exist.

$H_0^{(11)}$: No interactions between Max Length and Shift Start Time exist.

$H_0^{(12)}$: No interactions between Max Length and Method exist.

$H_0^{(13)}$: No interactions between Shift Increment and Shift Start Time exist.

$H_0^{(14)}$: No interactions between Shift Increment and Method exist.

$H_0^{(15)}$: No interactions between Shift Start Time and Method exist.

If any of the hypotheses $H_0^{(6)}$ through $H_0^{(15)}$ is rejected, it will be concluded that the two factors under question are interacting. In addition to the above null hypotheses, other ones should be tested; ten three-factor, five four-factor, and one five-factor interactions.

According to the small P-values in the last column of the ANOVA table for Increasing pattern (Table 24), A&C, A&E, B&C, B&E, C&D, C&E, and D&E are interacting. On the other hand, P-values which are significantly greater than 0.05 show that there are no interactions between A&B, A&D, and B&D. In total, just for Increasing Pattern, there are 188 two-factor interaction graphs. All Sixty interaction graphs for AE, BE, CE, and DE for every combination of levels of factors are depicted in Appendix III. The reason that these graphs have been selected is they all include Method, which is the focus of this study, as one of the interacting factors.

Table 24: Analysis of Variance for Increasing Pattern

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	p-value
A: Min Length	18140.83	1	18140.83	203.68	< 0.0001
B: Max Length	14775.84	1	14775.84	165.9	< 0.0001
C: Shift Increment	2.56E+05	2	1.28E+05	1439.54	< 0.0001
D: Shift Start Time	18460.4	2	9230.2	103.63	< 0.0001
E: Method	97405.67	7	13915.1	156.24	< 0.0001
AB	0.094	1	0.094	1.05E-03	0.9741
AC	3882.4	2	1941.2	21.8	< 0.0001
AD	144.23	2	72.12	0.81	0.4455
AE	2247.95	7	321.14	3.61	0.0008
BC	4123	2	2061.5	23.15	< 0.0001
BD	91	2	45.5	0.51	0.6002
BE	2100.91	7	300.13	3.37	0.0016
CD	7021.05	4	1755.26	19.71	< 0.0001
CE	37678.29	14	2691.31	30.22	< 0.0001
DE	21122.12	14	1508.72	16.94	< 0.0001
ABC	0	2	0	0	1
ABD	0	2	0	0	1
ABE	0.66	7	0.094	1.05E-03	1
ACD	95.88	4	23.97	0.27	0.8978
ACE	744.34	14	53.17	0.6	0.8684
ADE	70.29	14	5.02	0.056	1
BCD	196	4	49	0.55	0.699
BCE	413	14	29.5	0.33	0.9899
BDE	157	14	11.21	0.13	1
CDE	1243.32	28	44.4	0.5	0.9864
ABCD	0	4	0	0	1
ABCE	0	14	0	0	1
ABDE	0	14	0	0	1
ACDE	41.82	28	1.49	0.017	1
BCDE	28	28	1	0.011	1
ABCDE	0	28	0	0	1
Residual	51301.33	576	89.06		
Total	5.379E+005	863			

According to the interaction graphs for Increasing Pattern, Method 9 in many cases performs far more efficient than other methods. As a preliminary observation, this can be an indication for superiority of M9 over other methods.

The ANOVA table for Decreasing Pattern confirms that except A and B, every other two factors are interacting significantly. Interaction for more than two factors only among the highlighted three factors exist.

Table 25: Analysis of Variance for Decreasing Pattern

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	p-value
A: Min Length	6722.34	1	6722.34	700.78	< 0.0001
B: Max Length	10347.34	1	10347.34	1078.68	< 0.0001
C: Shift Increment	90835.29	2	45417.65	4734.66	< 0.0001
D: Shift Start Time	31813.82	2	15906.91	1658.25	< 0.0001
E: Method	67793.31	7	9684.76	1009.61	< 0.0001
AB	0.12	1	0.12	0.012	0.9126
AC	998.96	2	499.48	52.07	< 0.0001
AD	321.16	2	160.58	16.74	< 0.0001
AE	776.98	7	111	11.57	< 0.0001
BC	2417.09	2	1208.54	125.99	< 0.0001
BD	165.36	2	82.68	8.62	0.0002
BE	928.98	7	132.71	13.83	< 0.0001
CD	15965.38	4	3991.34	416.09	< 0.0001
CE	4590.39	14	327.89	34.18	< 0.0001
DE	21121.97	14	1508.71	157.28	< 0.0001
ABC	2.32E-03	2	1.16E-03	1.21E-04	0.9999
ABD	2.32E-03	2	1.16E-03	1.21E-04	0.9999
ABE	0.64	7	0.092	9.58E-03	1
ACD	244.8	4	61.2	6.38	< 0.0001
ACE	88.72	14	6.34	0.66	0.8133
ADE	102.64	14	7.33	0.76	0.7085
BCD	501.17	4	125.29	12.85	< 0.0001
BCE	112.27	14	8.02	0.84	0.6299
BDE	233.77	14	16.7	1.74	0.0444
CDE	5654.32	28	201.94	21.05	< 0.0001
ABCD	4.63E-03	4	1.16E-03	1.21E-04	1
ABCE	0.016	14	1.16E-03	1.21E-04	1
ABDE	0.016	14	1.16E-03	1.21E-04	1
ACDE	99.91	28	3.57	0.37	0.9988
BCDE	75.2	28	2.69	0.28	0.9999
ABCDE	0.032	28	1.16E-03	1.21E-04	1
Residual	5525.33	576	9.59		
Total	2.67E+05	863			

For One-Hump and Two-Hump Patterns, the effect of interactions for more than two factors is not significant (Table 26 and Table 27) but, for One-Hump, A&C, B&C, C&D, C&E, and D&E are interacting.

Table 26: Analysis of Variance for One-Hump Pattern

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	p-value
A: Min Length	8066.67	1	8066.67	72.32	< 0.0001
B: Max Length	14081.19	1	14081.19	126.24	< 0.0001
C: Shift Increment	1.28E+05	2	64024.02	573.99	< 0.0001
D: Shift Start Time	26183.28	2	13091.64	117.37	< 0.0001
E: Method	51601.83	7	7371.69	66.09	< 0.0001
AB	0.042	1	0.042	3.74E-04	0.9846
AC	1466.19	2	733.1	6.57	0.0015
AD	223.76	2	111.88	1	0.3674
AE	690.02	7	98.57	0.88	0.5189
BC	3498.12	2	1749.06	15.68	< 0.0001
BD	196.99	2	98.49	0.88	0.4141
BE	1399.98	7	200	1.79	0.0861
CD	11967.88	4	2991.97	26.82	< 0.0001
CE	8409.52	14	600.68	5.39	< 0.0001
DE	20095.72	14	1435.41	12.87	< 0.0001
ABC	0.58	2	0.29	2.62E-03	0.9974
ABD	0.13	2	0.066	5.92E-04	0.9994
ABE	0.64	7	0.092	8.24E-04	1
ACD	151.86	4	37.97	0.34	0.8507
ACE	158.62	14	11.33	0.1	1
ADE	70.28	14	5.02	0.045	1
BCD	326.69	4	81.67	0.73	0.5702
BCE	185.66	14	13.26	0.12	1
BDE	265.12	14	18.94	0.17	0.9998
CDE	2925.45	28	104.48	0.94	0.5608
ABCD	0.39	4	0.097	8.72E-04	1
ABCE	1.34	14	0.096	8.60E-04	1
ABDE	2.35	14	0.17	1.51E-03	1
ACDE	60.44	28	2.16	0.019	1
BCDE	102.76	28	3.67	0.033	1
ABCDE	4.02	28	0.14	1.29E-03	1
Residual	64248	576	111.54		
Total	3.44E+05	863			

ANOVA table for Two-Hump (Table 27) shows that between A&C, B&C, B&E, C&D, C&E, and D&E interactions exist. This is due to the very small P-values which appear in the last column of the table.

Table 27: Analysis of Variance for Two-Hump Pattern

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	p-value
A: Min Length	15956.45	1	15956.45	117.63	< 0.0001
B: Max Length	18900.09	1	18900.09	139.33	< 0.0001
C: Shift Increment	3.37E+05	2	1.69E+05	1242.45	< 0.0001
D: Shift Start Time	28845.97	2	14422.99	106.33	< 0.0001
E: Method	60590.43	7	8655.78	63.81	< 0.0001
AB	65.01	1	65.01	0.48	0.489
AC	6496.91	2	3248.45	23.95	< 0.0001
AD	170.45	2	85.22	0.63	0.5339
AE	1223.71	7	174.82	1.29	0.2534
BC	6132.34	2	3066.17	22.6	< 0.0001
BD	58.19	2	29.09	0.21	0.807
BE	2221.14	7	317.31	2.34	0.0232
CD	13450.84	4	3362.71	24.79	< 0.0001
CE	39932.99	14	2852.36	21.03	< 0.0001
DE	21170.34	14	1512.17	11.15	< 0.0001
ABC	382.51	2	191.25	1.41	0.245
ABD	135.02	2	67.51	0.5	0.6082
ABE	18.67	7	2.67	0.02	1
ACD	1118.84	4	279.71	2.06	0.0844
ACE	760.3	14	54.31	0.4	0.9749
ADE	94.53	14	6.75	0.05	1
BCD	1195.06	4	298.76	2.2	0.0674
BCE	573.34	14	40.95	0.3	0.9937
BDE	160.61	14	11.47	0.085	1
CDE	4825.68	28	172.35	1.27	0.1615
ABCD	781.64	4	195.41	1.44	0.2191
ABCE	41.4	14	2.96	0.022	1
ABDE	38.66	14	2.76	0.02	1
ACDE	117.9	28	4.21	0.031	1
BCDE	176.31	28	6.3	0.046	1
ABCDE	84.84	28	3.03	0.022	1
Residual	78133.33	576	135.65		
Total	6.41E+05	863			

Values greater than 0.15 for the P-value in Table 28 are interpreted as follows.

There is not enough evidence to reject null hypotheses (6) and (11).

As a result, these hypotheses cannot be rejected. Therefore, for Mixed Pattern, $H_0^{(6)}$ is accepted, that means Min Length and Max Length are not interacting. Also, acceptance of $H_0^{(11)}$ is an indication for no interaction between Max Length and Shift Start Time.

For more than two factors, interactions among B, C, and D as well as C, D, and E exist.

To this point, it is evident that methods are different, but the question of “which one is the best method?” is still unanswered.

Table 28: Analysis of Variance for Mixed Pattern

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	p-value
A: Min Length	22183.89	1	22183.89	724.07	< 0.0001
B: Max Length	14097.34	1	14097.34	460.13	< 0.0001
C: Shift Increment	3.46E+05	2	1.73E+05	5641.7	< 0.0001
D: Shift Start Time	34386.84	2	17193.42	561.18	< 0.0001
E: Method	46800.79	7	6685.83	218.22	< 0.0001
AB	0.23	1	0.23	7.40E-03	0.9315
AC	4699.59	2	2349.8	76.7	< 0.0001
AD	351.62	2	175.81	5.74	0.0034
AE	1102.31	7	157.47	5.14	< 0.0001
BC	3944.34	2	1972.17	64.37	< 0.0001
BD	182.48	2	91.24	2.98	0.0517
BE	1263.75	7	180.54	5.89	< 0.0001
CD	17900.52	4	4475.13	146.07	< 0.0001
CE	36015.14	14	2572.51	83.97	< 0.0001
DE	24019.42	14	1715.67	56	< 0.0001
ABC	0.12	2	0.06	1.96E-03	0.998
ABD	0.45	2	0.23	7.40E-03	0.9926
ABE	0.57	7	0.081	2.66E-03	1
ACD	284.94	4	71.23	2.33	0.0553
ACE	557.93	14	39.85	1.3	0.2016
ADE	181.45	14	12.96	0.42	0.9676
BCD	318.13	4	79.53	2.6	0.0355
BCE	249.62	14	17.83	0.58	0.88
BDE	252.93	14	18.07	0.59	0.8741
CDE	9231.78	28	329.71	10.76	< 0.0001
ABCD	0.49	4	0.12	4.00E-03	1
ABCE	1.47	14	0.11	3.43E-03	1
ABDE	1.92	14	0.14	4.47E-03	1
ACDE	174.77	28	6.24	0.2	1
BCDE	96.24	28	3.44	0.11	1
ABCDE	4.25	28	0.15	4.95E-03	1
Residual	17647.33	576	30.64		
Total	5.82E+05	863			

4.5.2 Pairwise Comparison

In the previous section, it was determined that the level of factors affect a dependent variable of interest (number of decision variables). Concluding that the factors under study have an impact on the response variable is not where the analysis should be

stopped. F tests in analysis of variance led to rejection of H_0 's in favour of H_1 's (not all means are equal), but there is still no indication of the way in which they are not equal. Particularly for this study, it is not clear yet if all methods are different or just one is different from the rest. If the latter is the case, the question is which method is the different one. The procedure to answer these types of inquiries is referred to as multiple-comparison tests.

Pairwise comparison tests are the most frequently used type of multiple-comparison tests. Fisher's least significant difference (LSD) test as one of the most popular pairwise comparison tests involves performing a series of pairwise t tests, each with a specified Type I error (α). Here, the probability to reject a null hypothesis when it is true is set equal to 0.05 and for each pair of levels (methods) hypotheses are:

$$H_0: \mu_i = \mu_j$$

$$H_1: \mu_i \neq \mu_j$$

If Fisher's least significant difference for any pair falls outside of the acceptance region, the null hypothesis will be rejected. In this case, if μ_i is greater than μ_j , it will be concluded that Method j is superior to Method i as its mean number of variables is less than that of Method i . As an example, Method's pairwise comparisons for Increasing Pattern are demonstrated in this section.

Referring to Table 23, for Increasing Pattern when Min Length = 3 hrs, Max Length = 8 hrs, Shift Increment = 1hr, and Shift Start Time = 1hr, the following table shows the response variable for the three replications of each method.

Table 29: Response Values for the Specified Case of Increasing Pattern

M2	M3	M4	M5	M6	M7	M8	M9
18	20	21	23	18	18	15	9
22	23	25	23	22	22	19	11
27	27	29	23	27	27	25	13

Just for this case, $8(8-1) / 2 = 28$ pairwise comparisons are required, but this number might be cut if the potential superior method is selected as the start point of comparison. For clarity, the following example is presented.

Figure 4 illustrates the one factor plot of Method for the previously specified case of Increasing Pattern (Min and Max Lengths are 3 and 8 hours, respectively and Shift Increment as well as Shift Start Time is hourly). The graph shows that M9 has the lowest mean number of decision variables, so it is chosen as the start point of comparison which means that the pairwise t test between M9 and other methods are conducted first.

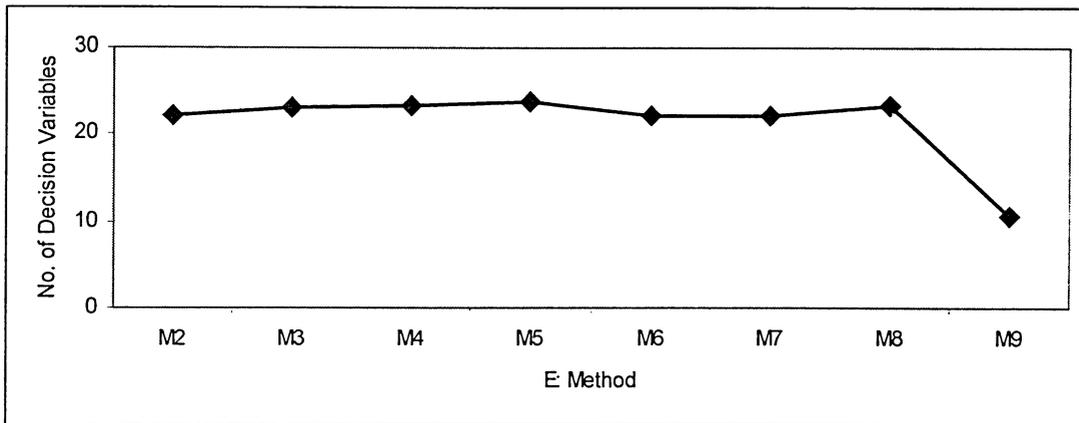


Figure 4: One Factor Plot of Method for the Specified Case

The first pairwise comparison is between M2 and M9. To conduct the t test, the data in the first and last columns of Table 29 is fed into Excel as the input for Data analysis. The outcome is displayed in the following table (Table 30).

Since the least significant difference (t Stat) is greater than “t Critical two-tail”, the null hypothesis ($H_0: \mu_{M2} = \mu_{M9}$) is rejected. Such a conclusion is also supported by the P-value (0.016041564) as it is less than $\alpha = 0.05$. As a result, Method 9 is significantly different from Method 2 and also superior. The positive mean difference ($\mu_{M2} - \mu_{M9} = 7.800$) indicates $\mu_{M2} > \mu_{M9}$, so on average, M9 generates fewer number of shifts.

Table 30: Output of the *t* Test for M2 and M9

	<i>Variable 1 (M2)</i>	<i>Variable 2 (M9)</i>
Mean	22.33333333	11
Variance	20.33333333	4
Observations	3	3
Pearson Correlation	0.997948716	
Hypothesized Mean Difference	0	
df	2	
t Stat	7.800134952	
P(T<=t) one-tail	0.008020782	
t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.016041564	
t Critical two-tail	4.302655725	

Table 31 represents the paired *t* test between Method 9 and the remaining methods. All comparisons result in rejecting the null hypothesis due to P-values which are less than $\alpha = 0.05$. So, it is concluded that there is a significant difference between M9's mean number of variables and every other methods'. The positive value for "t Stat" in all cases specifies the advantage of Method 9 over other methods in terms of efficiency.

It is clear that there is no point to continue the pairwise comparison for this case, as M9 is already known as the best method for that particular combination. Thus, by looking at plots (one factor plots or interaction plots) and selecting the potential superior method (M9) as the point at which pairwise comparison should start (all other methods are compared with M9 first), the number of comparisons (28) was reduced to 7.

In total, with choosing the proper start point for comparison, 1689 pairwise *t* tests were conducted as summarized in Table 32 and Table 33.

Table 32 consists of all 36 possible combinations of levels of the first four factors. For instance, the first combination is Min Length = 3 hrs, Max Length = 8 hrs, Shift Increment = 1/4 hr, and Shift Start Time = 1/4 hr. For this case, according to the Fisher's LSD test, both M8 and M9 are recognized equally good in terms of efficiency, because

there is not enough evidence to reject the difference between their means. However, the remaining methods' means are significantly different from means of these two.

Table 31: Pairwise Comparison between M9 and Other Methods for the Specified Case

Paired <i>t</i> Test (M3 & M9)			Paired <i>t</i> Test (M6& M9)		
	<i>Variable 1</i>	<i>Variable 2</i>		<i>Variable 1</i>	<i>Variable 2</i>
Mean	23.3333333	11	Mean	22.3333333	11
Variance	12.3333333	4	Variance	20.3333333	4
Observations	3	3	Observations	3	3
Pearson Correlation	0.9966159		Pearson Correlation	0.99794872	
Hypothesized Mean Difference	0		Hypothesized Mean Difference	0	
df	2		df	2	
t Stat	13.9846855		t Stat	7.80013495	
P(T<=t) one-tail	0.00253717		P(T<=t) one-tail	0.00802078	
t Critical one-tail	2.91998731		t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.00507434		P(T<=t) two-tail	0.01604156	
t Critical two-tail	4.30265573		t Critical two-tail	4.30265573	

Paired <i>t</i> Test (M4 & M9)		
	<i>Variable 1</i>	<i>Variable 2</i>
Mean	25	11
Variance	16	4
Observations	3	3
Pearson Correlation	1	
Hypothesized Mean Difference	0	
df	2	
t Stat	12.1243557	
P(T<=t) one-tail	0.00336704	
t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.00673408	
t Critical two-tail	4.30265573	

Paired <i>t</i> Test (M7 & M9)		
	<i>Variable 1</i>	<i>Variable 2</i>
Mean	22.3333333	11
Variance	20.3333333	4
Observations	3	3
Pearson Correlation	0.99794872	
Hypothesized Mean Difference	0	
df	2	
t Stat	7.80013495	
P(T<=t) one-tail	0.00802078	
t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.01604156	
t Critical two-tail	4.30265573	

Paired <i>t</i> Test (M5 & M9)		
	<i>Variable 1</i>	<i>Variable 2</i>
Mean	23	11
Variance	0	4
Observations	3	3
Pearson Correlation		
Hypothesized Mean Difference	0	
df	2	
t Stat	10.3923048	
P(T<=t) one-tail	0.00456631	
t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.00913261	
t Critical two-tail	4.30265573	

Paired <i>t</i> Test (M8 & M9)		
	<i>Variable 1</i>	<i>Variable 2</i>
Mean	19.6666667	11
Variance	25.3333333	4
Observations	3	3
Pearson Correlation	0.99339927	
Hypothesized Mean Difference	0	
df	2	
t Stat	4.91353815	
P(T<=t) one-tail	0.01950617	
t Critical one-tail	2.91998731	
P(T<=t) two-tail	0.03901235	
t Critical two-tail	4.30265573	

Table 32 confirms the preliminary observation of interaction graphs that suggests M9 as an efficient method for Increasing Pattern. Table 33 reveals the same information as of Table 32 for Decreasing, One-Hump, Two-Hump, and Mixed patterns.

Table 32: Superior Method(s) for Every Combination of Levels of Factors for Increasing Pattern

MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/4 hr	M8 M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/4 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/4 hr	M8 M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/4 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/4 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/4 hr	M8 M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/2 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/2 hr	M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/2 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1/2 hr	M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/2 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/2 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/2 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1/2 hr	M8 M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/2 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/2 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/2 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1/2 hr	M8 M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/4 hr SHIFT START TIME = 1 hr	M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1/2 hr SHIFT START TIME = 1 hr	M9
MIN LENGTH = 3 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1 hr	M9	MIN LENGTH = 4 hrs MAX. LENGTH = 8 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1 hr	M8 M9	MIN LENGTH = 3 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1 hr	M8 M9	MIN LENGTH = 4 hrs MAX. LENGTH = 10 hrs SHIFT INCREMENT = 1 hr SHIFT START TIME = 1 hr	M8 M9

* Superior Method(s)

The first part of Table 33, which is assigned to Decreasing Pattern, presents M8, M9, and sometimes M2, as the most efficient methods. This conclusion is true for One-Hump Pattern too if M3 is also added to the other three methods. As shown in the table, M8 and M9 are always the best while M2 and M3 only appear in a few combinations.

For Two-Hump as well as Mixed Pattern, any method except M6 can be found in Table 33; however, M8 and M9 are the ones which appear most frequently.

4.6 Discussion

It was observed that for all combinations Method 8 and Method 9 are the best methods in terms of efficiency. By having a closer look, it can be viewed that most of the time for Increasing Pattern M9 is selected as the most efficient method. On the other hand, when the staffing requirement follows a decreasing trend, M8 is usually the best method. This is always true due to the nature of the rules on which these two methods have been built. Therefore, a combination of Method 8 and Method 9 behaves efficiently for any problem. Since each problem can be broken to some increasing and decreasing parts, when the staffing requirement is following an increasing trend, shifts are generated by M9 and at the time that the staffing requirement switches to decreasing trend M8 generates the shifts.

Effectiveness, as mentioned earlier, is another criterion that must also be taken into consideration when announcing the best method. Through the statistical analysis, M8 and M9 are recognized as the candidate methods in terms of efficiency. In order to find out if they are also effective, Dominance Analysis is conducted. All objective function values from Table 22 are illustrated in the form of a graph in Figure 5.

Since the objective is to minimize the total hours scheduled, those methods that always produce objective function values as low as M1's are called effective. In the above case, M2 and M5 are not effective as in some points their values are greater than that of M1. In other words, they are strongly dominated by Method 1.

For all other objective function values listed in Appendix II, similar graphs are depicted in Appendix IV. These graphs show that for all combinations of Min Length, Max Length, Shift Increment, and Shift Start Time, Method 6 to Method 9 are effective.

Hence, it can be concluded that among the proposed methods M8 and M9 are the best ones in terms of both efficiency and effectiveness.

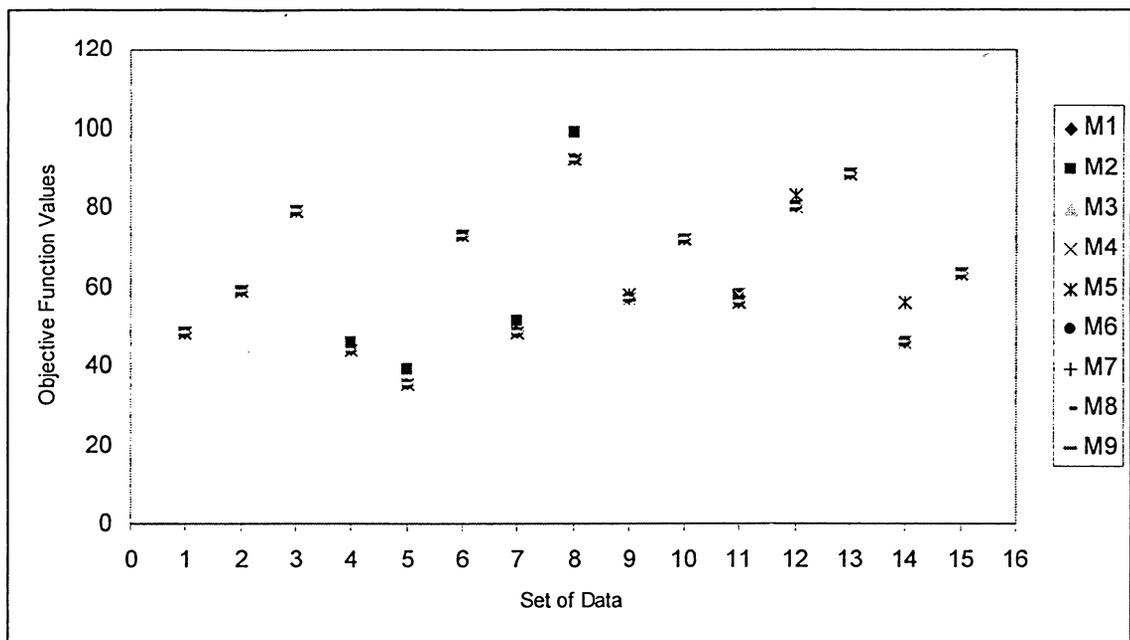


Figure 5: Dominance Analysis for Data in Table 22

All the above conclusions are right when there is 100% availability. For other percentage of availability, the best method can be a different one due to the trade off between efficiency and effectiveness. Suppose there is a given method (Method A) that reduces the size of a problem by say, 10%, but at the same time it eliminates some decision variables which can potentially be part of the optimal solution. Therefore, by applying this particular method to the problem, the objective function value will be say, 25% below the optimal value. If, on the other hand, another method (Method B) exists which just reduces the size of the problem by say, 5%, but sacrifices only 10% of the optimal solution, it might be a reasonable decision to pick Method B over Method A.

According to the above discussion, Method 6 has the potential to be chosen as the best method when there is less than 100% availability; however, it did not appear as an efficient method in Table 33. A relatively large number of shifts generated by M6 qualify this method as the best candidate in terms of effectiveness.

CHAPTER 5: CONCLUSIONS & RESEARCH EXTENSIONS

5.1 Research Contributions

In this thesis, for improving shift generation in the labour scheduling problem, eight heuristics have been introduced. Two of these methods were taken from the industry while the other six heuristics were developed through this research. Each method has been defined by some rules whose specifications along with an example were discussed. On the other hand, this thesis classified the trend of staffing requirements into five popular patterns as Increasing, Decreasing, One-Hump, Two-Hump, and Mixed. This helps to have a sound judgement about the proposed methods.

In the statistical analysis part, an experimental design was developed; factors and their levels as well as the response variable were determined, and for every individual pattern, a multifactor ANOVA with three replications was conducted. To identify the best methods in terms of efficiency, in this study Fisher's LSD test was performed and the result of the paired comparisons between methods revealed Method 8 and Method 9 as the best. Finally, dominance analysis proved that the candidate methods could be referred to as the best in terms of effectiveness, as well, since their objective function values were frequently optimal.

5.2 Future Research

The labour scheduling problem is an open area to investigate, and based on the work which has been done in this thesis, the following research extensions are suggested.

- a) In a preliminary analysis, it has been noted that, if the first assumption is relaxed, there is a small possibility that the proposed methods do not work for some specific staffing requirements. In other words, when Max Length is not at

least twice as much as the difference between Closing Time and Opening Time, the problem might end up with an infeasible solution. The reason is that there is at least one interval in the scheduling day which is not covered by any of the generated shifts. As this can be the case in stores with longer hours of operation, for example the ones that are running 24 hrs, it is recommended to find an efficient method to overcome this problem. This new method must generate a subset of shifts which cover all intervals. As an initial idea, combination of the methods' rules can be examined.

- b) The second assumption in this research guarantees 100% availability of employees all the time, and based on this idea, M8 and M9 were chosen as the best methods. It would be more realistic to compare the efficiency and effectiveness of the proposed methods when different levels of availability, such as 75% or 50%, exist.

REFERENCES

Aggarwal, S. C. (1982), A Focused Review of Scheduling in Services, *European Journal of Operational Research*, 9, pp. 114-121.

Alfares, H. K. (2004), Survey, Categorization, and Comparison of Recent Tour Scheduling Literature, *Annals of Operations Research*, 127, pp. 145-175.

Baker, K. R. (1976), Workforce Allocation in Cyclical Scheduling Problems: A Survey, *Operational Research Quarterly*, 27 (1ii), pp. 155-167.

Bechtold, S. E., M. J. Brusco, and M. J. Showalter (1991), A Comparative Evaluation of Labour Tour Scheduling Methods, *Decision Sciences*, 22 (4), pp. 683-699.

Berger, P. D., R. E. Maurer (2002), Experimental Design with Applications in Management, Engineering, and the Sciences, *Duxbury*

Dantzig, G. B. (1954), A Comment on Edie's Traffic Delays at Toll Booths, *Operations Research*, 2 (3), pp. 339-341.

Khoong, C. M. (1993), A Simple but Effective Heuristic for Work-Shift Assignment, *Omega*, 21 (3), pp. 393-395.

Lauer, J., L. W. Jacobs, M. J. Brusco, and S. E. Bechtold (1994), An Interactive Optimization-Based Decision Support System for Scheduling Part-Time Computer Lab Attendants, *Omega*, 22 (6), pp. 613-626.

Litchfield, J. A., A. Ingolfsson, and K. J. Cheng (2003), Rostering for a Restaurant, *INFOR*, 41 (3), pp. 287-300.

Loucks, J. S. and F. R. Jacobs (1991), Tour Scheduling and Task Assignment of a Heterogeneous Work Force: A Heuristic Approach, *Decision Sciences*, 22 (4), pp. 719-738.

Love, R. R. and J. M. Hoey (1990), Management Science Improves Fast-Food Operations, *Interfaces*, 20 (2), pp. 21-29.

Mabert, V. A. and C. A. Watts (1982), A Simulation Analysis of Tour-Shift Construction Procedures, *Management Science*, 28 (5), pp. 520-532.

Mason, A. J., D. M. Ryan, and D. M. Panton (1998), Integrated Simulation, Heuristic, and Optimization Approaches to Staff Scheduling, *Operations Research*, 46 (2), pp. 161-175.

Melachrinoudis, E. and M. Olafsson (1992), A Scheduling System for Supermarket Cashiers, *Computers and Industrial Engineering*, 23 (1-4), pp. 121-124.

Morris, J. G. and M. J. Showalter (1983), Simple Approaches to Shift, Days-off, and Tour Scheduling Problems, *Management Science*, 29 (8), pp. 942-950.

Quan, V. (2004), Retail Labour Scheduling, *ORMS Today*, 31 (6), pp. 32-36.

Thompson, G. M. (1993), Accounting for the Multi-Period Impact of Service When Determining Employee Requirements for Labour Scheduling, *Journal of Operations Management*, 11, pp. 269-287.

Thompson, G. M. (1995), Labour Scheduling Using NPV Estimates of the Marginal Benefit of Additional Labour Capacity, *Journal of Operations Management*, 13 (1), pp. 67-86.

Thompson, G. M. (1998), Labour Scheduling, Part 1: Forecasting Demand, *Cornell Hotel and Restaurant Administration Quarterly*, 39 (5), pp. 22-31.

APPENDIX I
(COMPUTER PROGRAMMES)

I.1 Computer Programmes for M1

I.1.1 First Programme for M1

```
model 'shifts'
uses 'mmxprs'
parameters
DataFile = 'E:\Banafsheh\back up\generator\first.dat'
end-parameters
declarations
openingtime:      real
closingtime:      real
interval1:        real
interval2:        real
minlength:        real
maxlength:        real
end-declarations
initializations from DataFile
openingtime closingtime interval1 interval2 minlength maxlength
end-initializations
fopen("out.dat",F_OUTPUT)

!generating all shifts in an array of real
a:= openingtime
i:=1
write("shifts : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlist(a+maxlength , closingtime)) do
        !write("(",a,"",b,"",c,"")
        write(a,"",b,"",c,"")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
end-do
writeln("]")
writeln

writeln("openingtime:",openingtime)
writeln("closingtime:",closingtime)
writeln("interval1:",interval1)
writeln("interval2:",interval2)
writeln("minlength:",minlength)
writeln("maxlength:",maxlength)
writeln("n :",i-1)
writeln

!generating all shifts in an array of string
a:= openingtime
i:=1
write("SHIFTS : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlist(a+maxlength , closingtime)) do
        write(" ",a,"",b,"",c,"")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
```

```

end-do

writeln("]")
writeln
!shift length
a:= openingtime
i:=1
write("LENGTH : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlist(a+maxlength , closingtime)) do
        write(b,",")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
end-do

writeln("]")
writeln

end-model

```

I.1.2 Second Programme for M1

```

model 'shifts'
uses 'mmxprs'
parameters
DataFile = 'out.dat'
DataFile1 = 'E:\Banafsheh\back up\generator\req.dat'
end-parameters
declarations
openingtime:    real
closingtime:    real
interval1:      real
interval2:      real
minlength:     real
maxlength:     real
shifts:         dynamic array(1..10000) of real
req:            dynamic array(1..100) of integer
end-declarations
initializations from DataFile
shifts openingtime closingtime interval1 interval2 minlength maxlength
end-initializations
initializations from DataFile1
req
end-initializations

```

```
fopen("out.mos",F_OUTPUT)
```

```

writeln("model 'shifts'")
writeln("uses 'mmxprs'")
writeln("parameters")
writeln("DataFile = 'out.dat'")
!writeln("DataFile1 = 'req.dat'")
writeln("end-parameters")
writeln("declarations")
writeln("SHIFTS: set of string")
writeln("LENGTH: array(SHIFTS) of real")
writeln("SCHEDULE: array(SHIFTS) of mpvar ")

```

```

!writeln("req: dynamic array(1..100) of integer")
writeln("end-declarations")
writeln("initializations from DataFile")
writeln("SHIFTS LENGTH")
writeln("end-initializations")
!writeln("initializations from DataFile1")
!writeln("req")
!writeln("end-initializations")
writeln("forall(i in SHIFTS) create (SCHEDULE(i))")
writeln("forall(i in SHIFTS) SCHEDULE(i)is_integer")
writeln("TotalHoursWorked :=sum(i in SHIFTS) (LENGTH(i)*SCHEDULE(i))")

M:=getsize(shifts)
N:=floor((((closingtime-openingtime-minlength)/interval2)+1)/3)
a:= openingtime
j:=a
k:=1
while(a<closingtime)do
    i:=0
    !if(j<=(((closingtime-openingtime)/interval2)+openingtime))then
        !write("0")
    !end-if
    while(i<=M) do
        if(shifts(3*i+1)<=a and shifts(3*i+3)>a) then
            write("+SCHEDULE(",shifts(3*i+1),",",shifts(3*i+2),",",shifts(3*i+3),")")
        end-if
        i+=1
    end-do
    j+=interval2
    write(">=",req(k))
    writeln
    a+=0.25
    k+=1
end-do
writeln("minimize(TotalHoursWorked)")
write("writeln(")
write("objective value is")
writeln(",getobjval")
write("forall (i in SHIFTS |getsol(SCHEDULE(i))>0) writeln(")
write("SCHEDULE(")
write(",i,")
write(")")
writeln(",getsol (SCHEDULE(i))")
writeln("end-model")
end-model

```

I.2 Computer Programmes for M2

I.2.1 First Programme for M2

```

model 'shifts'
uses 'mmxprs'
parameters
DataFile = 'E:\Banafsheh\back up\generator\first.dat'
end-parameters
declarations
openingtime:    real
closingtime:    real
interval1:      real
interval2:      real
minlength:      real
maxlength:      real
end-declarations

```

```

initializations from DataFile
openingtime closingtime interval1 interval2 minlength maxlength
end-initializations

```

```
fopen("out.dat",F_OUTPUT)
```

```
!generating all shifts in an array of real
```

```

a:= openingtime
i:=1
write("shifts : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlength(a+maxlength, closingtime)) do
        !write(" ",a," ",b," ",c,"")
        write(a," ",b," ",c,"")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
end-do
writeln("]")
writeln

```

```

writeln("openingtime:",openingtime)
writeln("closingtime:",closingtime)
writeln("interval1:",interval1)
writeln("interval2:",interval2)
writeln("minlength:",minlength)
writeln("maxlength:",maxlength)
writeln("n :",i-1)
writeln

```

```
!generating all shifts in an array of string
```

```

a:= openingtime
i:=1
write("SHIFTS : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlength(a+maxlength, closingtime)) do
        write(" ",a," ",b," ",c,"")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
end-do

```

```

writeln("]")
writeln
!shift length
a:= openingtime
i:=1
write("LENGTH : [")
while (closingtime-a >= minlength) do
    b:= minlength
    c:=a+b
    while (c <= minlength(a+maxlength, closingtime)) do
        write(b,"")
        b+=interval1
        c:=a+b
        i+=1
    end-do
    a+=interval2
end-do

```

```
writeln("]")
writeln

end-model
```

I.2.2 Second Programme for M2

```
model 'shifts'
uses 'mmxprs'
parameters
DataFile = 'out.dat'
DataFile1 = 'E:\Banafsheh\back up\generator\req.dat'
end-parameters
declarations
openingtime:      real
closingtime:      real
interval1:        real
interval2:        real
minlength:        real
maxlength:        real
shifts:           dynamic array(1..10000) of real
req:              dynamic array(1..100) of integer
end-declarations
initializations from DataFile
openingtime closingtime interval1 interval2 minlength maxlength shifts
end-initializations
initializations from DataFile1
req
end-initializations

fopen("out1.dat",F_OUTPUT)

M:=getsize(shifts)
N:=floor((((closingtime-openingtime-minlength)/interval2)+1)/3)
a:= openingtime

!generating the shifts on the jump in an array of real
i:=0
l:=1
write("F : (")
while (i<=M)do
    if (shifts(3*i+1)= openingtime) then
        write(shifts(3*i+1),",",shifts(3*i+2),",",shifts(3*i+3),",")
    end-if
    i+=1
end-do
while(l<=((closingtime-openingtime)/0.25)) do
    i:=0
    A:=openingtime+l*0.25
    if(req(l+1)>req(l)) then
        while(i<=M) do
            if(shifts(3*i+1)=A) then
                !write("(",shifts(3*i+1),",",shifts(3*i+2),",",shifts(3*i+3),")")
                write(shifts(3*i+1),",",shifts(3*i+2),",",shifts(3*i+3),",")
            end-if
            i+=1
        end-do
        !write("=0")
        !writeln
    end-if
    l+=1
end-do
```

```

writeln("")
writeln

!generating the shifts on the jump in an array of string
i:=0
l:=1
write("FF : [")
while (i<=M)do
    if (shifts(3*i+1)= openingtime) then
        write(" ",shifts(3*i+1)," ",shifts(3*i+2)," ",shifts(3*i+3),"")
    end-if
    i+=1
end-do
while(l<=((closingtime-openingtime)/0.25)) do
    i:=0
    A:=openingtime+l*0.25
    if(req(l+1)>req(l)) then
        while(j<=M) do
            if(shifts(3*i+1)=A) then
                write(" ",shifts(3*i+1)," ",shifts(3*i+2)," ",shifts(3*i+3),"")
            end-if
            i+=1
        end-do
    end-if
    l+=1
end-do
writeln("]")
writeln

```

```

!generating the shifts on the falls in an array of real
i:=0
l:=1
write("B : (")
while (i<=M)do
    if (shifts(3*i+3)= closingtime) then
        write(shifts(3*i+1)," ",shifts(3*i+2)," ",shifts(3*i+3),"")
    end-if
    i+=1
end-do
writeln(")")
writeln

```

```

!generating the shifts on the falls in an array of string
i:=0
l:=1
write("BB : [")
while (i<=M)do
    if (shifts(3*i+3)= closingtime) then
        write(" ",shifts(3*i+1)," ",shifts(3*i+2)," ",shifts(3*i+3),"")
    end-if
    i+=1
end-do
writeln("]")
writeln

end-model

```

I.2.3 Third Programme for M2

```

model 'shifts'
uses 'mmxprs'
parameters
DataFile = 'out.dat'
DataFile1 = 'out1.dat'
DataFile2 = 'E:\Banafsheh\back up\generator\req.dat'

```

```

end-parameters
declarations
openingtime:      real
closingtime:      real
interval1:        real
interval2:        real
minlength:        real
maxlength:        real
shifts:           dynamic array(1..10000) of real
req:              dynamic array(1..100) of integer
FF : set of string
BB : set of string
SHIFTS : set of string
ZERO : set of string
end-declarations
initializations from DataFile
shifts openingtime closingtime interval1 interval2 minlength maxlength  SHIFTS
end-initializations
initializations from DataFile1
FF      BB
end-initializations
initializations from DataFile2
req
end-initializations

fopen("out.mos",F_OUTPUT)

writeln("model 'shifts'")
writeln("uses 'mxxprs'")
writeln("parameters")
writeln("DataFile = 'out.dat'")
!writeln("DataFile1 = 'out1.dat'")
!writeln("DataFile2 = 'req.dat'")
writeln("end-parameters")
writeln("declarations")
writeln("SHIFTS: set of string")
writeln("LENGTH: array(SHIFTS) of real")
writeln("SCHEDULE: array(SHIFTS) of mpvar ")
!writeln("req: dynamic array(1..100) of integer")
writeln("ZERO : set of string")
writeln("end-declarations")
writeln("initializations from DataFile")
writeln("SHIFTS LENGTH")
writeln("end-initializations")
!writeln("initializations from DataFile2")
!writeln("req")
!writeln("end-initializations")
writeln("forall(i in SHIFTS) create (SCHEDULE(i))")
writeln("forall(i in SHIFTS) SCHEDULE(i)is_integer")
writeln("TotalHoursWorked :=sum(i in SHIFTS) (LENGTH(i)*SCHEDULE(i))")
SHIFTS1 := FF+BB
ZERO := (SHIFTS-SHIFTS1)
forall(i in ZERO) writeln("SCHEDULE(",ZERO(i),")=0")
M:=getsize(shifts)
N:=floor((((closingtime-openingtime-minlength)/interval2)+1)/3)
a:= openingtime
j:=a
k:=1
while(a<closingtime)do
  i:=0
  !if(j<=(((closingtime-openingtime)/interval2)+openingtime))then
    !write("0")
  !end-if
  while(i<=M) do
    if(shifts(3*i+1)<=a and shifts(3*i+3)>a) then
      write("+SCHEDULE(",shifts(3*i+1),",",shifts(3*i+2),",",shifts(3*i+3),")")
    end-if
    i+=1
  end-while
  j+=interval2
end-while

```

```
        end-do
        j+=interval2
        write(">=",req(k))
        writeln
        a+=0.25
        k+=1
    end-do
    writeln("minimize(TotalHoursWorked)")
    write("writeln(")
    write("objective value is")
    writeln(",getobjval)")
    write("forall (i in SHIFTS |getsol(SCHEDULE(i))>0) writeln(")
    write("SCHEDULE(")
    write(",i,")
    write(")=")
    writeln(",getsol (SCHEDULE(i)))")
    writeln("end-model")
end-model
```

APPENDIX II

OBJECTIVE FUNCTION VALUE AND NUMBER OF DECISION VARIABLES FOR ALL COMBINATIONS OF LEVELS OF FACTORS

Results for Min Length=3, Max Length=8, Shift Increment=1/4, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	567	567	567	567	567	567	567	567	567	567	567	567	567	567	567
	M2	66	79	99	42	42	42	42	42	75	100	63	99	83	79	99
	M3	74	81	99	58	58	58	58	58	79	102	76	99	87	79	101
	M4	81	94	113	58	58	58	58	58	90	114	79	111	97	93	112
	M5	92	92	92	92	92	92	92	92	92	92	92	92	92	92	92
	M6	66	79	99	91	84	70	63	63	95	133	84	138	135	135	135
	M7	66	79	99	46	46	44	44	44	77	102	65	101	86	83	99
	M8	48	61	82	27	27	24	25	25	59	84	46	85	67	66	81
	M9	24	26	28	73	65	52	44	44	47	65	46	72	84	89	74

Results for Min Length=4, Max Length=8, Shift Increment=1/4, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	425	425	425	425	425	425	425	425	425	425	425	425	425	425	425
	M2	50	63	75	34	34	34	34	34	59	80	51	75	67	59	79
	M3	50	63	75	42	42	42	42	42	59	80	57	75	67	59	79
	M4	57	70	82	42	42	42	42	42	67	87	59	82	74	67	85
	M5	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80
	M6	50	63	75	71	68	54	51	51	75	105	68	104	105	100	112
	M7	50	63	75	38	38	36	36	36	61	81	53	77	71	61	80
	M8	35	49	61	23	23	21	21	21	47	67	38	65	56	47	66
	M9	20	22	23	57	53	40	36	36	39	52	38	56	65	66	59

Results for Min Length=3, Max Length=10, Shift Increment=1/4, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	667	667	667	667	667	667	667	667	667	667	667	667	667	667	667
	M2	82	102	122	58	58	58	58	58	98	127	87	118	102	98	118
	M3	90	104	122	74	74	74	74	74	102	129	100	118	106	98	120
	M4	97	117	136	74	74	74	74	74	113	141	103	130	116	112	131
	M5	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	M6	82	102	122	114	111	86	87	87	125	167	115	164	157	169	160
	M7	82	102	122	61	61	60	60	60	100	129	88	119	104	102	118
	M8	57	76	98	34	34	33	33	33	74	104	61	96	79	78	94
	M9	33	33	35	89	85	61	60	60	62	80	61	87	95	112	82

Results for Min Length=4, Max Length=10, Shift Increment=1/4, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	525	525	525	525	525	525	525	525	525	525	525	525	525	525	525
	M2	66	86	98	50	50	50	50	50	82	107	75	94	86	78	98
	M3	66	86	98	58	58	58	58	58	82	107	81	94	86	78	98
	M4	73	93	105	58	58	58	58	58	90	114	83	101	93	86	104
	M5	88	88	88	88	88	88	88	88	88	88	88	88	88	88	88
	M6	66	86	98	94	95	70	75	75	105	139	99	130	127	134	131
	M7	66	86	98	53	53	52	52	52	84	108	76	95	89	80	99
	M8	44	64	77	30	30	29	29	29	62	87	53	76	68	59	79
	M9	28	29	30	73	73	49	52	52	54	67	53	71	76	89	67

Results for Min Length=3,Max Length=8,Shift Increment=1/4, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	289	289	289	289	289	289	289	289	289	289	289	289	289	289	289
	M2	56	69	89	32	32	32	32	32	65	90	53	89	73	69	89
	M3	60	70	89	40	40	40	40	40	67	91	60	89	75	69	90
	M4	63	76	95	40	40	40	40	40	72	96	61	93	79	75	94
	M5	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
	M6	56	69	89	57	54	46	43	43	75	105	64	106	97	95	107
	M7	56	69	89	36	36	34	34	34	67	92	55	91	76	73	89
	M8	48	61	82	27	27	24	25	25	59	84	46	85	67	66	81
	M9	14	16	18	39	35	28	24	24	27	37	26	40	46	49	40

Results for Min Length=4, Max Length=8,Shift Increment=1/4, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	217	217	217	217	217	217	217	217	217	217	217	217	217	217	217
	M2	42	55	67	26	26	26	26	26	51	72	43	67	59	51	71
	M3	42	55	67	30	30	30	30	30	51	72	46	67	59	51	71
	M4	45	58	70	30	30	30	30	30	55	75	47	70	62	55	73
	M5	48	48	48	48	48	48	48	48	48	48	48	48	48	48	48
	M6	42	55	67	45	44	36	35	35	59	83	52	80	77	70	86
	M7	42	55	67	30	30	28	28	28	53	73	45	69	63	53	72
	M8	35	49	61	23	23	21	21	21	47	67	38	65	56	47	66
	M9	12	14	15	31	29	22	20	20	23	30	22	32	37	36	33

Results for Min Length=3, Max Length=10,Shift Increment=1/4, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	341	341	341	341	341	341	341	341	341	341	341	341	341	341	341
	M2	68	88	108	44	44	44	51	44	84	113	73	104	88	84	104
	M3	72	89	108	52	52	52	52	52	86	114	80	104	90	84	105
	M4	75	95	114	52	52	52	52	52	91	119	81	108	94	90	109
	M5	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
	M6	68	88	108	72	71	58	59	59	97	131	87	124	113	113	122
	M7	68	88	108	47	47	46	46	46	86	115	74	105	90	88	104
	M8	57	76	98	34	34	33	33	33	74	104	61	96	79	78	94
	M9	19	19	21	47	45	33	32	32	34	44	33	47	51	60	44

Results for Min Length=4, Max Length=10,Shift Increment=1/4, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	269	269	269	269	269	269	269	269	269	269	269	269	269	269	269
	M2	54	74	86	38	38	38	38	38	70	95	63	82	74	66	86
	M3	54	74	86	42	42	42	42	42	70	95	66	82	74	66	86
	M4	57	77	89	42	42	42	42	42	74	98	67	85	77	70	88
	M5	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
	M6	54	74	86	60	60	48	51	51	81	109	75	98	93	92	101
	M7	54	74	86	41	41	40	40	40	72	96	64	83	77	68	87
	M8	44	64	77	30	30	29	29	29	62	87	53	76	68	59	79
	M9	16	17	18	39	39	27	28	28	30	37	29	39	42	47	37

Results for Min Length=3, Max Length=8,Shift Increment=1/4, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
	M2	51	64	84	27	27	27	27	27	60	85	48	84	68	64	84
	M3	53	65	84	31	31	31	31	31	61	86	52	84	69	64	84
	M4	54	67	86	31	31	31	31	31	63	87	52	85	70	66	85
	M5	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
	M6	51	64	84	40	39	34	33	33	65	91	54	90	78	75	90
	M7	51	64	84	31	31	29	29	29	62	87	50	86	71	68	84
	M8	48	61	82	27	27	24	25	25	59	84	46	85	67	66	81
	M9	9	11	13	22	20	16	14	14	17	23	16	24	27	29	23

Results for Min Length=4, Max Length=8,Shift Increment=1/4, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	38	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	38	76	52	104	61	75	62	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	113	113	113	113	113	113	113	113	113	113	113	113	113	113	113
	M2	38	51	63	22	22	22	22	22	47	68	39	63	55	47	67
	M3	38	51	63	24	24	24	24	24	47	68	41	63	55	47	67
	M4	39	52	64	24	24	24	24	24	49	69	41	64	56	49	67
	M5	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
	M6	38	51	63	32	32	27	27	27	51	72	44	68	68	55	73
	M7	38	51	63	26	26	24	24	24	49	69	41	65	59	49	68
	M8	35	49	61	23	23	21	21	21	47	67	38	65	56	47	66
	M9	8	10	11	18	17	13	12	12	15	19	14	20	23	21	20

Results for Min Length=3, Max Length=10,Shift Increment=1/4, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	178	178	178	178	178	178	178	178	178	178	178	178	178	178	178
	M2	61	81	101	37	37	37	37	37	77	106	66	97	81	77	97
	M3	63	82	101	41	41	41	41	41	78	107	70	97	82	77	98
	M4	64	84	103	41	41	41	41	41	80	108	70	98	83	79	98
	M5	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
	M6	61	81	101	51	51	44	45	45	83	113	73	104	91	91	103
	M7	61	81	101	40	40	39	39	39	79	108	67	98	83	81	97
	M8	57	76	98	34	34	33	33	33	74	104	61	96	79	78	94
	M9	12	12	14	26	25	19	18	18	20	26	19	27	29	34	25

Results for Min Length=4, Max Length=10,Shift Increment=1/4, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	141	141	141	141	141	141	141	141	141	141	141	141	141	141	141
	M2	48	68	80	32	32	32	32	32	64	89	57	76	68	60	80
	M3	48	68	80	34	34	34	34	34	64	89	59	76	68	60	80
	M4	49	69	81	34	34	34	34	34	66	90	59	77	69	62	80
	M5	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
	M6	48	68	80	43	44	37	34	39	69	94	63	82	76	71	86
	M7	48	68	80	35	35	34	34	34	66	90	58	77	71	62	81
	M8	44	64	77	30	30	29	29	29	62	87	53	76	68	59	79
	M9	10	11	12	22	22	16	16	16	18	22	17	23	25	26	22

Results for Min Length=3, Max Length=8,Shift Increment=1/2, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	297	297	297	297	297	297	297	297	297	297	297	297	297	297	297
	M2	34	41	51	22	22	22	22	22	39	52	33	51	43	41	51
	M3	38	42	51	30	30	30	30	30	41	53	40	51	45	41	52
	M4	41	48	57	30	30	30	30	30	46	58	41	55	49	47	56
	M5	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72
	M6	34	41	51	47	44	36	33	33	49	67	44	68	67	67	69
	M7	34	41	51	26	26	24	24	24	41	54	35	53	46	45	45
	M8	26	33	44	17	17	14	15	15	33	46	26	47	37	38	43
	M9	14	16	18	39	35	28	24	24	27	37	26	40	46	49	40

Results for Min Length=4, Max Length=8,Shift Increment=1/2, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	225	225	225	225	225	225	225	225	225	225	225	225	225	225	225
	M2	26	33	39	18	18	18	18	18	31	42	27	39	35	31	41
	M3	26	33	39	22	22	22	22	22	31	42	30	39	35	31	41
	M4	29	36	42	22	22	22	22	22	35	45	31	42	38	35	43
	M5	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
	M6	26	33	39	37	36	28	27	27	39	53	36	52	53	50	56
	M7	26	33	39	22	22	20	20	20	33	43	29	41	39	33	42
	M8	19	27	33	15	15	13	13	13	27	37	22	37	32	27	36
	M9	12	14	15	31	29	22	20	20	23	30	22	32	37	36	33

Results for Min Length=3, Max Length=10,Shift Increment=1/2, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	345	345	345	345	345	345	345	345	345	345	345	345	345	345	345
	M2	42	52	62	30	30	30	30	30	50	65	45	60	52	50	60
	M3	46	53	62	38	38	38	38	38	52	66	52	60	54	50	61
	M4	49	59	68	38	38	38	38	38	57	71	53	64	58	56	65
	M5	72	72	72	72	72	72	72	72	72	72	72	72	72	72	72
	M6	42	52	62	58	57	44	45	45	62	83	59	80	77	83	78
	M7	42	52	62	33	33	32	32	32	53	67	46	61	54	54	60
	M8	31	40	52	20	20	19	19	19	40	56	33	52	43	44	50
	M9	19	19	21	47	45	33	32	32	34	44	33	47	51	60	44

Results for Min Length=4, Max Length=10,Shift Increment=1/2, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	273	273	273	273	273	273	273	273	273	273	273	273	273	273	273
	M2	34	44	50	26	26	26	26	26	42	55	39	48	44	40	50
	M3	34	44	50	30	30	30	30	30	42	55	42	48	44	40	50
	M4	37	47	53	30	30	30	30	30	46	58	43	51	47	44	52
	M5	64	64	64	64	64	64	64	64	64	64	64	64	64	64	64
	M6	34	44	50	48	49	36	39	39	53	69	51	64	63	66	65
	M7	34	44	50	29	29	28	28	28	44	56	40	49	47	42	51
	M8	24	34	41	18	18	17	17	17	34	47	29	42	38	33	43
	M9	16	17	18	39	39	27	28	28	30	37	29	39	42	47	37

Results for Min Length=3, Max Length=8,Shift Increment=1/2, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	154	154	154	154	154	154	154	154	154	154	154	154	154	154	154
	M2	34	41	51	22	22	22	22	22	39	52	33	51	43	41	51
	M3	38	42	51	30	30	30	30	30	41	53	40	51	45	41	52
	M4	41	48	57	30	30	30	30	30	46	58	41	55	49	47	56
	M5	46	46	46	46	46	46	46	46	46	46	46	46	46	46	46
	M6	34	41	51	47	44	36	33	33	49	67	44	68	67	67	69
	M7	34	41	51	26	26	24	24	24	41	54	35	53	46	45	51
	M8	26	33	44	17	17	14	15	15	33	46	26	47	37	38	43
	M9	14	16	18	39	35	28	24	24	27	37	26	40	46	49	40

Results for Min Length=4, Max Length=8,Shift Increment=1/2, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	117	117	117	117	117	117	117	117	117	117	117	117	117	117	117
	M2	26	33	39	18	18	18	18	18	31	42	27	39	35	31	41
	M3	26	33	39	22	22	22	22	22	31	42	30	39	35	31	41
	M4	29	36	42	22	22	22	22	22	35	45	31	42	38	35	43
	M5	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
	M6	26	33	39	37	36	28	27	27	39	53	36	52	53	50	56
	M7	26	33	39	22	22	20	20	20	33	43	29	41	39	33	42
	M8	19	27	33	15	15	13	13	13	27	37	22	37	32	27	36
	M9	12	14	15	31	29	22	20	20	23	30	22	32	37	36	33

Results for Min Length=3, Max Length=10,Shift Increment=1/2, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M4	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	180	180	180	180	180	180	180	180	180	180	180	180	180	180	180
	M2	42	52	62	30	30	30	30	30	50	65	45	60	52	50	60
	M3	46	53	62	38	38	38	38	38	52	66	52	60	54	50	61
	M4	49	59	68	38	38	38	38	38	57	71	53	64	58	56	65
	M5	50	50	50	50	50	50	50	50	50	50	50	50	50	50	50
	M6	42	52	62	58	57	44	45	45	62	83	59	80	77	83	78
	M7	42	52	62	33	33	32	32	32	53	67	46	61	54	54	60
	M8	31	40	52	20	20	19	19	19	40	56	33	52	43	44	50
	M9	19	19	21	47	45	33	32	32	34	44	33	47	51	60	44

Results for Min Length=4, Max Length=10,Shift Increment=1/2, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M4	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	143	143	143	143	143	143	143	143	143	143	143	143	143	143	143
	M2	34	44	50	26	26	26	26	26	42	55	39	48	44	40	50
	M3	34	44	50	30	30	30	30	30	42	55	42	48	44	40	50
	M4	37	47	53	30	30	30	30	30	46	58	43	51	47	44	52
	M5	44	44	44	44	44	44	44	44	44	44	44	44	44	44	44
	M6	34	44	50	48	49	36	39	39	53	69	51	64	63	66	65
	M7	34	44	50	29	29	28	28	28	44	56	40	49	47	42	51
	M8	24	34	41	18	18	17	17	17	34	47	29	42	38	33	43
	M9	16	17	18	39	39	27	28	28	30	37	29	39	42	47	37

Results for Min Length=3, Max Length=8, Shift Increment=1/2, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	80	80	80	80	80	80	80	80	80	80	80	80	80	80	80
	M2	29	36	46	17	17	17	17	17	34	47	28	46	38	36	46
	M3	31	37	46	21	21	21	21	21	35	48	32	46	39	36	47
	M4	32	39	48	21	21	21	21	21	37	49	32	47	40	38	47
	M5	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
	M6	29	36	46	30	29	24	23	23	39	53	34	52	48	47	52
	M7	29	36	46	21	21	19	19	19	36	49	30	48	41	40	46
	M8	26	33	44	17	17	14	15	15	33	46	26	47	37	38	43
	M9	9	11	13	22	20	16	14	14	17	23	16	24	27	29	23

Results for Min Length=4, Max Length=8, Shift Increment=1/2, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	38	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	38	76	52	104	61	75	62	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	82	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	61	61	61	61	61	61	61	61	61	61	61	61	61	61	61
	M2	22	29	35	14	14	14	14	14	27	38	23	35	31	27	37
	M3	22	29	35	16	16	16	16	16	27	38	25	35	31	27	37
	M4	23	30	36	16	16	16	16	16	29	39	25	36	32	29	37
	M5	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24
	M6	22	29	35	24	24	19	19	19	31	42	28	40	39	35	43
	M7	22	29	35	18	18	16	16	16	29	39	25	37	35	29	38
	M8	19	27	33	15	15	13	13	13	27	37	22	37	32	27	36
	M9	8	10	11	18	17	13	12	12	15	19	14	20	23	21	20

Results for Min Length=3, Max Length=10,Shift Increment=1/2, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	94	94	94	94	94	94	94	94	94	94	94	94	94	94	94
	M2	35	45	55	23	23	23	23	23	43	58	35	53	45	43	53
	M3	37	46	55	27	27	27	27	27	44	59	42	53	46	43	54
	M4	38	48	57	27	27	27	27	27	46	60	42	54	47	45	54
	M5	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
	M6	35	45	55	37	37	30	31	31	49	65	45	60	55	57	59
	M7	35	45	55	26	26	25	25	19	45	60	39	54	47	47	53
	M8	31	40	52	20	20	19	19	19	40	56	33	52	43	44	50
	M9	12	12	14	26	25	19	18	18	20	26	19	27	29	34	25

Results for Min Length=4, Max Length=10,Shift Increment=1/2, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	75	75	75	75	75	75	75	75	75	75	75	75	75	75	75
	M2	28	38	44	20	20	20	20	20	36	49	33	42	38	34	44
	M3	28	38	44	22	22	22	22	22	36	49	35	42	38	34	44
	M4	29	39	45	22	22	22	22	22	38	50	35	43	39	36	44
	M5	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
	M6	28	38	44	31	32	25	27	27	41	54	39	48	46	45	50
	M7	28	38	44	23	23	22	22	22	38	50	34	43	41	36	45
	M8	24	34	41	18	18	17	17	17	34	47	29	42	38	33	43
	M9	10	11	12	22	22	16	16	16	18	22	17	23	25	26	22

Results for Min Length=3, Max Length=8,Shift Increment=1, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	162	162	162	162	162	162	162	162	162	162	162	162	162	162	162
	M2	18	22	27	12	12	12	12	12	21	28	18	27	23	22	27
	M3	20	23	27	16	16	16	16	16	22	29	22	27	24	22	28
	M4	21	25	29	16	16	16	16	16	24	30	22	28	25	24	28
	M5	62	62	62	62	62	62	62	62	62	62	62	62	62	62	62
	M6	18	22	27	25	24	19	18	18	26	34	24	33	33	33	33
	M7	18	22	27	16	16	14	14	14	23	30	20	29	26	26	27
	M8	15	19	25	12	12	9	10	10	20	27	16	28	22	24	24
	M9	9	11	13	22	20	16	14	14	17	23	16	24	27	29	23

Results for Min Length=4, Max Length=8,Shift Increment=1, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M11	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	38	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	38	76	52	104	61	75	62	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
	M2	14	18	21	10	10	10	10	10	17	23	15	21	19	17	22
	M3	14	18	21	12	12	12	12	12	17	23	17	21	19	17	22
	M4	15	19	22	12	12	12	12	12	19	24	17	22	20	19	22
	M5	56	56	56	56	56	56	56	56	56	56	56	56	56	56	56
	M6	14	18	21	20	20	15	15	15	21	27	20	26	27	25	28
	M7	14	18	21	14	14	12	12	12	19	24	17	23	23	19	23
	M8	11	16	19	11	11	9	9	9	17	22	14	23	20	17	21
	M9	8	10	11	18	17	13	12	12	15	19	14	20	23	21	20

Results for Min Length=3, Max Length=10,Shift Increment=1, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	184	184	184	184	184	184	184	184	184	184	184	184	184	184	184
	M2	22	27	32	16	16	16	16	16	26	34	24	31	27	26	31
	M3	24	28	32	20	20	20	20	20	27	35	28	31	28	26	32
	M4	25	30	34	20	20	20	20	20	29	36	28	32	29	28	32
	M5	58	58	58	58	58	58	58	58	58	58	58	58	58	58	58
	M6	22	27	32	30	30	23	24	24	32	41	31	38	37	40	37
	M7	22	27	32	19	19	18	18	18	28	36	25	32	29	30	31
	M8	18	22	29	13	13	12	12	12	23	32	19	30	25	27	28
	M9	12	12	14	26	25	19	18	18	20	26	19	27	29	34	25

Results for Min Length=4, Max Length=10,Shift Increment=1, Shift Start Time=1/4

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	147	147	147	147	147	147	147	147	147	147	147	147	147	147	147
	M2	18	23	26	14	14	14	14	14	22	29	21	25	23	21	26
	M3	18	23	26	16	16	16	16	16	22	29	23	25	23	21	26
	M4	19	24	27	16	16	16	16	16	24	30	23	26	24	23	26
	M5	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52
	M6	18	23	26	25	26	19	21	21	27	34	27	31	31	32	32
	M7	18	23	26	17	17	16	16	16	24	30	22	26	26	23	27
	M8	14	19	23	12	12	11	11	11	20	27	17	25	23	20	25
	M9	10	11	12	22	22	16	16	16	18	22	17	23	25	26	22

Results for Min Length=3, Max Length=8,Shift Increment=1, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	84	84	84	84	84	84	84	84	84	84	84	84	84	84	84
	M2	18	22	27	12	12	12	12	12	21	28	18	27	23	22	27
	M3	20	23	27	16	16	16	16	16	22	29	22	27	24	22	28
	M4	21	25	29	16	16	16	16	16	24	30	22	28	25	24	28
	M5	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	M6	18	22	27	25	24	19	18	18	26	34	24	33	33	33	33
	M7	18	22	27	16	16	14	14	14	23	30	20	29	26	26	27
	M8	15	19	25	12	12	9	10	10	20	27	16	28	22	24	24
	M9	9	11	13	22	20	16	14	14	17	23	16	24	27	29	23

Results for Min Length=4, Max Length=8,Shift Increment=1, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	38	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	38	76	52	104	61	75	62	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	65	65	65	65	65	65	65	65	65	65	65	65	65	65	65
	M2	14	18	21	10	10	10	10	10	17	23	15	21	19	17	22
	M3	14	18	21	12	12	12	12	12	17	23	17	21	19	17	22
	M4	15	19	22	12	12	12	12	12	19	24	17	22	20	19	22
	M5	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
	M6	14	18	21	20	20	15	15	15	21	27	20	26	27	25	28
	M7	14	18	21	14	14	12	12	12	19	24	17	23	23	19	23
	M8	11	16	19	11	11	9	9	9	17	22	14	23	20	17	21
	M9	8	10	11	18	17	13	12	12	15	19	14	20	23	21	20

Results for Min Length=3, Max Length=10,Shift Increment=1, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	96	96	96	96	96	96	96	96	96	96	96	96	96	96	96
	M2	22	27	32	16	16	16	16	16	26	34	24	31	27	26	31
	M3	24	28	32	20	20	20	20	20	27	35	28	31	28	26	32
	M4	25	30	34	20	20	20	20	20	29	36	28	32	29	28	32
	M5	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36
	M6	22	27	32	30	30	23	24	24	32	41	31	38	37	40	37
	M7	22	27	32	19	19	18	18	18	28	36	25	32	29	30	31
	M8	18	22	29	13	13	12	12	12	23	32	19	30	25	27	28
	M9	12	12	14	26	25	19	18	18	20	26	19	27	29	34	25

Results for Min Length=4, Max Length=10,Shift Increment=1, Shift Start Time=1/2

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	77	77	77	77	77	77	77	77	77	77	77	77	77	77	77
	M2	18	23	26	14	14	14	14	14	22	29	21	25	23	21	26
	M3	18	23	26	16	16	16	16	16	22	29	23	25	23	21	26
	M4	19	24	27	16	16	16	16	16	24	30	23	26	24	23	26
	M5	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
	M6	18	23	26	25	26	19	21	21	27	34	27	31	31	32	32
	M7	18	23	26	17	17	16	16	16	24	30	22	26	26	23	27
	M8	14	19	23	12	12	11	11	11	20	27	17	25	23	20	25
	M9	10	11	12	22	22	16	16	16	18	22	17	23	25	26	22

Results for Min Length=3, Max Length=8, Shift Increment=1, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	46	39	73	51	99	57	72	58	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	58	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	56	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	45	45	45	45	45	45	45	45	45	45	45	45	45	45	45
	M2	18	22	27	12	12	12	12	12	21	28	18	27	23	22	27
	M3	20	23	27	16	16	16	16	16	22	29	22	27	24	22	28
	M4	21	25	29	16	16	16	16	16	24	30	22	28	25	24	28
	M5	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23
	M6	18	22	27	25	24	19	18	18	26	34	24	33	33	33	33
	M7	18	22	27	16	16	14	14	14	23	30	20	29	26	26	27
	M8	15	19	25	12	12	9	10	10	20	27	16	28	22	24	24
	M9	9	11	13	22	20	16	14	14	17	23	16	24	27	29	23

Results for Min Length=4, Max Length=8, Shift Increment=1, Shift Start Time=1

	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	49	40	76	52	104	61	75	62	82	92	47	72
	M3	48	61	80	47	38	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	38	76	52	104	61	75	62	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	77	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	-61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	35	35	35	35	35	35	35	35	35	35	35	35	35	35	35
	M2	14	18	21	10	10	10	10	10	17	23	15	21	19	17	22
	M3	14	18	21	12	12	12	12	12	17	23	17	21	19	17	22
	M4	15	19	22	12	12	12	12	12	19	24	17	22	20	19	22
	M5	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
	M6	14	18	21	20	20	15	15	15	21	27	20	26	27	25	28
	M7	14	18	21	14	14	12	12	12	19	24	17	23	23	19	23
	M8	11	16	19	11	11	9	9	9	17	22	14	23	20	17	21
	M9	8	10	11	18	17	13	12	12	15	19	14	20	23	21	20

Results for Min Length=3, Max Length=10,Shift Increment=1, Shift Start Time=1

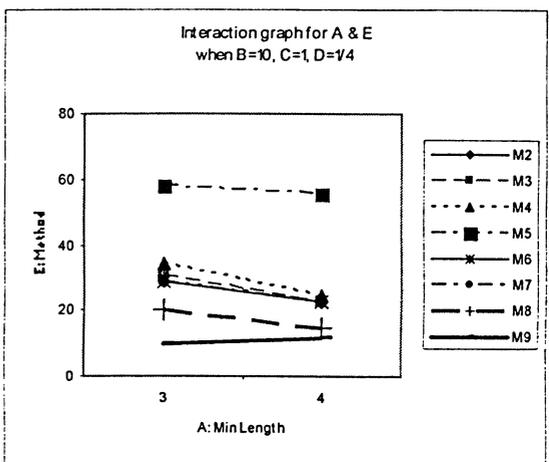
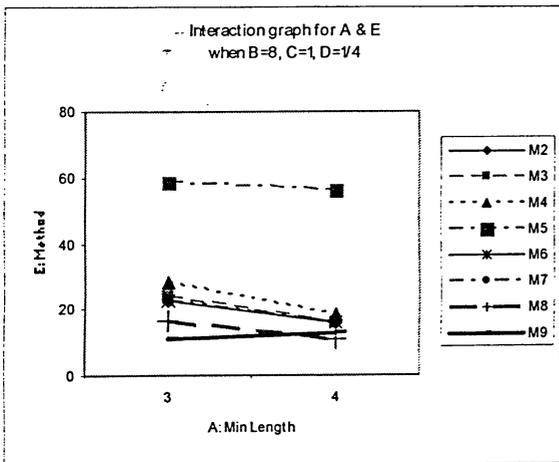
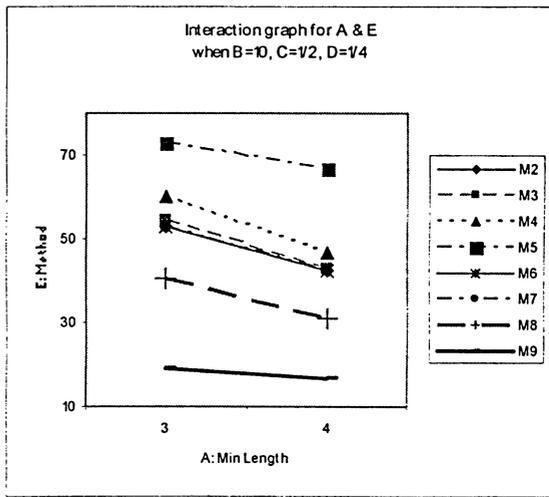
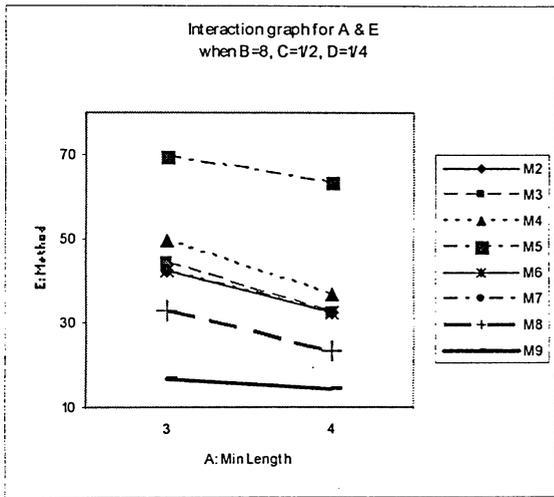
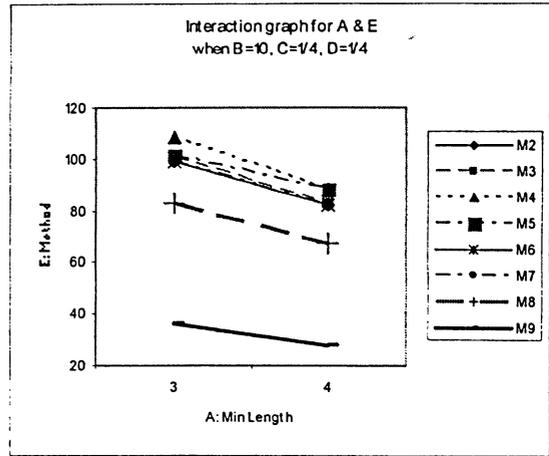
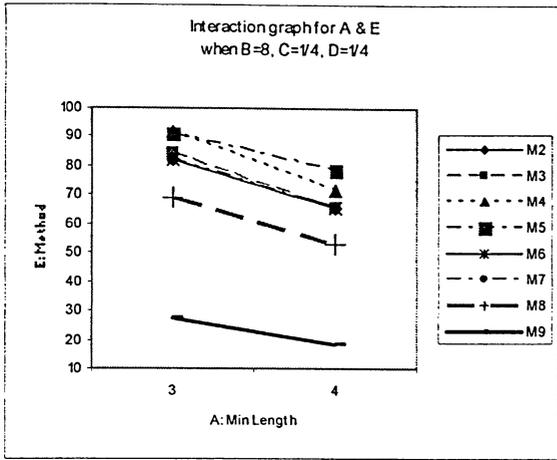
	INC.	INC.	INC.	DEC.	DEC.	DEC.	ONE-HUMP	ONE-HUMP	ONE-HUMP	TWO-HUMP	TWO-HUMP	TWO-HUMP	MIXED	MIXED	MIXED	
OBJECTIVE FUNCTION VALUE#2	M1	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M2	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M3	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M4	48	59	79	44	36	73	51	99	57	72	56	80	88	46	63
	M5	48	59	79	44	35	73	48	92	58	72	56	83	88	59	63
	M6	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M7	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M8	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
	M9	48	59	79	44	35	73	48	92	57	72	56	80	88	46	63
NUMBER OF VARIABLES	M1	52	52	52	52	52	52	52	52	52	52	52	52	52	52	52
	M2	22	27	32	16	16	16	16	16	26	34	24	31	27	26	31
	M3	24	28	32	20	20	20	20	20	27	35	28	31	28	26	32
	M4	25	30	34	20	20	20	20	20	29	36	28	32	29	28	32
	M5	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
	M6	22	27	32	30	30	23	24	24	32	41	31	38	37	40	37
	M7	22	27	32	19	19	18	18	18	28	36	25	32	29	30	31
	M8	18	22	29	13	13	12	12	12	23	32	19	30	25	27	28
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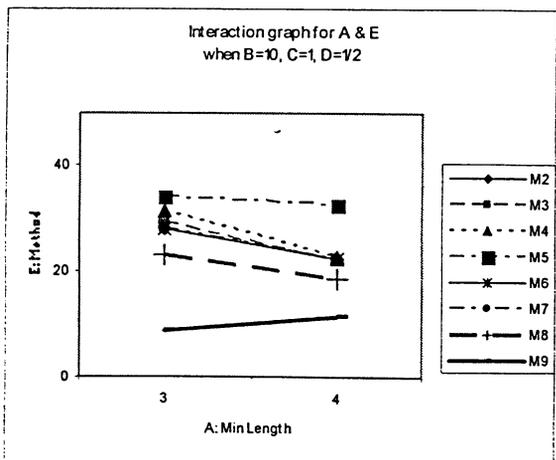
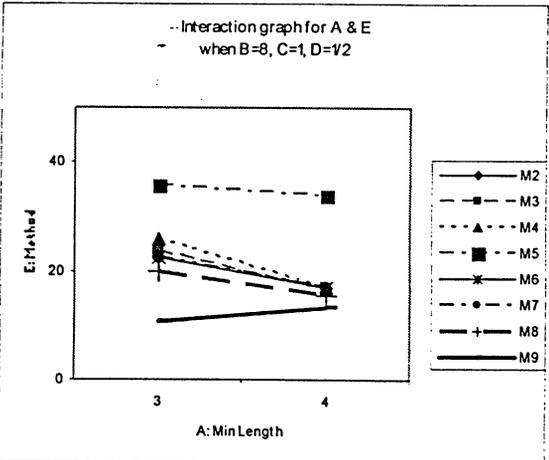
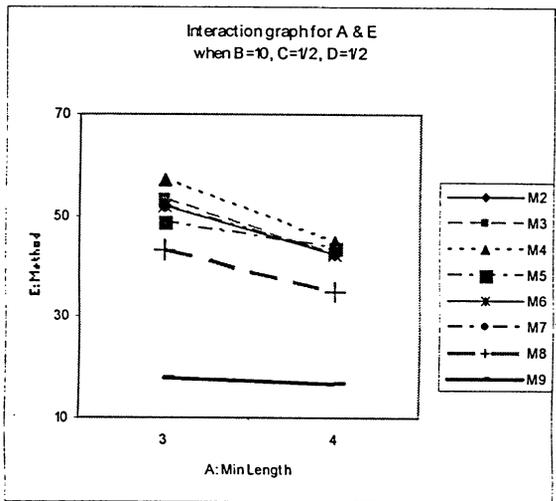
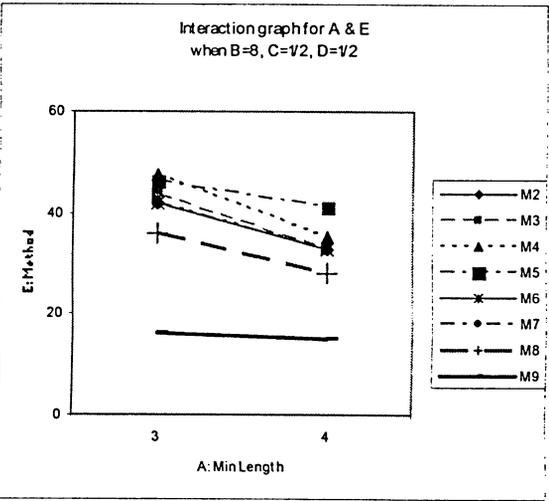
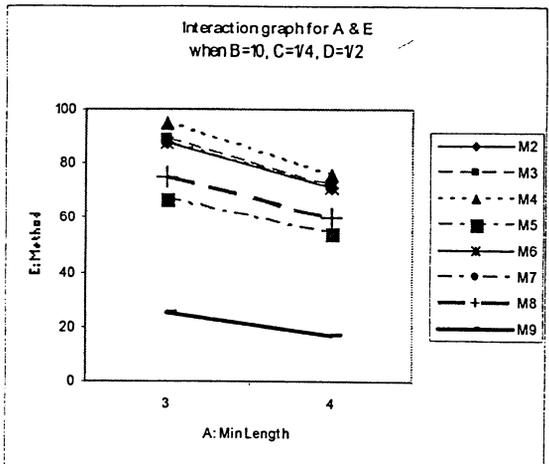
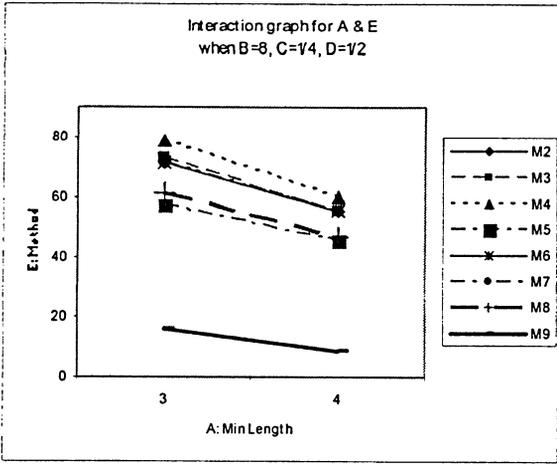
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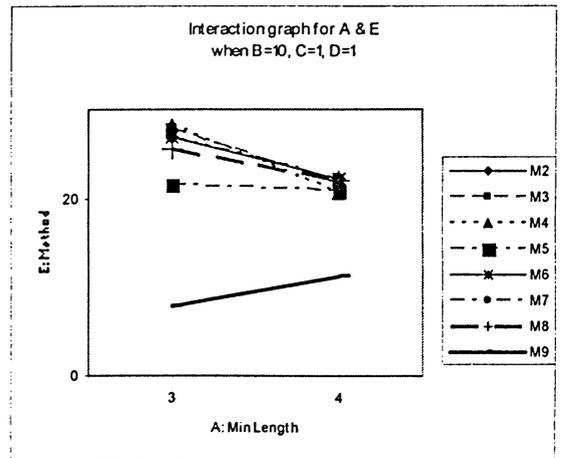
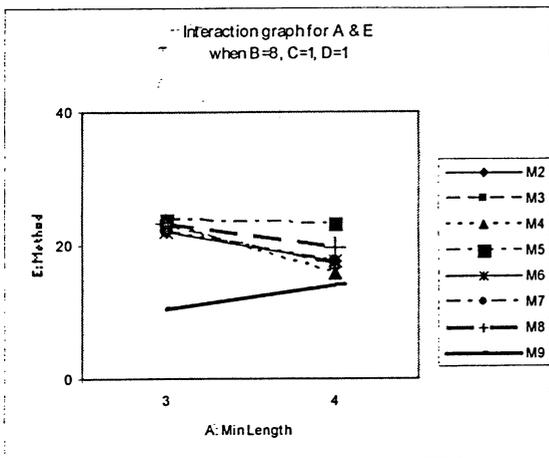
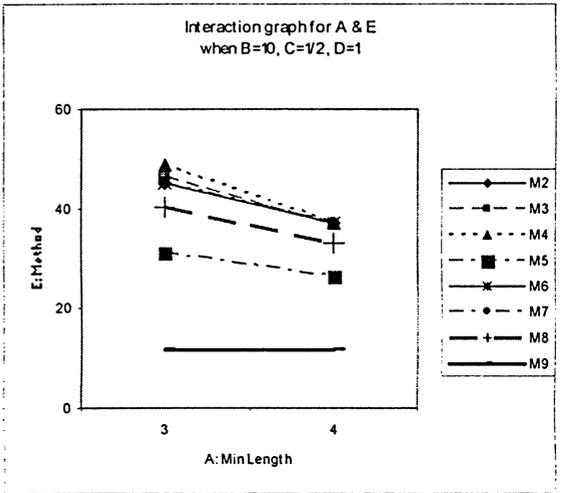
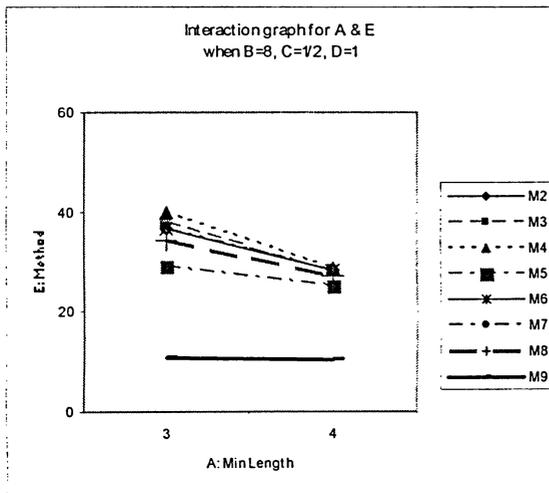
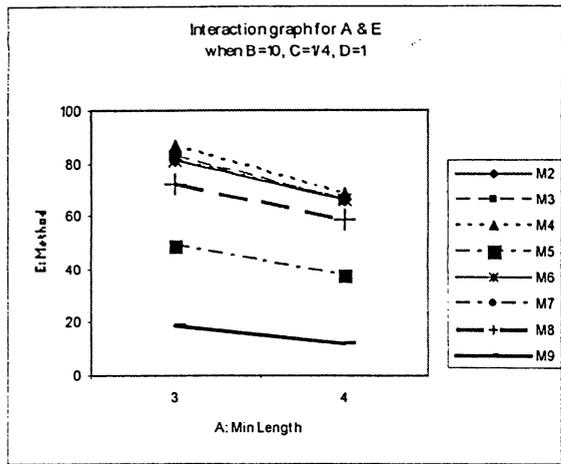
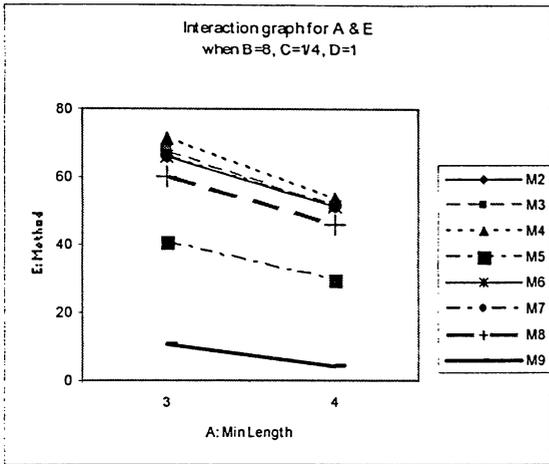
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OBJECTIVE FUNCTION VALUE#2	M1	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M2	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M3	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M4	48	61	80	47	37	76	52	104	61	75	60	82	92	47	72
	M5	48	61	80	47	36	76	49	97	64	75	60	92	92	48	72
	M6	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M7	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M8	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
	M9	48	61	80	47	36	76	49	97	61	75	60	82	92	47	72
NUMBER OF VARIABLES	M1	42	42	42	42	42	42	42	42	42	42	42	42	42	42	42
	M2	18	23	26	14	14	14	14	14	22	29	21	25	23	21	26
	M3	18	23	26	16	16	16	16	16	22	29	23	25	23	21	26
	M4	19	24	27	16	16	16	16	16	24	30	23	26	24	23	26
	M5	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22
	M6	18	23	26	25	26	19	21	21	27	34	27	31	31	32	32
	M7	18	23	26	17	17	16	16	16	24	30	22	26	26	23	27
	M8	14	19	23	12	12	11	11	11	20	27	17	25	23	20	25
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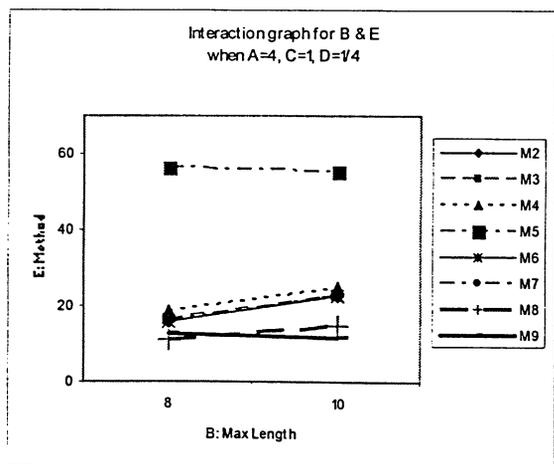
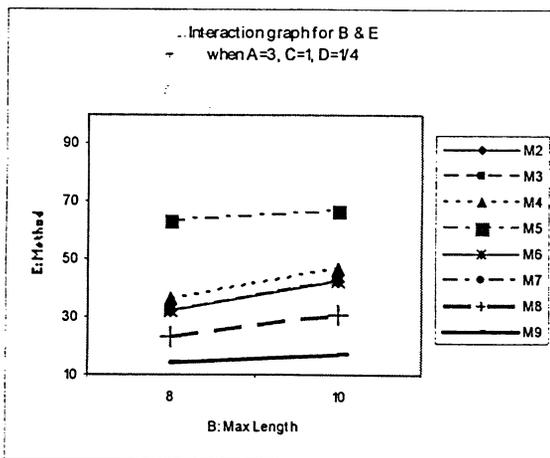
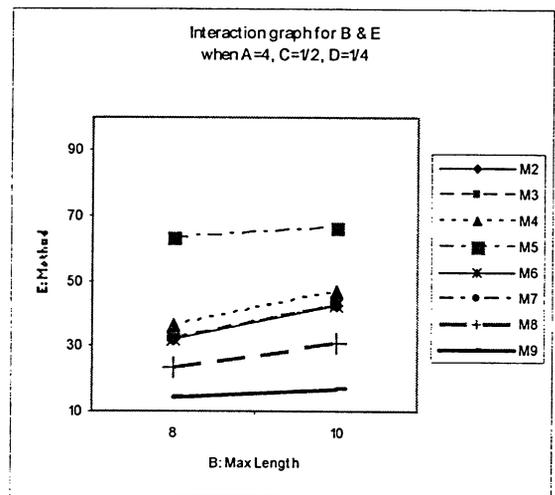
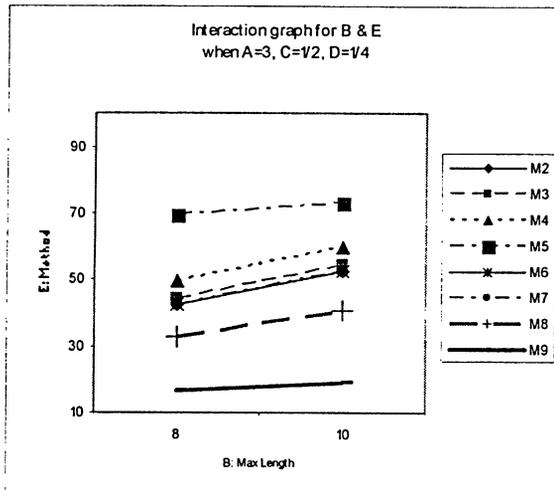
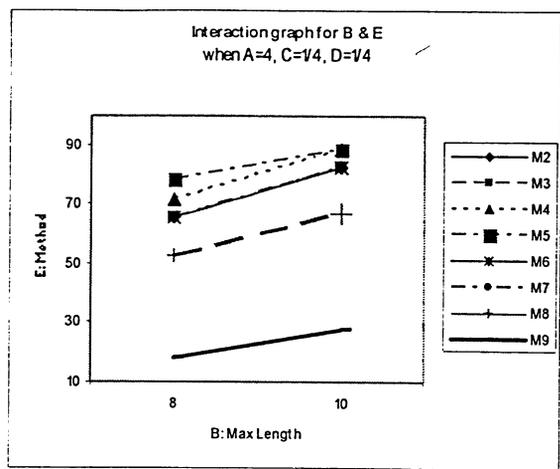
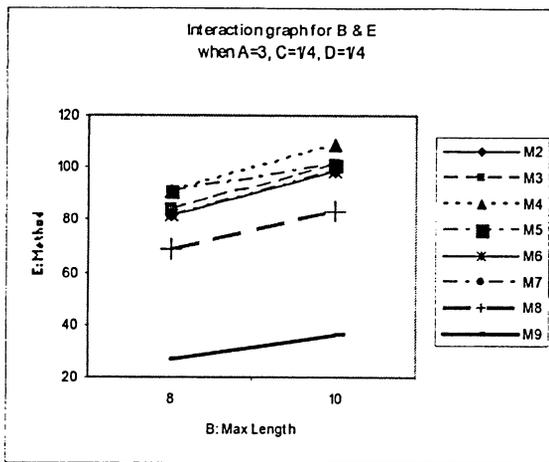
APPENDIX III

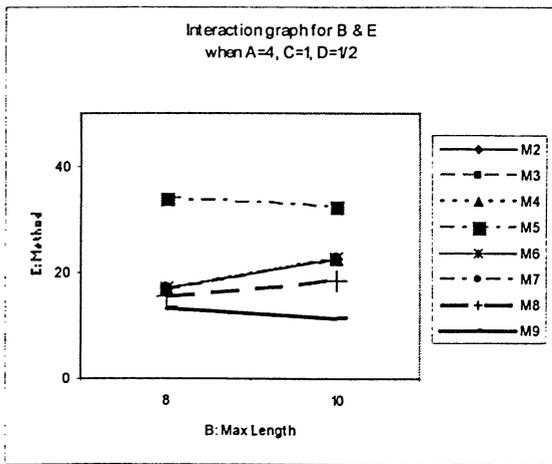
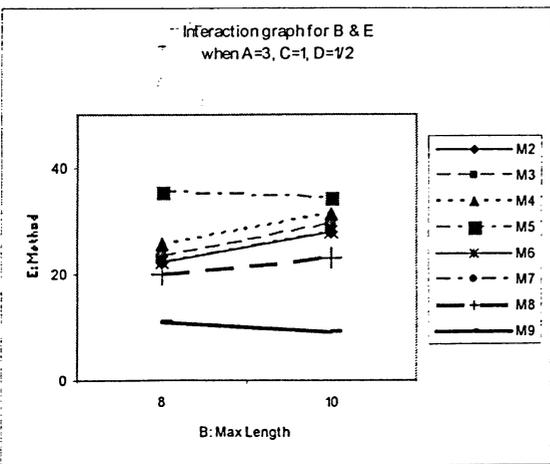
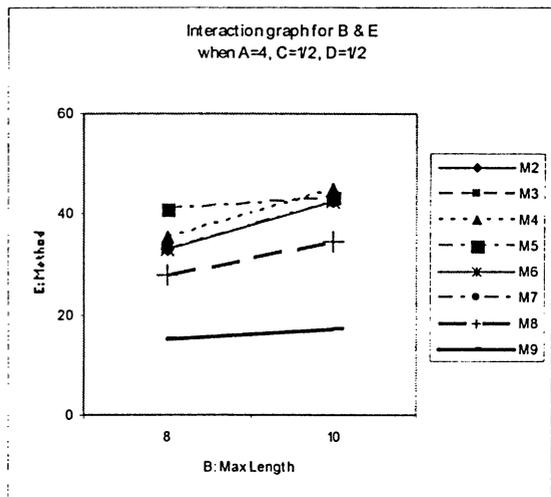
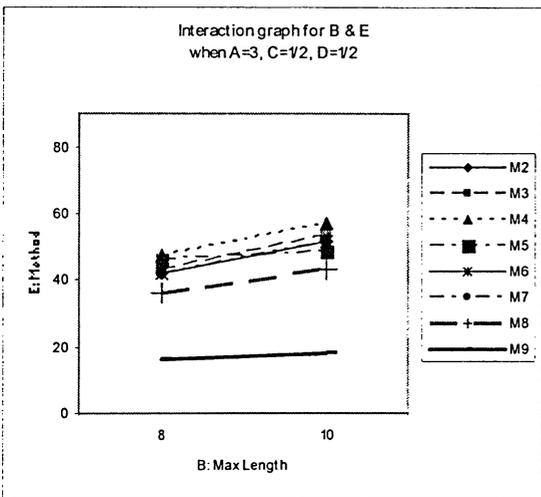
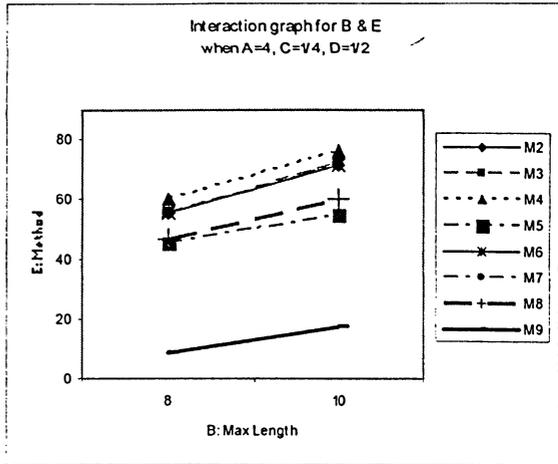
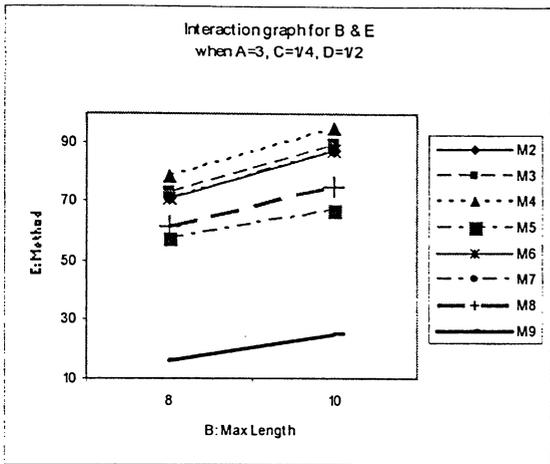
**INTERACTION GRAPHS (AE, BE, CE, and DE) FOR ALL
COMBINATIONS OF LEVELS OF FACTORS FOR INCREASING
PATTERN**

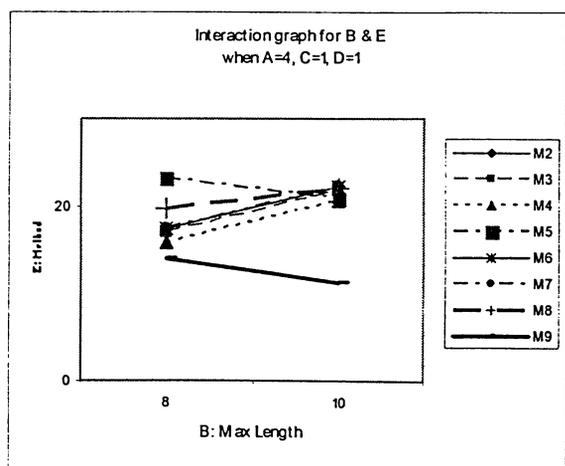
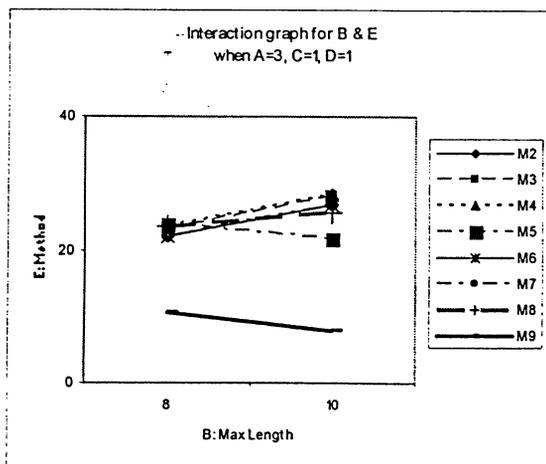
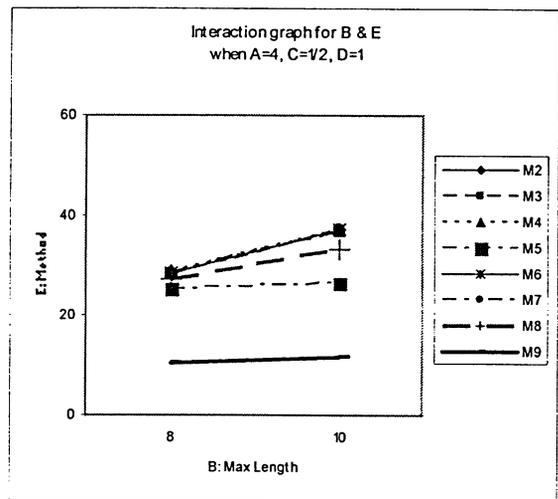
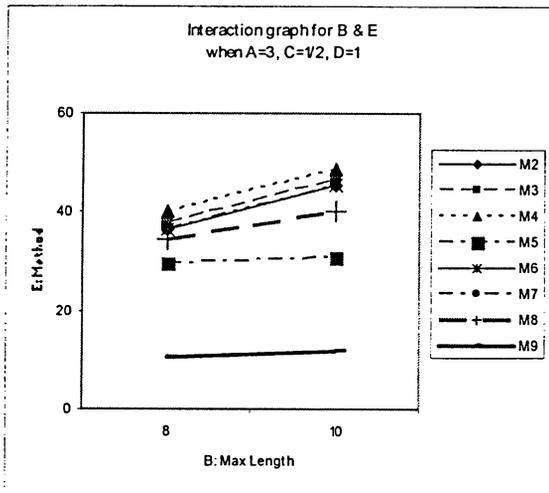
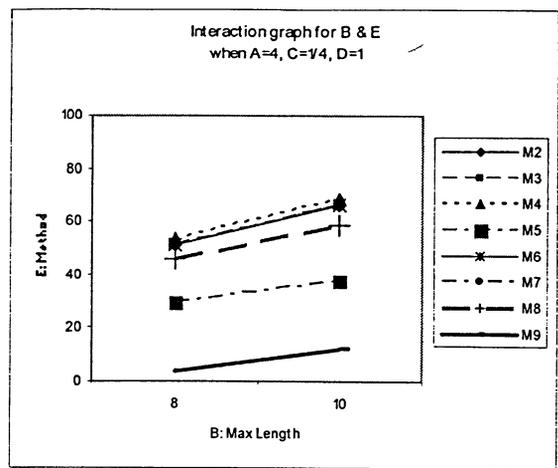
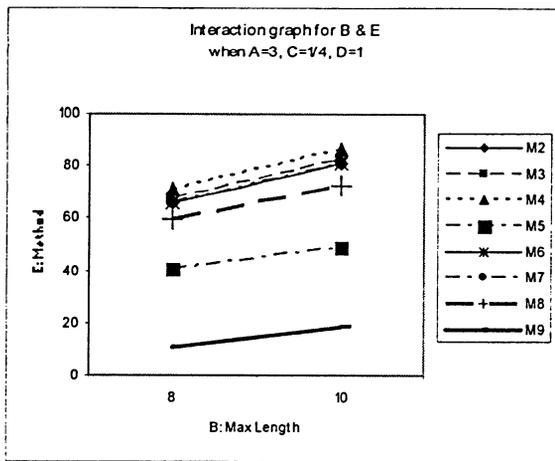


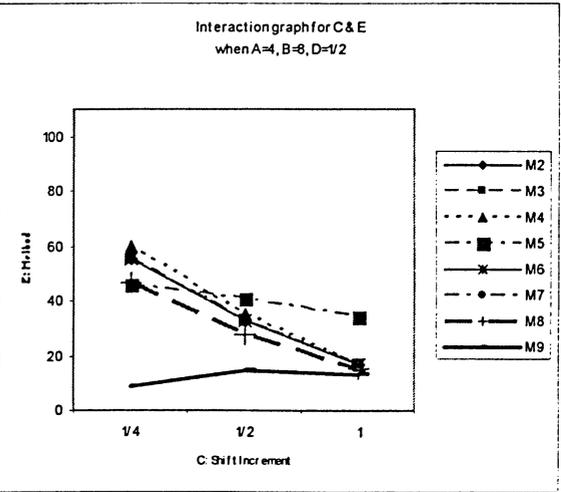
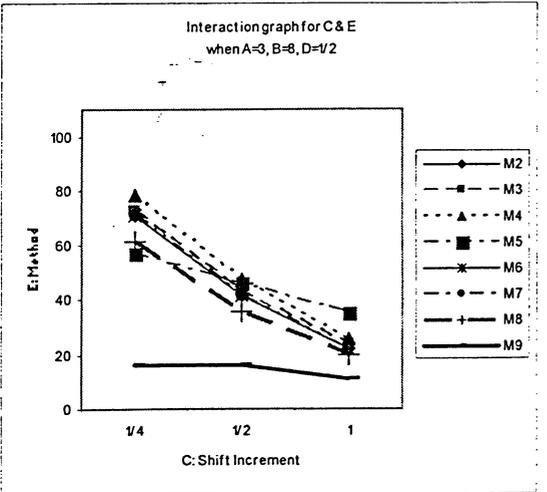
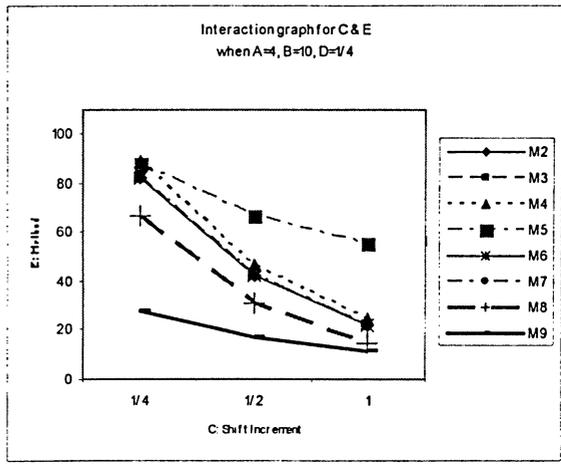
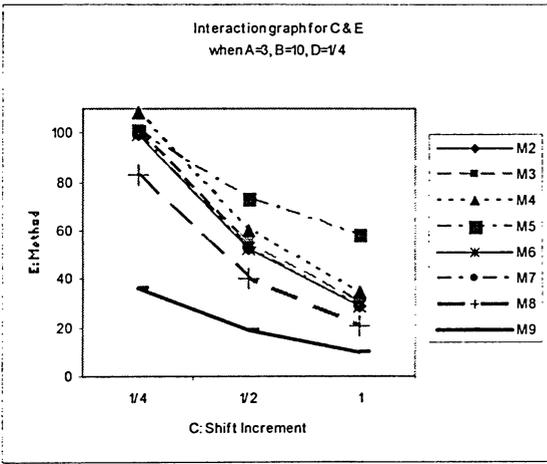
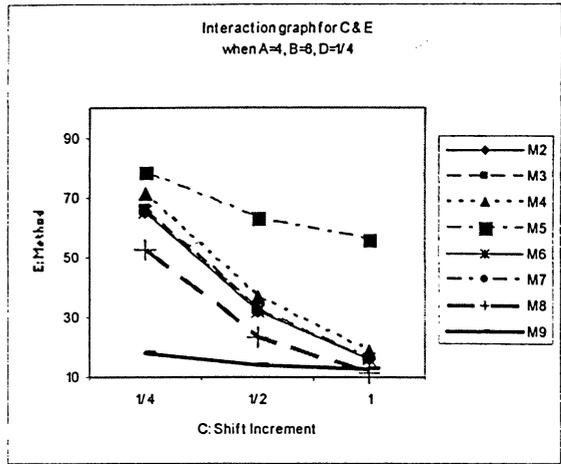
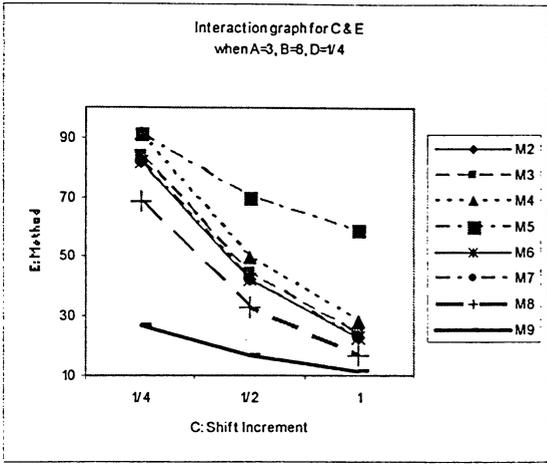


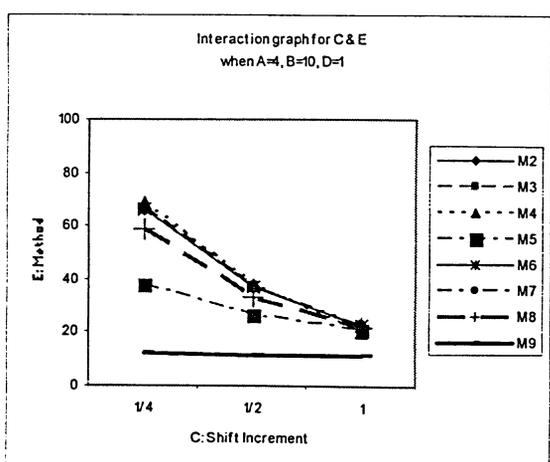
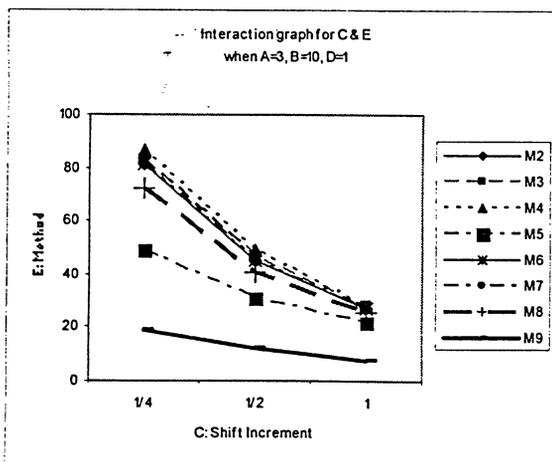
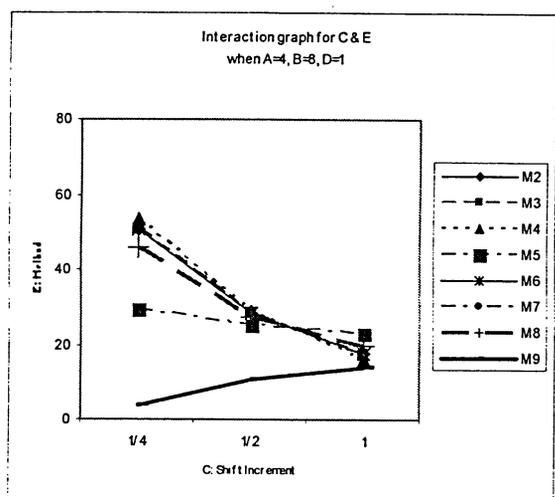
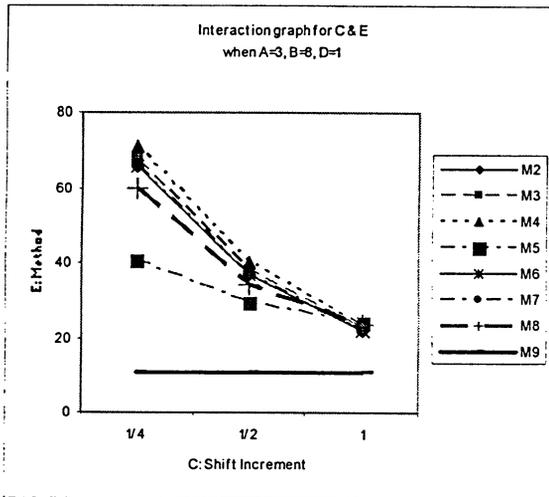
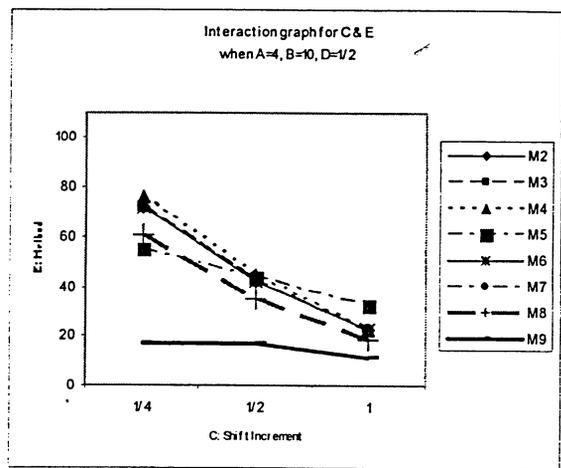
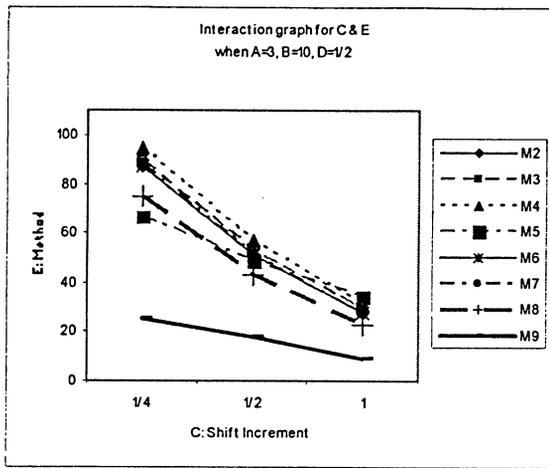


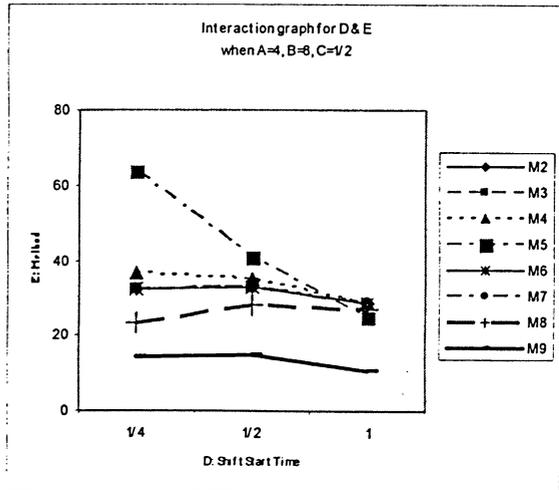
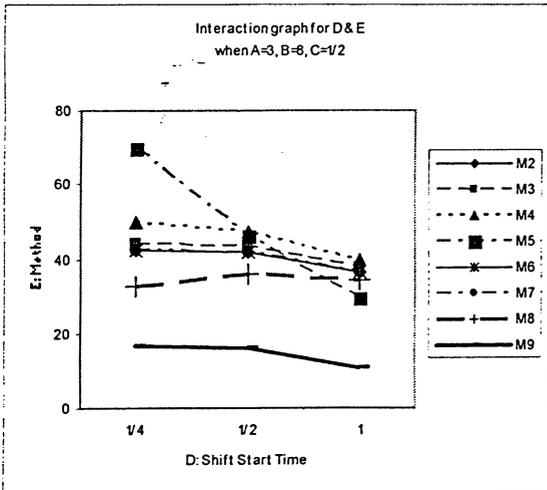
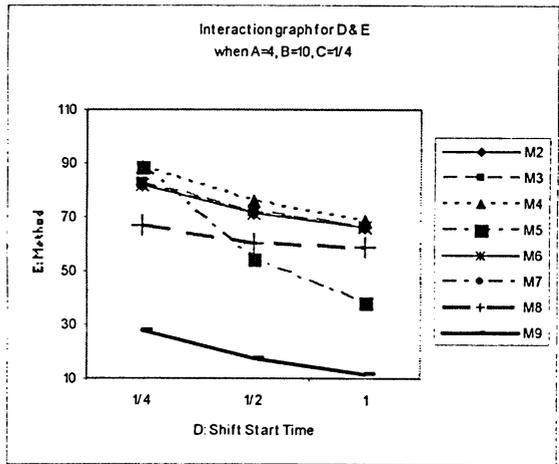
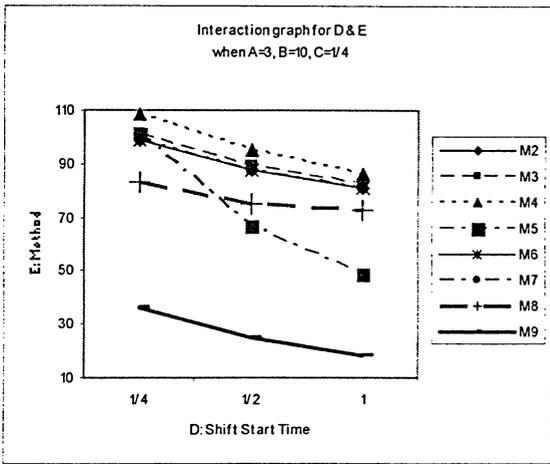
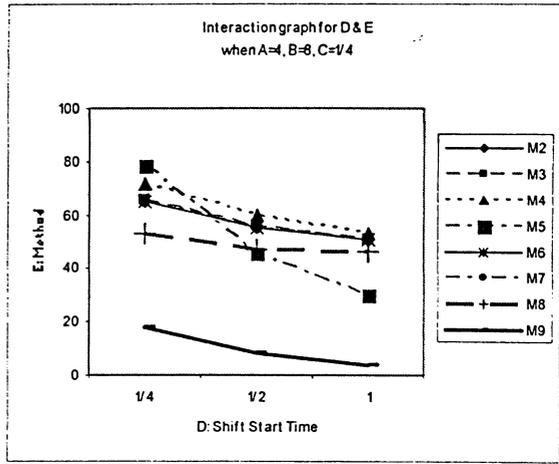
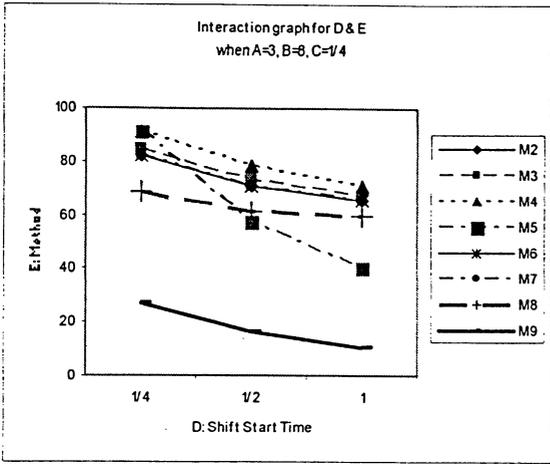


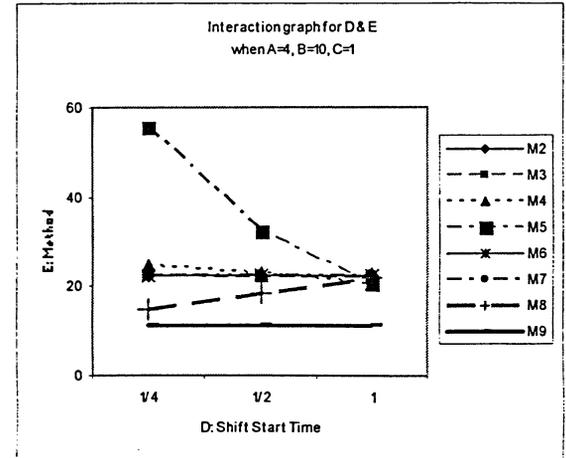
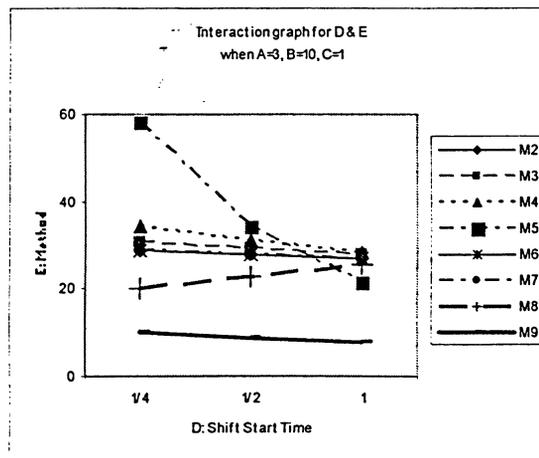
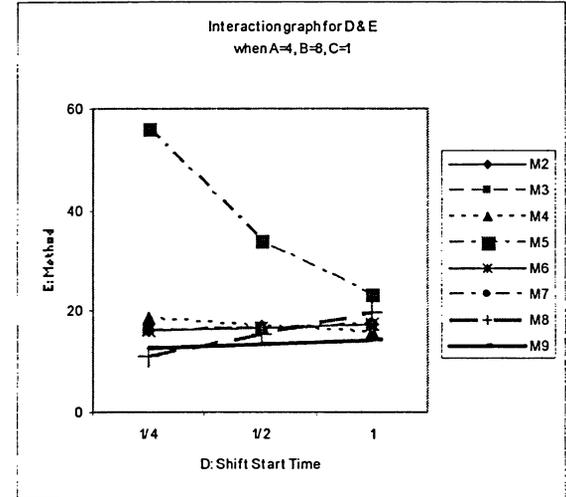
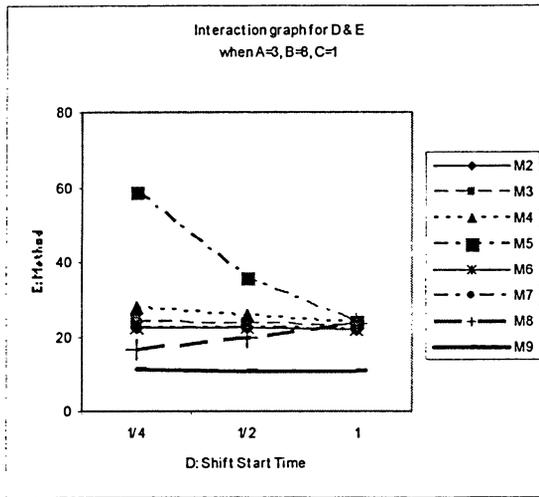
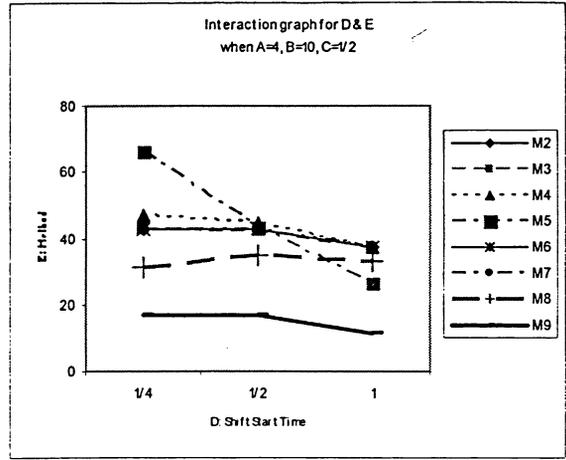
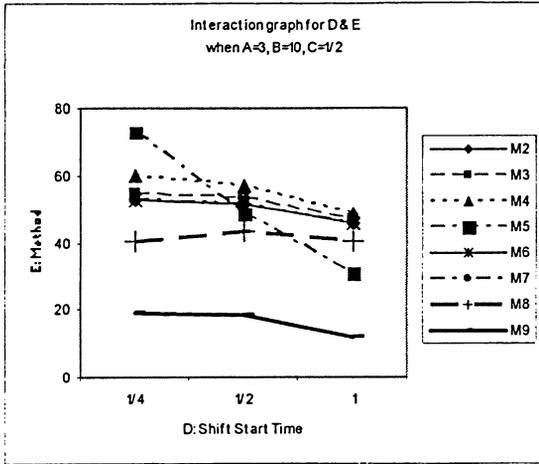






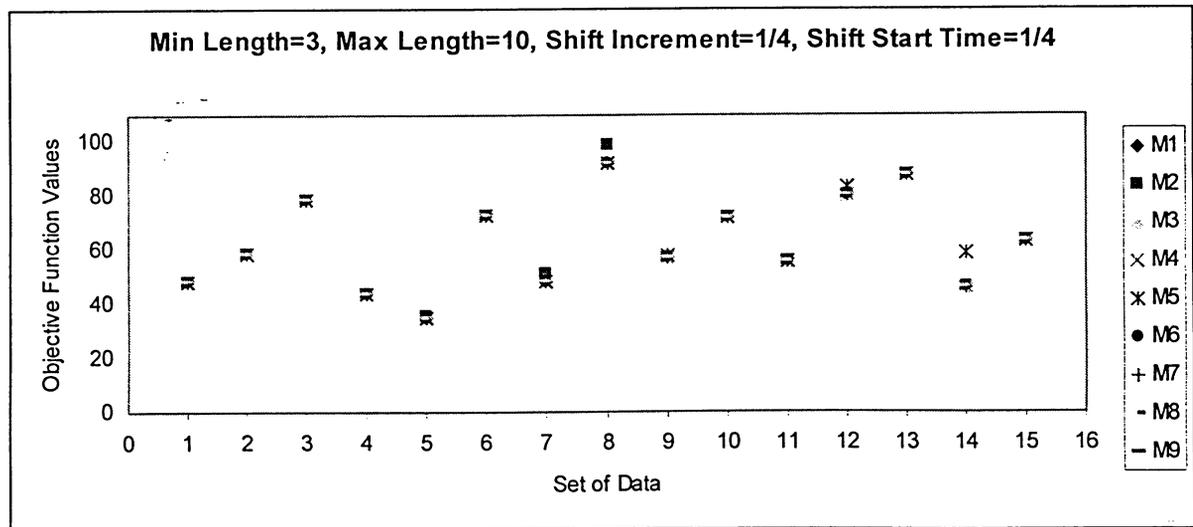
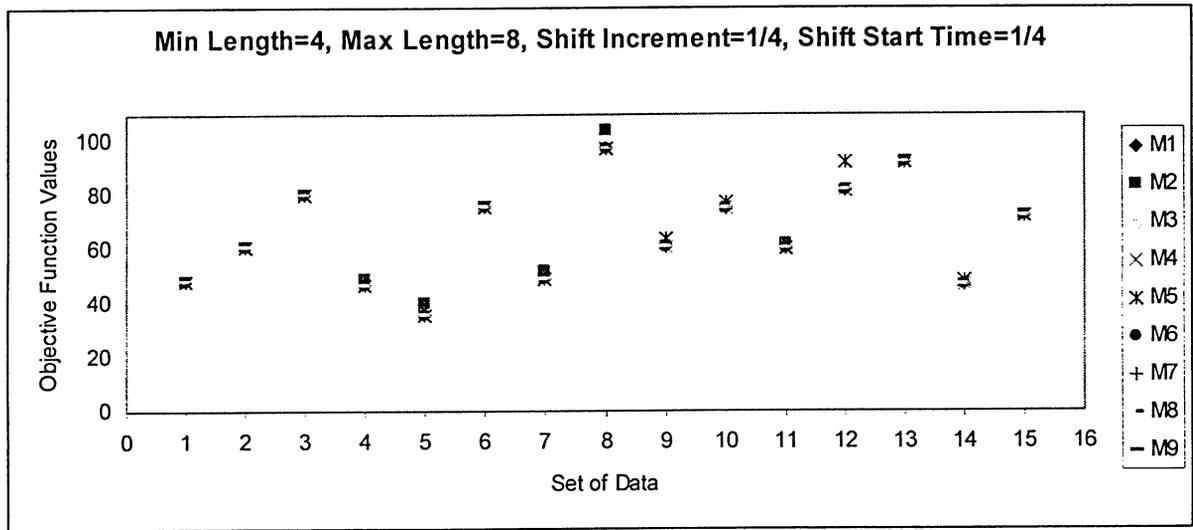
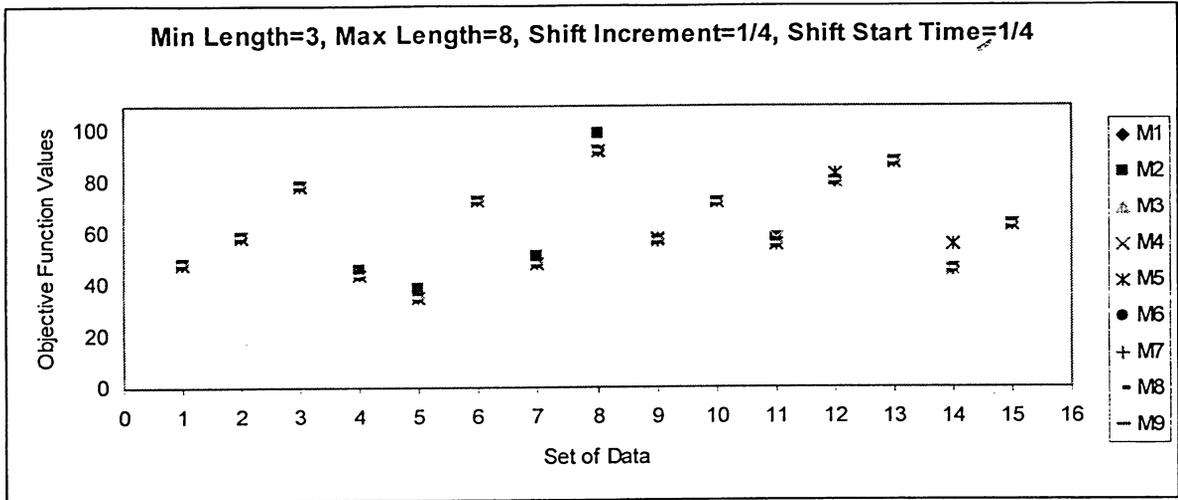


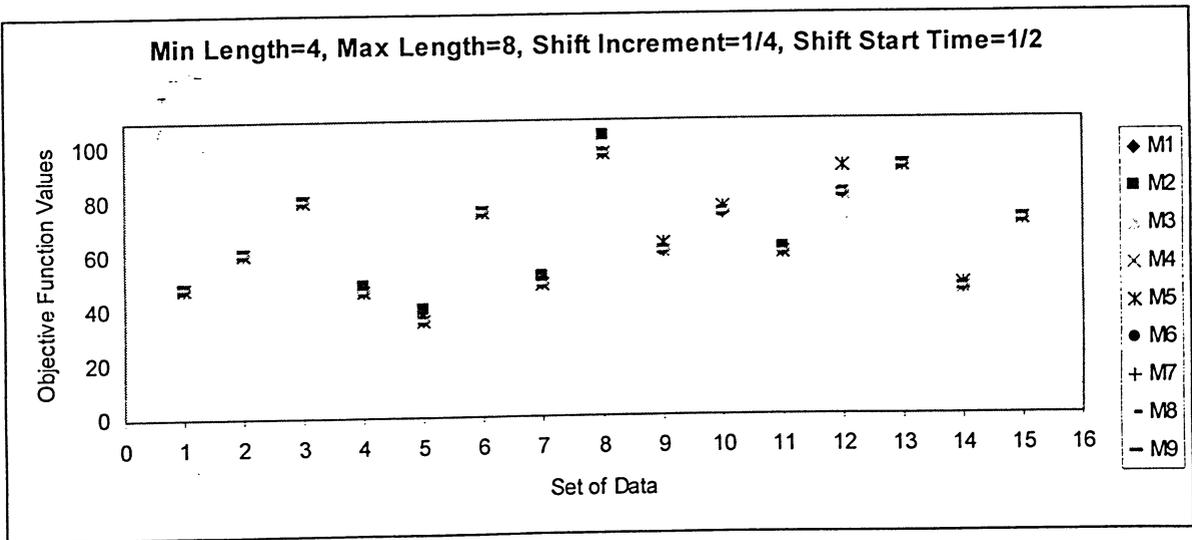
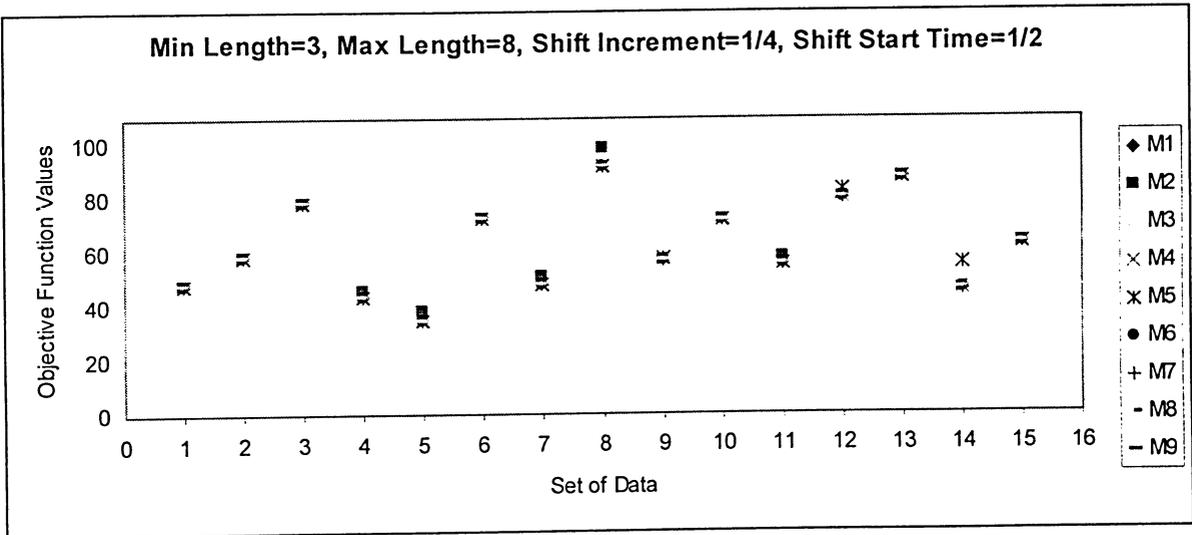
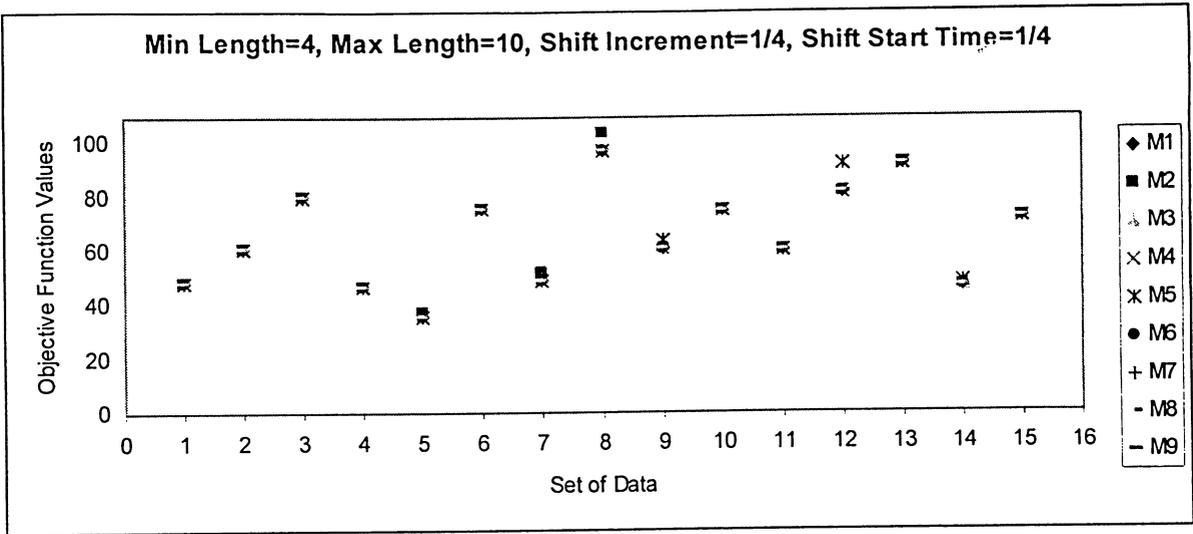


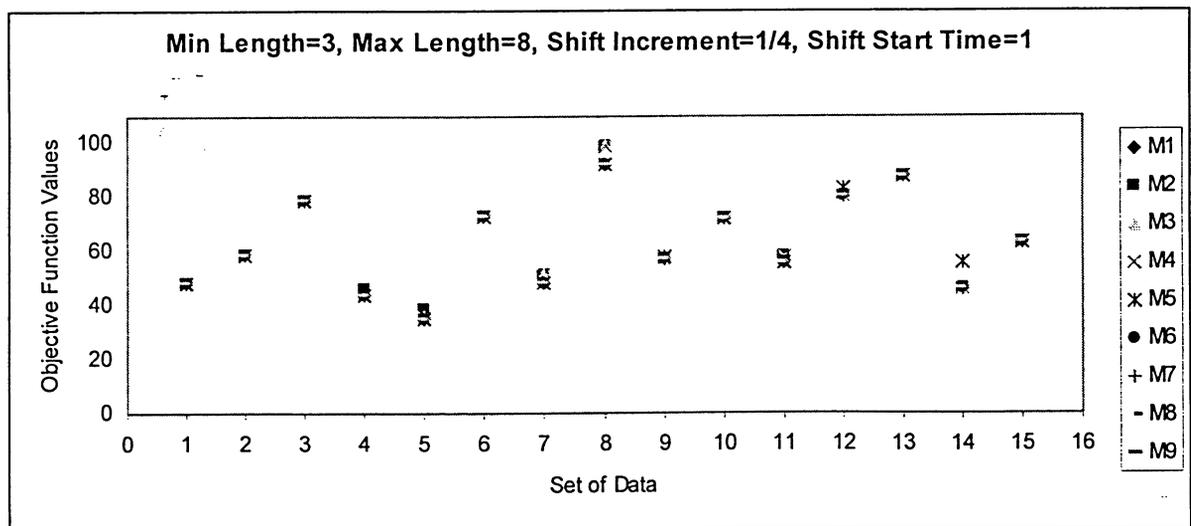
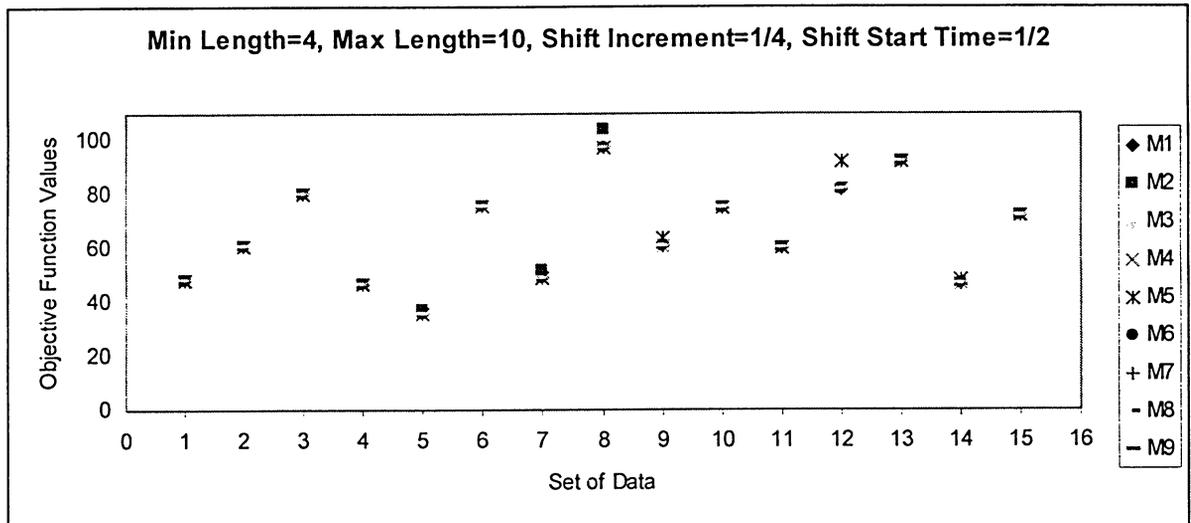
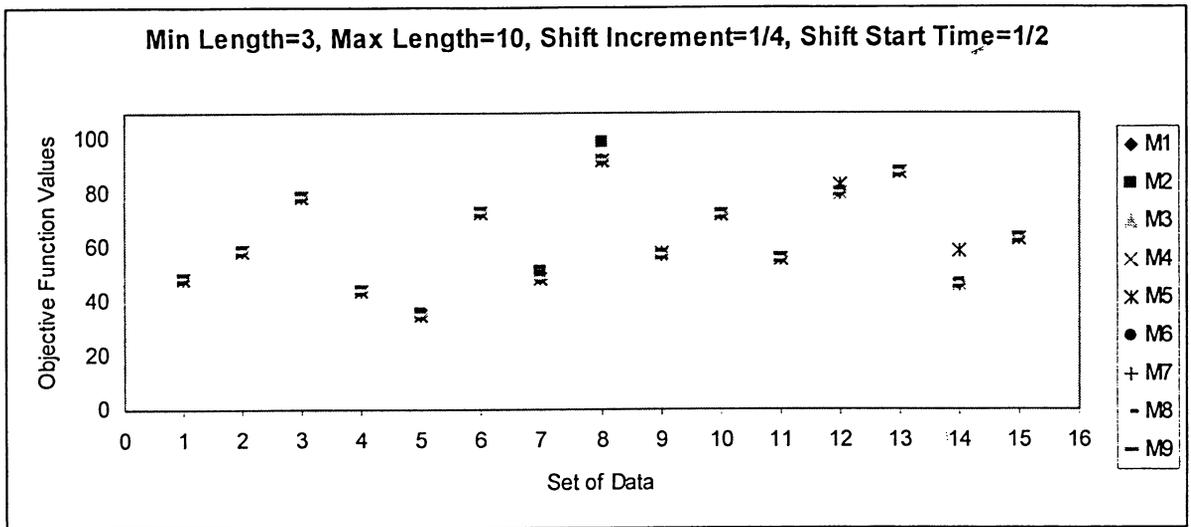


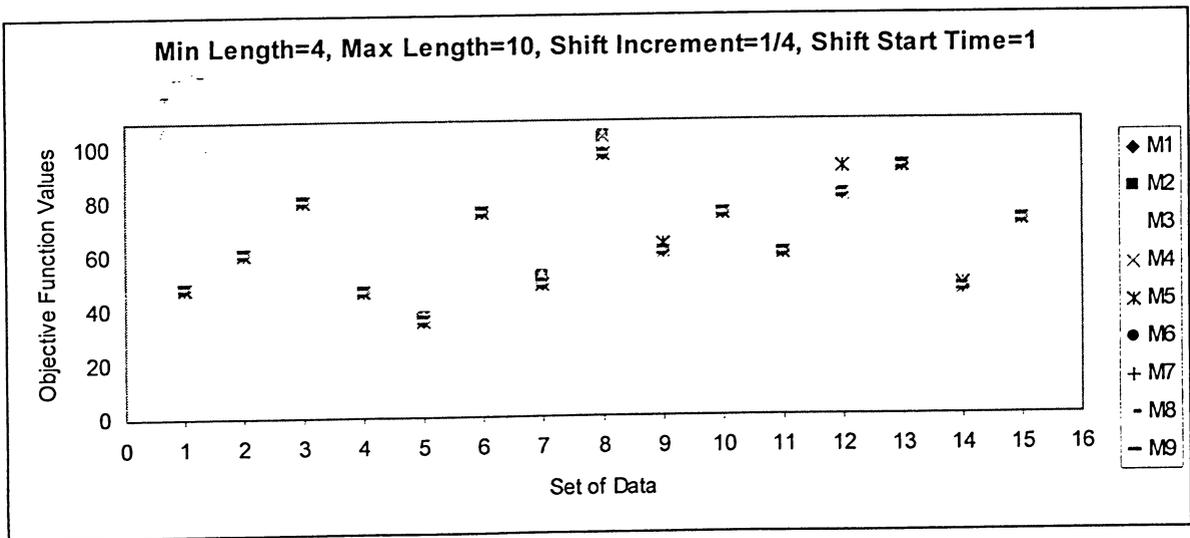
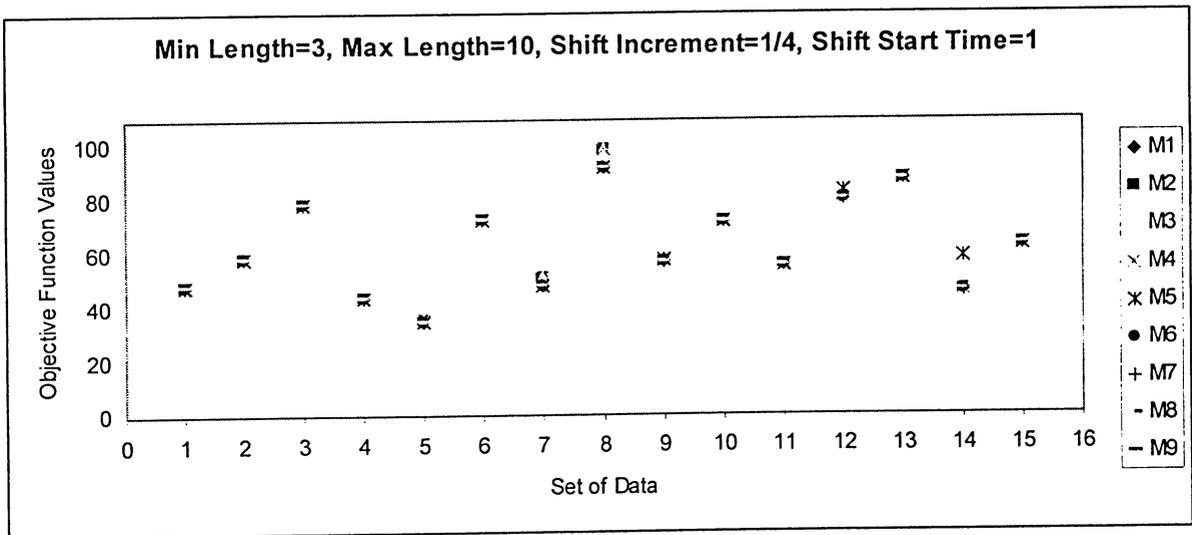
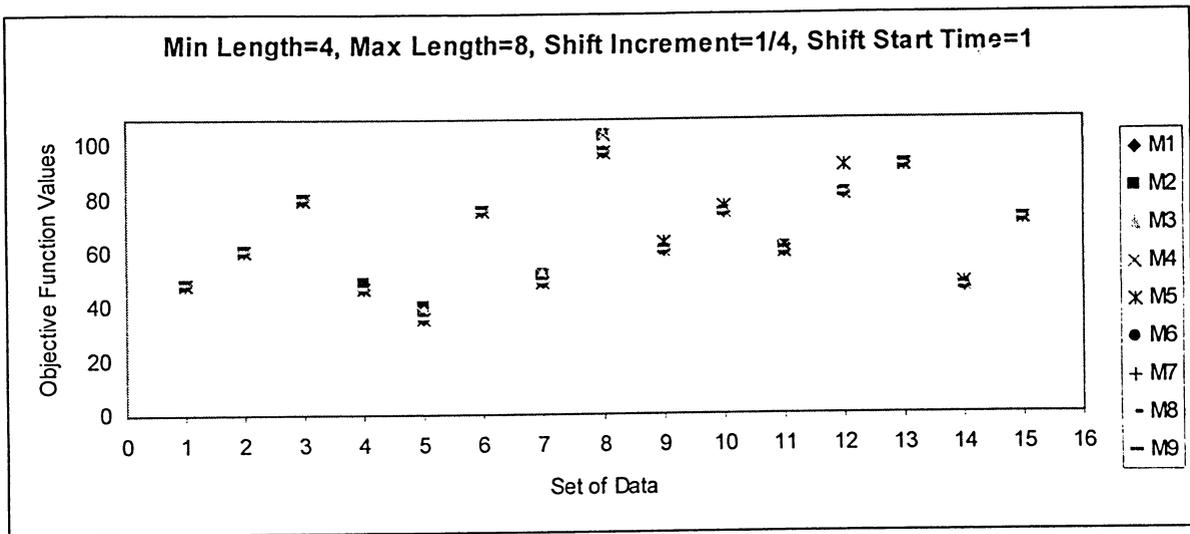
APPENDIX IV

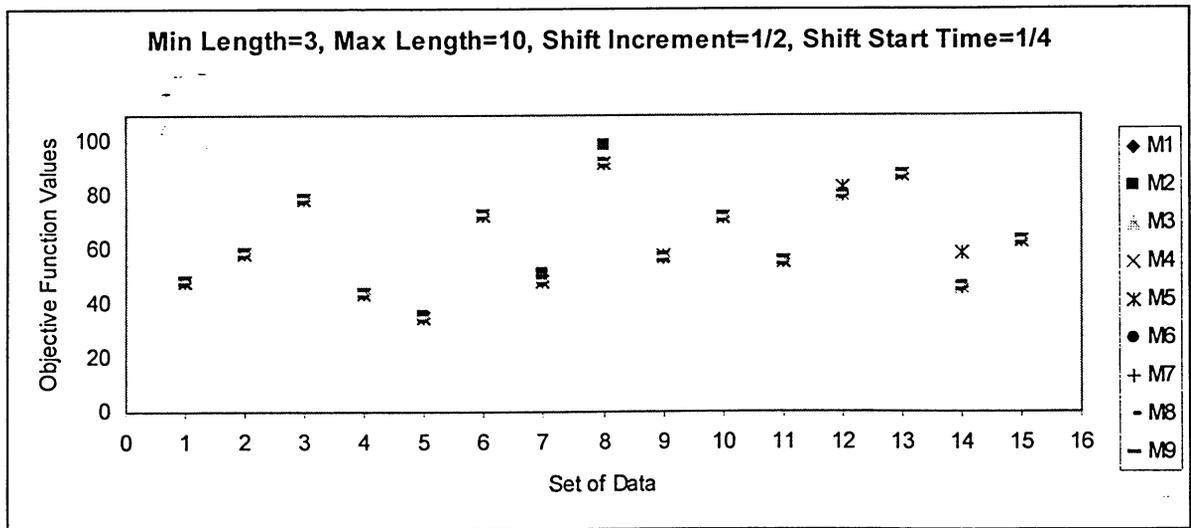
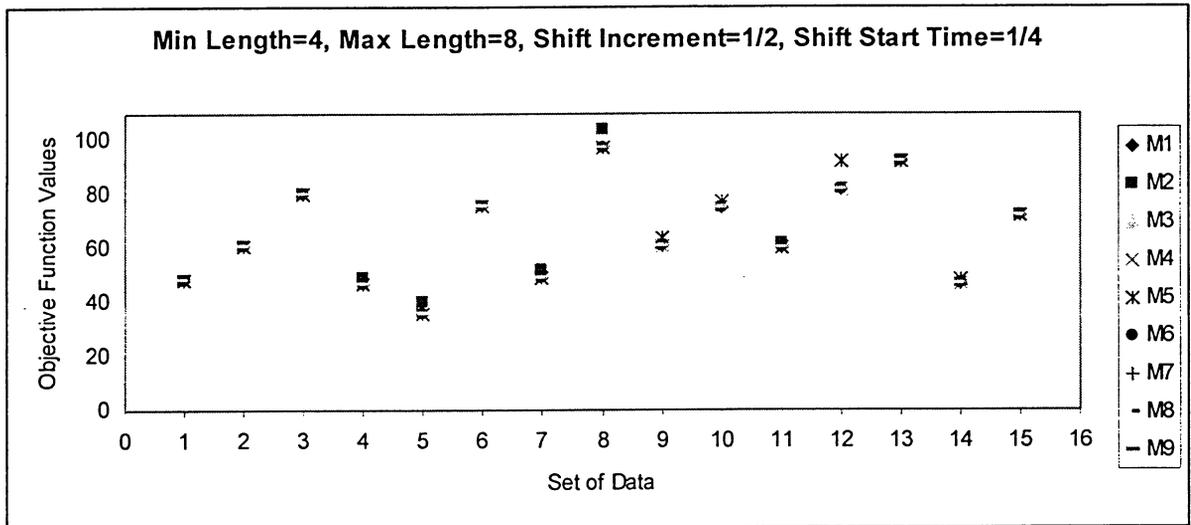
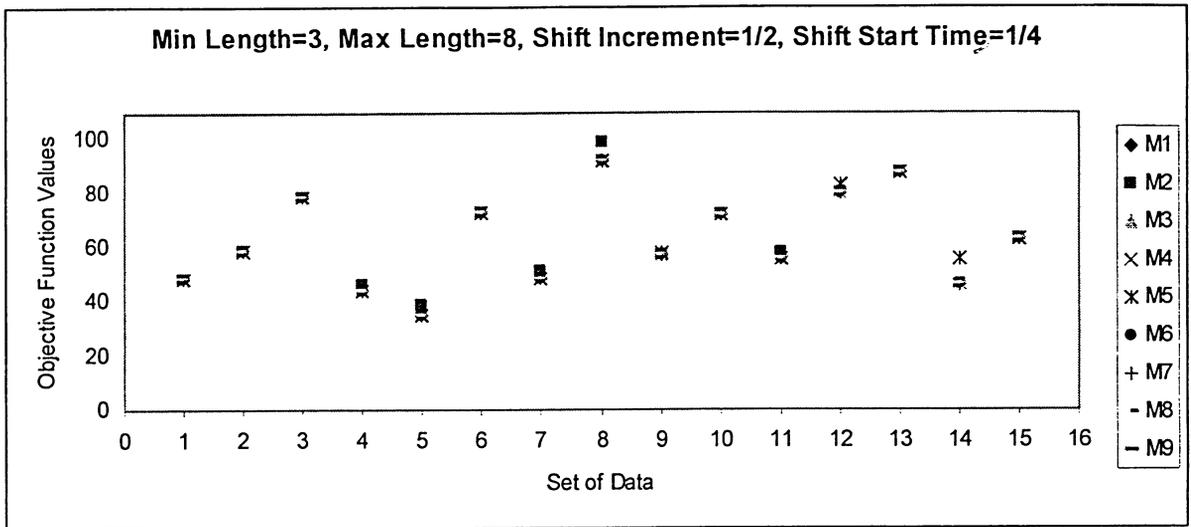
REQUIRED GRAPHS FOR DOMINANCE ANALYSIS



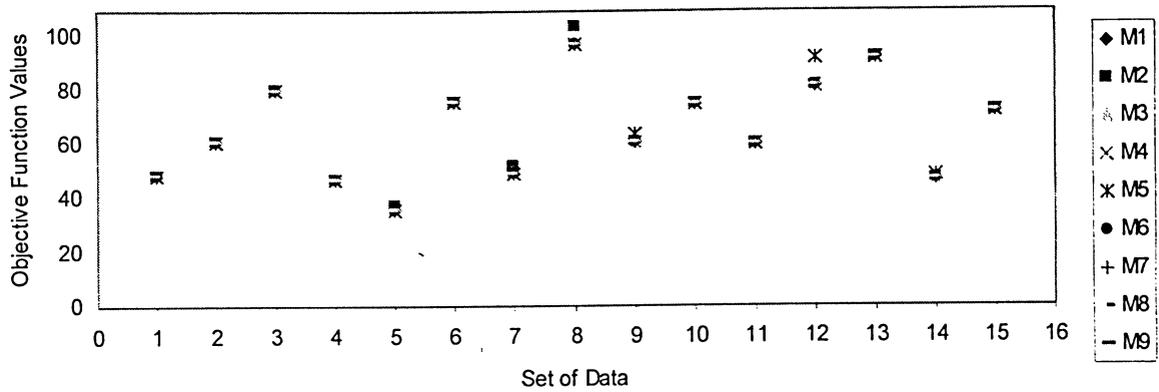




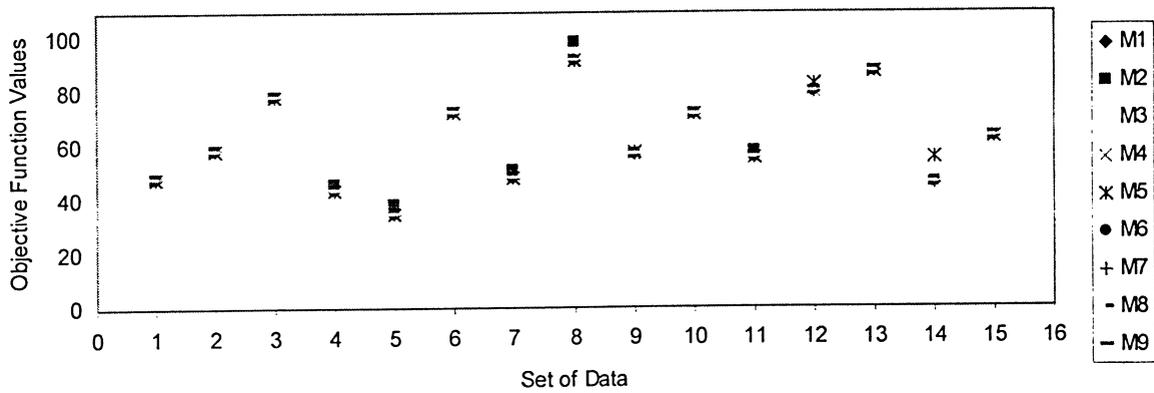




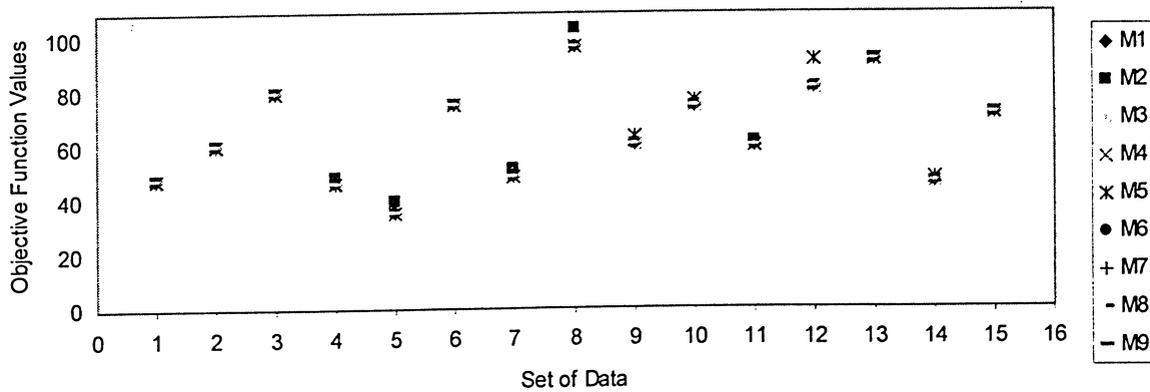
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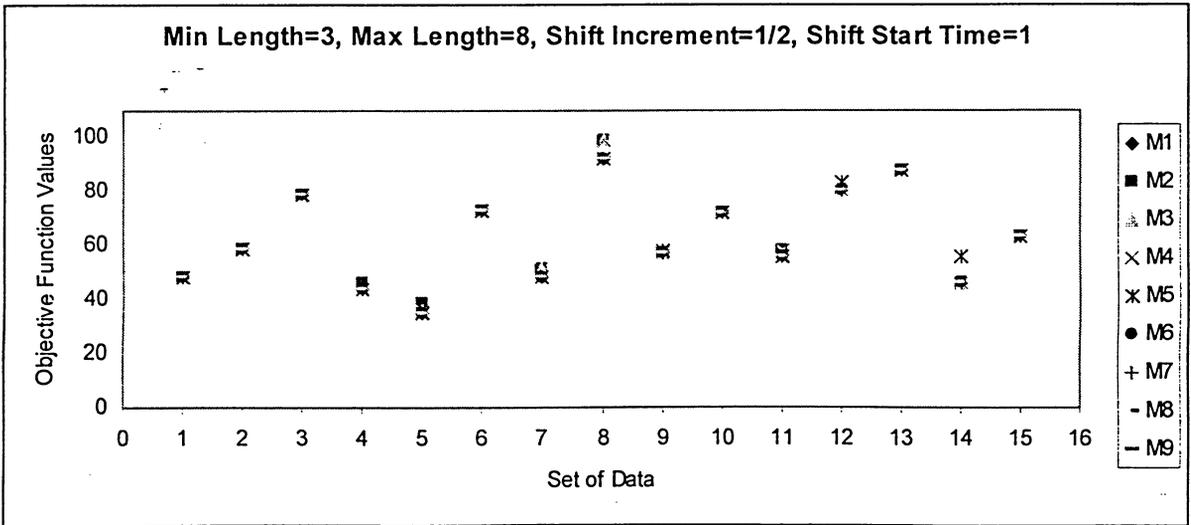
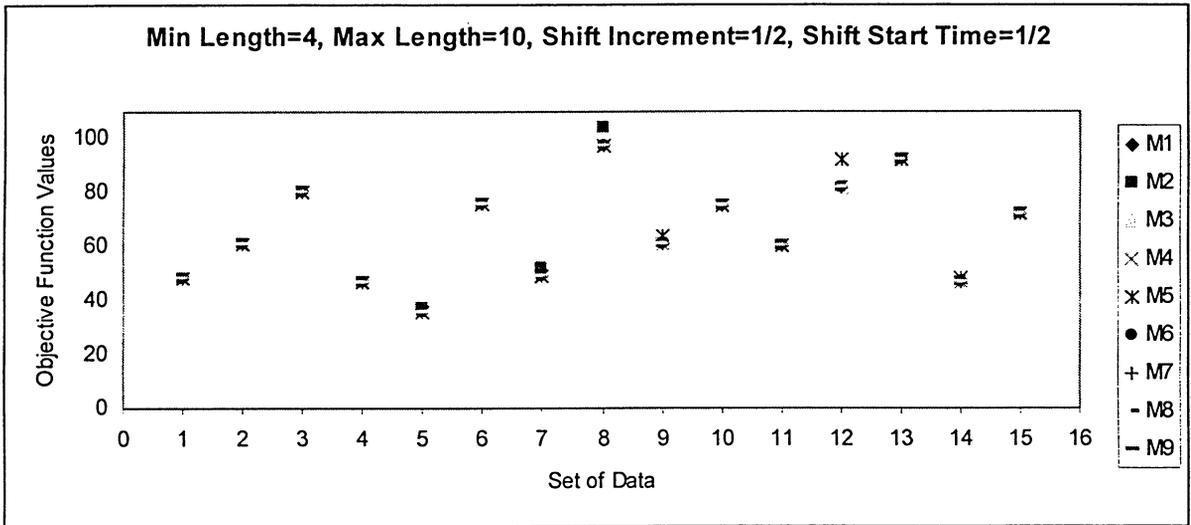
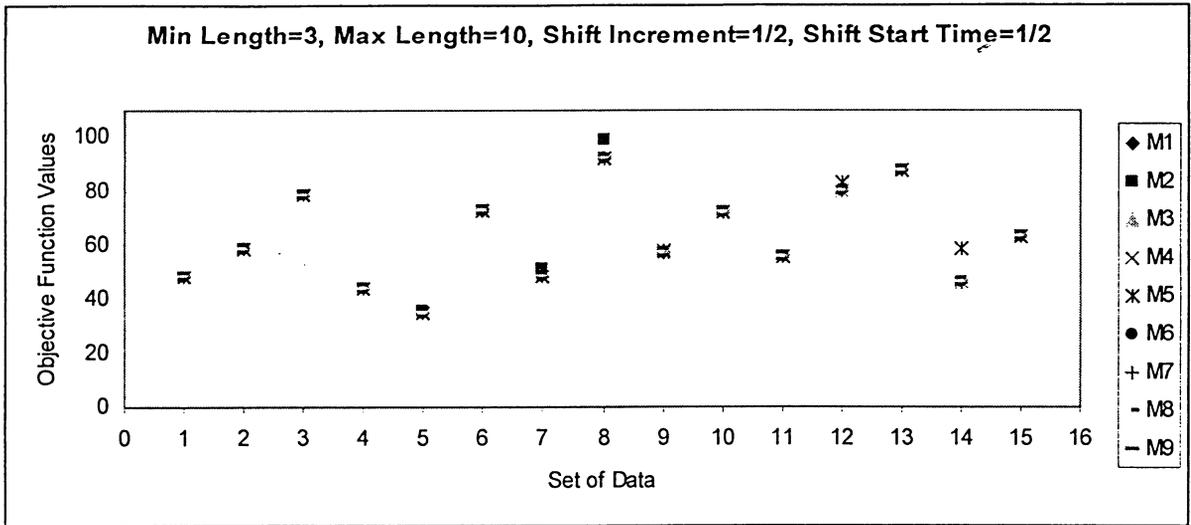


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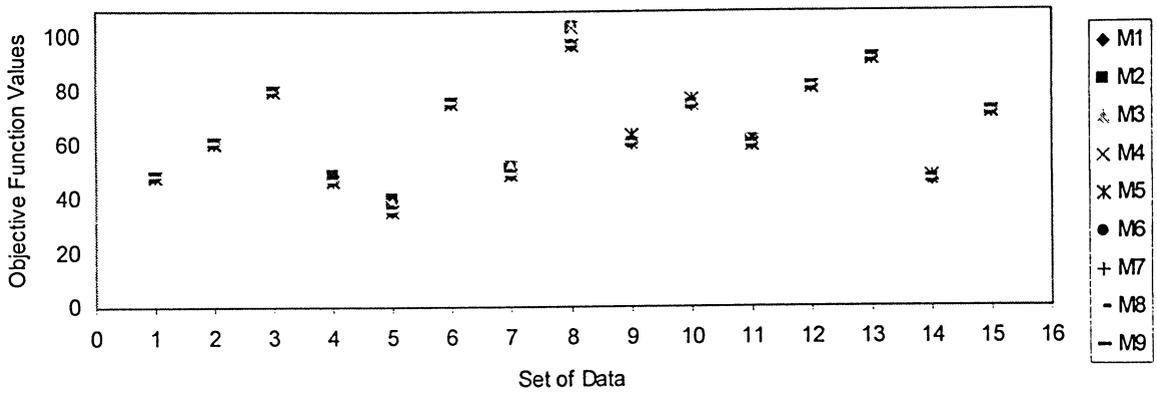


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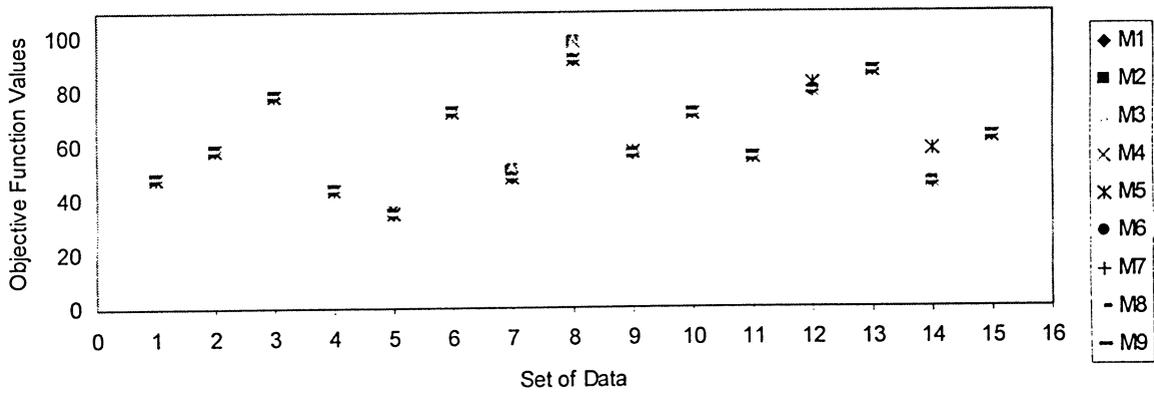




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