

# **APPLICATION OF NEWTON-RAPHSON METHOD IN THREE-PHASE UNBALANCED POWER FLOW**

By

Ali Khadem Sameni, B.Eng.  
Ferdowsi University of Mashad, 1994

A project  
presented to Ryerson University  
in partial fulfillment of the  
requirements for the degree of  
Master of Engineering  
in the Program of  
Electrical and Computer Engineering

Toronto, Ontario, Canada, 2010  
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# **APPLICATION OF NEWTON-RAPHSON METHOD IN THREE-PHASE UNBALANCED POWER FLOW**

Ali Khadem Sameni

Master of Engineering

Department of Electrical and Computer Engineering

Ryerson University, Toronto, 2010

## **ABSTRACT**

Renewable energy sources are in the forefront of new energy in power systems. They are predominantly connected to distribution systems at lower voltage levels. Distribution systems have three phases and are largely unbalanced in their line parameters and loads. A large percentage of these systems suffer from severe imbalance in their phases.

In the past, two methods of analysis were commonly used. The first is the ‘ladder iterative technique’ that consider unbalanced systems. However, this method does not possess information about the distribution system. The second method uses the ‘Newton Raphson Technique’ with  $2N$  equations. It is superior since it computes a Jacobian that holds information about the distribution system. However, predominant implementations of Newton Raphson method assume that the system is balanced and they suffer from poor convergence properties.

In order to overcome these difficulties, this research furthers a recent development of single phase  $3N$  equation model of distribution systems by enlarging it to model three phase distribution systems. The Jacobian models each of the three phases using a set of  $3N$  equations. Modeling of network components and formulation of three phase power flow equations are presented. The important characteristics of the Jacobin matrix are also presented.

The method is developed and coded. It is tested on standard IEEE distribution systems with 4, 13, 34, 37, and 123 nodes. The results are compared with published IEEE data.

## **ACKNOWLEDGEMENT**

I wish to express my deep gratitude to my supervisor, Professor Bala Venkatesh for his support and guidance and the knowledge he shared during my graduate studies at Ryerson University.

Also, I would like to thank Chandrabhanu opathella, Ph.D student of electrical and computer department of Ryerson university, for his helpful recommendations and supports during working on this project.

And also I would like to give my deepest thanks to my wife Nooshin, her constant support and understanding made this work possible.

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## **Chapter 1**

### **Introduction**

With increasing integration of renewable energy sources to low voltage distribution systems, their analysis methods are being relooked into and analyzed. Distribution systems operate at a lower kV. Their lines are inherently unbalanced and their connected loads are unbalanced. Hence, an analysis method devised for distribution systems should be able to account for this unbalanced nature.

Distribution systems have lines that have a high R/X ratio. Consequently, transmission system's  $2N$  equation based Newton Raphson technique shows poor convergence characteristic. The cause for this poor convergence characteristic is due to the Jacobian being ill-conditioned [1], [2], [3].

A common method used popularly in the past uses a ladder-iterative technique. This technique determines load currents and sums it sequential in the upward direction. This method and its variants show good convergence characteristics. However, ladder iterative technique does not compute a Jacobian matrix. This is a drawback as a Jacobian provides an invaluable insight into the state of the system – through matrix methods.

In this research, a newly reported model using a set of  $3N$  equations for single-phase equivalent of a three-phase system is used. This model is expanded to accommodate all the three phases.

This report presents three-phase modeling of all the network elements such as lines, transformers and loads. The method is developed and coded. It is tested on standard IEEE distribution systems with 4, 13, 34, 37, and 123 nodes. The results are compared with published IEEE data.

## Chapter 2

### Modeling of elements

All elements of the system are modeled as an electrical three-phase two-port network element. Detailed modeling can be found in [3]. Modeling of components in [3] is done based on impedance matrix of each element. To form admittance matrix required by Newton Raphson, in this study, modeling is done based on admittance matrix of each segment.

General form of presenting a segment is shown in Fig. 2.1

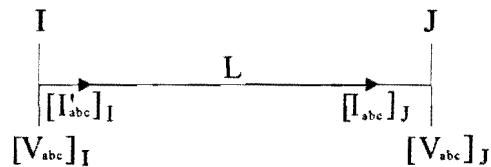


Fig. 2-1: General form of presenting a segment

$$[I_{abc}]_J = [Y_L]_L \times ([V_{abc}]_I - [a]_L \times [V_{abc}]_J) \quad (2-1)$$

$$[I'_{abc}]_I = [c]_L \times [V_{abc}]_J + [d]_L \times [I_{abc}]_J \quad (2-2)$$

L: Number of the segment

I: Sending end of segment L

J: Receiving end of segment L

$[V_{abc}]_I$ :  $3 \times 1$  matrix of voltage of node I

$[V_{abc}]_J$ :  $3 \times 1$  matrix of voltage of node J

$[I_{abc}]_J$ :  $3 \times 1$  matrix of output current of node J

$[I'_{abc}]_I$ :  $3 \times 1$  matrix of input current of node I

$[Y_L]_L$ :  $3 \times 3$  Admittance matrix of segment L

$[a]_L$ ,  $[c]_L$ ,  $[d]_L$ :  $3 \times 3$  matrices of segment L

### 2.1 Loads

Loads are categorized into two configurations ( $Y$  and  $\Delta$  connected) and three types (constant Power, impedance, current). Fig. 2.2 is showing  $Y$  and  $\Delta$  connected loads.

The loads on a distribution system are typically specified by the complex power that they consume. In this project active and reactive power of loads at nominal voltage of the node are considered as data. By changing voltages at the end of any iteration, active and reactive power of load must be updated.

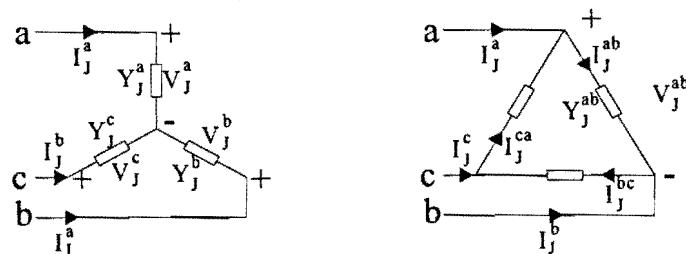


Fig. 2-2: Y and Δ connected loads

### 2.1.1 Y connected/ constant power loads

Active and reactive powers of these loads remain constant by changing the voltage of the node.

### 2.1.2 Y connected/ Constant Impedance

Because impedances of missing phases of two and single-phase loads, are infinity, admittance of loads is considered in this study. Equation 2-3 gives admittance of load “J” on phase “a”.

$$Y_J^a = \left( \frac{P_{J}^{oa} + jQ_{J}^{oa}}{|V_J^{oa}|^2} \right) \quad (2-3)$$

$Y_J^a$ : Admittance of load “J” on phase “a”

$P_J^{oa}, Q_J^{oa}$ : Initial active and reactive power of load “J” on phase “a”, at nominal line to neutral voltage

$|V_J^{oa}|$ : Nominal line to neutral voltage of load “J” on phase “a”

Equation 2-3 is used just once before starting the iterative process. Then  $Y_J^a$  is kept constant to update active and reactive powers of load “J” at the end of any iteration using updated voltage.

$$\begin{aligned} P_J^a &= \text{real} \left( |V_J^a|^2 (Y_J^a)^* \right) \\ Q_J^a &= \text{imag} \left( |V_J^a|^2 (Y_J^a)^* \right) \end{aligned} \quad (2-4)$$

$P_J^a, Q_J^a$ : Updated active and reactive power of load “J” on phase “a”

$|V_J^a|$ : Updated line to neutral voltage of load “J” on phase “a”

Similarly, power of phase b and c will be updated.

### 2.1.3 Y connected/ Constant Current

Magnitudes of line currents of load remain constant, but their angles change with angle of line to neutral voltages to keep the power factors constant.

$$|I_J^a| = \left( \frac{P_J^{oa} + jQ_J^{oa}}{V_J^{oa}} \right)^* \quad (2-5)$$

$$\square VI_J^{oa} = (\square V_J^{oa} - \square I_J^a) \quad (2-6)$$

$|I_J^a|$ : Magnitude of current of load “J” on phase “a”

$\square V_J^{oa}$ : Angle of nominal line to neutral voltage of phase “a” of load “J”

$\square I_J^a$ : Angle of current of phase “a” of load “J”

$\square VI_J^{oa}$ : Initial difference between angle of nominal voltage and current of phase “a” of load “J”

After an iteration, by changing angle of line to neutral voltage of a node, angle of current of load must be changed to keep  $\square VI_J^{oa}$  constant.

$$\square I_J^a = \square V_J^a - \square VI_J^{oa} \quad (2-7)$$

Then updated current of phase "a" of load "J" can be calculated as

$$I_J^a = |I_J^a| \square I_J^a \quad (2-8)$$

Load can be updated by using updated line to neutral voltage and current.

$$PI_J^a = \text{real}\left(V_J^a (I_J^a)^*\right) \quad (2-9)$$

$$QI_J^a = \text{imag}\left(V_J^a (I_J^a)^*\right)$$

Similarly, power of phase b and c will be updated.

#### 2.1.4 $\Delta$ connected/ constant power loads

As all calculations are based on line to neutral voltages, line to line active and reactive power of loads should be changed to line to neutral power. By definition, relation between "line to line" and "line to neutral" voltages are

$$\begin{aligned} V_{ab} &= V_a - V_b \\ V_{bc} &= V_b - V_c \\ V_{ca} &= V_c - V_a \end{aligned} \quad (2-10)$$

Using matrix form of eq. 2-10 for load "J"

$$\begin{bmatrix} V_{ab}^J \\ V_{bc}^J \\ V_{ca}^J \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a^J \\ V_b^J \\ V_c^J \end{bmatrix} \quad (2-11)$$

Eq. 2-12 gives updated current of load "J" between phases "a" and "b".

$$I_J^{ab} = \left( \frac{PI_J^{oab} + jQI_J^{oab}}{V_J^{ab}} \right) \quad (2-12)$$

$PI_J^{oab}, QI_J^{oab}$  : Initial active and reactive power of load "J" between phases "a" and "b", at nominal line to line voltage

$V_J^{ab}$  : Updated line to line voltage of load "J" between phases "a" and "b"

$I_J^{ab}$  : Updated current of load "J" between phases "a" and "b"

Applying KCL on  $\Delta$  connected load of Fig. 2-2 gives line current of load in matrix form of eq. 2-13.

$$\begin{bmatrix} I_J^a \\ I_J^b \\ I_J^c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_J^{ab} \\ I_J^{bc} \\ I_J^{ca} \end{bmatrix} \quad (2-13)$$

Updated line to neutral active and reactive power of load can be calculated by using updated line to neutral voltage, line current, and eq. 2-9.

Similarly, power of phase b and c will be updated.

### 2.1.5 $\Delta$ connected/ constant Impedance loads

Eq. 2-14 calculates admittance of load "J" between phases "a" and "b".

$$Y_J^{ab} = \left( \frac{P_{I_J^{ab}} + jQ_{I_J^{ab}}}{|V_J^{oab}|^2} \right)^* \quad (2-14)$$

$Y_J^{ab}$ : Admittance of load "J" between phases "a" and "b".

$V_J^{oab}$ : Nominal line to line voltage of load "J" between phases "a" and "b".

Updated current of load between two phases can be calculated by using updated voltage between these two phases and constant admittance of load.

$$I_J^{ab} = Y_J^{ab} \cdot V_J^{ab} \quad (2-15)$$

Updated line to neutral active and reactive power of load can be calculated by using updated line to neutral voltage, line current, and eq. 2-9.

### 2.1.6 $\Delta$ connected/ constant current loads

Again magnitudes of currents between phases remain constant but their angles change with angle of line to line voltages to keep the power factors constant.

$$|I_J^{ab}| = \left( \frac{P_{I_J^{ab}} + jQ_{I_J^{ab}}}{V_J^{oab}} \right)^* \quad (2-16)$$

$$\square VI_J^{oab} = (\square V_J^{oab} - \square I_J^{ab}) \quad (2-17)$$

$|I_J^{ab}|$ : Magnitude of current of load "J" between phases "a" and "b"

$\square V_J^{oab}$ : Angle of nominal line to line voltage between phases "a" and "b" of load "J"

$\square I_J^{ab}$ : Angle of current of load "J" between phases "a" and "b"

$\square VI_J^{oab}$ : Initial difference between angle of nominal line to line voltage and current between phases "a" and "b" of load "J"

After any iteration, by changing the angle of line to line voltage of node, angle of current of load must be changed to keep  $\square VI_J^{oab}$  constant.

$$\square I_J^{ab} = \square V_J^{ab} - \square VI_J^{oab} \quad (2-18)$$

Then updated current between phases "a" and "b" of load "J" can be calculated as

$$I_J^{ab} = |I_J^{ab}| \square I_J^{ab} \quad (2-19)$$

Using eq. 2-13 line current of load can be calculated, then using eq. 2-9 updated active and reactive powers are calculated.

## 2.2 Capacitors

Shunt capacitor banks are commonly used in distribution systems to help in voltage regulation and to provide reactive power support. The capacitor banks are in either "Y" or " $\Delta$ " connection.

All capacitor banks are modeled as three-phase constant impedance/admittance load with active power equal to zero and negative consumption of reactive power. Admittance of the missing phases are considered zero for single-phase and two-phase banks.

## 2.3 Distributed loads

Uniform distribution is considered for modeling of distributed loads as Fig. 2.3. This figure shows "n" uniformly spaced loads with distance of  $dL$  between two adjacent distributed loads. The loads are all equal and will be treated as constant current loads with a value of " $dI$ ". The total current into the feeder is "I". First a model to determine the total voltage drop from the source node (S) to the last node "R" is developed.

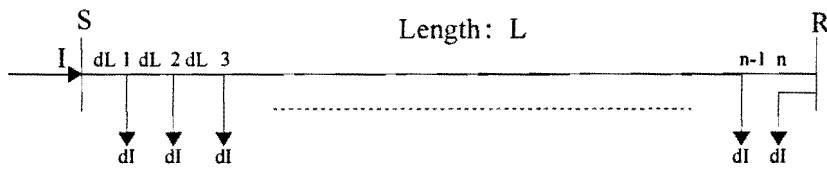


Fig. 2-3: Uniform distributed loads

$L$  = length of the segment

$z = r + jx$  = impedance of the line in  $\Omega/\text{mile}$

$dL$  = length of each line section

$dI$  = load currents at each node

$n$  = number of nodes and number of line sections

$I$  = total current into the feeder

### 2.3.1 Calculating voltage drop

The voltage drop in the first and second line segments are:

$$V_{\text{drop}1} = \text{real}\left((z.dL)(n.dI)\right) \quad (2-20)$$

$$V_{\text{drop}2} = \text{real}\left((z.dL)((n-1).dI)\right)$$

The total voltage drop from the source node to the last node is:

$$V_{\text{drop}} = \text{real}\left((z.dL)([n+(n-1)+...+3+2+1].dI)\right) \quad (2-21)$$

Substituting the series expansion and incremental values of current and distance in eq. 2-21

$$V_{\text{drop}} = \text{real}\left(\frac{1}{2} z.L.I\left(\frac{n+1}{n}\right)\right) \quad (2-22)$$

If  $Z$  is considered as the total impedance of segment in ohm and "n" goes to infinity then

$$V_{\text{drop}} = \text{real}\left(\frac{1}{2} Z.I\right) \quad (2-23)$$

### 2.3.2 Calculating power loss

Power loss calculation is as important as voltage calculation in the analysis of a distribution feeder. To calculate power loss, reference is made to Fig. 2-3 and the definitions for the parameters in that figure. The total three-phase power loss down the line will be the sum of the power losses in each short segment of the line. The three-phase power losses in the first and second segments are:

$$\begin{aligned} P_{\text{loss1}} &= 3(r.dL)|n.dl|^2 \\ P_{\text{loss2}} &= 3(r.dL)|(n-1).dl|^2 \end{aligned} \quad (2-24)$$

The total power loss over the length of the line is

$$P_{\text{loss}} = 3(r.dL)|dl|^2 \left( n^2 + (n-1)^2 + \dots + 2^2 + 1^2 \right) \quad (2-25)$$

The series inside the parenthesis of eq. 2-25 is the sum of the squares of "n" numbers and is equal to  $\frac{n(n+1)(2n+1)}{6}$

Substituting the series expansion and incremental values of current and distance in eq. 2-25

$$P_{\text{loss}} = 3R|I|^2 \frac{(n+1)(2n+1)}{6n^2} \quad (2-26)$$

Where  $R = r \times L$  is the total resistance per phase of the line segment in ohm. For a true uniformly distributed load, the number of nodes goes to infinity. Then the final equation for computing the total three-phase power loss down the line is:

$$P_{\text{loss}} = 3 \left( \frac{1}{3} R |I|^2 \right) \quad (2-27)$$

### 2.3.3 The Exact Lumped Load Model

In this section, one model that will work for both voltage drop and power loss calculations will be developed.

Fig. 2-4 shows the general configuration of the exact model that will give correct results for voltage drop and power loss. In Fig. 2.4 a portion ( $I_x$ ) of the total line current ( $I$ ) will be modeled  $kL$  miles from the source end, and the remaining current ( $cI$ ) will be modeled at the end of the line.

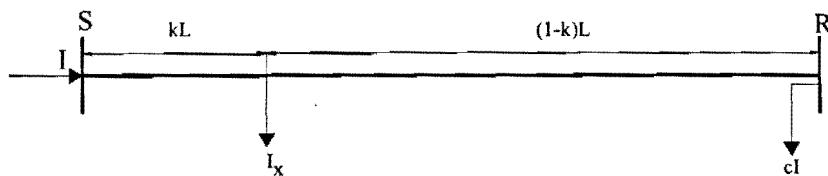


Fig. 2-4: General configuration of the exact model of distributed load

The values of  $k$  and  $c$  need to be derived.

In Fig. 2-4 the total voltage drop down the line is

$$V_{\text{drop}} = \text{real}(k.Z.I + (1-k).Z.c.I) \quad (2-28)$$

where

$Z$  = total line impedance in ohms

$k$  = factor of the total line length where the first part of the load current is modeled

$c$  = factor of the total current to place at the end of the line such that  $I = I_x + c.I$

Voltage drop of eq. 2-23 and 2-28 must be equal

$$V_{\text{drop}} = \text{real}(k.Z.I + (1-k).Z.c.I) = \text{real}\left(\frac{1}{2}ZI\right) \quad (2-29)$$

This equation gives first relation between  $k$  and  $c$ .

$$k = \frac{0.5 - c}{1 - c} \quad (2-30)$$

The same procedure can be followed for the power loss model. The total three-phase power loss in Fig. 2-4 is:

$$P_{\text{loss}} = 3(k.R.|I|^2 + (1-k).R.|c.I|^2) \quad (2-31)$$

Power loss of eq. 2-27 and 2-31 must be equal

$$P_{\text{loss}} = 3(k.R.|I|^2 + (1-k).R.|c.I|^2) = 3\left(\frac{1}{3}R|I|^2\right) \quad (2-32)$$

This equation gives second relation between  $k$  and  $c$ .

$$c^2 + k.(1 - c^2) = \frac{1}{3} \quad (2-33)$$

Solving equations 2-30 and 2-33 results in  $c=1/3$  and  $k=1/4$ .

The interpretation is that one-third of the load should be placed at the end of the line, and two-thirds of the load placed one-fourth of the way from the source end. Fig. 2.5 gives the final exact lumped load model.

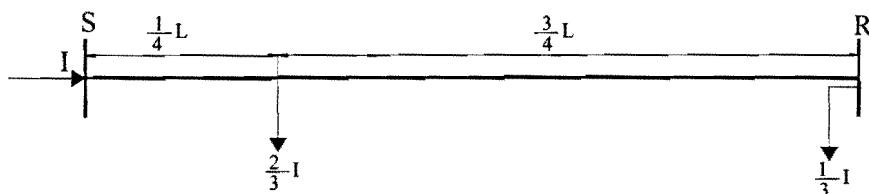


Fig. 2-5: Exact lumped load model

## 2.4 Overhead and Underground Distribution Lines

### 2.4.1 Series Admittance

#### Overhead line

If there are “ $n$ ” conductors, which the sum of their currents is equal to zero, self and mutual inductance between conductors “ $i$ ” and “ $n$ ” will be:

$$L_{ii} = \frac{\lambda_{ii}}{I_i} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{\text{GMR}_i} \text{ H/m} \quad (2-34)$$

$$L_{in} = \frac{\lambda_{in}}{l_n} = 2 \cdot 10^{-7} \cdot \ln \frac{1}{D_{in}} \text{ H/m}$$

GMR<sub>i</sub>: Geometric Mean Radius of conductor "i"

D<sub>in</sub>: distance between conductor "i" and "n"

Because distribution systems consist of single-phase, two-phase, and untransposed three-phase lines serving unbalanced loads, it is necessary to retain the identity of the self and mutual impedance terms of the conductors and take into account the ground return path for the unbalanced currents.

To calculate self and mutual impedance of conductor "i", AC resistance of conductor is taken from conductor data sheet. If frequency is 60 Hz and length of conductor is 1 mile by using eq. 2-34, self and mutual impedance will be

$$\bar{z}_{ii} = r_i + j0.12134 \cdot \ln \frac{1}{\text{GMR}_i} \Omega/\text{mile} \quad (2-35)$$

$$\bar{z}_{ij} = j0.12134 \cdot \ln \frac{1}{D_{ij}} \Omega/\text{mile}$$

Fig. 2.6 shows a line consisting of two conductors (i and j) carrying currents (I<sub>i</sub> and I<sub>j</sub>) with the remote ends of the conductors tied to ground. A fictitious "dirt" conductor carrying current I<sub>d</sub> is used to represent the return path for the currents. In Fig. 2.6, KVL and KCL are used to write the equations for the voltage between conductor "i" and ground, and currents of conductors and ground.

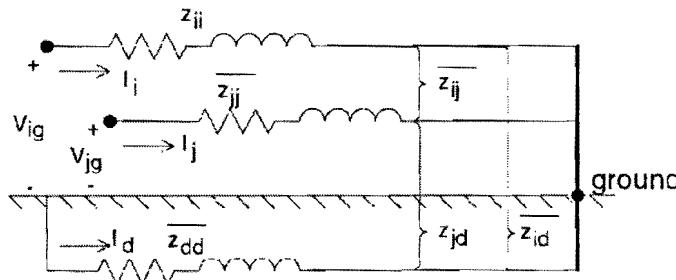


Fig. 2-6: Two conductor line

$$V_{ig} = \bar{z}_{ii} \cdot I_i + \bar{z}_{ij} \cdot I_j + \bar{z}_{id} \cdot I_d - (\bar{z}_{dd} \cdot I_d + \bar{z}_{di} \cdot I_i + \bar{z}_{dj} \cdot I_j) \quad (2-36)$$

$$I_d = -I_i - I_j$$

Then V<sub>ig</sub> can be expressed based on I<sub>i</sub> and I<sub>j</sub>

$$V_{ig} = (\bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id}) \cdot I_i + (\bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id}) \cdot I_j \quad (2-37)$$

By defining coefficient of currents in eq. 2-37 as self and mutual primitive impedances, these impedances can be written as:

$$\bar{z}_{ii} = \bar{z}_{ii} + \bar{z}_{dd} - \bar{z}_{di} - \bar{z}_{id} \quad (2-38)$$

$$\bar{z}_{ij} = \bar{z}_{ij} + \bar{z}_{dd} - \bar{z}_{dj} - \bar{z}_{id}$$

Self and mutual impedances of eq. 2-38 can be replaced by eq. 2-35

$$\hat{z}_{ii} = r_i + r_d + j0.12134 \cdot \left( \ln \frac{1}{\text{GMR}_i} + \ln \frac{D_{id} \cdot D_{di}}{\text{GMR}_d} \right) \quad (2-39)$$

$$\hat{z}_{ij} = r_d + j0.12134 \left( \ln \frac{1}{D_{ij}} + \ln \frac{D_{id} \cdot D_{id'}}{\text{GMR}_d} \right)$$

In Equation 2-39, the values of the resistance of dirt ( $r_d$ ), the Geometric Mean Radius of dirt ( $\text{GMR}_d$ ), and the distances from the conductors to dirt ( $D_{id}$ ,  $D_{di}$ ,  $D_{jd}$ ,  $D_{dj}$ ) are not known. By using modified carson's method, assumption of Earth Resistivity ( $\rho$ ) equal to  $100\Omega\text{-m}$ , and frequency ( $f$ ) equal to 60 Hz, equation 2-39 is simplified to

$$\hat{z}_{ii} = r_i - 0.09530 + j0.12134 \left( \ln \frac{1}{\text{GMR}_i} + 7.93402 \right) \Omega/\text{mile} \quad (2-40)$$

$$\hat{z}_{ij} = 0.09530 + j0.12134 \left( \ln \frac{1}{D_{ij}} + 7.93402 \right) \Omega/\text{mile}$$

An overhead four-wire distribution line segment (three phases and neutral) will result in a  $4 \times 4$  Primitive matrix. Using Fig. 2.7

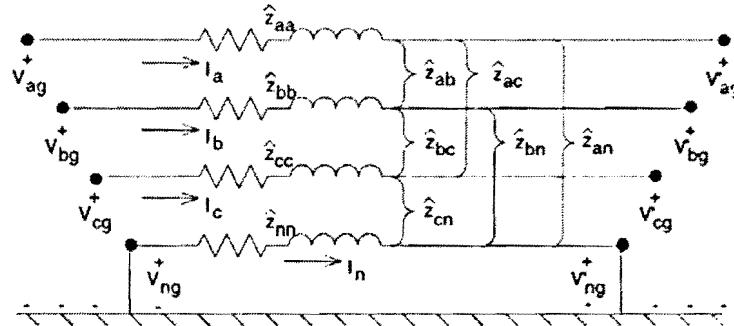


Fig. 2-7: Overhead four-wire distribution line segment

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \\ V_{ng} \end{bmatrix} = \begin{bmatrix} V'_{ag} \\ V'_{bg} \\ V'_{cg} \\ V'_{ng} \end{bmatrix} + \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn} \\ \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \quad (2-41)$$

In partition form

$$\begin{bmatrix} [V_{abc}] \\ [V_{ng}] \end{bmatrix} = \begin{bmatrix} [V'_{abc}] \\ [V'_{ng}] \end{bmatrix} + \begin{bmatrix} [\hat{z}_{ij}] & [\hat{z}_{in}] \\ [\hat{z}_{nj}] & [\hat{z}_{nn}] \end{bmatrix} \cdot \begin{bmatrix} [I_{abc}] \\ [I_n] \end{bmatrix} \quad (2-42)$$

To eliminate effect of neutral, Kron reduction method is employed. In this method  $V_{ng}$  and  $V'_{ng}$  are equal to zero because neutral conductor is grounded. Then from second row of eq. 2-42,  $I_n$  is calculated as

$$[I_n] = -[\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \cdot [I_{abc}] \quad (2-43)$$

Replacing eq. 2-43 in the first row of eq. 2-42, results in final phase impedance matrix.

$$[z_{abc}] = [\hat{z}_{ij}] - [\hat{z}_{in}] \cdot [\hat{z}_{nn}]^{-1} \cdot [\hat{z}_{nj}] \quad (2-44)$$

The phase impedance matrix for a three-wire delta line is determined by the application of Carson's equations without the Kron reduction step.

## *Underground lines*

### *Concentric neutral cable*

Fig. 2.8 shows a concentric neutral cable.

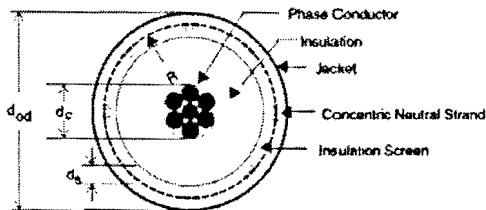


Fig. 2-8: Concentric neutral cable

Characteristics which are shown in Fig. 2.8 should be extracted from a table of underground cables. [3],[4] Appendixes A and B. These are

$d_c$  : phase conductor diameter (inches)

$d_{od}$  : nominal diameter over the concentric neutrals of the cable (inches)

$d_s$  : diameter of a concentric neutral strand (inches)

$GMR_c$  : geometric mean radius of the phase conductor (ft.)

$GMR_s$  : geometric mean radius of a neutral strand (ft.)

$r_c$  : resistance of the phase conductor ( $\Omega/\text{mile}$ )

$r_s$  : resistance of a solid neutral strand ( $\Omega/\text{mile}$ )

$k$  : number of concentric neutral strands

$R$ : radius of circle passing through the center of the concentric neutral strands

The equivalent geometric mean radius of the concentric neutral is computed using the equation for the geometric mean radius of bundled conductors used in high voltage transmission lines.[6]

$$GMR_{cn} = \sqrt[k]{GMR_s \cdot K \cdot R^{k-1}} \quad (2-45)$$

The equivalent resistance of the concentric neutral is

$$r_{cn} = \frac{r_s}{K} \quad (2-46)$$

Equivalent distance of concentric neutral conductors to their own phase conductor is  $R$ .

The geometric mean distance between a concentric neutral and an adjacent phase conductor, by assumption of laying conductors in trench not conduit, is equal to distance of phases.

Also, the geometric mean distance between concentric neutrals is equal to distance of phases.

By considering three phases in trench, primitive matrix for three phases and their three neutral conductors are given in (2-47). In this matrix the left top sub-matrix is representing self and mutual impedances of phase conductors. The right below sub-matrix is representing self and mutual impedances of concentric neutrals. The other two sub-matrices are representing mutual impedances between phase conductors and concentric neutrals.

$$[\hat{z}_{\text{primitive}}] = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & | & \hat{z}_{an1} & \hat{z}_{an2} & \hat{z}_{ann} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & | & \hat{z}_{bn1} & \hat{z}_{bn2} & \hat{z}_{bnn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & | & \hat{z}_{cn1} & \hat{z}_{cn2} & \hat{z}_{cnn} \\ --- & --- & --- & | & --- & --- & --- \\ \hat{z}_{n1a} & \hat{z}_{n1b} & \hat{z}_{n1c} & | & \hat{z}_{n1n1} & \hat{z}_{n1n2} & \hat{z}_{n1nn} \\ \hat{z}_{n2a} & \hat{z}_{n2b} & \hat{z}_{n2c} & | & \hat{z}_{n2n1} & \hat{z}_{n2n2} & \hat{z}_{n2nn} \\ \hat{z}_{nnn} & \hat{z}_{nnb} & \hat{z}_{nnn} & | & \hat{z}_{nnn1} & \hat{z}_{nnn2} & \hat{z}_{nnnn} \end{bmatrix} \quad (2-47)$$

Eq. 2-40 is deployed to calculate elements of matrix (2-47). Applying eq. 2-44 (Kron reduction method) to primitive matrix of eq. 2-47, gives final impedance matrix of phases.

### Tape shield cable

Fig. 2-9 shows a tape-shielded cable.

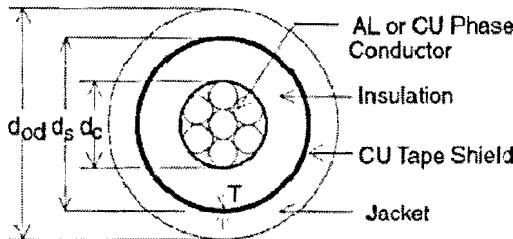


Fig. 2-9: Tape-shielded cable

Again eq. 2-40 is used to calculate elements of Primitive matrix.

Related values of tape shield are:

$$r_{\text{shield}} = 7.9385 \times 10^8 \frac{\rho}{d_s \cdot T} \quad (2-48)$$

$$\text{GMR}_{\text{shield}} = \frac{\frac{d_s}{2} - \frac{T}{12}}{1000} \quad (2-49)$$

$\rho$  :  $100 \Omega \cdot \text{m}$  at  $50^\circ\text{C}$

$d_s$ : outside diameter of the tape shield in miles

$T$ : thickness of tape shield in miles

Equivalent distance of tape shield and its own phase conductor, is  $\text{GMR}_{\text{shield}}$ .

### 2.4.2 Shunt admittance

If there are  $N$  solid and round conductors, and each conductor has a unique uniform charge density of “ $q$ ”, the voltage drop between conductor “ $i$ ” and conductor “ $j$ ”, which has uniform charge density of “ $-q$ ”, is:

$$V_{ij} = \frac{1}{2\pi\epsilon} \sum_{n=1}^N q_n \ln \frac{D_{nj}}{D_{ni}} \quad (2-50)$$

Where,

$\epsilon = \epsilon_0 \epsilon_r$  = permittivity of the medium

$\epsilon_0$  = permittivity of free space =  $8.85 \times 10^{-12} \mu\text{F/meter}$ ,

$\epsilon_r$  = relative permittivity of the medium

$q_n$  = charge density on Conductor  $n$  cb/meter

$D_{ni}$ = distance between Conductor n and Conductor i (ft.)

$D_{nj}$ = distance between Conductor n and Conductor j (ft.)

$RD_n$ = radius of Conductor n

### Overhead lines

The method of conductors and their images is employed in the calculation of the shunt capacitance of overhead lines. Fig. 2.10 shows conductors and their images.

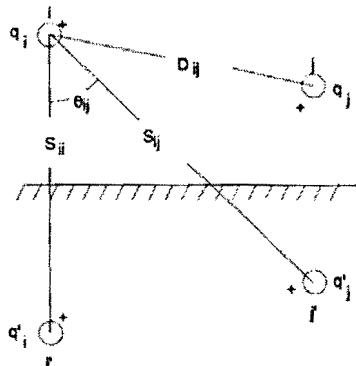


Fig. 2-10: conductors and their images

In this method, the image conductors have negative charge with respect to their conductors.

$$q'_i = -q_i \text{ and } q'_j = -q_j$$

Applying eq. 2-50 to Fig. 2.10 and the fact that voltage drop between Conductor i and ground will be one-half of the voltage drop between Conductor i and its image

$$V_{ig} = \frac{1}{2\pi\epsilon} \left( q_i \ln \frac{S_{ii}}{RD_i} + q_j \ln \frac{S_{ij}}{D_{ij}} \right) \quad (2-51)$$

In general form:

$$V_{ig} = \hat{P}_{ii} \cdot q_i + \hat{P}_{ij} \cdot q_j \quad (2-52)$$

where  $\hat{P}_{ii}$  and  $\hat{P}_{ij}$  are the self and mutual potential coefficients. For overhead lines the relative permittivity of air is assumed to be 1.0 so that

$$\epsilon_{air} = 1.0 \times 8.85 \times 10^{-12} \text{ F/meter} = 1.4240 \times 10^{-2} \mu\text{F}/\text{mile} \quad (2-53)$$

Then  $\hat{P}_{ii}$  and  $\hat{P}_{ij}$  can be written as:

$$\begin{aligned} \hat{P}_{ii} &= 11.17689 \cdot \ln \frac{S_{ii}}{RD_i} \text{ mile}/\mu\text{F} \\ \hat{P}_{ij} &= 11.17689 \cdot \ln \frac{S_{ij}}{D_{ij}} \text{ mile}/\mu\text{F} \end{aligned} \quad (2-54)$$

Primitive potential coefficient matrix of overhead lines is shown in eq. 2-55

$$[\hat{P}_{\text{primitive}}] = \begin{bmatrix} \hat{P}_{aa} & \hat{P}_{ab} & \hat{P}_{ac} & \cdots & \hat{P}_{an} \\ \hat{P}_{ba} & \hat{P}_{bb} & \hat{P}_{bc} & \cdots & \hat{P}_{bn} \\ \hat{P}_{ca} & \hat{P}_{cb} & \hat{P}_{cc} & \cdots & \hat{P}_{cn} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \hat{P}_{na} & \hat{P}_{nb} & \hat{P}_{nc} & \cdots & \hat{P}_{nn} \end{bmatrix} \quad (2-55)$$

In partition form, eq. 2-55 is:

$$[\hat{P}_{\text{primitive}}] = \begin{bmatrix} [\hat{P}_{ij}] & [\hat{P}_{in}] \\ [\hat{P}_{nj}] & [\hat{P}_{nn}] \end{bmatrix} \quad (2-56)$$

Again, Korn reduction method is applied to reduce  $\hat{P}_{\text{primitive}}$  matrix to a  $3 \times 3$  matrix and calculate a final potential coefficient matrix.

$$[P_{abc}] = [\hat{P}_{ij}] - [\hat{P}_{in}] \cdot [\hat{P}_{nn}]^{-1} \cdot [\hat{P}_{nj}] \quad (2-57)$$

Shunt admittance matrix can be calculated from eq. 2-57.

$$[y_{abc}] = j(2\pi f)[P_{abc}]^{-1} \quad (2-58)$$

### *Underground line*

#### *Concentric neutral cable*

Because of stranding, it is assumed that the electric field created by the charge on the phase conductor will be confined to the boundary of the concentric neutral strands.

Cross-linked polyethylene is assumed to be the insulation material and the minimum value of its relative permittivity is taken (2.3).

Off diagonal elements of shunt admittance matrix are zero, and diagonal elements can be calculated by eq. 2-59.

$$y = j \left( \frac{77.3619}{\ln\left(\frac{R_b}{RD_c}\right) - \left(\frac{1}{K}\right) \ln\left(\frac{K.RD_s}{R_b}\right)} \right)^{\mu\text{s}/\text{mile}} \quad (2-59)$$

$R_b$ : Radius of circle passing through the center of the neutral strands

$RD_c$ : Radius of phase conductor

$RD_s$ : Radius of strand conductor

K: number of strands

#### *Tape shield*

As with the concentric neutral cable, the electric field is confined to the insulation. Also the relative permittivity of 2.3 will apply.

Off diagonal elements of shunt admittance matrix are zero, and diagonal elements can be calculated by eq. 2-60.

$$y = j \left( \frac{77.3619}{\ln \left( \frac{R_b}{RD_c} \right)} \right) \mu\text{s/mile} \quad (2-60)$$

$R_b$ : Radius of circle passing through the center of tape shield

$RD_c$ : Radius of phase conductor

#### 2.4.3 Line complete model

Fig. 2.11 shows  $\pi$  model of a line.

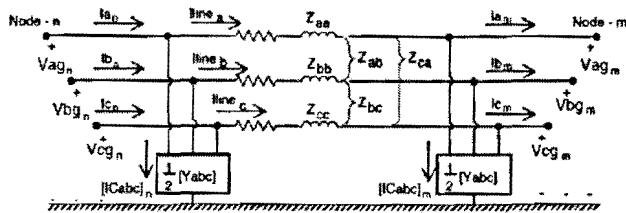


Fig. 2-11:  $\pi$  model of a line.

By applying KCL on node "m" and KVL to segment model

$$[I_{line}]_n = [I_{abc}]_m + \frac{1}{2}[Y_{abc}][V_{abc}]_m \quad (2-61)$$

$$[V_{abc}]_n = [V_{abc}]_m + [Z_{abc}] [I_{line}]_m \quad (2-62)$$

By replacing 2-61 into 2-62

$$[V_{abc}]_n = \left( U + \frac{1}{2}[Z_{abc}][Y_{abc}] \right) [V_{abc}]_m + [Z_{abc}] [I_{abc}]_m \quad (2-63)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where  $U$  is

By applying KCL on node n and using eq. 2-62

$$[I_{abc}]_n = \left( [Y_{abc}] + \frac{1}{4}[Y_{abc}][Z_{abc}][Y_{abc}] \right) [V_{abc}]_m + \left( U + \frac{1}{2}[Z_{abc}][Y_{abc}] \right) [I_{abc}]_m \quad (2-64)$$

If equations 2-64 and 2-63 are compared with 2-1 and 2-2, matrices  $[YL]$ ,  $[a]$ ,  $[c]$ , and  $[d]$  of a distribution line can be calculated as below.

$$[YL] = [Z_{abc}]^{-1}$$

$$[a] = [U] + 1/2[Z_{abc}][Y_{abc}]$$

$$[c] = [y_{abc}] + 1/4[y_{abc}][Z_{abc}][Y_{abc}]$$

$$[d] = [U] + 1/2[y_{abc}][Z_{abc}] \quad (2-65)$$

## 2.5 Transformers

Three-phase transformer banks are found in the distribution substation where the voltage is transformed from the transmission or sub-transmission level. In most cases, the substation transformer will be a three-phase unit, perhaps with high-voltage no-load taps and, perhaps, low-voltage load tap changing. For a four-wire Y feeder, the most common substation transformer connection is the  $\Delta$ -grounded Y.

A three-wire delta feeder will typically have a  $\Delta$ - $\Delta$  transformer connection in the substation. Configurations of transformers on distribution feeder are:

Grounded Y- Grounded Y

Grounded Y-  $\Delta$

Ungrounded Y-  $\Delta$

$\Delta$ - Grounded Y

$\Delta$ -  $\Delta$

Open Grounded Y- Open  $\Delta$

All variations of the Y- $\Delta$  connections are “American Standard Thirty-Degree” connection.

Based on this, positive sequence voltages and currents are:

Step down transformer

Phase to phase voltage and line current of primary are leading secondary by  $30^\circ$

Step up transformer

Phase to phase voltage and line current of primary are lagging secondary by  $30^\circ$

Before going through developing models for transformers, relations between voltages and currents of  $\Delta$  side are reviewed.

Fig. 2-12 shows a  $\Delta$  connection.

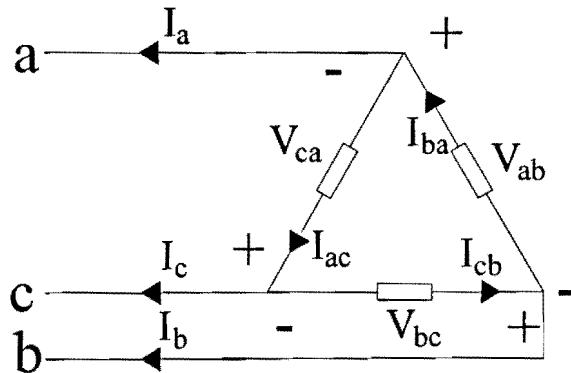


Fig. 2-12:  $\Delta$  connection

Applying KVL on Fig. 2-12 results in

$$\begin{aligned} V_{ab} &= V_a - V_b \\ V_{bc} &= V_b - V_c \quad \text{or in matrix form} \\ V_{ca} &= V_c - V_a \end{aligned} \quad \left[ \begin{array}{c} V_{ab} \\ V_{bc} \\ V_{ca} \end{array} \right] = \left[ \begin{array}{ccc} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{array} \right] \left[ \begin{array}{c} V_a \\ V_b \\ V_c \end{array} \right] \quad (2-66)$$

Applying KCL on Fig. 2-12 results in

$$\begin{aligned} I_a &= I_{ba} - I_{ac} \\ I_b &= I_{cb} - I_{ba} \quad \text{or in matrix form} \\ I_c &= I_{ac} - I_{cb} \end{aligned} \quad \left[ \begin{array}{c} I_a \\ I_b \\ I_c \end{array} \right] = \left[ \begin{array}{ccc} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{array} \right] \left[ \begin{array}{c} I_{ba} \\ I_{cb} \\ I_{ac} \end{array} \right] \quad (2-67)$$

As it can be seen, the matrices in equations 2-66 and 2-67 are singular. It means that these equations do not obtain a unique relation between variables of two side of equation.

For the sake of unique relations, voltages and currents transferred to sequence planes.

$$\begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix}_{LL} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \text{ and } \begin{bmatrix} I_0 \\ I_+ \\ I_- \end{bmatrix}_L = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a_s^2 & a_s \\ 1 & a_s & a_s^2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-68)$$

$a_s = 1 \square 120$

By definition, the zero sequence line-to-line voltage is always zero. The relationship between the positive and negative sequence “line to neutral” and “line to line” voltages is known.

Also, the zero sequence line current is always zero. The relationship between the positive and negative sequence “line” and “line to line” current is known.

$$\begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix}_L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & t_s^* & 0 \\ 0 & 0 & t_s \end{bmatrix} \begin{bmatrix} V_0 \\ V_+ \\ V_- \end{bmatrix}_{LL} \text{ and } \begin{bmatrix} I_0 \\ I_+ \\ I_- \end{bmatrix}_L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & g_s^* & 0 \\ 0 & 0 & g_s \end{bmatrix} \begin{bmatrix} I_0 \\ I_+ \\ I_- \end{bmatrix}_L \quad (2-69)$$

$t_s = \frac{1}{\sqrt{3}} \square 30 \qquad \qquad g_s = \frac{1}{\sqrt{3}} \square -30$

Replacing 2-68 into 2-69 results in:

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}, [WV] = \frac{1}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}, [WV]^{-1} = \frac{1}{3} \begin{bmatrix} 4 & -2 & 1 \\ 1 & 4 & -2 \\ -2 & 1 & 4 \end{bmatrix} \quad (2-70)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix}, [WI] = \frac{1}{3} \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix}, [WI]^{-1} = \frac{1}{3} \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad (2-71)$$

In all cases, turns-ratio between two windings of primary and secondary is called “ $n_t$ ”. Also, all impedances are transferred to secondary side.

### 2.5.1 Grounded Y- Grounded Y transformers

#### Step down

Fig. 2-13 shows step down Grounded Y- Grounded Y transformer connection and voltage vectors.

Induced line voltage on the secondary side is equal to line voltage of terminals of secondary, plus voltage drop on transformer impedances.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{induced} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} + \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = n_t \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{induced} \quad (2-72)$$

Then,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix}^{-1} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - n_t \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-73)$$

Considering turns-ratio of transformer, relation between primary and secondary currents will be:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-74)$$

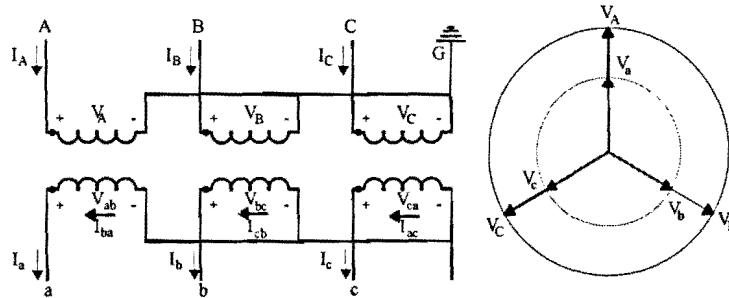


Fig. 2-13: Grounded Y-Grounded Y transformer

Comparing equations 2-73 and 2-74 with equations 2-1 and 2-2

$$[a] = n_t \cdot U_{3 \times 3}, [c] = 0, [d] = \frac{1}{n_t} \cdot U_{3 \times 3}, [YL] = \frac{1}{n_{trans}} \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix}^{-1} \quad (2-75)$$

*Step up*

Model developed for step down will be used for step up as well.

### 2.5.2 Grounded Y- $\Delta$ transformers

*Step Down*

Fig. 2-14 shows step down Grounded Y- $\Delta$  transformer connection and voltage vectors.

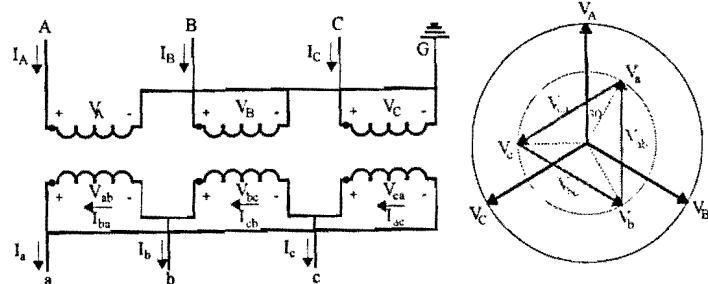


Fig. 2-14: Step down Grounded Y- $\Delta$  transformer

Considering turns-ratio of transformer and equation 2-71

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{3n_t} \begin{bmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-76)$$

Induced line to line voltage on the secondary side is equal to line to line voltage of terminals of secondary, plus voltage drop on transformer impedances.

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_{\text{induced}} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} Z_{ab} & 0 & 0 \\ 0 & Z_{bc} & 0 \\ 0 & 0 & Z_{ca} \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix}, \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = n_{\text{trans}} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_{\text{induced}} \quad (2-77)$$

Using equation 2-67, equation 2-77 can be written

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_{\text{trans}}} \begin{bmatrix} \frac{1}{Z_{ab}} & 0 & \frac{-1}{Z_{ca}} \\ \frac{-1}{Z_{ab}} & \frac{1}{Z_{bc}} & 0 \\ 0 & \frac{-1}{Z_{bc}} & \frac{1}{Z_{ca}} \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - n_{\text{trans}} [WV]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-78)$$

Comparing equations 2-76 and 2-78 with equations 2-1 and 2-2

$$[a] = n_t [WV]^{-1}, [c] = 0, [d] = \frac{1}{n_t} [WI]^{-1}, [YL] = \frac{1}{n_{\text{trans}}} \begin{bmatrix} \frac{1}{Z_{ab}} & 0 & \frac{-1}{Z_{ca}} \\ \frac{-1}{Z_{ab}} & \frac{1}{Z_{bc}} & 0 \\ 0 & \frac{-1}{Z_{bc}} & \frac{1}{Z_{ca}} \end{bmatrix} \quad (2-79)$$

### Step up

Fig. 2-15 shows step up Grounded Y- $\Delta$  transformer connection and voltage vectors.

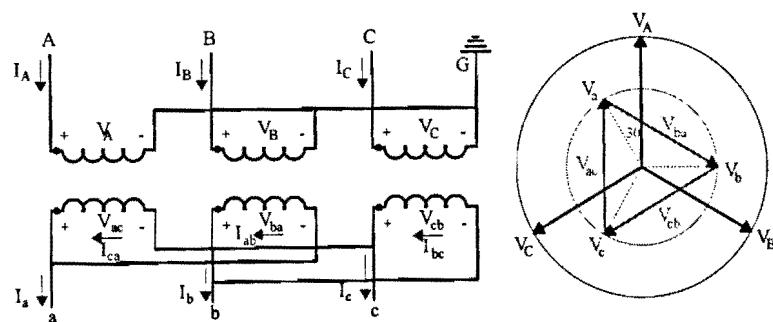


Fig. 2-15: Step up Grounded Y- $\Delta$  transformer

Rearranging equations 2-67, 2-70, and 2-71 for Fig. 2-15

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{ca} \\ I_{ab} \\ I_{bc} \end{bmatrix} \quad (2-80)$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} \rightarrow \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \frac{-1}{3} \begin{bmatrix} -2 & 1 & 4 \\ 4 & -2 & 1 \\ 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (2-81)$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 4 & 1 & -2 \\ -2 & 4 & 1 \\ 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{-1}{3} \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-82)$$

Considering turns-ratio of transformer and equation 2-82

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{-1}{3n_t} \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-83)$$

Induces Line to line voltage on the secondary side is equal to line to line voltage of terminals of secondary, plus voltage drop on transformer impedances.

$$\begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_{\text{indu}} = \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + \begin{bmatrix} Z_{ab} & 0 & 0 \\ 0 & Z_{bc} & 0 \\ 0 & 0 & Z_{ca} \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix}, \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = n_{\text{trans}} \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix}_{\text{induced}} \quad (2-84)$$

Using equation 2-80 and 2-81, equation 2-84 can be written

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} \frac{1}{Z_{ab}} & \frac{-1}{Z_{bc}} & 0 \\ 0 & \frac{1}{Z_{bc}} & \frac{-1}{Z_{ca}} \\ \frac{-1}{Z_{ab}} & 0 & \frac{1}{Z_{ca}} \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \frac{-n_t}{3} \begin{bmatrix} -2 & 1 & 4 \\ 4 & -2 & 1 \\ 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-85)$$

Comparing equations 2-83 and 2-85 with equations 2-1 and 2-2

$$[a] = \frac{-n_t}{3} \begin{bmatrix} -2 & 1 & 4 \\ 4 & -2 & 1 \\ 1 & 4 & -2 \end{bmatrix}, [c] = 0, [d] = \frac{-1}{3n_t} \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}, [YL] = \frac{1}{n_t} \begin{bmatrix} \frac{1}{Z_{ab}} & \frac{-1}{Z_{bc}} & 0 \\ 0 & \frac{1}{Z_{bc}} & \frac{-1}{Z_{ca}} \\ \frac{-1}{Z_{ab}} & 0 & \frac{1}{Z_{ca}} \end{bmatrix} \quad (2-86)$$

### 2.5.3 Unrounded Y-Δ transformers

*Step Down*

Fig. 2-16 shows step down ungrounded Y- $\Delta$  transformer connection and voltage vectors. In this configuration, there is no passage to ground, so there is no zero sequence current in primary and secondary.

$$I_{ba} + I_{cb} + I_{ac} = 0 \text{ or } \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (2-87)$$

Adding eq. 2-87 and eq. 2-67 and considering turns-ratio, results in:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3n_t} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-88)$$

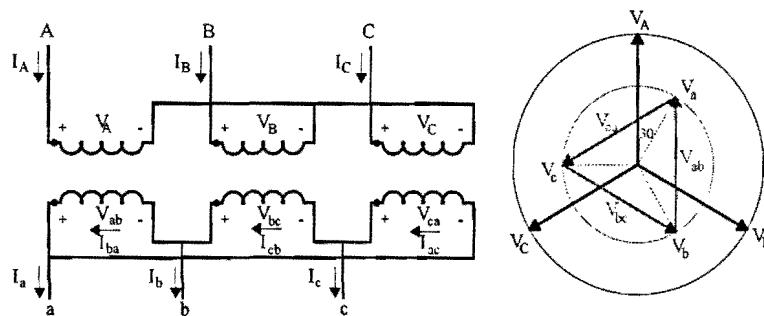


Fig. 2-16: Step down ungrounded Y- $\Delta$  transformer

Equation 2-77 can also be written for this configuration. Then using equation 2-88, this equation can be written as follows:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_{\text{trans}}} \begin{bmatrix} \frac{1}{Z_{ab}} & 0 & -\frac{1}{Z_{ca}} \\ -\frac{1}{Z_{ab}} & \frac{1}{Z_{bc}} & 0 \\ \frac{1}{Z_{ab}} & 0 & \frac{2}{Z_{ca}} \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - n_{\text{trans}} [WV]^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-89)$$

Comparing equations 2-88 and 2-89 with equations 2-1 and 2-2

$$[a] = n_t [WV]^{-1}, [c] = 0, [d] = \frac{1}{3n_t} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & 1 \end{bmatrix}, [YL] = \frac{1}{n_{\text{trans}}} \begin{bmatrix} \frac{1}{Z_{ab}} & 0 & -\frac{1}{Z_{ca}} \\ -\frac{1}{Z_{ab}} & \frac{1}{Z_{bc}} & 0 \\ \frac{1}{Z_{ab}} & 0 & \frac{2}{Z_{ca}} \end{bmatrix} \quad (2-90)$$

### Step up

Fig. 2-17 shows step up Ungrounded Y- $\Delta$  transformer connection and voltage vectors.

Adding eq. 2-80 and eq. 2-87 and considering turns-ratio

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{3n_t} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-91)$$

Using equation 2-91 and 2-81, equation 2-84 can be written

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} \frac{1}{Z_{ab}} & \frac{-1}{Z_{bc}} & 0 \\ 0 & \frac{1}{Z_{bc}} & \frac{-1}{Z_{ca}} \\ 0 & \frac{1}{Z_{ca}} & \frac{2}{Z_{ba}} \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \frac{-n_t}{3} \begin{bmatrix} -2 & 1 & 4 \\ 4 & -2 & 1 \\ 1 & 4 & -2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-92)$$

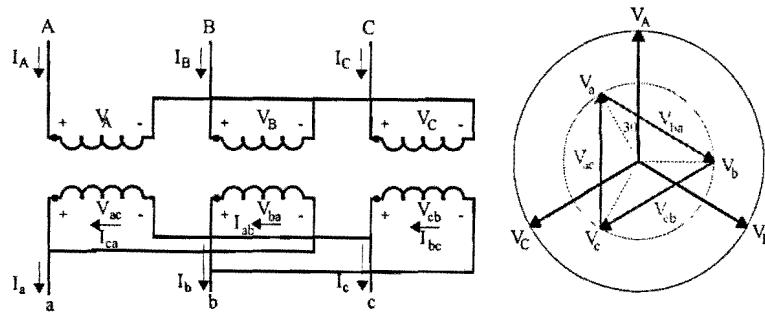


Fig. 2-17: Step up Ungrounded Y-Δ transformer

Comparing equations 2-91 and 2-92 with equations 2-1 and 2-2

$$[a] = \frac{-n_t}{3} \begin{bmatrix} -2 & 1 & 4 \\ 4 & -2 & 1 \\ 1 & 4 & -2 \end{bmatrix}, [c] = 0, [d] = \frac{1}{3n_t} \begin{bmatrix} 3 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 1 \end{bmatrix}, [YL] = \frac{1}{n_t} \begin{bmatrix} \frac{1}{Z_{ab}} & \frac{-1}{Z_{bc}} & 0 \\ 0 & \frac{1}{Z_{bc}} & \frac{-1}{Z_{ca}} \\ 0 & \frac{1}{Z_{ca}} & \frac{2}{Z_{ba}} \end{bmatrix} \quad (2-93)$$

#### 2.5.4 Δ-Grounded Y transformers

##### Step Down

Fig. 2-18 shows a step down Δ-Grounded Y transformer connection and voltage vectors. Applying KCL on primary side and considering turns-ratio:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{AC} \\ I_{BA} \\ I_{CB} \end{bmatrix} \rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-94)$$

Induced line voltage on the secondary side is equal to line voltage of terminals of secondary plus voltage drop on transformer impedances.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{\text{induced}} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} + \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{\text{induced}} \quad (2-95)$$

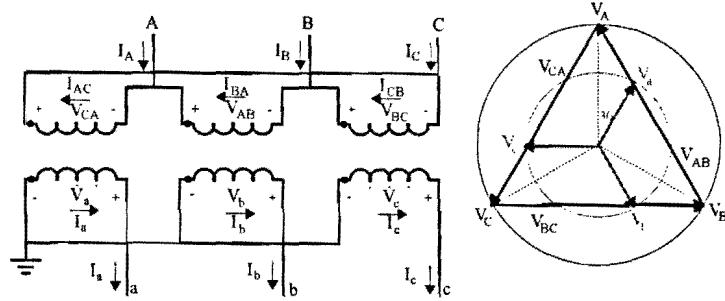


Fig. 2-18: Step down Δ-Grounded Y transformer

Using 2-70

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{-1}{3n_t} \begin{bmatrix} -2 & \frac{1}{Z_a} & \frac{4}{Z_a} \\ \frac{1}{Z_a} & \frac{4}{Z_a} & \frac{1}{Z_a} \\ \frac{4}{Z_a} & \frac{1}{Z_a} & -2 \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \frac{-n_t}{3} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-96)$$

Comparing equations 2-94 and 2-96 with equations 2-1 and 2-2

$$[a] = \frac{-n_t}{3} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}, [c] = 0, [d] = \frac{1}{n_t} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, [YL] = \frac{-1}{3n_t} \begin{bmatrix} -2 & \frac{1}{Z_a} & \frac{4}{Z_a} \\ \frac{1}{Z_a} & \frac{4}{Z_a} & \frac{1}{Z_a} \\ \frac{4}{Z_a} & \frac{1}{Z_a} & -2 \end{bmatrix} \quad (2-97)$$

### Step up

Fig. 2-19 shows step up Δ-Grounded Y transformer connection and voltage vectors. Applying KCL on primary side and considering turns-ratio

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-98)$$

Induces line voltage on the secondary side is equal to line voltage of terminals of secondary plus voltage drop on transformer impedances.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{\text{induced}} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} + \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}, \quad \begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}_{\text{induced}} \quad (2-99)$$

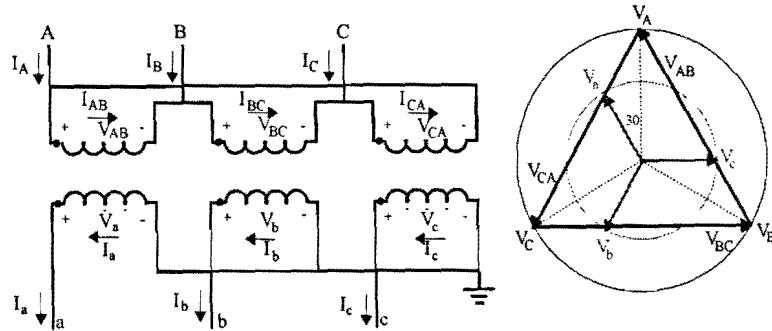


Fig. 2-19: Step up  $\Delta$ -Ungrounded Y transformer

Using 2-70

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3n_t} \begin{bmatrix} \frac{4}{z_a} & -\frac{2}{z_a} & \frac{1}{z_a} \\ \frac{1}{z_b} & \frac{4}{z_b} & -\frac{2}{z_b} \\ -\frac{2}{z_c} & \frac{1}{z_c} & \frac{4}{z_c} \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - \frac{n_t}{3} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-100)$$

Comparing equations 2-98 and 2-100 with equations 2-1 and 2-2

$$[a] = n_t [WV], [c] = 0, [d] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, [YL] = \frac{1}{3n_t} \begin{bmatrix} \frac{4}{z_a} & -\frac{2}{z_a} & \frac{1}{z_a} \\ \frac{1}{z_b} & \frac{4}{z_b} & -\frac{2}{z_b} \\ -\frac{2}{z_c} & \frac{1}{z_c} & \frac{4}{z_c} \end{bmatrix} \quad (2-101)$$

### 2.5.5 $\Delta$ - $\Delta$ transformers

#### Step Down

Fig. 2-20 shows  $\Delta$ - $\Delta$  transformer connection and voltage vectors. Applying KCL on primary and secondary side of transformer

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (2-102)$$

Considering turns-ratio and equation 2-102 results in an equation between primary and secondary current

$$\begin{bmatrix} I_{AB} \\ I_{BC} \\ I_{CA} \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} U_{3 \times 3} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-103)$$

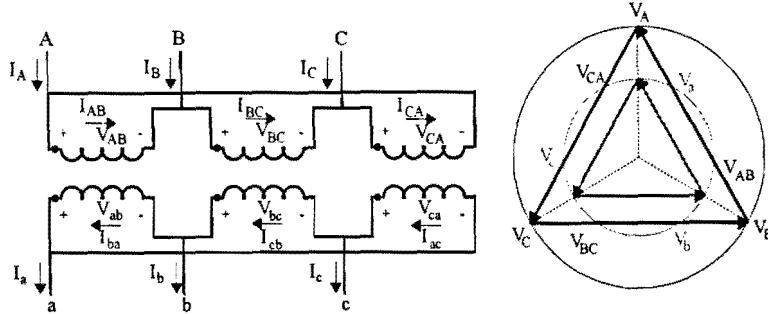


Fig. 2-20:  $\Delta$ - $\Delta$  transformer

Equation 2-77 can be written for the secondary side of transformer.

$$\begin{bmatrix} V_{AB} \\ V_{BC} \\ V_{CA} \end{bmatrix} = n_t \begin{bmatrix} V_{ab} \\ V_{bc} \\ V_{ca} \end{bmatrix} + n_t \begin{bmatrix} Z_{ab} & 0 & 0 \\ 0 & Z_{bc} & 0 \\ 0 & 0 & Z_{ca} \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (2-104)$$

Applying KVL on secondary side of transformer

$$V_{ab} + V_{bc} + V_{ca} = 0 \rightarrow (V_{ab} + V_{bc} + V_{ca})_{\text{induced}} = z_{ab}I_{ba} + z_{bc}I_{cb} + z_{ca}I_{ac} \quad (2-105)$$

Zero sequence line to line voltage of delta side is zero, so eq. 2-105 can be written as

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ z_{ab} & z_{bc} & z_{ca} \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (2-106)$$

Adding eq. 2-106 to KCL part of equation 2-102

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ z_{ab} & z_{bc} - 1 & z_{ca} + 1 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \quad (2-107)$$

Replacing eq. 2-107 into eq. 2-104 and using eq. 2-70

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ z_a & z_b - 1 & z_c + 1 \end{bmatrix} [Z_{abc}]^{-1} [WV]^{-1} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - n_t \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-108)$$

Comparing equations 2-103 and 2-108 with equations 2-1 and 2-2

$$[a] = n_t U_{3 \times 3}, [c] = 0, [d] = \frac{1}{n_t} U_{3 \times 3}, [YL] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ z_a & z_b - 1 & z_c + 1 \end{bmatrix} [Z_{abc}]^{-1} [WV]^{-1} \quad (2-109)$$

*Step up*

Model developed for step down will be used for step up as well.

### 2.5.6 Open Grounded Y-Open Δ transformers

*Step Down*

Fig. 2-21 shows step down Open Grounded Y-Open Δ transformer connection and voltage vectors. Applying KCL on secondary side

$$\begin{aligned} I_a &= I_{ba} \\ I_b &= -(I_a + I_c) \\ I_c &= -I_{cb} \end{aligned} \quad (2-110)$$

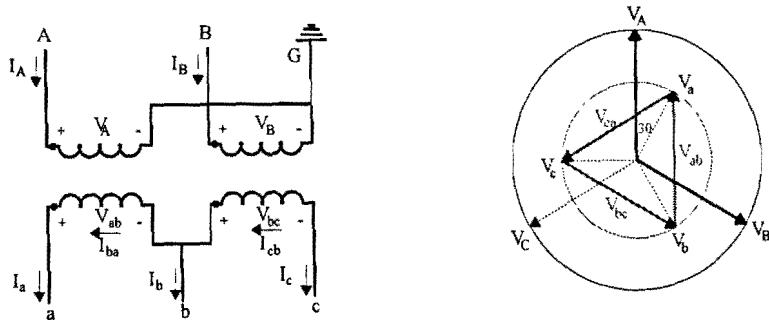


Fig. 2-21: Step down Open Grounded Y- Open Δ transformer

Considering turns-ratio

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{ba} \\ I_{cb} \\ I_{ac} \end{bmatrix} \rightarrow \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (2-111)$$

Induced voltage on secondary side is equal to line to line voltage of secondary terminals plus voltage drop on impedance of transformer.

$$\begin{aligned} (V_{ab})_{\text{induced}} &= V_{ab} + z_{ab} I_{ba}, \quad (V_{ab})_{\text{induced}} = \frac{1}{n_t} V_A \\ (V_{bc})_{\text{induced}} &= V_{bc} + z_{bc} I_{cb}, \quad (V_{bc})_{\text{induced}} = \frac{1}{n_t} V_B \end{aligned} \quad (2-112)$$

From eq. 2-110 and 2-112

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{n_t} \begin{bmatrix} \frac{1}{z_a} & 0 & 0 \\ -\frac{1}{z_a} & \frac{1}{z_b} & 0 \\ 0 & -\frac{1}{z_b} & 0 \end{bmatrix} \left( \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} - n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \right) \quad (2-113)$$

Comparing equations 2-111 and 2-113 with equations 2-1 and 2-2

$$[a] = n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, [c] = 0, [d] = \frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}, [YL] = \frac{1}{n_t} \begin{bmatrix} \frac{1}{z_a} & 0 & 0 \\ -\frac{1}{z_a} & \frac{1}{z_b} & 0 \\ 0 & -\frac{1}{z_b} & 0 \end{bmatrix} \quad (2-114)$$

*Step up*

Fig. 2-21 shows step up Open Grounded Y-Open Δ transformer connection and voltage vectors.

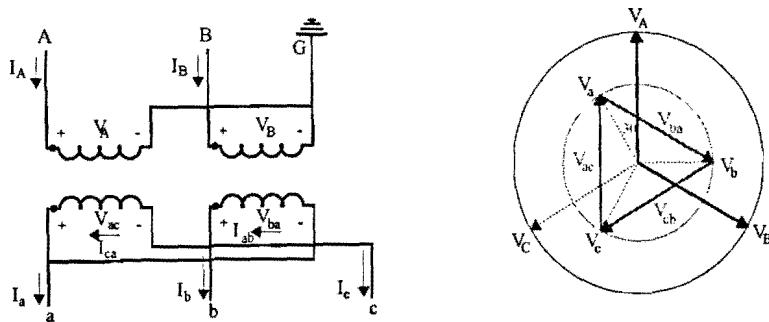


Fig. 2-22: Step up Grounded Y-Δ transformer

Similar to step down transformer

$$[a] = n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}, [c] = 0, [d] = \frac{1}{n_t} \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, [YL] = \frac{1}{n_t} \begin{bmatrix} \frac{1}{z_a} & -\frac{1}{z_b} & 0 \\ \frac{1}{z_a} & \frac{1}{z_b} & 0 \\ -\frac{1}{z_a} & 0 & 0 \end{bmatrix} \quad (2-115)$$

Model of different step down transformers in the form of eq. 2-1 and 2-2 are given in Table 1. In all models matrix [c] is zero, so this matrix is not shown in the table.

Table 1: Step down transformers

Config.	[YL]	[a]	[d]
GY-GY	$\frac{1}{n_{trans}} \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix}^{-1}$	$n_t \cdot U_{3 \times 3}$	$\frac{1}{n_t} \cdot U_{3 \times 3}$
GY- $\Delta$	$\frac{1}{n_{trans}} \begin{bmatrix} \frac{1}{z_{ab}} & 0 & -\frac{1}{z_{ca}} \\ -\frac{1}{z_{ab}} & \frac{1}{z_{bc}} & 0 \\ 0 & -\frac{1}{z_{bc}} & \frac{1}{z_{ca}} \end{bmatrix}$	$n_t [WV]^{-1}$	$\frac{1}{n_t} [WI]^{-1}$
Y- $\Delta$	$\frac{1}{n_t} \begin{bmatrix} \frac{1}{z_{ab}} & 0 & -\frac{1}{z_{ca}} \\ -\frac{1}{z_{ab}} & \frac{1}{z_{bc}} & 0 \\ \frac{1}{z_{ab}} & 0 & \frac{2}{z_{ca}} \end{bmatrix}$	$n_t [WV]^{-1}$	$\frac{1}{3n_t} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 3 & 1 \\ -1 & 0 & 1 \end{bmatrix}$
$\Delta$ -GY	$\frac{-1}{3n_t} \begin{bmatrix} -2 & \frac{1}{z_a} & \frac{4}{z_a} \\ \frac{4}{z_b} & -2 & \frac{1}{z_b} \\ \frac{1}{z_c} & \frac{4}{z_c} & -2 \end{bmatrix}$	$\frac{-n_t}{3} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}$	$\frac{1}{n_t} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$
$\Delta$ - $\Delta$	$\frac{1}{n_t} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ z_a & z_b - 1 & z_c + 1 \end{bmatrix} [Z_{abc}]^{-1} [WV]^{-1}$	$n_t \cdot U_{3 \times 3}$	$\frac{1}{n_t} \cdot U_{3 \times 3}$
Open Y- Open $\Delta$	$\frac{1}{n_t} \begin{bmatrix} \frac{1}{z_a} & 0 & 0 \\ -\frac{1}{z_a} & \frac{1}{z_b} & 0 \\ 0 & -\frac{1}{z_b} & 0 \end{bmatrix}$	$n_t \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$	$\frac{1}{n_t} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

Where,  $[Z_{abc}] = \begin{bmatrix} z_a & 0 & 0 \\ 0 & z_b & 0 \\ 0 & 0 & z_c \end{bmatrix}$

## 2.6 Regulators

A regulator is combination of an autotransformer, a tap changer and maybe a compensator. Tap is considered on the secondary winding, with 32 steps (-16 to 16) to regulate the voltage between 0.9 pu and 1.1 pu (each step 0.00625 pu or at base of 120 volt, 0.75 volt). Fig. 2-23 shows type A and type B step regulator in rising situation.

As shunt winding in type A is directly connected to feeder, core excitation will vary with change in voltage of feeder. However, in type B this winding is connected to regulated output voltage, so the core excitation is constant. This is the reason that type B is more common than type A.

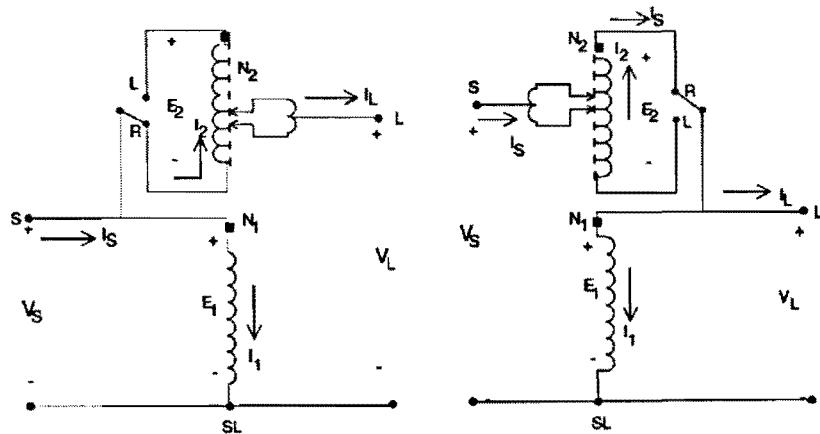


Fig. 2-23: Type A (left) and type B (right) step regulator

Based on this, transformer ratio of regulator can be calculated as below:

$$n_r = \frac{1}{1 + 0.00625 \times \text{Tap}} \quad (2-116)$$

A compensator is combination of a CT, a PT, and a relay. While CT and PT are reading output current and voltage of regulator, relay impedance must be equal to equivalent impedance to load center. As impedance of auto transformer is neglected, matrix  $[Y_L]$  is infinity. Equations 2-1 and 2-2 can be written

$$[V]_{ABC} = [a][V]_{abc} \quad (2-117)$$

$$[I]_{ABC} = [d][I]_{abc} \quad (2-118)$$

$$[d] = [a]^{-1} \quad (2-119)$$

Here, for those regulators that are connected directly to substation, a very big value is considered as admittance (i.e.  $1e4$ ). Effect of regulators, which are not connected directly to substation, will be considered in forming  $Y_{Bus}$  later. Table 2 shows  $[a]$  matrix of  $Y$  and open delta connection of regulator.

### 2.6.1 Setting regulators

This study is considered as two situations to analyze a network. First, by computing load flow in the network by knowing tap number of regulators, and second, calculating tap

numbers by knowing setting of relay of a regulator. As load center of regulators are not known in IEEE test feeders, calculation of relay setting has not been considered yet.

In the first situation where tap numbers are known, as explained before, matrices [a] and [d] of Table 2 are formed by using eq. 2-116.

In the second situation where tap numbers must be calculated by known relay setting, regardless of the voltage of load center, the impedance of compensator is kept constant during operation. Fig. 2-24 shows regulator and its compensator.

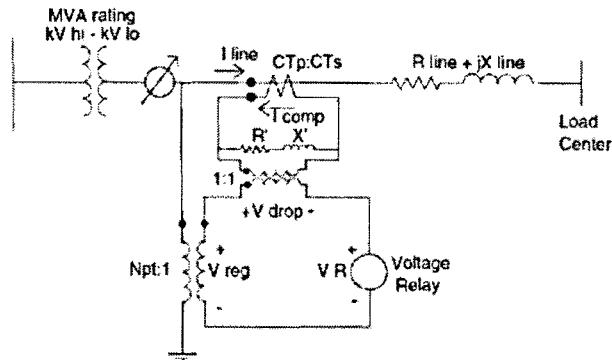


Fig. 2-24: Regulator and its compensator

To calculate tap number of regulator in this study, first, regulators are classified based on the number of regulators they feed. The more regulators they feed the higher priority number they take. That one which is closer to substation has a higher priority number and its setting has a bigger effect on the others, so it must be put in the first class. The others are put in appropriate classes based on their effect on the others.

To calculate tap numbers, all of them are set to zero to find out the profile of voltages without any regulation. Then Tap numbers of regulators, which are in the first class, are calculated. Calculations of tap number are discussed for two situations: Y and open  $\Delta$  regulators.

#### *Y connected regulators*

Output voltages and currents of regulator are calculated after first load flow study. These voltages and currents are transferred to secondary of PT and CT to know actual value of input voltage and current of relay. On Fig. 2-24

$$[V]_{\text{reg}} = \frac{[V]_J}{n_{\text{PT}}} \quad (2-120)$$

$$[I]_{\text{relay}} = \frac{[I]_{\text{Line}}}{n_{\text{CT}}} = \frac{\left( \sum_{K=1}^N [I]_{JK} \right)}{n_{\text{CT}}} \quad (2-121)$$

J: Receiving end of regulator

K: nodes after node "J" which are directly connected to it

$n_{\text{PT}}$ : turns ratio of PT

$n_{\text{CT}}$ : turns ratio of CT

To know the output voltage of the relay, the voltage drop on the impedance of relay should be calculated. Setting of relay is given by converting impedance of compensator to volt.

$$(R' + jX')^{\text{Volt}} = (R + jX)^{\text{Ohm}} \times (\text{nominal current of secondary of CT}) \quad (2-122)$$

$$[V]_{\text{drop}}^{\text{relay}} = [I]_{\text{relay}} [R + jX]^{\text{Ohm}} \quad (2-123)$$

Using 2-121, 2-122, and 2-123

$$[V]_{\text{drop}} = \frac{[I]_{\text{line}}}{n_{\text{CT}}} [R + jX]^{\text{Ohm}} = [I]_{\text{line}} \frac{[I_{\text{CT}}]_{\text{secondary}}^{\text{nominal}}}{[I_{\text{CT}}]_{\text{primary}}^{\text{nominal}}} [R + jX]^{\text{Ohm}} = [I]_{\text{line}} \frac{[R' + jX']^{\text{Volt}}}{[I_{\text{CT}}]_{\text{primary}}^{\text{nominal}}} \quad (2-124)$$

Applying KVL on relay circuit of Fig. 2-24

$$[V]_{\text{relay}} = [V]_{\text{reg}} - [V]_{\text{drop}} \quad (2-125)$$

As it was mentioned before, each step is equal to 0.75 volt.

$$\Delta \text{Tap} = \frac{[V_{\text{ref}}] - [V]_{\text{relay}}}{0.75} \quad (2-126)$$

$\Delta \text{Tap}$  should be added to present number of tap to correct it. This change in tap number is not an integer number, so

$$\text{Tap}^{\text{new}} = \text{round}(\text{Tap}^{\text{old}} + \Delta \text{Tap}) \quad (2-127)$$

In this process, attention must be paid to those regulators with two and single-phases. Also, in some cases regulator has all three phases but compensator is monitoring one or two phases. In this case the other phases without compensator, must obey those phases with compensator.

Second load flow is performed to calculate Tap number of regulators of second class and so on.

Whole process can be repeated until the change in Tap numbers reach zero.

### *Open Δ regulators*

In this configuration the input voltage of the relay is line to line voltage of node "J" which is the receiving end of the regulator segment. The same process of Y connected regulators is performed except in eq. 2-120. In this equation, line-to-line voltage must be used.

$$[V]_{\text{relay}} = \frac{[V_{\text{LL}}]_J}{n_{\text{PT}}} \quad (2-128)$$

Table 2: [a] matrix of Y and open delta connection of regulator

Configuration	[a]
Y	$\begin{bmatrix} n_r^a & 0 & 0 \\ 0 & n_r^b & 0 \\ 0 & 0 & n_r^c \end{bmatrix}$
Open $\Delta$	$[W] \begin{bmatrix} n_r^{ab} & 0 & 0 \\ 0 & n_r^{bc} & 0 \\ -n_r^{ab} & -n_r^{bc} & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$

## 2.7 Switches

Switches are considered as a big admittance (i.e. 1e4) with [a] and [d] matrix equal to  $U_{3 \times 3}$ .

### Chapter 3

#### Newton Raphson Method

To apply Newton Raphson method,

Bus admittance matrix is formed,

Based on this bus admittance matrix power balance equations are developed,

Jacobian matrix calculated,

System of linear equations (SOE) is solved to update voltages.

#### 3.1 Bus Admittance Matrix

Fig. 3-1 shows a section of a distribution feeder that has a radial structure. Because of the radial structure of the network, there is just one node before node "J" (node "I") and "N" number of nodes after it. (K: 1, 2... N)

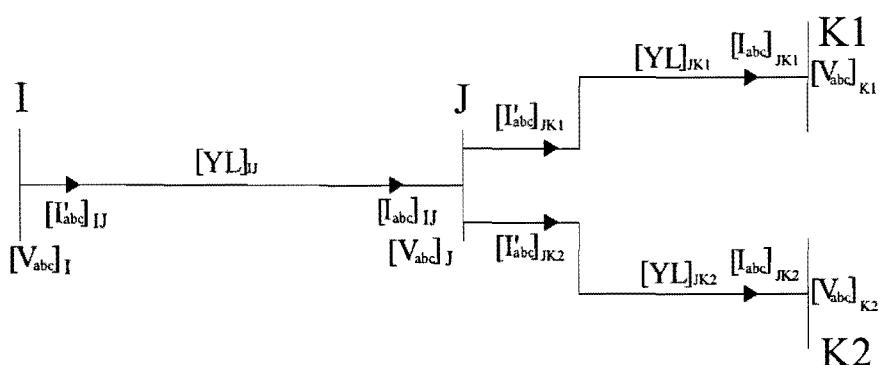


Fig. 3-1: A section of a distribution feeder

Injected current to node "J" is equal to:

$$I_J = -I_{IJ} + I_{JK1} + I_{JK2} \quad (3-1)$$

Using eq. 2-2

$$\begin{aligned} [I]_{JK1} &= [c]_{JK1} [V]_{K1} + [d]_{JK1} [I]_{JK1} \\ [I]_{JK2} &= [c]_{JK2} [V]_{K2} + [d]_{JK2} [I]_{JK2} \end{aligned} \quad (3-2)$$

Replacing 3-2 into 3-1

$$[I]_J = -[I]_{IJ} + [c]_{JK1} [V]_{K1} + [d]_{JK1} [I]_{JK1} + [c]_{JK2} [V]_{K2} + [d]_{JK2} [I]_{JK2} \quad (3-3)$$

Using eq. 2-1

$$\begin{aligned} [I]_{IJ} &= [YL]_{IJ} ([V]_I - [a]_{IJ} [V]_J) \\ [I]_{JK1} &= [YL]_{JK1} ([V]_J - [a]_{JK1} [V]_{K1}) \\ [I]_{JK2} &= [YL]_{JK2} ([V]_J - [a]_{JK2} [V]_{K2}) \end{aligned} \quad (3-4)$$

By replacing  $I_{IJ}$ ,  $I_{JK1}$ , and  $I_{JK2}$  from eq. 3-4,  $I_J$  in eq. 3-3 can be expressed based on the voltage of nodes connected to node "J".

$$[I]_J = -[YL]_{IJ} [V]_I + \left( [YL]_{IJ} [a]_I + \sum_{K=1}^N [d]_{JK} [YL]_{JK} \right) [V]_J + \sum_{K=1}^N \left( (-[d]_{JK} [YL]_{JK} [a]_K + [c]_{JK}) [V]_K \right) \quad (3-5)$$

where,

K: counter for nodes after node "J"

N: number of nodes after node "J"

Let's call coefficient of voltage matrices of nodes as equivalent admittance between two nodes. These are  $3 \times 3$  matrices and denote by "y".

$$[I]_J = [y]_{IJ} [V]_I + [y]_{JJ} [V]_J + \sum_{K=1}^N ([y]_{JK} [V]_K) \quad (3-6)$$

where,

$[y]_{IJ}$ : The equivalent admittance matrix between nodes "I" and "J"

$[y]_{JJ}$ : The equivalent admittance of the network seen from node "J".

Equation 3-6 can be written in matrix form for NB number of nodes.

$$[I]_{NB \times 1} = [Y_{bus}]_{NB \times NB} [V]_{NB \times 1} \quad (3-7)$$

Each element of matrix  $Y_{bus}$  is a  $3 \times 3$  matrix and elements of matrices V and I are  $3 \times 1$  matrix. In the other word, in these three matrices every three consecutive rows belong to one node, for example rows 3J-2, 3J-1, and 3J belong to phases a, b, and c of node "J", respectively. Eq. 3-7 can be rewritten in form of eq. 3-8.

$$[I]_{3NB \times 1} = [Y_{bus}]_{3NB \times 3NB} [V]_{3NB \times 1} \quad (3-8)$$

### 3.1.1 Effect of regulator

As mentioned before, admittance of regulators is considered as a relatively big value for those connected to a substation. Effect of other regulators is considered in  $Y_{bus}$  matrix. Two situations are discussed, first counter of the nodes, "J", is the sending end of regulator, and second, "J" is the receiving end of regulator.

"J", sending end of regulator

Fig. 3-2 shows this situation.

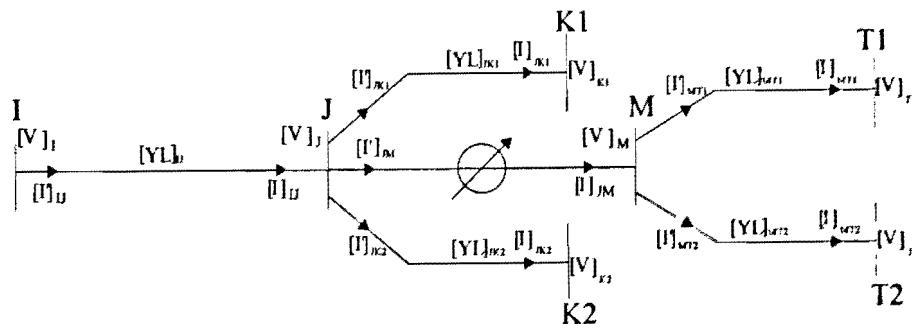


Fig. 3-2: "J", sending end of regulator

Injected current to node "J" is equal to:

$$I_J = -[I]_{JJ} + [I]_{JK1} + [I]_{JK2} + [I]_{JM} \quad (3-9)$$

Using eq. 2-118 and concept of eq. 3-5,  $I_J$  can be expressed based on the voltage of the nodes connected to node "J" except last term.

$$[I]_J = -[YL]_{JJ}[V]_J + \left( [YL]_{JU}[a]_J + \sum_{\substack{K=1 \\ K \neq M}}^N [d]_{JK} [YL]_{JK} \right) [V]_J + \sum_{\substack{K=1 \\ K \neq M}}^N ((-[d]_{JK} [YL]_{JK}[a]_{JK} + [c]_{JK}) [V]_K) + [d]_{JM} [I]_{JM} \quad (3-10)$$

K: counter for nodes after node "J" except "M"

N: number of nodes after node "J"

Last term in eq. 3-10 (named A) is replaced by KCL on node M,

$$A = [d]_{JM} ([I]_{MT1} + [I]_{MT2}) \quad (3-11)$$

Eq. 2-2 for  $[I]_{MT1}$  and  $[I]_{MT2}$  is replaced in 3-11

$$A = [d]_{JM} ([c]_{MT1} [V]_{T1} + [d]_{MT1} [I]_{MT1} + [c]_{MT2} [V]_{T2} + [d]_{MT2} [I]_{MT2}) \quad (3-12)$$

Using eq. 2-1 for  $I_{MT1}$  and  $I_{MT2}$  and replacing in 3-12, term "A" can be expressed based on the voltage of the nodes after node "M".

$$A = \left( [d]_{JM} \sum_{T=1}^{N1} [d]_{MT} [YL]_{MT} \right) [V]_M + \sum_{T=1}^{N1} ([d]_{JM} (-[d]_{MT} [YL]_{MT} [a]_{MT} + [c]_{MT})) [V]_T \quad (3-13)$$

T: counter for nodes after node "M"

N1: number of nodes after node "M"

Eq. 2-117 and 2-119 for  $V_M$  is replaced in 3-13

$$A = \left( \left( [d]_{JM} \sum_{T=1}^{N1} [d]_{MT} [YL]_{MT} \right) [d]_{JM} \right) [V]_J + \sum_{T=1}^{N1} ([d]_{JM} (-[d]_{MT} [YL]_{MT} [a]_{MT} + [c]_{MT})) [V]_T \quad (3-14)$$

Replacing 3-14 in 3-10 expresses  $[I]_J$  based on voltages of nodes except "M".

$$\begin{aligned} [I]_J &= -[YL]_{JJ}[V]_J + \left( [YL]_{JU}[a]_J + \sum_{\substack{K=1 \\ K \neq M}}^{N-1} [d]_{JK} [YL]_{JK} + [d]_{JM} \left( \sum_{T=1}^{N1} [d]_{MT} [YL]_{MT} \right) [d]_{JM} \right) [V]_J \\ &+ \sum_{\substack{K=1 \\ K \neq M}}^{N-1} ((-[d]_{JK} [YL]_{JK}[a]_{JK} + [c]_{JK}) [V]_K) + \sum_{T=1}^{N1} ([d]_{JM} (-[d]_{MT} [YL]_{MT}[a]_{MT} + [c]_{MT})) [V]_T \end{aligned} \quad (3-15)$$

Elements of row number J of  $Y_{Bus}$  will be:

$$\begin{aligned}
y_{JI} &= -[YL]_{IJ} \\
y_{JK} &= -[d]_{JK} [YL]_{JK} [a]_{JK} + [c]_{JK} \quad K : 1 \dots N \text{ & } K \neq M \\
y_{JJ} &= [YL]_{JJ} [a]_{JJ} + \sum_{\substack{K=1, \\ K \neq M}}^{N-1} [d]_{JK} [YL]_{JK} + [d]_{JM} \left( \sum_{T=1}^{N-1} [d]_{MT} [YL]_{MT} \right) [d]_{JM} \\
y_{JT} &= ([d]_{JM} (-[d]_{MT} [YL]_{MT} [a]_{MT} + [c]_{MT})) \quad T : 1 \dots N-1
\end{aligned} \tag{3-16}$$

Because all connections of node "J" to node "M" are eliminated, if there is any load on node "M", it is not seen by node "J". To have this effect, equivalent admittance of this load is added to  $y_{JJ}$ .

In KCL of eq. 3-11, current seen by node "M" because of Load, should be added on the left side

$$[I]_{JM}^{\text{LoadM}} = [y]_M^{\text{Load}} [V]_M \tag{3-17}$$

While  $[y]_M^{\text{Load}}$  is a  $3 \times 3$  diagonal matrix and elements are calculated by

$$y = \frac{P_M^{\text{Load}} - jQ_M^{\text{Load}}}{|V_M|^2} \tag{3-18}$$

Transferring this current to node "J" using eq. 2-118

$$[I]_{JM}^{\text{LoadM}} = [d]_{JM} [I]_{JM}^{\text{LoadM}} \tag{3-19}$$

Replacing 3-17 into 3-19 and using 2-117 for  $[V]_M$

$$[I]_{JM}^{\text{LoadM}} = ([d]_{JM} [y]_M^{\text{Load}} [d]_{JM}) [V]_J \tag{3-20}$$

The effect of the load should be added to  $y_{JJ}$ .

"J", Receiving end of regulator

Fig. 3-3 shows this situation.

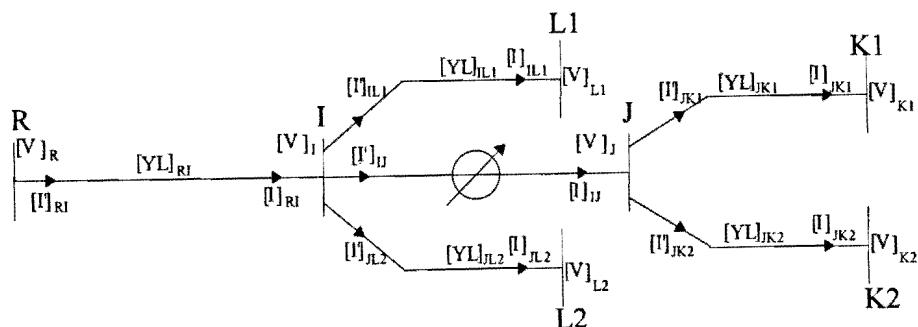


Fig. 3-3: "J", Receiving end of regulator

Injected current to node "J" is equal to:

$$I_J = -[I]_{IJ} + [I]_{JK1} + [I]_{JK2} \quad (3-21)$$

Using concept of eq. 3-5

$$[I]_J = -[I]_{IJ} + \sum_{K=1}^N \left( (-[d]_{JK} [YL]_{JK} [a]_{JK} + [c]_{JK}) [V]_K \right) + \left( \sum_{K=1}^N [d]_{JK} [YL]_{JK} \right) [V]_J \quad (3-22)$$

K: counter for nodes after node "J"

N: number of nodes after node "J"

Using eq. 2-118 and 2-119 for  $[I]_{IJ}$

$$[I]_U = [a]_U [I]_U \quad (3-23)$$

KCL on node "I" results in

$$[I]_U = [I]_{RI} - [I]_{IL1} - [I]_{IL2} \quad (3-24)$$

Replacing 3-24 into 3-23

$$[I]_{IJ} = [a]_{IJ} \left( [I]_{RI} - [I]_{IL1} - [I]_{IL2} \right) \quad (3-25)$$

By using concept of eq. 3-5

$$\begin{aligned} [I]_{IJ} &= ([a]_{IJ} [YL]_{RI}) [V]_R - \sum_{\substack{L=1 \\ L \neq J}}^{N2} \left( [a]_U [c]_{IL1} - [a]_U [d]_{IL1} [YL]_{IL1} [a]_{IL1} \right) [V]_L \\ &\quad - \left( [a]_U [YL]_{RI} [a]_{RI} + [a]_U \sum_{\substack{L=1 \\ L \neq J}}^{N2} [d]_{IL1} [YL]_{IL1} \right) [V]_I \end{aligned} \quad (3-26)$$

where,

L: counter for nodes after node "I" except "J"

N2: number of nodes after node "I"

Using eq. 2-117 for  $V_I$  and replacing 3-26 into 3-22

$$\begin{aligned} [I]_J &= -([a]_{IJ} [YL]_{RI}) [V]_R + \sum_{\substack{L=1 \\ L \neq J}}^{N2-1} \left( [a]_U [c]_{IL1} - [a]_U [d]_{IL1} [YL]_{IL1} [a]_{IL1} \right) [V]_L \\ &\quad + \left( [a]_U [YL]_{RI} [a]_{RI} [a]_U + [a]_U \left( \sum_{\substack{L=1 \\ L \neq J}}^{N2-1} [d]_{IL1} [YL]_{IL1} \right) [a]_U + \sum_{K=1}^N [d]_{JK} [YL]_{JK} \right) [V]_J \quad (3-27) \\ &\quad + \sum_{K=1}^N \left( -[d]_{JK} [YL]_{JK} [a]_{JK} + [c]_{JK} \right) [V]_K \end{aligned}$$

Elements of row number J of  $Y_{Bus}$  will be

$$y_{JR} = -[a]_{JJ} [YL]_{RI}$$

$$y_{JL} = [a]_{JJ} [c]_{IL1} - [a]_{JJ} [d]_{IL1} [YL]_{IL1} [a]_{IL1} \quad L:1...N2 \& L \neq J$$

$$y_{JJ} = [a]_{JJ} [YL]_{RI} [a]_{RI} [a]_{JJ} + [a]_{JJ} \left( \sum_{\substack{L=1 \\ L \neq J}}^{N2-1} [d]_{IL1} [YL]_{IL1} \right) [a]_{JJ} + \sum_{K=1}^N [d]_{JK} [YL]_{JK} \quad (3-28)$$

$$y_{JK} = -[d]_{JK} [YL]_{JK} [a]_{JK} + [c]_{JK} \quad K:1...N$$

Because all connections to node "I" are eliminated, if there is any load on node "I", it is not seen by node "J". To have the effect of load connected to node "I" on node "J", equivalent admittance of this load is added to  $y_{JJ}$ .

Current seen by node "I" because of Load on this node is

$$[I]_I^{Load} = [y]_I^{Load} [V]_I \quad (3-29)$$

While  $[y]_I^{Load}$  is a  $3 \times 3$  diagonal matrix and elements are calculated by

$$y_{ii} = \frac{P_i^{Load} - jQ_i^{Load}}{|V_i|^2} \quad (3-30)$$

Transferring this current to node "J"

$$[I]_J^{Load} = [a]_J [I]_I^{Load} \quad (3-31)$$

Replacing 3-29 into 3-31 and using eq. 2-117 for  $[V]_I$

$$[I]_J^{LoadM} = ([a]_J [y]_I^{Load} [a]_I) [V]_J \quad (3-32)$$

The effect of the load should be added to  $y_{JJ}$ .

### 3.2 Power Balance Equations

Injected power to each node depends on the injected current to the node and the voltage of the node. For node "J"

$$[FS]_J = [I]_J^* \otimes [V]_J \quad (3-33)$$

Operator " $\otimes$ " is used to show element wise multiplication of two matrices.

But as it was shown in forming  $Y_{Bus}$ , injected current to a node is a function of voltages of nodes of network, so by replacing 3-7 into 3-33

$$[FS]_J = ([Y_{bus}] [V])_J^* \otimes [V]_J \quad (3-34)$$

As it was mentioned before rows 3J-2, 3J-1, and 3J belong to phases a, b, and c of node "J", respectively. For phase "a" of node "J"

$$[FS]_J^1 = \left( \sum_{K=1}^{NB} [y]_{JK}^1 [V]_K \right) V_J^1 \quad (3-35)$$

$[FS]_J^1$  : Power balance for phase "a" of node "J" (element (1,1))

$[V]_K$  : Matrices of voltages of nodes connected to node "J"

$V_J^1$  : Voltage of phase "a" of node J (element (1,1))

$[y]_{JK}$  : Equivalent admittance matrix between nodes "J" and "K"

$[y]_{JK}^1$  : First row of  $[y]_{JK}$  (phase a)

Power balance equations of phase "b" and "c" can be developed in the same way.

$$\begin{aligned} [FS]_J^2 &= \left( \sum_{K=1}^{NB} [y]_{JK}^2 [V]_K \right) V_J^2 \\ [FS]_J^3 &= \left( \sum_{K=1}^{NB} [y]_{JK}^3 [V]_K \right) V_J^3 \end{aligned} \quad (3-36)$$

### 3.3 Newton-Raphson method

By applying the concept of first order Newton-Raphson method to solve set of power balance equations of 3-35 and 3-36:

$$[D] = \left[ \frac{\partial FS}{\partial V} \right] [\Delta V] \quad (3-37)$$

$[D]$  : Target vector

$[\Delta V]$  : Variable vector correction

$\left[ \frac{\partial FS}{\partial V} \right]$  : Jacobian matrix

In this equation, row and column related to first node (substation) are eliminated. So dimension of the system of equations is reduced to  $n=NB-1$ .

#### 3.3.1 Target vector

Target vector is the matrix of difference between net apparent power injected to nodes by generators and loads, and the output value of power balance equations, which is calculated by updated value of variable vectors (voltages).

$$[D] = ([SG] - [SD]) - [FS] \quad (3-38)$$

As no generation is considered for the purpose of this study, target vector will be:

$$[D] = -[FS] - [SD] \quad (3-39)$$

Target vector should be updated at the end of every iteration with updating variable vector. The process will be end when Target vector is less than 0.0001.

### 3.3.2 Variable vector

Variable vector is a matrix of voltages of nodes of the network. In this study, rectangular form of voltage is considered.

$$V = u + jw \quad (3-40)$$

To form variable vector, real part  $[u]_{3n \times 1}$  and imaginary part  $[w]_{3n \times 1}$  of voltage are located in a matrix which is shown in 3-41.

$$[V]_{6n \times 1} = \begin{bmatrix} [u]_{3n \times 1} \\ [w]_{3n \times 1} \end{bmatrix}_{6n \times 1} \quad (3-41)$$

Dimension of variable vector is  $6n \times 1$ .

Variable vector correction is a matrix that should be added to variable vector at the end of every iteration in a way that leads the target vector to zero

$$[\Delta V]_{6n \times 1} = \begin{bmatrix} [\Delta u]_{3n \times 1} \\ [\Delta w]_{3n \times 1} \end{bmatrix}_{6n \times 1} \quad (3-42)$$

### 3.3.3 Jacobian

The Jacobian is a matrix whose elements are first partial derivative of the matrix power balance equations with respect to a variable vector. General form of eq. 3-37 shows the structure of the Jacobian.

$$[D]_{3n \times 1} = \left[ \begin{bmatrix} \frac{\partial FS}{\partial u} \end{bmatrix}_{3n \times 3n} \quad \begin{bmatrix} \frac{\partial FS}{\partial w} \end{bmatrix}_{3n \times 3n} \right]_{3n \times 6n} \begin{bmatrix} \Delta u \\ \Delta w \end{bmatrix}_{6n \times 1} \quad (3-43)$$

The first section on the left is partial derivative of "FS" with respect to "u" and the second one is partial derivative of "FS" with respect to "w".

$$\begin{bmatrix} D_{3 \times 2-2} \\ D_{3 \times 2-1} \\ D_{3 \times 2} \\ \vdots \\ D_{3 \times J-2} \\ D_{3 \times J-1} \\ D_{3 \times J} \\ \vdots \\ D_{3 \times NB-2} \\ D_{3 \times NB-2} \\ D_{3 \times NB-2} \end{bmatrix} = \begin{bmatrix} \vdots & \vdots \\ \dots & a_1 & a_2 & a_3 & \dots & a_4 & a_5 & a_6 & \dots & a_7 & a_8 & a_9 & \dots & a_{10} & a_{11} & a_{12} & \dots \\ \dots & b_1 & b_2 & b_3 & \dots & b_4 & b_5 & b_6 & \dots & b_7 & b_8 & b_9 & \dots & b_{10} & b_{11} & b_{12} & \dots \\ \dots & c_1 & c_2 & c_3 & \dots & c_4 & c_5 & c_6 & \dots & c_7 & c_8 & c_9 & \dots & c_{10} & c_{11} & c_{12} & \dots \\ \vdots & \vdots \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \vdots \\ u_{3 \times K-2} \\ u_{3 \times K-1} \\ u_{3 \times K} \\ \vdots \\ u_{3 \times J-2} \\ u_{3 \times J-1} \\ u_{3 \times J} \\ \vdots \\ w_{3 \times K-2} \\ w_{3 \times K-1} \\ w_{3 \times K} \\ \vdots \\ w_{3 \times J-2} \\ w_{3 \times J-1} \\ w_{3 \times J} \\ \vdots \end{bmatrix} \quad (3-44)$$

In this equation, three rows of Jacobian related to three phases of node "J" are shown. Node "K" is directly connected to node "J".

To clarify the way elements of Jacobian are calculated, row number 3J-2 of Jacobian will be formed. This row belongs to phase "a" of node number "J".

$$\begin{aligned}
 a_4 &= JB_{(3J-2),(3J-2)} = \frac{\partial FS_J^1}{\partial u_J^1} = \left( \sum_{K=1}^{NB} [y]_{JK}^1 [V]_K \right)^* + (y_{JJ}^{1,1})^* \cdot V_J^1 \\
 a_5 &= JB_{(3J-2),(3J-1)} = \frac{\partial FS_J^1}{\partial u_J^2} = (y_{JJ}^{1,2})^* \cdot V_J^1 \\
 a_6 &= JB_{(3J-2),(3J)} = \frac{\partial FS_J^1}{\partial u_J^3} = (y_{JJ}^{1,3})^* \cdot V_J^1 \\
 a_{10} &= JB_{(3J-2),(6J-2)} = \frac{\partial FS_J^1}{\partial w_J^1} = -j \left( \sum_{K=1}^{NB} [y]_{JK}^1 [V]_K \right)^* + j(y_{JJ}^{1,1})^* \cdot V_J^1 \\
 a_{11} &= JB_{(3J-2),(6J-1)} = \frac{\partial FS_J^1}{\partial w_J^2} = j(y_{JJ}^{1,2})^* \cdot V_J^1 \\
 a_{12} &= JB_{(3J-2),(6J)} = \frac{\partial FS_J^1}{\partial w_J^3} = j(y_{JJ}^{1,3})^* \cdot V_J^1 \\
 a_1 &= JB_{(3J-2),(3K-2)} = \frac{\partial FS_J^1}{\partial u_K^1} = (y_{JK}^{1,1})^* \cdot V_J^1 \\
 a_2 &= JB_{(3J-2),(3K-1)} = \frac{\partial FS_J^1}{\partial u_K^2} = (y_{JK}^{1,2})^* \cdot V_J^1 \\
 a_3 &= JB_{(3J-2),(3K)} = \frac{\partial FS_J^1}{\partial u_K^3} = (y_{JK}^{1,3})^* \cdot V_J^1 \\
 a_7 &= JB_{(3J-2),(3J+3K-2)} = \frac{\partial FS_J^1}{\partial w_K^1} = j(y_{JK}^{1,1})^* \cdot V_J^1 \\
 a_8 &= JB_{(3J-2),(3J+3K-1)} = \frac{\partial FS_J^1}{\partial w_K^2} = j(y_{JK}^{1,2})^* \cdot V_J^1 \\
 a_9 &= JB_{(3J-2),(3J+3K)} = \frac{\partial FS_J^1}{\partial w_K^3} = j(y_{JK}^{1,3})^* \cdot V_J^1
 \end{aligned} \tag{3-45}$$

As all elements of this row are function of voltage of phase "a" of node "J" ( $V_J^1$ ), if this row is divided by  $V_J^1$ , just two diagonal elements of second and forth blocks will need to be updated at the end of every iteration.

$$a_4 = JB_{(3J-2),(3J-2)} = \frac{\partial FS_J^1}{\partial u_J^1} = \left( \frac{\sum_{K=1}^{NB} [y]_{JK}^1 [V]_K}{(V_J^1)^*} \right)^* + (y_{JJ}^{1,1})^*$$

$$a_5 = JB_{(3J-2),(3J-1)} = \frac{\partial FS_J^1}{\partial u_J^2} = (y_{JJ}^{1,2})^*$$

$$a_6 = JB_{(3J-2),(3J)} = \frac{\partial FS_J^1}{\partial u_J^3} = (y_{JJ}^{1,3})^*$$

$$a_{10} = JB_{(3J-2),(6J-2)} = \frac{\partial FS_J^1}{\partial w_J^1} = -j \left( \frac{\sum_{K=1}^{NB} [y]_{JK}^1 [V]_K}{(V_J^1)^*} \right) + j(y_{JJ}^{1,1})^*$$

$$a_{11} = JB_{(3J-2),(6J-1)} = \frac{\partial FS_J^1}{\partial w_J^2} = j(y_{JJ}^{1,2})^*$$

$$a_{12} = JB_{(3J-2),(6J)} = \frac{\partial FS_J^1}{\partial w_J^3} = j(y_{JJ}^{1,3})^*$$

$$a_1 = JB_{(3J-2),(3K-2)} = \frac{\partial FS_J^1}{\partial u_K^1} = (y_{JK}^{1,1})^*$$

$$a_2 = JB_{(3J-2),(3K-1)} = \frac{\partial FS_J^1}{\partial u_K^2} = (y_{JK}^{1,2})^*$$

$$a_3 = JB_{(3J-2),(3K)} = \frac{\partial FS_J^1}{\partial u_K^3} = (y_{JK}^{1,3})^*$$

$$a_7 = JB_{(3J-2),(3J+3K-2)} = \frac{\partial FS_J^1}{\partial w_K^1} = j(y_{JK}^{1,1})^*$$

$$a_8 = JB_{(3J-2),(3J+3K-1)} = \frac{\partial FS_J^1}{\partial w_K^2} = j(y_{JK}^{1,2})^*$$

$$a_9 = JB_{(3J-2),(3J+3K)} = \frac{\partial FS_J^1}{\partial w_K^3} = j(y_{JK}^{1,3})^* \quad (3-46)$$

Also the other side of the equation 3-43 must be divided by  $V_J^1$ . Then eq. 3-39 is corrected to

$$[D] = \frac{[-FS] - [SD]}{V_J^1} \quad (3-47)$$

To solve the eq. 3-43, the Jacobian must have equal number of rows and columns. To form the new Jacobian, real and imaginary part of the existing Jacobian are separated and put in the form of eq. 3-48. Also real and imaginary part of target vector [D] must be separated to keep the balance of equation.

$$\begin{bmatrix} \text{real}(D) \\ \text{imag}(D) \end{bmatrix}_{6n \times 1} = \begin{bmatrix} \text{real}\left(\left[\frac{\partial \text{FS}}{\partial u}\right]_{3n \times 3n}\right) & \text{real}\left(\left[\frac{\partial \text{FS}}{\partial w}\right]_{3n \times 3n}\right) \\ \text{imag}\left(\left[\frac{\partial \text{FS}}{\partial u}\right]_{3n \times 3n}\right) & \text{imag}\left(\left[\frac{\partial \text{FS}}{\partial w}\right]_{3n \times 3n}\right) \end{bmatrix}_{6n \times 6n} \begin{bmatrix} \Delta u \\ \Delta w \end{bmatrix}_{6n \times 1} \quad (3-48)$$

In distribution networks, there are sections with two and single-phase lines and nodes. Related row and column of missing phases of these nodes appear as zeros in Jacobian, which makes it singular. To solve the problem, related row and columns of these missing phases must be eliminated from Jacobian, target vector, and variable vector.

## Chapter 4

### Proposed Algorithm

#### 1. Reading data of feeder:

Data files are developed on notepad. These files are made based on the information of reference [4].

First data file contains the specifications of conductors, concentric neutral cables, and tape shield cables and is common for all test feeders. Appendixes A, and B are tables of these specifications.

Also, information about different types of spacing of conductors is included in this data file. Figures 4-1 and 4-2 show these configurations. For three-phase, three-wire configuration of  $\Delta$  side, ID-500 has been chosen. However, to calculate the impedance matrix of lines, Kron's reduction method explained in chapter 2 has not been applied.

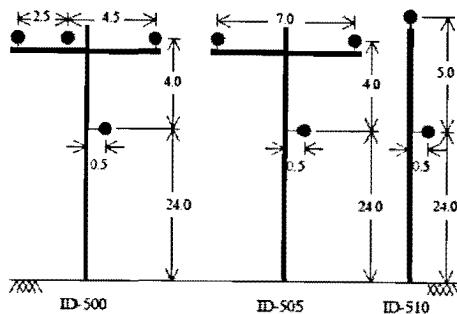


Fig. 4-1: Overhead line spacing

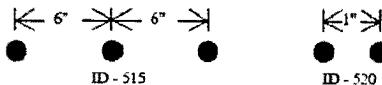


Fig. 4-2: Underground concentric neutral (left) and tape shield (right) spacing

Sub-program “*Line\_spec*” reads this data. Also distances between different conductors and their images required for Series and Shunt admittances are calculated in this sub-program except those ones which depend on the diameter (size) of the conductor.

Second data file contains information of IEEE test feeders. Overhead, concentric neutral and tape shield underground configurations which exist in test feeders are given in three tables. In these tables one code is assigned for each configuration. Next table gives two-end nodes, length, and configuration of each segment. There is no significance which node is closer to substation (real sending/receiving end). If the segment is a line then the configuration will be a code that matches with the configuration codes of the three tables mentioned earlier. If the segment is a transformer, a regulator, or a switch the configuration code will start with XFM-, REG-, and SWITCH respectively. Last digit is the number of these devices. The next two tables contain the data of transformers and compensators. In transformer section, three-phase power of substation transformer is converted to one-phase and is taken as base of power.

Specification of each regulator is given in separate table. In some cases (for example 123-node test feeder) regulator is installed on all phases but compensator is monitoring just one

phase. In these situations regulation of other phases obey the phase with compensator monitoring. Data of spot loads, distributed loads, and switches are given in last three tables.

Sub-program “*Data*” reads this information.

### *2. Adding nodes to the feeder:*

Sub-program Interface adds appropriate nodes and segments for regulators and distributed loads. As regulators can connect to a bus or a distribution line, in latter situation another node must be created between the regulator and the line. This node is called REG-(no. of regulator). The segment is separated into two segments. First segment has length of zero and configuration of REG-, and second one has configuration and length of original segment.

Same procedure is performed for distributed loads. Related segment is divided into two segments and created node is called DL-(no. of distributed load). Configurations of the two segments stay the same but their length will be corrected later in “*Load*” sub-program, after recognizing real sending and receiving end of the segment.

Each segment is known by the names of two ends. Defining a number for each node is performed in this sub-program. Now all segments, transformers, regulators, distributed loads can be known by numbers of two end nodes.

### *3. Form data structure*

As there is no significance in the order of the data of each line segment, and whether node A or B is closer to substation [4], true order of nodes of the feeder is made using algorithm proposed in reference [5]. Output of sub-program “*Data\_Str*” is two variables of “ARR” and “CON”. The first one keeps the number of the node and the segment behind the present node, while the second one stores the numbers of nodes and segments in front of the present node.

### *4. Calculate base values/Initial guess*

Sub-program “*Base*” builds up base values for voltage, current, and admittance. First, voltage magnitude of all nodes are considered equal to secondary voltage of substation transformer and voltage angles set to zero for phase “a”, -120 for phase “b”, and 120 for phase “c”. By passing over any transformer, the voltage magnitudes and the angles of the nodes after the transformer are changed based on the secondary voltage and the phase shift of that transformer.

By knowing base power and voltage of the whole feeder, base current and admittance are calculated.

Initial values of voltage magnitudes of nodes all over the feeder are 1 p.u. but initial voltage angles must obey angles that were made for base voltage. If one or two phases are missing, initial value of voltage magnitudes and angles of these phases are set to zero.

### *5. Load modeling*

Modeling distributed loads as spot load is the first part of sub-program “*Load*”. As it was mentioned in part 2, the segment of an individual distributed load was divided into two

segments and a node was added in between, but the length of each segment and allocation of load on appropriate nodes is not performed yet. As actual sending and receiving ends of the distributed load are recognized in part 3, the length of the segment which is closer to substation is considered 1/4 and the other one 3/4 of the length of the main segment.

Also load is separated into two parts and 2/3 of it is placed on the new node and 1/3 of it is placed on actual receiving end as spot load.

The following part of this sub-program defines capacitors and reactors as constant impedance, negative spot loads which can be either Y or Δ connected.

Last part of this sub-program adds updated active and reactive power of all different combinations and types of loads connected to the nodes. (Section 2.1)

#### *6. Calculate series and shunt admittance of lines*

Matrices of series admittance of distribution lines are calculated in Sub-program "*Line\_Y*".

First step is finding specification of phase and neutral conductors from general data file of conductors explained in part 1.

Next step is calculating the elements of impedance and potential coefficient matrix based on appropriate equations of section 2.4.1 and 2.4.2. This step must be done based on the existing phase of the segment which resulted from phase sequence of segment. Primitive matrices are formed and then if neutral conductor exists, Kron's reduction method is applied to primitive matrices, to reduce their dimension to 3×3.

Final impedance and potential coefficient matrices are calculated for phase locations shown in Figures 4-1 and 4-2. However, real matrices must be formed by rearranging these matrices based on phase sequence of segment. For example if phase sequence is "CBAN" first, rows 1 and 3 and then columns 1 and 3 must be substituted.

Admittance matrices are calculated by using impedance matrices. These values are in ohm/mile but lengths of segments are given in feet. After converting feet to mile and multiplying them by admittance matrices, these matrices change to per unit value by base admittance calculated in part 4.

Sub-program "*config\_y*", is developed to calculate shunt admittance matrix of lines. Similarly, appropriate equations of section 2.4.2 are deployed along with potential coefficient matrices calculated by "*Line\_Y*" sub-program, to calculate shunt admittance matrices. Rearranging these matrices based on phase sequence of segment and making them per unit are other parts of this sub-program.

#### *7. Model Transformers*

Impedance values of transformers are given in per unit in transformer's bases. At the beginning of sub-program "*Trans\_Model*", these values are changed to actual values. Based on the configuration and step down/up type of transformer, one of the models explained in section 2.5 is chosen. Matrices [a], [d], and [YL] are changed to per unit.

#### *8. Pre-set or calculate regulator Taps*

As it was mentioned before, this study considers two situations for regulators. First, status of a feeder is required by specific setting of regulator Taps (pre-set taps), and second, tap numbers is needed to be calculated by specific setting of the compensator of a regulator. In

latter situation, tap numbers are set to zero to see the voltage profile without any regulation.

#### *9. Model and classify regulators*

Classification of regulators is performed based on the number of other regulators they feed. For example in 123-node feeder, there are 4 regulators and regulator 1 is feeding the other three. Therefore, this regulator is put in the first class and the other three are put in the second class. First class has one member and second one has three members. Modeling of regulators is carried out for two types of Y and open  $\Delta$  connected. Sub-program "Regulators" obtains this information.

#### *10. [a], [c], [d], and [YL] matrices and $Y_{Bus}$*

Using [a], [c], [d], and [YL] matrices of each element, general matrices of feeder are formed by sub-program "a\_y\_matrix". Next step is forming  $Y_{Bus}$  with the method explained in section 3.1.

#### *11. Power balance equations*

Sub-program "Compts" calculates net injected power of each node as a function of voltage. Equations 3-34 and 3-35 are deployed to calculate these powers.

#### *12. Form target and variable vectors*

Sub-program "Target\_Variable\_Vectors" forms target vector using eq. 3-46 and variable vector using eq. 3-40. It also eliminates non-existing phases.

#### *13. Newton-Raphson iterative process*

Sub-program is "NR\_loop"

Set criteria for NR loop. To end the process, the maximum of the absolute value of the target vector elements must be less than 0.0001.

Form Jacobian matrix. Elements of Jacobian matrix are calculated using equation 3-45. Rows and columns of non-existing phases are eliminated.

Sub-program "Jacob" solves eq. 3-47 for  $\Delta u$  and  $\Delta w$  and updates u and w. Zeros are added for u and w of not existing phases.

Update loads.

Update mismatches for power balance equations.

Check if criteria are satisfied.

#### *14. Pre-set or calculate regulator Taps*

If pre-set tap numbers were used, display results. Otherwise, check if any tap changing is required.

#### *15. Calculate Taps*

If changes in tap numbers are required, a sub-program "Reg\_set" will calculate tap numbers. As explained before in order to calculate the tap numbers, first, all of

them should be set to zero to find a voltage profile without any regulation. On changing tap numbers of regulators in the first class the output voltage of regulators of downstream classes change dramatically, therefore just tap number of regulators in first class is calculated based on this voltage profile.

New turns-ratios of regulators in first class are calculated using eq. 2-116 and update [a] and [d] matrix of regulator. Next step is updating appropriate rows of  $Y_{Bus}$  matrix.

Updating loads

Updating mismatches

Forming target and variable vectors

Performing load flow to calculate tap numbers of the next class of regulators by using new tap numbers of previous class of regulators.

Downstream regulators also have minor effect on upstream ones. After changing tap setting of these regulators, their input current changes. Effect of this change in current will be a change in current of upstream regulators, which causes a change in voltage drop of relay of regulator. This may disturb setting of upstream regulators.

Because of this fact, this step is an iterative process until no changes happen for taps.

Fig. 4-3 presents flowchart of the proposed algorithm.

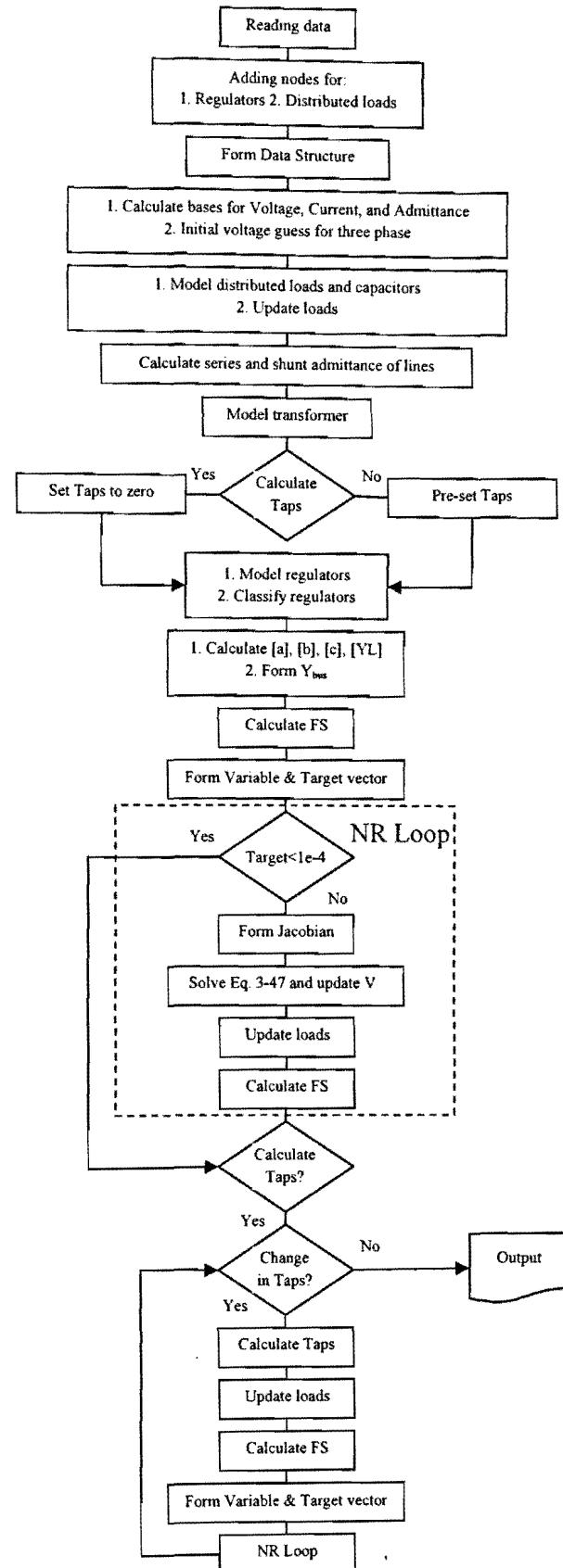


Fig. 4-3: Flowchart of proposed method

## Chapter 5

### Results

Proposed method has been tested on IEEE test feeders [4].

#### 5.1 Test feeders

##### 5.1.1 4-node feeder

4-node feeder is a good test for proper modeling of transformers. Fig. 5.1 shows this feeder.

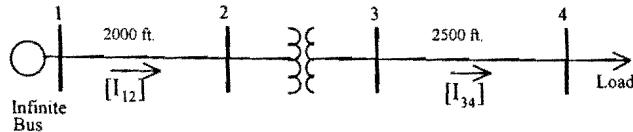


Fig. 5-1: 4-node feeder

If primary or secondary of transformer is  $\Delta$ , distribution line at that side is three-phase three-wire (without neutral).

Two types of balance and unbalanced loads are considered for this test. Also for Open grounded Y-open  $\Delta$  transformer, and balance and unbalance open load has been considered.

##### 5.1.2 ) 13-node feeder

This short highly loaded feeder contains most of the features that a power flow program should handle. Main features of this feeder are:

- Overhead/Underground (concentric neutral and tape shield) conductors
- Three, two, and single phase lines
- spot/distributed unbalance loads
- Capacitor banks
- Transformer
- Regulator

##### 5.1.3 ) 34-node feeders

This feeder is an actual feeder located in Arizona. It is characterized by:

- Very long and lightly loaded
- Two in-line regulators required to maintain a good voltage profile
- An in-line transformer
- Unbalanced loading with both "spot" and "distributed" loads. There are 19 Distributed loads which are considered to be connected at the center of the line segment in IEEE analysis. But in this study model of section 2.1 is considered.
- Shunt capacitors

Because of the length of the feeder and the unbalanced loading it can have a convergence problem.

#### **5.1.4 ) 37-node feeders**

This feeder is an actual feeder located in California. The characteristics of the feeder are:

- Three-wire delta operating at a nominal voltage of 4.8 kV
- All line segments are underground
- Substation voltage regulator consisting of two single phase units connected in open delta
- All loads are “spot” loads and consist of constant PQ, constant current and constant impedance
- The loading is very unbalanced

#### **5.1.5 ) 123-node feeder**

This is the most comprehensive feeder and is characterized by:

- Overhead and underground line segments with various phasing.
- Unbalanced loading with all combinations of load types (PQ, constant I, constant Z)
- All loads are “spot loads” located at a node
- Four step-type voltage regulators
- Shunt capacitor banks
- Switching to provide alternate paths of power-flow

This feeder is well behaved and does not have a convergence problem. It provides a test of the modeling of the phasing of the lines.

## **5.2 Results with pre-set regulator taps**

Table 3 shows the number of the iteration and the time taken for all test feeders with pre-set tap numbers. Just 4-node step down test feeders are shown in this table.

The procedure stops when the maximum of the absolute value of the target vector elements is less than 0.0001. Times are given in seconds.

Table 3: Iteration number and time taken for IEEE test feeders

Feeder	Iteration No.	Time for Data	Time for procedure
13 Node	4	1.295	0.473
34 Node	8	1.317	1.794
37 Node	3	1.229	0.58
123 Node	5	1.855	19.911
4 Node, GY-GY, Balance Load	4	0.760	0.325
4 Node, GY-GY, Unbalance Load	4	0.760	0.325
4 Node, D-GY, Balance Load	4	0.780	0.235
4 Node, D-GY, Unbalance Load	4	0.780	0.235
4 Node, D-D, Balance Load	11	0.760	0.340
4 Node, D-D, Unbalance Load	13	0.760	0.340
4 Node, GY-D, Balance Load	14	0.780	0.260
4 Node, GY-D, Unbalance Load	10	0.780	0.250
4 Node, Y-D, Balance Load	13	0.760	0.260
4 Node, Y-D, Unbalance Load	16	0.760	0.269
4 Node, Open Y- Open D, Balance Load	11	0.780	0.340
4 Node, Open Y- Open D, Unbalance Load	10	0.780	0.336

### 5.2.1 13-node feeder

#### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
650	1.0000<	0.00	1.0000<-120.00		1.0000<120.00		LG
645	0.0000<	0.00	1.0328<-121.90		1.0155<117.86		LG
633	1.0180<	-2.55	1.0401<-121.76		1.0149<117.83		LG
634	0.9940<	-3.23	1.0217<-122.22		0.9960<117.35		LG
646	0.0000<	0.00	1.0311<-121.97		1.0134<117.90		LG
Reg-1	1.0625<	0.00	1.0500<-120.00		1.0687<120.00		LG
632	1.0210<	-2.49	1.0420<-121.72		1.0175<117.83		LG
652	0.9825<	-5.24	0.0000<0.00		0.0000<0.00		LG
671	0.9900<	-5.30	1.0529<-122.34		0.9779<116.03		LG
684	0.9881<	-5.32	0.0000<0.00		0.9758<115.93		LG
680	0.9900<	-5.30	1.0529<-122.34		0.9779<116.03		LG
692	0.9900<	-5.30	1.0529<-122.34		0.9779<116.03		LG
611	0.0000<	0.00	0.0000<0.00		0.9738<115.78		LG
675	0.9835<	-5.55	1.0553<-122.52		0.9759<116.04		LG

#### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 650						
To Node Reg-1   593.29<-28.57   435.59<-140.91   626.84<93.60						
<hr/>						
Node 645						
From Node 632	0.00<0.00	143.03<-142.66	65.21<57.83			
To Node 646	0.00<0.00	65.21<-122.17	65.21<57.83			
<hr/>						
Node 633						
From Node 632	81.33<-37.74	61.12<-159.09	62.70<80.48			
To Node 634	81.33<-37.74	61.12<-159.09	62.70<80.48			
<hr/>						
Node 634						
From Node 633	704.85<-37.74	529.75<-159.09	543.42<80.48			
<hr/>						
Node 646						
From Node 645	0.00<0.00	65.21<-122.17	65.21<57.83			
<hr/>						
Node Reg-1						
From Node 650	558.39<-28.57	414.84<-140.91	586.52<93.60			
To Node 632	558.39<-28.57	414.84<-140.91	586.52<93.60			
<hr/>						
Node 632						
From Node Reg-1	558.39<-28.57	414.85<-140.91	586.52<93.60			
To Node 671	478.28<-27.02	215.10<134.66	475.44<99.92			
To Node 633	81.33<-37.74	61.12<-159.09	62.70<80.48			
To Node 645	0.00<0.00	143.02<142.66	65.21<57.83			
<hr/>						
Node 652						
From Node 684	63.09<-39.14	0.00<0.00	0.00<0.00			
<hr/>						
Node 671						
From Node 632	472.92<-26.94	195.84<-132.91	439.13<100.99			
To Node 692	229.09<-18.16	69.60<-55.19	178.36<109.41			
To Node 680	0.00<0.00	0.00<0.00	0.00<0.00			
To Node 684	63.07<-39.12	0.00<0.00	71.15<121.62			
<hr/>						
Node 684						
From Node 671	63.07<-39.12	0.00<0.00	0.00<0.00			
To Node 652	63.07<-39.12	0.00<0.00	0.00<0.00			
To Node 611	0.00<0.00	0.00<0.00	71.15<121.62			
<hr/>						
Node 680						
From Node 671	0.00<0.00	0.00<0.00	0.00<0.00			
<hr/>						
Node 692						
From Node 671	229.09<-18.16	69.60<-55.19	178.36<109.41			
To Node 675	205.32<-5.13	69.60<-55.19	124.06<111.81			
<hr/>						
Node 611						
From Node 684	0.00<0.00	0.00<0.00	71.15<121.62			
<hr/>						
Node 675						
From Node 692	205.32<-5.13	69.58<-55.20	124.06<111.80			
<hr/>						

#### 3.Losses (KW)

TOTAL	PHASE A	PHASE B	PHASE C
-------	---------	---------	---------

NODE 632	To NODE 645		3.592		0.000		2.658		0.933
NODE 632	To NODE 633		1.435		0.585		0.421		0.429
NODE 633	To NODE 634		5.438		2.518		1.422		1.497
NODE 645	To NODE 646		0.741		0.000		0.371		0.370
NODE 650	To NODE Reg-1		0.000		0.000		0.000		0.000
NODE Reg-1	To NODE 632		205.249		71.682		60.416		73.150
NODE 684	To NODE 652		0.810		0.810		0.000		0.000
NODE 632	To NODE 671		115.896		43.944		30.136		41.815
NODE 671	To NODE 684		0.788		0.359		0.000		0.429
NODE 671	To NODE 680		0.000		0.000		0.000		0.000
NODE 671	To NODE 692		0.000		0.000		0.000		0.000
NODE 684	To NODE 611		0.382		0.000		0.000		0.382
NODE 692	To NODE 675		8.296		3.748		2.101		2.447
TOTAL					123.647		97.526		121.452

TOTAL FEEDER POWER LOSS= 342.6251

#### 4. Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C			
			P	Q		P	Q		P	Q		
SPOT LOADS												
634		Y		PQ		160.00		110.00		120.00		90.00
645		Y		PQ		0.00		0.00		170.00		125.00
646		D		Z		0.00		0.00		240.66		138.12
652		Y		Z		123.56		63.02		0.00		0.00
671		D		PQ		385.00		220.00		385.00		220.00
675		Y		PQ		485.00		190.00		68.00		60.00
692		D		I		0.00		0.00		0.00		168.37
611		Y		I		0.00		0.00		0.00		149.55
												77.91
DISTRIBUTED LOADS												
DL-1		Y		PQ		11.33		6.67		44.00		25.33
671		Y		PQ		5.67		3.33		22.00		12.67
SHUNT CAPACITORS												
675		Y		Z		0.00		-193.46		0.00		-222.73
611		Y		Z		0.00		0.00		0.00		0.00
												-190.49
												-94.84

#### 5. Voltage Regulators Setting

REGULATOR ID.		NODE A		NODE B		PT RATIO		CT RATE		BAND WIDTH		PHASES		CONNECTION
Reg-1		650		632		20.00		700.00		2.00		A-B-C		AG-BG-CG
PHASE		R-VOLT		X-VOLT		VOLTAGE LEVEL		RELAY VOLTAGE		TAP				
A		3.000		9.000		122.000		122.166		10				
B		3.000		9.000		122.000		122.604		8				
C		3.000		9.000		122.000		122.867		11				

#### 5.2.2 34-node feeder

1. Voltage Profile											
NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG				
600		1.0500<	0.00		1.0500<-120.00		1.0500< 120.00		LG		
802		1.0475<	-0.05		1.0484<-120.07		1.0484< 119.95		LG		
806		1.0457<	-0.08		1.0474<-120.11		1.0474< 119.91		LG		
808		1.0137<	-0.75		1.0296<-120.95		1.0286< 119.30		LG		
810		0.0000<	0.00		1.0295<-120.95		0.0000< 0.00		LG		
812		0.9763<	-1.58		1.0100<-121.92		1.0066< 118.58		LG		
814		0.9467<	-2.26		0.9945<-122.70		0.9893< 118.00		LG		
Reg-1		1.0177<	-2.26		1.0256<-122.70		1.0202< 118.00		LG		
650		1.0177<	-2.26		1.0256<-122.70		1.0202< 118.00		LG		
818		1.0164<	-2.27		0.0000< 0.00		0.0000< 0.00		LG		
824		1.0082<	-2.38		1.0159<-122.93		1.0115< 117.74		LG		
820		0.9927<	-2.33		0.0000< 0.00		0.0000< 0.00		LG		
822		0.9896<	-2.34		0.0000< 0.00		0.0000< 0.00		LG		
826		0.0000<	0.00		1.0157<-122.93		0.0000< 0.00		LG		
828		1.0075<	-2.39		1.0152<-122.95		1.0108< 117.72		LG		
830		0.9895<	-2.65		0.9983<-123.39		0.9937< 117.22		LG		
854		0.9890<	-2.66		0.9979<-123.40		0.9933< 117.21		LG		
858		1.0337<	-3.19		1.0323<-124.27		1.0337< 116.20		LG		
888		0.9996<	-4.65		0.9985<-125.73		0.9998< 114.78		LG		
860		1.0305<	-3.25		1.0291<-124.38		1.0308< 116.06		LG		
842		1.0309<	-3.26		1.0295<-124.39		1.0312< 116.06		LG		
840		1.0303<	-3.25		1.0287<-124.38		1.0307< 116.06		LG		
862		1.0303<	-3.25		1.0288<-124.38		1.0307< 116.06		LG		
844		1.0307<	-3.29		1.0292<-124.41		1.0309< 116.03		LG		
846		1.0310<	-3.33		1.0292<-124.46		1.0312< 115.98		LG		
848		1.0310<	-3.34		1.0292<-124.46		1.0313< 115.98		LG		
816		1.0173<	-2.27		1.0253<-122.71		1.0200< 117.99		LG		

Reg-2		1.0360<-3.13		1.0346<-124.18		1.0359<116.30		LG
832		1.0360<-3.13		1.0346<-124.18		1.0359<116.30		LG
856		0.0000<0.00		0.9978<-123.41		0.0000<0.00		LG
852		0.9581<-3.13		0.9681<-124.18		0.9636<116.30		LG
864		1.0336<-3.19		0.0000<0.00		0.0000<0.00		LG
834		1.0310<-3.26		1.0296<-124.38		1.0312<116.06		LG
836		1.0303<-3.25		1.0286<-124.38		1.0307<116.06		LG
838		0.0000<0.00		1.0286<-124.39		0.0000<0.00		LG
890		0.9167<-5.21		0.9237<-126.77		0.9175<113.94		LG

## 2. Current Flow (AMP)

	MAG	ANGLE		MAG	ANGLE		MAG	ANGLE
Node 800								
To Node 802		51.55<-12.75		44.56<-127.70		40.93<117.35		
Node 802								
From Node 800		51.57<-12.80		44.56<-127.76		40.93<117.29		
To Node 806		51.57<-12.80		44.56<-127.76		40.93<117.29		
Node 806								
From Node 802		51.57<-12.84		43.16<-127.16		39.80<118.06		
To Node 808		51.57<-12.84		42.46<-126.83		39.24<118.49		
Node 808								
From Node 806		51.75<-13.48		42.46<-127.59		39.28<117.74		
To Node 812		51.75<-13.48		41.30<-127.10		39.28<117.74		
To Node 810		0.00<0.00		1.22<-144.61		0.00<0.00		
Node 810								
From Node 808		0.00<0.00		0.41<-147.52		0.00<0.00		
Node 812								
From Node 808		51.94<-14.19		41.29<-127.99		39.33<116.88		
To Node 814		51.94<-14.19		41.29<-127.99		39.33<116.88		
Node 814								
From Node 812		52.09<-14.74		41.28<-128.69		39.37<116.21		
To Node Reg-1		52.09<-14.74		41.28<-128.69		39.37<116.21		
Node Reg-1								
From Node 814		48.46<-14.74		40.03<-128.69		38.18<116.21		
To Node 850		48.46<-14.74		40.03<-128.69		38.18<116.21		
Node 850								
From Node Reg-1		48.46<-14.74		40.03<-128.69		38.18<116.21		
To Node 816		48.46<-14.74		40.03<-128.69		38.18<116.21		
Node 818								
From Node 816		13.02<-26.77		0.00<0.00		0.00<0.00		
To Node 820		13.02<-26.77		0.00<0.00		0.00<0.00		
Node 824								
From Node 816		35.87<-10.71		39.89<-128.99		38.10<116.14		
To Node 828		35.87<-10.71		36.92<-127.38		38.05<116.22		
To Node 826		0.00<0.00		3.10<-148.91		0.00<0.00		
Node 820								
From Node 818		11.48<-28.98		0.00<0.00		0.00<0.00		
To Node 822		10.61<-28.99		0.00<0.00		0.00<0.00		
Node 822								
From Node 820		3.56<-29.74		0.00<0.00		0.00<0.00		
Node 826								
From Node 824		0.00<0.00		1.04<-149.50		0.00<0.00		
Node 828								
From Node 824		35.87<-10.73		36.92<-127.41		37.87<116.33		
To Node 830		35.87<-10.73		36.92<-127.41		37.78<116.40		
Node 830								
From Node 828		35.61<-11.14		36.91<-127.92		37.80<115.94		
To Node 854		34.22<-9.99		36.19<-127.47		36.49<116.24		
Node 854								
From Node 830		34.22<-10.00		36.19<-127.48		36.49<116.23		
To Node 852		34.22<-10.00		35.92<-127.72		36.49<116.23		
To Node 856		0.00<0.00		0.31<-98.60		0.00<0.00		
Node 858								
From Node 832		21.01<0.63		23.21<-116.62		24.13<128.35		

To Node 864		0.14< -22.82		0.00< 0.00		0.00< 0.00
To Node 834		20.73< 1.00		23.13<-116.39		24.03< 128.46
<hr/>						
Node 888						
From Node 832		69.89< -32.30		70.03<-152.74		69.51< 87.37
To Node 890		69.89< -32.30		70.03<-152.74		69.51< 87.37
<hr/>						
Node 860						
From Node 834		7.61< -38.30		8.15<-155.96		7.02< 92.63
To Node 836		4.16< -30.20		5.96<-154.63		3.60< 90.23
<hr/>						
Node 842						
From Node 834		14.74< 34.65		16.30< -95.64		15.12< 151.01
To Node 844		14.74< 34.65		16.30< -95.64		15.12< 151.01
<hr/>						
Node 840						
From Node 836		1.01< -31.13		1.30<-156.33		1.11< 72.68
<hr/>						
Node 862						
From Node 836		0.00< 0.00		2.09<-149.49		0.00< 0.00
To Node 838		0.00< 0.00		2.09<-149.49		0.00< 0.00
<hr/>						
Node 844						
From Node 842		14.55< 36.25		16.29< -95.70		15.11< 150.94
To Node 846		9.83< 78.86		9.40< -63.87		9.40<-170.70
<hr/>						
Node 846						
From Node 844		9.76< 78.79		9.34< -56.34		9.61<-164.82
To Node 848		9.76< 78.79		9.40< -52.54		9.79<-161.96
<hr/>						
Node 848						
From Node 846		9.76< 78.78		9.61< -45.74		9.78<-161.97
<hr/>						
Node 816						
From Node 850		48.46< -14.74		40.03<-128.70		38.18< 116.20
To Node 824		35.83< -10.43		40.03<-128.70		38.18< 116.20
To Node 818		13.01< -26.69		0.00< 0.00		0.00< 0.00
<hr/>						
Node Reg-2						
From Node 852		31.77< -11.01		33.59<-128.66		33.98< 115.38
To Node 832		31.77< -11.01		33.59<-128.66		33.98< 115.38
<hr/>						
Node 832						
From Node Reg-2		31.77< -11.02		33.59<-128.66		33.98< 115.38
To Node 858		21.31< 0.46		23.40<-116.89		24.35< 128.33
To Node 888		11.68< -32.30		11.70<-152.74		11.61< 87.37
<hr/>						
Node 856						
From Node 854		0.00< 0.00		0.10<-149.97		0.00< 0.00
<hr/>						
Node 852						
From Node 854		34.35< -11.01		35.90<-128.66		36.53< 115.38
To Node Reg-2		34.35< -11.01		35.90<-128.66		36.53< 115.38
<hr/>						
Node 864						
From Node 858		0.05< -29.75		0.00< 0.00		0.00< 0.00
<hr/>						
Node 834						
From Node 858		20.44< 1.68		22.61<-116.25		23.49< 129.42
To Node 842		14.75< 34.67		16.30< -95.62		15.12< 151.02
To Node 860		11.16< -43.07		9.09<-154.82		10.60< 99.32
<hr/>						
Node 836						
From Node 860		2.38< -26.22		4.93<-152.47		2.33< 79.06
To Node 840		1.50< -20.02		2.33<-151.97		1.75< 67.98
To Node 862		0.00< 0.00		2.09<-149.37		0.00< 0.00
<hr/>						
Node 838						
From Node 862		0.00< 0.00		0.71<-150.95		0.00< 0.00
<hr/>						
Node 890						
From Node 888		69.91< -32.33		70.04<-152.76		69.53< 87.35
<hr/>						

### 3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 800	To NODE 802		5.407		2.115   1.727   1.565
NODE 802	To NODE 806		3.534		1.407   1.113   1.014
NODE 806	To NODE 808		64.288		26.097   19.936   18.255
NODE 808	To NODE 810		0.002		0.000   0.002   0.000
NODE 808	To NODE 812		73.875		30.405   22.310   21.159

NODE 812	To NODE 814		58.770		24.242		17.707		16.821
NODE 814	To NODE Reg-1		0.000		0.000		0.000		0.000
NODE Reg-1	To NODE 850		0.024		0.010		0.007		0.007
NODE 816	To NODE 818		0.154		0.154		0.000		0.000
NODE 816	To NODE 824		19.992		6.176		7.126		6.690
NODE 818	To NODE 820		3.571		3.571		0.000		0.000
NODE 820	To NODE 822		0.274		0.274		0.000		0.000
NODE 824	To NODE 826		0.005		0.000		0.005		0.000
NODE 824	To NODE 828		1.552		0.499		0.515		0.537
NODE 828	To NODE 830		37.536		12.029		12.516		12.991
NODE 830	To NODE 854		0.896		0.284		0.304		0.309
NODE 832	To NODE 858		3.479		1.041		1.182		1.256
NODE 832	To NODE 888		9.615		3.212		3.225		3.178
NODE 834	To NODE 860		0.192		0.067		0.064		0.061
NODE 834	To NODE 842		0.090		0.028		0.032		0.029
NODE 836	To NODE 840		0.002		0.001		0.001		0.001
NODE 836	To NODE 862		0.001		0.000		0.000		0.000
NODE 842	To NODE 844		0.432		0.135		0.156		0.141
NODE 844	To NODE 846		0.453		0.157		0.146		0.151
NODE 846	To NODE 848		0.068		0.023		0.022		0.023
NODE 850	To NODE 816		0.759		0.308		0.232		0.219
NODE 852	To NODE Reg-2		0.000		0.000		0.000		0.000
NODE Reg-2	To NODE 832		0.015		0.005		0.005		0.005
NODE 854	To NODE 856		0.000		0.000		0.000		0.000
NODE 854	To NODE 852		63.242		20.112		21.267		21.863
NODE 858	To NODE 864		0.000		0.000		0.000		0.000
NODE 858	To NODE 834		3.944		1.175		1.344		1.425
NODE 860	To NODE 836		0.052		0.012		0.028		0.011
NODE 862	To NODE 838		0.003		0.000		0.003		0.000
NODE 888	To NODE 890		51.178		17.183		17.036		16.959
TOTAL					150.721		128.013		124.669

TOTAL FEEDER POWER LOSS= 403.4029

#### 4. Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C		
			P	Q	P	Q	P	Q			

#### SPOT LOADS

860		Y		PQ		20.00		16.00		20.00		16.00
840		Y		I		9.27		7.21		9.26		7.20
844		Y		Z		143.42		111.55		142.99		111.22
848		D		PQ		20.00		16.00		20.00		16.00
890		D		I		139.11		69.55		137.59		68.80
830		D		Z		9.95		4.98		9.86		4.93
												24.55
												9.62

#### DISTRIBUTED LOADS

DL-1		Y		PQ		0.00		0.00		20.00		10.00
806		Y		PQ		0.00		0.00		10.00		5.00
DL-2		Y		I		0.00		0.00		10.98		5.49
810		Y		I		0.00		0.00		2.75		0.00
DL-3		Y		Z		23.11		11.56		0.00		0.00
820		Y		Z		11.17		5.58		0.00		0.00
DL-4		Y		PQ		90.00		46.67		0.00		0.00
822		Y		PQ		45.00		23.33		0.00		0.00
DL-5		D		I		0.00		0.00		3.39		1.36
824		D		I		0.00		0.00		1.68		0.67
DL-6		Y		I		0.00		0.00		27.09		13.54
826		Y		I		0.00		0.00		13.54		6.77
DL-7		Y		PQ		0.00		0.00		0.00		2.67
828		Y		PQ		0.00		0.00		0.00		1.33
DL-8		Y		PQ		4.67		2.00		0.00		0.00
830		Y		PQ		2.33		1.00		0.00		0.00
DL-9		Y		PQ		0.00		0.00		2.67		1.33
856		Y		PQ		0.00		0.00		1.33		0.67
DL-10		D		Z		5.05		2.16		1.42		0.71
858		D		Z		2.52		1.08		0.71		0.35
DL-11		Y		PQ		1.33		0.67		0.00		0.00
864		Y		PQ		0.67		0.33		0.00		0.00
DL-12		D		PQ		2.67		1.33		10.00		5.33
834		D		PQ		1.33		0.67		5.00		2.67
DL-13		D		Z		11.45		5.72		14.09		7.04
860		D		Z		5.72		2.86		7.04		3.52
DL-14		D		PQ		20.00		10.00		6.67		4.00
836		D		PQ		10.00		5.00		3.33		2.00
DL-15		D		I		12.42		6.21		15.07		7.53
840		D		I		6.21		3.11		7.53		3.77
DL-16		Y		PQ		0.00		0.00		18.67		9.33
838		Y		PQ		0.00		0.00		9.33		4.67
DL-17		Y		PQ		6.00		3.33		0.00		0.00
844		Y		PQ		3.00		1.67		0.00		0.00

DL-18	Y	PQ	0.00	0.00	16.67	8.00	13.33	7.33
846	Y	PQ	0.00	0.00	8.33	4.00	6.67	3.67
DL-19	Y	PQ	0.00	0.00	15.33	7.33	0.00	0.00
848	Y	PQ	0.00	0.00	7.67	3.67	0.00	0.00
SHUNT CAPACITORS								

844	Y	Z	0.00	-106.24	0.00	-105.92	0.00	-106.29
848	Y	Z	0.00	-159.45	0.00	-158.89	0.00	-159.53

#### 5. Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	814	850	120.00	100.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	2.700	1.600	122.000	120.477	12		
B	2.700	1.600	122.000	121.726	5		
C	2.700	1.600	122.000	121.175	5		
REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-2	852	832	120.00	100.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	2.500	1.500	124.000	123.261	13		
B	2.500	1.500	124.000	123.074	11		
C	2.500	1.500	124.000	123.247	12		

#### 5.2.3 37-node feeder

##### 1. Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
799	1.0000<	0.00	1.0000<-120.00		1.0000<120.00		LL
702	1.0248<	-0.14	1.0088<-120.58		1.0101<120.43		LL
705	1.0241<	-0.13	1.0075<-120.59		1.0088<120.46		LL
713	1.0234<	-0.14	1.0070<-120.60		1.0083<120.44		LL
703	1.0178<	-0.17	1.0050<-120.70		1.0034<120.20		LL
727	1.0167<	-0.15	1.0044<-120.69		1.0024<120.19		LL
730	1.0127<	-0.12	1.0021<-120.73		0.9981<120.10		LL
714	1.0214<	-0.17	1.0043<-120.60		1.0064<120.46		LL
720	1.0205<	-0.21	1.0011<-120.66		1.0040<120.53		LL
742	1.0238<	-0.15	1.0067<-120.59		1.0086<120.48		LL
712	1.0240<	-0.11	1.0073<-120.61		1.0082<120.46		LL
725	1.0202<	-0.22	1.0003<-120.66		1.0038<120.55		LL
724	1.0184<	-0.32	0.9950<-120.61		1.0023<120.69		LL
722	1.0165<	-0.30	0.9954<-120.62		1.0023<120.68		LL
733	1.0063<	-0.05	0.9993<-120.73		0.9925<119.96		LL
732	1.0086<	-0.07	1.0001<-120.74		0.9941<120.02		LL
731	1.0109<	-0.13	1.0004<-120.74		0.9964<120.10		LL
708	1.0087<	-0.08	1.0002<-120.73		0.9945<120.02		LL
735	1.00023<	0.03	0.9966<-120.78		0.9873<119.91		LL
736	1.0019<	-0.02	0.9951<-120.75		0.9875<119.96		LL
741	0.9981<	0.07	0.9962<-120.75		0.9849<119.76		LL
740	0.9981<	0.07	0.9961<-120.75		0.9846<119.76		LL
704	1.0217<	-0.17	1.0044<-120.61		1.0065<120.46		LL
718	1.0201<	-0.16	1.0041<-120.57		1.0060<120.42		LL
707	1.0187<	-0.30	0.9959<-120.62		1.0025<120.67		LL
706	1.0204<	-0.22	1.0007<-120.66		1.0039<120.54		LL
744	1.0160<	-0.16	1.0040<-120.68		1.0020<120.18		LL
709	1.0111<	-0.11	1.0012<-120.73		0.9967<120.08		LL
734	1.0029<	-0.01	0.9978<-120.74		0.9693<119.88		LL
737	0.9996<	0.02	0.9968<-120.71		0.9872<119.79		LL
710	1.0024<	0.01	0.9967<-120.77		0.9878<119.91		LL
738	0.9985<	0.04	0.9965<-120.71		0.9661<119.76		LL
711	0.9982<	0.06	0.9963<-120.74		0.9852<119.76		LL
728	1.0156<	-0.15	1.0037<-120.68		1.0017<120.18		LL
729	1.0157<	-0.15	1.0040<-120.67		1.0019<120.17		LL
Reg-1	1.0437<	0.00	1.0250<-120.00		1.0345<120.90		LL
701	1.0317<	-0.08	1.0144<-120.39		1.0183<120.61		LL
775	1.0111<	-0.11	1.0012<-120.73		0.9967<120.08		LL

##### 2. Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<b>Node 799</b>						
To Node Reg-1	382.15<-32.30	283.94<-149.64	356.36<102.65			
<b>Node 702</b>						
From Node 701	267.60<-29.40	218.84<-149.92	248.59<109.69			
To Node 703	189.65<-25.64	134.27<-161.61	131.80<109.32			
To Node 713	59.46<-34.43	72.38<-133.22	86.37<69.64			
To Node 705	20.46<-50.17	20.69<-120.30	33.69<94.93			

Node 705  
 From Node 702 | 20.47< -50.19 | 20.69<-120.34 | 33.70< 94.51  
 To Node 712 | 19.40< -54.74 | 0.01< 0.00 | 19.42< 125.28  
 To Node 742 | 1.91< 3.58 | 20.69<-120.36 | 19.70< 64.23

---

Node 713  
 From Node 702 | 59.47< -34.44 | 72.39<-133.23 | 86.38< 89.63  
 To Node 704 | 41.82< -25.16 | 72.39<-133.23 | 71.49< 80.54

---

Node 703  
 From Node 702 | 189.68< -25.66 | 134.33<-161.64 | 131.82< 109.28  
 To Node 730 | 147.56< -27.25 | 99.35<-161.23 | 106.22< 110.47  
 To Node 727 | 42.38< -20.11 | 34.99<-162.78 | 25.72< 104.36

---

Node 727  
 From Node 703 | 42.38< -20.12 | 34.99<-162.79 | 25.72< 104.34  
 To Node 744 | 34.99< -10.62 | 34.99<-162.79 | 16.82< 93.29

---

Node 730  
 From Node 703 | 147.57< -27.26 | 99.37<-161.24 | 106.22< 110.46  
 To Node 709 | 130.63< -23.26 | 99.37<-161.24 | 87.44< 107.26

---

Node 714  
 From Node 704 | 23.87< 4.66 | 26.66<-166.34 | 4.83< 64.03  
 To Node 718 | 19.96< 4.67 | 19.95<-175.32 | 0.02<-148.09

---

Node 720  
 From Node 704 | 24.26< -54.59 | 52.12<-117.07 | 66.91< 81.68  
 To Node 706 | 0.03< 88.60 | 9.78<-117.00 | 9.76< 62.87  
 To Node 707 | 4.80< -54.39 | 42.34<-117.09 | 44.76< 68.36

---

Node 742  
 From Node 705 | 1.91< 3.28 | 20.69<-120.39 | 19.71< 64.21

---

Node 712  
 From Node 705 | 19.41< -54.75 | 0.00< 0.00 | 19.42< 125.26

---

Node 725  
 From Node 706 | 0.00< 0.00 | 9.78<-117.20 | 9.78< 62.78

---

Node 724  
 From Node 707 | 0.00< 0.00 | 9.73<-117.16 | 9.74< 62.82

---

Node 722  
 From Node 707 | 4.84< -54.80 | 32.61<-117.17 | 35.13< 69.84

---

Node 733  
 From Node 708 | 122.55< -20.76 | 86.78<-170.51 | 64.60< 116.71  
 To Node 734 | 105.22< -25.35 | 67.30<-169.14 | 64.60< 116.71

---

Node 732  
 From Node 708 | 9.84< -56.56 | 0.00< 0.00 | 9.85< 123.45

---

Node 731  
 From Node 709 | 0.00< 0.00 | 19.58<-115.92 | 19.59< 64.06

---

Node 708  
 From Node 709 | 130.64< -23.27 | 86.76<-170.50 | 74.38< 117.61  
 To Node 733 | 122.54< -20.75 | 86.77<-170.50 | 64.60< 116.72  
 To Node 732 | 9.83< -56.53 | 0.01< -30.60 | 9.85< 123.51

---

Node 735  
 From Node 710 | 19.82< -55.31 | 0.00< 0.00 | 19.83< 124.71

---

Node 736  
 From Node 710 | 0.00< 0.00 | 9.73<-117.30 | 9.74< 62.68

---

Node 741  
 From Node 711 | 9.78< -56.83 | 0.00< 0.00 | 9.79< 123.19

---

Node 740  
 From Node 711 | 19.87< -55.46 | 0.00< 0.00 | 19.89< 124.56

---

Node 704  
 From Node 713 | 41.83< -25.18 | 72.39<-133.25 | 71.50< 80.52  
 To Node 720 | 24.24< -54.54 | 52.12<-117.04 | 66.89< 81.70  
 To Node 714 | 23.87< 4.67 | 26.66<-166.33 | 4.83< 64.05

---

Node 718  
 From Node 714 | 19.96< 4.62 | 19.96<-175.34 | 0.00< 0.00

---

Node 707

From Node 720		4.82< -54.60		42.34<-117.13		44.79< 68.34			
To Node 724		0.02< 88.62		9.73<-117.02		9.72< 62.89			
To Node 722		4.84< -54.77		32.61<-117.16		35.13< 69.84			
<hr/>									
Node 706									
From Node 720		0.01< 0.00		9.78<-117.15		9.78< 62.80			
To Node 725		0.01< 0.00		9.78<-117.15		9.78< 62.80			
<hr/>									
Node 744									
From Node 727		34.99< -10.64		35.00<-162.81		16.82< 93.26			
To Node 729		9.78< 3.32		9.77<-176.67		0.01< 0.00			
To Node 728		16.82< -26.78		16.82<-146.75		16.83< 93.24			
<hr/>									
Node 709									
From Node 730		130.63< -23.26		99.37<-161.25		87.44< 107.26			
To Node 708		130.64< -23.27		86.75<-170.49		74.38< 117.62			
To Node 731		0.02< 88.37		19.58<-115.85		19.57< 64.10			
To Node 775		0.00< 0.00		0.00< 0.00		0.00< 0.00			
<hr/>									
Node 734									
From Node 733		105.23< -25.36		67.32<-169.15		64.60< 116.69			
To Node 737		81.04< -14.70		61.83<-176.30		29.67< 124.25			
To Node 710		19.77< -55.20		9.74<-116.91		25.86< 105.42			
<hr/>									
Node 737									
From Node 734		81.04< -14.72		61.86<-176.31		29.67< 124.21			
To Node 738		51.09< -26.19		29.25<-176.08		29.67< 124.21			
<hr/>									
Node 710									
From Node 734		19.78< -55.23		9.74<-117.01		25.86< 105.39			
To Node 736		0.04< 88.34		9.74<-117.05		9.71< 62.80			
To Node 735		19.81< -55.30		0.01< 0.00		19.83< 124.73			
<hr/>									
Node 738									
From Node 737		51.10< -26.21		29.27<-176.10		29.67< 124.18			
To Node 711		29.62< -55.87		0.04< -30.43		29.67< 124.18			
<hr/>									
Node 711									
From Node 738		29.63< -55.89		0.02< -30.43		29.67< 124.15			
To Node 741		9.77< -56.78		0.02< -30.43		9.79< 123.28			
To Node 740		19.87< -55.45		0.01< 0.00		19.89< 124.58			
<hr/>									
Node 728									
From Node 744		16.82< -26.79		16.83<-146.77		16.83< 93.22			
<hr/>									
Node 729									
From Node 744		9.78< 3.27		9.78<-176.70		0.00< 0.00			
<hr/>									
Node Reg-1									
From Node 799		366.13< -32.30		274.00<-148.39		347.67< 102.65			
To Node 701		366.12< -32.30		273.99<-148.39		347.69< 102.65			
<hr/>									
Node 701									
From Node Reg-1		366.20< -32.32		274.07<-148.42		347.74< 102.63			
To Node 702		267.57< -29.39		218.81<-148.90		248.56< 100.61			
<hr/>									
Node 775									
From Node 709		0.00< 0.00		0.00< 0.00		0.00< 0.00			
<hr/>									
3.Losses (KW)									
		TOTAL		PHASE A		PHASE B		PHASE C	
<hr/>									
NODE 701	To NODE 702		25.210		8.990		7.859		8.362
NODE 702	To NODE 705		0.468		0.126		0.130		0.213
NODE 702	To NODE 713		2.492		0.719		0.830		0.943
NODE 702	To NODE 703		13.648		5.543		4.197		3.908
NODE 703	To NODE 727		0.522		0.215		0.175		0.132
NODE 703	To NODE 730		10.966		4.335		3.291		3.340
NODE 704	To NODE 714		0.062		0.024		0.027		0.011
NODE 704	To NODE 720		2.655		0.627		0.910		1.118
NODE 705	To NODE 742		0.155		0.026		0.067		0.063
NODE 705	To NODE 712		0.106		0.044		0.018		0.044
NODE 706	To NODE 725		0.032		0.005		0.013		0.013
NODE 707	To NODE 724		0.085		0.014		0.036		0.036
NODE 707	To NODE 722		0.165		0.028		0.066		0.072
NODE 708	To NODE 733		3.643		1.516		1.161		0.967
NODE 708	To NODE 732		0.036		0.015		0.006		0.015
NODE 709	To NODE 731		0.197		0.041		0.078		0.078
NODE 709	To NODE 708		4.105		1.714		1.261		1.130
NODE 710	To NODE 735		0.093		0.038		0.015		0.039
NODE 710	To NODE 736		0.143		0.023		0.060		0.060

NODE 711	To NODE 741		0.032		0.013		0.007		0.013
NODE 711	To NODE 740		0.093		0.039		0.016		0.039
NODE 713	To NODE 704		2.698		0.705		1.001		0.982
NODE 714	To NODE 718		0.245		0.103		0.103		0.040
NODE 720	To NODE 707		2.081		0.343		0.842		0.896
NODE 720	To NODE 706		0.049		0.010		0.019		0.019
NODE 727	To NODE 744		0.327		0.123		0.124		0.081
NODE 730	To NODE 709		2.947		1.151		0.943		0.853
NODE 733	To NODE 734		4.715		1.956		1.413		1.345
NODE 734	To NODE 737		3.077		1.305		1.044		0.729
NODE 734	To NODE 710		0.354		0.118		0.074		0.162
NODE 737	To NODE 738		0.740		0.318		0.213		0.209
NODE 738	To NODE 711		0.298		0.117		0.065		0.117
NODE 744	To NODE 728		0.100		0.033		0.034		0.033
NODE 744	To NODE 729		0.032		0.013		0.013		0.005
NODE 775	To NODE 709		0.000		0.000		0.000		0.000
NODE 799	To NODE Reg-1		0.000		0.000		0.000		0.000
NODE Reg-1	To NODE 701		45.667		16.943		12.975		15.749
TOTAL					47.331		39.086		41.812

TOTAL FEEDER POWER LOSS= 128.2291

#### 4. Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C		
			P	Q	P	Q	P	Q	P	Q	
SPOT LOADS											
701	D	PQ	140.00	70.00	140.00	70.00	350.00	175.00			
712	D	PQ	0.00	0.00	0.00	0.00	85.00	40.00			
713	D	PQ	0.00	0.00	0.00	0.00	85.00	40.00			
714	D	I	17.36	8.17	21.09	10.04	0.00	0.00			
718	D	Z	88.45	41.62	0.00	0.00	0.00	0.00			
720	D	PQ	0.00	0.00	0.00	0.00	85.00	40.00			
722	D	I	0.00	0.00	139.36	69.68	21.05	10.02			
724	D	Z	0.00	0.00	41.58	20.79	0.00	0.00			
725	D	PQ	0.00	0.00	42.00	21.00	0.00	0.00			
727	D	PQ	0.00	0.00	0.00	0.00	42.00	21.00			
728	D	PQ	42.00	21.00	42.00	21.00	42.00	21.00			
729	D	I	42.66	21.33	0.00	0.00	0.00	0.00			
730	D	Z	0.00	0.00	0.00	0.00	84.68	39.85			
731	D	Z	0.00	0.00	85.06	40.03	0.00	0.00			
732	D	PQ	0.00	0.00	0.00	0.00	42.00	21.00			
733	D	I	85.54	40.25	0.00	0.00	0.00	0.00			
734	D	PQ	0.00	0.00	0.00	0.00	42.00	21.00			
735	D	PQ	0.00	0.00	0.00	0.00	85.00	40.00			
736	D	Z	0.00	0.00	41.59	20.80	0.00	0.00			
737	D	I	139.95	69.97	0.00	0.00	0.00	0.00			
738	D	PQ	126.00	62.00	0.00	0.00	0.00	0.00			
740	D	PQ	0.00	0.00	0.00	0.00	85.00	40.00			
741	D	I	0.00	0.00	0.00	0.00	41.36	20.68			
742	D	Z	8.39	4.19	86.14	40.54	0.00	0.00			
744	D	PQ	42.00	21.00	0.00	0.00	0.00	0.00			

#### 5. Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	799	701	40.00	350.00	2.00	A-B-C	AB-CB
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
AB	1.500	3.000	122.000	121.741	7		
CB	1.500	3.000	122.000	122.240	4		

#### 5.2.4 123-node feeder

1.Voltage Profile								
NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG	
150	1.0000<	0.00	1.0000<-120.00		1.0000<	120.00		LG
2	0.0000<	0.00	1.0410<-120.31		0.0000<	0.00		LG
3	0.0000<	0.00	0.0000<	0.00	1.0332<	119.59		LG
7	1.0220<	-1.10	1.0395<-120.56		1.0292<	119.37		LG
4	0.0000<	0.00	0.0000<	0.00	1.0327<	119.58		LG
5	0.0000<	0.00	0.0000<	0.00	1.0319<	119.57		LG
6	0.0000<	0.00	0.0000<	0.00	1.0313<	119.55		LG
8	1.0160<	-1.41	1.0383<-120.72		1.0254<	119.20		LG
12	0.0000<	0.00	1.0380<-120.73		0.0000<	0.00		LG
9	1.0145<	-1.44	0.0000<	0.00	0.0000<	0.00		LG
13	1.0080<	-1.84	1.0361<-120.96		1.0198<	118.92		LG
Reg-2	1.0081<	-1.44	0.0000<	0.00	0.0000<	0.00		LG
14	1.0065<	-1.47	0.0000<	0.00	0.0000<	0.00		LG
34	0.0000<	0.00	0.0000<	0.00	1.0188<	118.90		LG

18		0.9990<-2.26		1.0319<-121.21		1.0125<118.85		LG
11		1.0058<-1.49		0.0000<0.00		0.0000<0.00		LG
10		1.0061<-1.48		0.0000<0.00		0.0000<0.00		LG
16		0.0000<0.00		0.0000<0.00		1.0175<118.87		LG
17		0.0000<0.00		0.0000<0.00		1.0180<118.88		LG
19		0.9976<-2.29		0.0000<0.00		0.0000<0.00		LG
21		0.9984<-2.31		1.0320<-121.21		1.0113<118.83		LG
20		0.9968<-2.30		0.0000<0.00		0.0000<0.00		LG
22		0.0000<0.00		1.0306<-121.23		0.0000<0.00		LG
23		0.9980<-2.36		1.0324<-121.19		1.0102<118.81		LG
24		0.0000<0.00		0.0000<0.00		1.0088<118.78		LG
25		0.9973<-2.42		1.0328<-121.18		1.0093<118.81		LG
Reg-3		0.9973<-2.42		0.0000<0.00		1.0030<118.81		LG
26		0.9971<-2.44		0.0000<0.00		1.0025<118.80		LG
28		0.9970<-2.44		1.0330<-121.18		1.0090<118.82		LG
27		0.9967<-2.46		0.0000<0.00		1.0025<118.81		LG
31		0.0000<0.00		0.0000<0.00		1.0020<118.79		LG
33		0.9954<-2.49		0.0000<0.00		0.0000<0.00		LG
29		0.9968<-2.47		1.0332<-121.18		1.0085<118.81		LG
30		0.9971<-2.47		1.0331<-121.16		1.0081<118.79		LG
250		0.9971<-2.47		1.0331<-121.16		1.0081<118.79		LG
32		0.0000<0.00		0.0000<0.00		1.0015<118.78		LG
15		0.0000<0.00		0.0000<0.00		1.0184<118.89		LG
36		0.9953<-2.36		1.0289<-121.34		0.0000<0.00		LG
40		0.9947<-2.38		1.0283<-121.34		1.0103<118.74		LG
37		0.9945<-2.38		0.0000<0.00		0.0000<0.00		LG
38		0.0000<0.00		1.0283<-121.35		0.0000<0.00		LG
39		0.0000<0.00		1.0279<-121.36		0.0000<0.00		LG
41		0.0000<0.00		0.0000<0.00		1.0099<118.73		LG
42		0.9931<-2.42		1.0271<-121.39		1.0095<118.71		LG
43		0.0000<0.00		1.0258<-121.42		0.0000<0.00		LG
44		0.9920<-2.45		1.0264<-121.42		1.0086<118.67		LG
45		0.9915<-2.46		0.0000<0.00		0.0000<0.00		LG
47		0.9910<-2.47		1.0253<-121.45		1.0076<118.63		LG
46		0.9911<-2.46		0.0000<0.00		0.0000<0.00		LG
48		0.9907<-2.47		1.0251<-121.46		1.0074<118.62		LG
49		0.9907<-2.47		1.0248<-121.46		1.0073<118.60		LG
50		0.9907<-2.48		1.0248<-121.45		1.0070<118.59		LG
51		0.9905<-2.49		1.0249<-121.45		1.0070<118.60		LG
151		0.9905<-2.49		1.0249<-121.45		1.0070<118.60		LG
53		0.9993<-2.39		1.0341<-121.32		1.0150<118.54		LG
54		0.9978<-2.49		1.0335<-121.39		1.0140<118.45		LG
55		0.9976<-2.50		1.0334<-121.40		1.0141<118.46		LG
57		0.9947<-2.79		1.0307<-121.59		1.0114<118.24		LG
56		0.9976<-2.49		1.0333<-121.41		1.0142<118.46		LG
58		0.0000<0.00		1.0300<-121.60		0.0000<0.00		LG
60		0.9882<-3.47		1.0257<-121.98		1.0054<117.79		LG
59		0.0000<0.00		1.0297<-121.61		0.0000<0.00		LG
61		0.9882<-3.47		1.0257<-121.98		1.0054<117.79		LG
62		0.9874<-3.46		1.0254<-121.95		1.0033<117.77		LG
63		0.9868<-3.45		1.0237<-121.94		1.0023<117.77		LG
64		0.9865<-3.43		1.0218<-121.91		1.0002<117.73		LG
65		0.9858<-3.44		1.0214<-121.87		0.9972<117.73		LG
66		0.9860<-3.47		1.0217<-121.84		0.9957<117.73		LG
68		1.0342<-3.74		0.0000<0.00		0.0000<0.00		LG
72		1.0361<-3.61		1.0302<-122.25		1.0345<117.53		LG
97		1.0347<-3.77		1.0306<-122.18		1.0340<117.63		LG
69		1.0325<-3.78		0.0000<0.00		0.0000<0.00		LG
70		1.0312<-3.80		0.0000<0.00		0.0000<0.00		LG
71		1.0305<-3.81		0.0000<0.00		0.0000<0.00		LG
73		0.0000<0.00		0.0000<0.00		1.0323<117.49		LG
76		1.0361<-3.87		1.0297<-122.35		1.0351<117.48		LG
74		0.0000<0.00		0.0000<0.00		1.0305<117.46		LG
75		0.0000<0.00		0.0000<0.00		1.0295<117.44		LG
77		1.0372<-3.94		1.0309<-122.43		1.0360<117.41		LG
86		1.0352<-3.90		1.0280<-122.51		1.0365<117.45		LG
78		1.0375<-3.96		1.0313<-122.45		1.0362<117.39		LG
79		1.0372<-3.97		1.0314<-122.45		1.0361<117.40		LG
80		1.0396<-4.02		1.0330<-122.51		1.0370<117.28		LG
81		1.0418<-4.09		1.0352<-122.54		1.0376<117.18		LG
82		1.0426<-4.13		1.0365<-122.57		1.0384<117.15		LG
84		0.0000<0.00		0.0000<0.00		1.0350<117.13		LG
83		1.0438<-4.15		1.0376<-122.60		1.0392<117.10		LG
85		0.0000<0.00		0.0000<0.00		1.0338<117.11		LG
87		1.0347<-3.92		1.0272<-122.60		1.0371<117.43		LG
88		1.0346<-3.95		0.0000<0.00		0.0000<0.00		LG
89		1.0342<-3.92		1.0269<-122.65		1.0375<117.42		LG
90		0.0000<0.00		1.0268<-122.69		0.0000<0.00		LG
91		1.0340<-3.92		1.0266<-122.66		1.0377<117.40		LG
92		0.0000<0.00		0.0000<0.00		1.0376<117.35		LG
93		1.0337<-3.92		1.0264<-122.68		1.0378<117.41		LG
94		1.0330<-3.94		0.0000<0.00		0.0000<0.00		LG

95		1.0337<-3.91		1.0260<-122.70		1.0380<117.41		LG
96		0.0000<0.00		1.0258<-122.70		0.0000<0.00		LG
98		1.0345<-3.78		1.0304<-122.18		1.0338<117.62		LG
99		1.0348<-3.77		1.0296<-122.19		1.0334<117.58		LG
100		1.0350<-3.77		1.0295<-122.18		1.0330<117.57		LG
450		1.0350<-3.77		1.0295<-122.18		1.0330<117.57		LG
102		0.0000<0.00		0.0000<0.00		1.0320<117.60		LG
105		1.0326<-3.85		1.0302<-122.24		1.0337<117.65		LG
103		0.0000<0.00		0.0000<0.00		1.0303<117.56		LG
104		0.0000<0.00		0.0000<0.00		1.0285<117.53		LG
106		0.0000<0.00		1.0291<-122.26		0.0000<0.00		LG
108		1.0311<-3.92		1.0309<-122.25		1.0335<117.69		LG
107		0.0000<0.00		1.0276<-122.29		0.0000<0.00		LG
109		1.0270<-4.00		0.0000<0.00		0.0000<0.00		LG
300		1.0311<-3.92		1.0309<-122.25		1.0335<117.69		LG
110		1.0250<-4.03		0.0000<0.00		0.0000<0.00		LG
111		1.0243<-4.05		0.0000<0.00		0.0000<0.00		LG
112		1.0244<-4.05		0.0000<0.00		0.0000<0.00		LG
113		1.0223<-4.09		0.0000<0.00		0.0000<0.00		LG
114		1.0219<-4.09		0.0000<0.00		0.0000<0.00		LG
35		0.9962<-2.34		1.0294<-121.29		1.0114<118.79		LG
1		1.0313<-0.63		1.0413<-120.31		1.0349<119.62		LG
52		1.0020<-2.22		1.0348<-121.20		1.0166<118.67		LG
Reg-4		1.0376<-3.47		1.0321<-121.98		1.0368<117.79		LG
67		1.0357<-3.72		1.0311<-122.15		1.0347<117.65		LG
160		0.9882<-3.47		1.0257<-121.98		1.0054<117.79		LG
101		1.0339<-3.81		1.0304<-122.19		1.0334<117.62		LG
152		1.0080<-1.84		1.0361<-120.96		1.0198<118.92		LG
135		0.9990<-2.26		1.0319<-121.21		1.0125<118.85		LG
610		1.7309<27.91		1.7611<-92.42		1.7374<146.88		LL
XFM-1		0.9882<-3.47		1.0257<-121.98		1.0054<117.79		LG
197		1.0347<-3.77		1.0306<-122.18		1.0340<117.63		LG
Reg-1		1.0437<0.00		1.0437<-120.00		1.0437<120.00		LG
149		1.0437<0.00		1.0437<-120.00		1.0437<120.00		LG

## 2. Current Flow (AMP)

	MAG	ANGLE		MAG	ANGLE		MAG	ANGLE
<hr/>								
Node 150								
To Node Reg-1		655.52<-21.68		425.90<-139.61		523.63<101.59		
<hr/>								
Node 2								
From Node 1		0.00<0.00		8.94<-146.88		0.00<0.00		
<hr/>								
Node 3								
From Node 1		0.00<0.00		0.00<0.00		46.54<93.00		
To Node 5		0.00<0.00		0.00<0.00		28.51<92.99		
To Node 4		0.00<0.00		0.00<0.00		18.03<93.02		
<hr/>								
Node 7								
From Node 1		610.07<-21.52		399.18<-139.44		455.71<102.46		
To Node 8		601.01<-21.42		399.18<-139.44		455.71<102.46		
<hr/>								
Node 4								
From Node 3		0.00<0.00		0.00<0.00		18.03<93.02		
<hr/>								
Node 5								
From Node 3		0.00<0.00		0.00<0.00		28.51<92.99		
To Node 6		0.00<0.00		0.00<0.00		19.20<92.99		
<hr/>								
Node 6								
From Node 5		0.00<0.00		0.00<0.00		19.20<92.99		
<hr/>								
Node 8								
From Node 7		601.01<-21.42		399.18<-139.44		455.71<102.46		
To Node 13		555.13<-20.88		390.30<-139.26		455.71<102.46		
To Node 9		46.22<-28.03		0.00<0.00		0.00<0.00		
To Node 12		0.00<0.00		8.97<-147.29		0.00<0.00		
<hr/>								
Node 12								
From Node 8		0.00<0.00		8.97<-147.29		0.00<0.00		
<hr/>								
Node 9								
From Node 8		46.22<-28.03		0.00<0.00		0.00<0.00		
To Node Reg-2		27.86<-28.04		0.00<0.00		0.00<0.00		
<hr/>								
Node 13								
From Node 8		555.13<-20.88		390.30<-139.26		455.71<102.46		
To Node 18		228.62<-30.51		155.56<-155.09		153.13<88.76		
To Node 34		0.00<0.00		0.00<0.00		46.42<92.32		
To Node 152		331.95<-14.26		244.33<-129.27		264.99<112.11		

Node Reg-2						
From Node 9   28.04< -28.04   0.00< 0.00   0.00< 0.00						
To Node 14   28.04< -28.04   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 14						
From Node Reg-2   28.04< -28.05   0.00< 0.00   0.00< 0.00						
To Node 10   9.31< -28.04   0.00< 0.00   0.00< 0.00						
To Node 11   18.73< -28.05   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 34						
From Node 13   0.00< 0.00   0.00< 0.00   46.42< 92.32						
To Node 15   0.00< 0.00   0.00< 0.00   27.45< 92.31						
<hr/>						
Node 18						
From Node 13   228.62< -30.51   155.57<-155.09   153.13< 88.76						
To Node 21   55.55< -29.48   19.19<-147.78   55.21< 92.62						
To Node 19   37.28< -28.86   0.00< 0.00   0.00< 0.00						
To Node 135   135.82< -31.39   136.56<-156.11   98.12< 86.60						
<hr/>						
Node 11						
From Node 14   18.73< -28.05   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 10						
From Node 14   9.31< -28.04   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 16						
From Node 15   0.00< 0.00   0.00< 0.00   18.30< 92.31						
<hr/>						
Node 17						
From Node 15   0.00< 0.00   0.00< 0.00   9.15< 92.32						
<hr/>						
Node 19						
From Node 18   37.28< -28.86   0.00< 0.00   0.00< 0.00						
To Node 20   18.62< -28.86   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 21						
From Node 18   55.55< -29.48   19.19<-147.78   55.21< 92.61						
To Node 23   55.55< -29.48   0.00< 0.00   55.21< 92.61						
To Node 22   0.00< 0.00   19.19<-147.80   0.00< 0.00						
<hr/>						
Node 20						
From Node 19   18.62< -28.87   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 22						
From Node 21   0.00< 0.00   19.19<-147.80   0.00< 0.00						
<hr/>						
Node 23						
From Node 21   55.55< -29.48   0.00< 0.00   55.21< 92.61						
To Node 25   55.55< -29.48   0.00< 0.00   36.75< 92.81						
To Node 24   0.00< 0.00   0.00< 0.00   18.46< 92.22						
<hr/>						
Node 24						
From Node 23   0.00< 0.00   0.00< 0.00   18.46< 92.22						
<hr/>						
Node 25						
From Node 23   55.55< -29.49   0.00< 0.00   36.75< 92.81						
To Node 28   37.18< -29.01   0.00< 0.00   18.47< 92.23						
To Node Reg-3   18.38< -30.44   0.00< 0.00   18.29< 93.40						
<hr/>						
Node Reg-3						
From Node 25   18.38< -30.44   0.00< 0.00   18.40< 93.40						
To Node 26   18.62< -29.04   0.00< 0.00   18.59< 92.23						
<hr/>						
Node 26						
From Node Reg-3   18.62< -29.05   0.00< 0.00   18.59< 92.23						
To Node 31   0.00< 0.00   0.00< 0.00   18.59< 92.22						
To Node 27   18.62< -29.05   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 28						
From Node 25   37.18< -29.02   0.00< 0.00   18.47< 92.23						
To Node 29   18.56< -29.02   0.00< 0.00   18.47< 92.23						
<hr/>						
Node 27						
From Node 26   18.62< -29.05   0.00< 0.00   0.00< 0.00						
To Node 33   18.62< -29.05   0.00< 0.00   0.00< 0.00						
<hr/>						
Node 31						
From Node 26   0.00< 0.00   0.00< 0.00   18.59< 92.22						
To Node 32   0.00< 0.00   0.00< 0.00   9.30< 92.22						
<hr/>						
Node 33						
From Node 27   18.62< -29.05   0.00< 0.00   0.00< 0.00						

Node 29
From Node 28   18.56< -29.02   0.00< 0.00   18.47< 92.23
To Node 30   0.00< 0.00   0.00< 0.00   18.47< 92.23
Node 30
From Node 29   0.00< 0.00   0.00< 0.00   18.47< 92.22
To Node 250   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 250
From Node 30   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 32
From Node 31   0.00< 0.00   0.00< 0.00   9.30< 92.22
Node 15
From Node 34   0.00< 0.00   0.00< 0.00   27.45< 92.31
To Node 17   0.00< 0.00   0.00< 0.00   9.15< 92.32
To Node 16   0.00< 0.00   0.00< 0.00   18.30< 92.31
Node 36
From Node 35   18.52< -28.94   18.37<-147.92   0.00< 0.00
To Node 38   0.00< 0.00   18.37<-147.92   0.00< 0.00
To Node 37   18.52< -28.94   0.00< 0.00   0.00< 0.00
Node 40
From Node 35   108.64< -34.92   108.47<-155.41   98.12< 86.60
To Node 42   108.64< -34.92   108.47<-155.41   88.95< 86.02
To Node 41   0.00< 0.00   0.00< 0.00   9.22< 92.17
Node 37
From Node 36   18.52< -28.94   0.00< 0.00   0.00< 0.00
Node 38
From Node 36   0.00< 0.00   18.37<-147.92   0.00< 0.00
To Node 39   0.00< 0.00   9.06<-147.92   0.00< 0.00
Node 39
From Node 38   0.00< 0.00   9.06<-147.92   0.00< 0.00
Node 41
From Node 40   0.00< 0.00   0.00< 0.00   9.22< 92.17
Node 42
From Node 40   108.64< -34.92   108.47<-155.41   88.95< 86.02
To Node 44   99.32< -35.48   89.57<-156.99   88.95< 86.02
To Node 43   0.00< 0.00   19.10<-147.98   0.00< 0.00
Node 43
From Node 42   0.00< 0.00   19.10<-147.98   0.00< 0.00
Node 44
From Node 42   99.32< -35.48   89.57<-156.99   88.95< 86.02
To Node 47   80.76< -36.97   89.57<-156.99   88.95< 86.02
To Node 45   18.70< -29.02   0.00< 0.00   0.00< 0.00
Node 45
From Node 44   18.70< -29.02   0.00< 0.00   0.00< 0.00
To Node 46   9.39< -29.02   0.00< 0.00   0.00< 0.00
Node 47
From Node 44   80.76< -36.97   89.57<-156.99   88.95< 86.02
To Node 49   27.40< -34.94   34.95<-156.99   35.14< 90.53
To Node 48   35.48< -38.01   36.71<-156.99   36.08< 83.08
Node 46
From Node 45   9.39< -29.03   0.00< 0.00   0.00< 0.00
Node 48
From Node 47   35.48< -38.01   36.71<-156.99   36.08< 83.08
Node 49
From Node 47   27.40< -34.95   34.95<-156.99   35.14< 90.53
To Node 50   9.40< -29.04   0.00< 0.00   18.49< 92.04
Node 50
From Node 49   9.40< -29.05   0.00< 0.00   18.49< 92.03
To Node 51   9.40< -29.05   0.00< 0.00   0.00< 0.00
Node 51
From Node 50   9.40< -29.05   0.00< 0.00   0.00< 0.00
To Node 151   0.00< 0.00   0.00< 0.00   0.00< 0.00

Node 151
From Node 51   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 53
From Node 52   314.00< -13.41   244.33<-129.27   264.99< 112.11
To Node 54   296.09< -12.44   244.33<-129.27   264.99< 112.11
Node 54
From Node 53   296.09< -12.44   244.33<-129.27   264.99< 112.11
To Node 57   287.20< -11.91   235.82<-128.57   264.99< 112.11
To Node 55   9.29< -29.05   9.01<-147.96   0.00< 0.00
Node 55
From Node 54   9.29< -29.06   9.01<-147.96   0.00< 0.00
To Node 56   0.00< 0.00   9.01<-147.96   0.00< 0.00
Node 57
From Node 54   287.20< -11.91   235.82<-128.57   264.99< 112.11
To Node 60   287.20< -11.91   218.62<-126.95   264.99< 112.11
To Node 58   0.00< 0.00   18.35<-148.17   0.00< 0.00
Node 56
From Node 55   0.00< 0.00   9.01<-147.97   0.00< 0.00
Node 58
From Node 57   0.00< 0.00   18.35<-148.17   0.00< 0.00
To Node 59   0.00< 0.00   9.04<-148.17   0.00< 0.00
Node 60
From Node 57   287.20< -11.91   218.62<-126.95   264.99< 112.11
To Node 62   45.37< -41.31   52.24<-150.52   80.73< 92.25
To Node 61   0.00< 0.00   0.00< 0.00   0.00< 0.00
To Node 160   240.05< -5.89   172.00<-119.98   191.05< 120.36
Node 59
From Node 58   0.00< 0.00   9.04<-148.17   0.00< 0.00
Node 61
From Node 60   0.00< 0.00   0.00< 0.00   0.00< 0.00
To Node XFM-1   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 62
From Node 60   45.37< -41.32   52.25<-150.53   80.73< 92.24
To Node 63   45.37< -41.32   52.25<-150.53   62.05< 92.55
Node 63
From Node 62   45.38< -41.33   52.25<-150.53   62.05< 92.55
To Node 64   27.13< -49.17   52.25<-150.53   62.05< 92.55
Node 64
From Node 63   27.13< -49.18   52.26<-150.54   62.06< 92.54
To Node 65   27.13< -49.18   18.00<-157.48   62.06< 92.54
Node 65
From Node 64   27.14< -49.20   18.00<-157.51   62.06< 92.53
To Node 66   0.01< 0.00   0.01< -31.85   34.60< 92.72
Node 66
From Node 65   0.00< 0.00   0.00< 0.00   34.61< 92.71
Node 68
From Node 67   54.13< -30.35   0.00< 0.00   0.00< 0.00
To Node 69   45.13< -30.36   0.00< 0.00   0.00< 0.00
Node 72
From Node 67   118.74< 22.75   126.11<-108.01   132.91< 134.10
To Node 76   118.74< 22.75   126.11<-108.01   100.03< 156.35
To Node 73   0.00< 0.00   0.00< 0.00   55.31< 90.90
Node 97
From Node 67   82.67< -30.55   54.29<-148.80   64.43< 91.01
To Node 98   18.00< -30.32   18.08<-148.74   19.23< 91.02
To Node 197   64.67< -30.61   36.21<-148.83   45.19< 91.00
Node 69
From Node 68   45.13< -30.36   0.00< 0.00   0.00< 0.00
To Node 70   27.10< -30.37   0.00< 0.00   0.00< 0.00
Node 70
From Node 69   27.10< -30.37   0.00< 0.00   0.00< 0.00
To Node 71   18.07< -30.38   0.00< 0.00   0.00< 0.00

Node 71
From Node 70   18.07< -30.38   0.00< 0.00   0.00< 0.00
-----
Node 73
From Node 72   0.00< 0.00   0.00< 0.00   55.31< 90.90
To Node 74   0.00< 0.00   0.00< 0.00   37.27< 90.89
-----
Node 76
From Node 72   118.74< 22.75   126.11<-108.01   100.03< 156.35
To Node 86   32.68< 5.63   57.67<-129.83   21.03< 157.60
To Node 77   77.84< 60.54   77.41<-57.27   77.46<-177.43
-----
Node 74
From Node 73   0.00< 0.00   0.00< 0.00   37.27< 90.88
To Node 75   0.00< 0.00   0.00< 0.00   18.09< 90.88
-----
Node 75
From Node 74   0.00< 0.00   0.00< 0.00   18.09< 90.87
-----
Node 77
From Node 76   77.84< 60.54   77.41<-57.27   77.46<-177.44
To Node 78   77.84< 60.54   80.02<-44.23   77.46<-177.44
-----
Node 86
From Node 76   32.68< 5.63   57.67<-129.84   21.03< 157.59
To Node 87   32.68< 5.63   49.21<-126.36   21.03< 157.59
-----
Node 78
From Node 77   77.84< 60.54   80.02<-44.23   77.46<-177.44
To Node 80   80.55< 74.41   80.02<-44.23   77.46<-177.44
To Node 79   19.31< -30.53   0.00< 0.00   0.00< 0.00
-----
Node 79
From Node 78   19.31< -30.54   0.00< 0.00   0.00< 0.00
-----
Node 80
From Node 78   80.55< 74.41   80.02<-44.23   77.46<-177.44
To Node 81   80.55< 74.41   86.41<-32.59   77.46<-177.44
-----
Node 81
From Node 80   80.54< 74.41   86.41<-32.59   77.46<-177.44
To Node 84   0.00< 0.00   0.00< 0.00   27.00< 90.55
To Node 82   80.54< 74.41   86.41<-32.59   82.92<-158.44
-----
Node 82
From Node 81   80.54< 74.41   86.41<-32.59   82.92<-158.44
To Node 83   86.92< 85.85   86.41<-32.59   82.92<-158.44
-----
Node 84
From Node 81   0.00< 0.00   0.00< 0.00   27.01< 90.55
To Node 85   0.00< 0.00   0.00< 0.00   18.01< 90.54
-----
Node 83
From Node 82   86.92< 85.85   86.41<-32.59   82.92<-158.44
-----
Node 85
From Node 84   0.00< 0.00   0.00< 0.00   18.01< 90.54
-----
Node 87
From Node 86   32.68< 5.63   49.21<-126.36   21.02< 157.59
To Node 88   21.00< 36.01   0.00< 0.00   0.00< 0.00
To Node 89   18.02< -30.49   33.25<-114.16   21.02< 157.59
-----
Node 88
From Node 87   21.00< 36.01   0.00< 0.00   0.00< 0.00
-----
Node 89
From Node 87   18.02< -30.49   33.25<-114.16   21.02< 157.58
To Node 90   0.00< 0.00   21.16< -84.61   0.00< 0.00
To Node 91   18.02< -30.49   18.15<-149.26   21.02< 157.58
-----
Node 90
From Node 89   0.00< 0.00   21.16< -84.61   0.00< 0.00
-----
Node 91
From Node 89   18.02< -30.49   18.15<-149.26   21.02< 157.58
To Node 92   0.00< 0.00   0.00< 0.00   21.02< 157.58
To Node 93   18.02< -30.49   18.15<-149.26   0.00< 0.00
-----
Node 92
From Node 91   0.00< 0.00   0.00< 0.00   21.02< 157.58

Node 93
From Node 91   18.02< -30.50   18.15<-149.26   0.00< 0.00
To Node 95   0.00< 0.00   18.15<-149.26   0.00< 0.00
To Node 94   18.02< -30.50   0.00< 0.00   0.00< 0.00
Node 94
From Node 93   18.02< -30.50   0.00< 0.00   0.00< 0.00
Node 95
From Node 93   0.00< 0.00   18.15<-149.26   0.00< 0.00
To Node 96   0.00< 0.00   9.08<-149.27   0.00< 0.00
Node 96
From Node 95   0.00< 0.00   9.08<-149.27   0.00< 0.00
Node 98
From Node 97   18.00< -30.33   18.08<-148.74   19.23< 91.02
To Node 99   0.00< 0.00   18.08<-148.74   19.23< 91.02
Node 99
From Node 98   0.00< 0.00   18.08<-148.75   19.23< 91.01
To Node 100   0.00< 0.00   0.00< 0.00   19.23< 91.01
Node 100
From Node 99   0.00< 0.00   0.00< 0.00   19.23< 91.01
To Node 450   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 450
From Node 100   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 102
From Node 101   0.00< 0.00   0.00< 0.00   45.20< 90.99
To Node 103   0.00< 0.00   0.00< 0.00   36.17< 90.99
Node 105
From Node 101   64.67< -30.61   36.21<-148.83   0.00< 0.00
To Node 106   0.00< 0.00   36.21<-148.83   0.00< 0.00
To Node 108   64.67< -30.61   0.00< 0.00   0.00< 0.00
Node 103
From Node 102   0.00< 0.00   0.00< 0.00   36.18< 90.98
To Node 104   0.00< 0.00   0.00< 0.00   18.10< 90.97
Node 104
From Node 103   0.00< 0.00   0.00< 0.00   18.10< 90.97
Node 106
From Node 105   0.00< 0.00   36.21<-148.84   0.00< 0.00
To Node 107   0.00< 0.00   18.12<-148.85   0.00< 0.00
Node 108
From Node 105   64.67< -30.61   0.00< 0.00   0.00< 0.00
To Node 109   64.67< -30.61   0.00< 0.00   0.00< 0.00
To Node 300   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 107
From Node 106   0.00< 0.00   18.12<-148.85   0.00< 0.00
Node 109
From Node 108   64.67< -30.61   0.00< 0.00   0.00< 0.00
To Node 110   46.54< -30.63   0.00< 0.00   0.00< 0.00
Node 300
From Node 108   0.00< 0.00   0.00< 0.00   0.00< 0.00
Node 110
From Node 109   46.54< -30.63   0.00< 0.00   0.00< 0.00
To Node 112   37.46< -30.64   0.00< 0.00   0.00< 0.00
To Node 111   9.09< -30.60   0.00< 0.00   0.00< 0.00
Node 111
From Node 110   9.09< -30.61   0.00< 0.00   0.00< 0.00
Node 112
From Node 110   37.46< -30.64   0.00< 0.00   0.00< 0.00
To Node 113   28.15< -30.65   0.00< 0.00   0.00< 0.00
Node 113
From Node 112   28.15< -30.65   0.00< 0.00   0.00< 0.00
To Node 114   9.11< -30.65   0.00< 0.00   0.00< 0.00

Node 114
From Node 113   9.11< -30.66   0.00< 0.00   0.00< 0.00
-----
Node 35
From Node 135   135.82< -31.39   136.56<-156.11   98.12< 86.60
To Node 40   108.64< -34.92   108.47<-155.41   98.12< 86.60
To Node 36   18.52< -28.94   18.37<-147.91   0.00< 0.00
-----
Node 1
From Node 149   628.04< -21.68   408.05<-139.61   501.68< 101.59
To Node 7   610.07< -21.52   399.18<-139.44   455.71< 102.46
To Node 3   0.00< 0.00   0.00< 0.00   46.54< 93.00
To Node 2   0.00< 0.00   8.94<-146.88   0.00< 0.00
-----
Node 52
From Node 152   331.95< -14.26   244.33<-129.27   264.99< 112.11
To Node 53   314.00< -13.41   244.33<-129.27   264.99< 112.11
-----
Node Reg-4
From Node 160   228.62< -5.89   170.93<-119.98   185.26< 120.36
To Node 67   228.62< -5.89   170.93<-119.98   185.26< 120.36
-----
Node 67
From Node Reg-4   228.62< -5.89   170.93<-119.98   185.26< 120.36
To Node 68   54.13< -30.35   0.00< 0.00   0.00< 0.00
To Node 97   82.67< -30.54   54.29<-148.80   64.43< 91.01
To Node 72   118.74< 22.75   126.11<-108.01   132.91< 134.10
-----
Node 160
From Node 60   240.05< -5.89   172.00<-119.98   191.05< 120.36
To Node Reg-4   240.05< -5.89   172.00<-119.98   191.05< 120.36
-----
Node 101
From Node 197   64.67< -30.61   36.21<-148.83   45.19< 91.00
To Node 102   0.00< 0.00   0.00< 0.00   45.20< 90.99
To Node 105   64.67< -30.61   36.21<-148.83   0.01< 0.00
-----
Node 152
From Node 13   331.95< -14.26   244.33<-129.27   264.99< 112.11
To Node 52   331.95< -14.26   244.33<-129.27   264.99< 112.11
-----
Node 135
From Node 18   135.82< -31.39   136.56<-156.11   98.12< 86.60
To Node 35   135.82< -31.39   136.56<-156.11   98.12< 86.60
-----
Node 610
From Node XFM-1   0.00< 0.00   0.00< 0.00   0.00< 0.00
-----
Node XFM-1
From Node 61   0.00< 0.00   0.00< 0.00   0.00< 0.00
To Node 610   0.00< 0.00   0.00< 0.00   0.00< 0.00
-----
Node 197
From Node 97   64.67< -30.61   36.21<-148.83   45.19< 91.00
To Node 101   64.67< -30.61   36.21<-148.83   45.19< 91.00
-----
Node Reg-1
From Node 150   628.04< -21.68   408.05<-139.61   501.68< 101.59
To Node 149   628.04< -21.68   408.05<-139.61   501.68< 101.59
-----
Node 149
From Node Reg-1   628.04< -21.68   408.05<-139.61   501.68< 101.59
To Node 1   628.04< -21.68   408.05<-139.61   501.68< 101.59
-----

### 3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 1	To NODE 2	0.004	0.000	0.004	0.000
NODE 1	To NODE 3	0.136	0.000	0.000	0.136
NODE 1	To NODE 7	32.417	12.905	9.389	10.123
NODE 3	To NODE 4	0.016	0.000	0.000	0.016
NODE 3	To NODE 5	0.067	0.000	0.000	0.067
NODE 5	To NODE 6	0.023	0.000	0.000	0.023
NODE 7	To NODE 8	21.293	8.413	6.194	6.685
NODE 8	To NODE 12	0.005	0.000	0.005	0.000
NODE 8	To NODE 9	0.122	0.122	0.000	0.000
NODE 8	To NODE 13	29.307	11.172	8.634	9.500
NODE 9	To NODE Reg-2	0.000	0.000	0.000	0.000
NODE Reg-2	To NODE 14	0.084	0.084	0.000	0.000
NODE 13	To NODE 34	0.081	0.000	0.000	0.081
NODE 13	To NODE 18	12.108	4.980	3.598	3.530

NODE 14	To NODE 11		0.022		0.022		0.000		0.000
NODE 14	To NODE 10		0.005		0.005		0.000		0.000
NODE 15	To NODE 16		0.032		0.000		0.000		0.032
NODE 15	To NODE 17		0.007		0.000		0.000		0.007
NODE 18	To NODE 19		0.087		0.087		0.000		0.000
NODE 18	To NODE 21		0.286		0.112		0.064		0.110
NODE 19	To NODE 20		0.028		0.028		0.000		0.000
NODE 21	To NODE 22		0.049		0.000		0.049		0.000
NODE 21	To NODE 23		0.225		0.091		0.045		0.089
NODE 23	To NODE 24		0.047		0.000		0.000		0.047
NODE 23	To NODE 25		0.179		0.086		0.036		0.057
NODE 25	To NODE Reg-3		0.000		0.000		0.000		0.000
NODE Reg-3	To NODE 26		0.028		0.014		0.000		0.014
NODE 25	To NODE 28		0.051		0.026		0.010		0.014
NODE 26	To NODE 27		0.011		0.008		0.000		0.003
NODE 26	To NODE 31		0.020		0.000		0.000		0.020
NODE 27	To NODE 33		0.044		0.044		0.000		0.000
NODE 28	To NODE 29		0.030		0.012		0.006		0.012
NODE 29	To NODE 30		0.017		0.004		0.003		0.010
NODE 30	To NODE 250		0.000		0.000		0.000		0.000
NODE 31	To NODE 32		0.007		0.000		0.000		0.007
NODE 34	To NODE 15		0.019		0.000		0.000		0.019
NODE 35	To NODE 36		0.051		0.026		0.026		0.000
NODE 35	To NODE 40		1.216		0.413		0.419		0.384
NODE 36	To NODE 37		0.026		0.026		0.000		0.000
NODE 36	To NODE 38		0.021		0.000		0.021		0.000
NODE 38	To NODE 39		0.007		0.000		0.007		0.000
NODE 40	To NODE 41		0.007		0.000		0.000		0.007
NODE 40	To NODE 42		1.153		0.400		0.406		0.347
NODE 42	To NODE 43		0.046		0.000		0.046		0.000
NODE 42	To NODE 44		0.755		0.264		0.247		0.244
NODE 44	To NODE 45		0.018		0.018		0.000		0.000
NODE 44	To NODE 47		0.823		0.258		0.285		0.280
NODE 45	To NODE 46		0.007		0.007		0.000		0.000
NODE 47	To NODE 48		0.086		0.028		0.029		0.028
NODE 47	To NODE 49		0.117		0.035		0.042		0.041
NODE 49	To NODE 50		0.016		0.004		0.003		0.008
NODE 50	To NODE 51		0.003		0.002		0.001		0.001
NODE 51	To NODE 151		0.000		0.000		0.000		0.000
NODE 52	To NODE 53		6.686		2.470		2.058		2.158
NODE 53	To NODE 54		3.980		1.425		1.246		1.309
NODE 54	To NODE 55		0.007		0.003		0.003		0.001
NODE 54	To NODE 57		10.687		3.825		3.252		3.611
NODE 55	To NODE 56		0.003		0.001		0.002		0.001
NODE 57	To NODE 58		0.021		0.000		0.021		0.000
NODE 57	To NODE 60		22.050		8.025		6.460		7.564
NODE 58	To NODE 59		0.005		0.000		0.005		0.000
NODE 60	To NODE 61		0.000		0.000		0.000		0.000
NODE 60	To NODE 62		1.361		0.367		0.409		0.584
NODE 62	To NODE 63		0.729		0.214		0.241		0.275
NODE 63	To NODE 64		1.236		0.294		0.435		0.506
NODE 64	To NODE 65		1.003		0.256		0.232		0.514
NODE 65	To NODE 66		0.187		0.036		0.038		0.112
NODE 67	To NODE 68		0.148		0.148		0.000		0.000
NODE 67	To NODE 72		1.921		0.611		0.635		0.675
NODE 67	To NODE 97		0.511		0.202		0.144		0.165
NODE 68	To NODE 69		0.141		0.141		0.000		0.000
NODE 69	To NODE 70		0.060		0.060		0.000		0.000
NODE 70	To NODE 71		0.023		0.023		0.000		0.000
NODE 72	To NODE 73		0.212		0.000		0.000		0.212
NODE 72	To NODE 76		1.171		0.399		0.417		0.355
NODE 73	To NODE 74		0.122		0.000		0.000		0.122
NODE 74	To NODE 75		0.033		0.000		0.000		0.033
NODE 76	To NODE 77		1.058		0.351		0.352		0.355
NODE 76	To NODE 86		0.493		0.142		0.233		0.118
NODE 77	To NODE 78		0.270		0.089		0.092		0.090
NODE 78	To NODE 79		0.012		0.007		0.002		0.002
NODE 78	To NODE 80		1.314		0.440		0.441		0.434
NODE 80	To NODE 81		1.388		0.454		0.485		0.449
NODE 81	To NODE 82		0.763		0.246		0.262		0.256
NODE 81	To NODE 84		0.124		0.000		0.000		0.124
NODE 82	To NODE 83		0.802		0.269		0.269		0.264
NODE 84	To NODE 85		0.039		0.000		0.000		0.039
NODE 86	To NODE 87		0.259		0.079		0.115		0.064
NODE 87	To NODE 88		0.019		0.019		0.000		0.000
NODE 87	To NODE 89		0.075		0.020		0.033		0.022
NODE 89	To NODE 90		0.025		0.000		0.025		0.000
NODE 89	To NODE 91		0.036		0.011		0.012		0.013
NODE 91	To NODE 92		0.033		0.000		0.000		0.033
NODE 91	To NODE 93		0.021		0.008		0.009		0.004
NODE 93	To NODE 94		0.022		0.022		0.000		0.000
NODE 93	To NODE 95		0.014		0.003		0.009		0.003

NODE 95	To NODE 96		0.004		0.000		0.004		0.000
NODE 97	To NODE 98		0.041		0.013		0.013		0.014
NODE 98	To NODE 99		0.056		0.011		0.022		0.023
NODE 99	To NODE 100		0.016		0.003		0.003		0.010
NODE 100	To NODE 450		0.000		0.000		0.000		0.000
NODE 101	To NODE 102		0.116		0.000		0.000		0.116
NODE 101	To NODE 105		0.221		0.111		0.065		0.045
NODE 102	To NODE 103		0.107		0.000		0.000		0.107
NODE 103	To NODE 104		0.058		0.000		0.000		0.058
NODE 105	To NODE 106		0.074		0.000		0.074		0.000
NODE 105	To NODE 108		0.199		0.119		0.040		0.041
NODE 106	To NODE 107		0.048		0.000		0.048		0.000
NODE 108	To NODE 109		0.474		0.474		0.000		0.000
NODE 108	To NODE 300		0.000		0.000		0.000		0.000
NODE 109	To NODE 110		0.164		0.164		0.000		0.000
NODE 110	To NODE 111		0.012		0.012		0.000		0.000
NODE 110	To NODE 112		0.044		0.044		0.000		0.000
NODE 112	To NODE 113		0.105		0.105		0.000		0.000
NODE 113	To NODE 114		0.007		0.007		0.000		0.000
NODE 135	To NODE 35		2.571		0.919		0.932		0.720
NODE 149	To NODE 1		47.516		18.574		13.561		15.381
NODE 152	To NODE 52		14.045		5.341		4.253		4.451
NODE 160	To NODE Reg-4		0.000		0.000		0.000		0.000
NODE Reg-4	To NODE 67		5.930		2.237		1.785		1.908
NODE 60	To NODE 160		0.000		0.000		0.000		0.000
NODE 197	To NODE 101		0.276		0.116		0.074		0.086
NODE 13	To NODE 152		0.000		0.000		0.000		0.000
NODE 18	To NODE 135		0.000		0.000		0.000		0.000
NODE XFM-1	To NODE 610		0.000		0.000		0.000		0.000
NODE 61	To NODE XFM-1		0.000		0.000		0.000		0.000
NODE 97	To NODE 197		0.000		0.000		0.000		0.000
NODE 150	To NODE Reg-1		0.000		0.000		0.000		0.000
NODE Reg-1	To NODE 149		0.001		0.000		0.000		0.000
TOTAL					88.607		68.378		75.374

TOTAL FEEDER POWER LOSS= 232.3583

#### 4. Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C		
			P	Q		P	Q		P	Q	
SPOT LOADS											
1	Y	PQ	40.00		20.00		0.00		0.00		0.00
2	Y	PQ	0.00		0.00		20.00		10.00		0.00
4	Y	PQ	0.00		0.00		0.00		0.00		40.00
5	Y	I	0.00		0.00		0.00		0.00		20.00
6	Y	Z	0.00		0.00		0.00		0.00		10.32
7	Y	PQ	20.00		10.00		0.00		0.00		42.54
9	Y	PQ	40.00		20.00		0.00		0.00		0.00
10	Y	I	20.12		10.06		0.00		0.00		0.00
11	Y	Z	40.46		20.23		0.00		0.00		0.00
12	Y	PQ	0.00		0.00		20.00		10.00		0.00
16	Y	PQ	0.00		0.00		0.00		40.00		20.00
17	Y	PQ	0.00		0.00		0.00		20.00		10.00
19	Y	PQ	40.00		20.00		0.00		0.00		0.00
20	Y	I	39.87		19.94		0.00		0.00		0.00
22	Y	Z	0.00		0.00		42.48		21.24		0.00
24	Y	PQ	0.00		0.00		0.00		40.00		20.00
28	Y	I	39.88		19.94		0.00		0.00		0.00
29	Y	Z	39.74		19.87		0.00		0.00		0.00
30	Y	PQ	0.00		0.00		0.00		40.00		20.00
31	Y	PQ	0.00		0.00		0.00		20.00		10.00
32	Y	PQ	0.00		0.00		0.00		20.00		10.00
33	Y	I	39.82		19.91		0.00		0.00		0.00
34	Y	Z	0.00		0.00		0.00		41.52		20.76
35	D	PQ	40.00		20.00		0.00		0.00		0.00
37	Y	Z	39.56		19.78		0.00		0.00		0.00
38	Y	I	0.00		0.00		20.57		10.28		0.00
39	Y	PQ	0.00		0.00		20.00		10.00		0.00
41	Y	PQ	0.00		0.00		0.00		20.00		10.00
42	Y	PQ	20.00		10.00		0.00		0.00		0.00
43	Y	Z	0.00		0.00		42.09		21.04		0.00
45	Y	I	19.83		9.91		0.00		0.00		0.00
46	Y	PQ	20.00		10.00		0.00		0.00		0.00
47	Y	I	34.68		24.77		35.89		25.63		35.27
48	Y	Z	68.70		49.07		73.55		52.54		71.04
49	Y	PQ	35.00		25.00		70.00		50.00		35.00
50	Y	PQ	0.00		0.00		0.00		40.00		20.00
51	Y	PQ	20.00		10.00		0.00		0.00		0.00
52	Y	PQ	40.00		20.00		0.00		0.00		0.00
53	Y	PQ	40.00		20.00		0.00		0.00		0.00

55		Y		Z		19.90		9.95		0.00		0.00		0.00		0.00
56		Y		PQ		0.00		0.00		20.00		10.00		0.00		0.00
58		Y		I		0.00		0.00		20.60		10.30		0.00		0.00
59		Y		PQ		0.00		0.00		20.00		10.00		0.00		0.00
60		Y		PQ		20.00		10.00		0.00		0.00		0.00		0.00
62		Y		Z		0.00		0.00		0.00		0.00		0.00		0.00
63		Y		PQ		40.00		20.00		0.00		0.00		40.27		20.13
64		Y		I		0.00		0.00		76.63		35.76		0.00		0.00
65		D		Z		34.69		24.78		35.80		25.57		69.62		49.73
66		Y		PQ		0.00		0.00		0.00		0.00		75.00		35.00
68		Y		PQ		20.00		10.00		0.00		0.00		0.00		0.00
69		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
70		Y		PQ		20.00		10.00		0.00		0.00		0.00		0.00
71		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
73		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
74		Y		Z		0.00		0.00		0.00		0.00		42.48		21.24
75		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
76		D		I		107.61		81.99		72.33		51.66		72.98		52.13
77		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
79		Y		Z		43.03		21.52		0.00		0.00		0.00		0.00
80		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
82		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
83		Y		PQ		0.00		0.00		0.00		0.00		20.00		10.00
84		Y		PQ		0.00		0.00		0.00		0.00		20.00		10.00
85		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
86		Y		PQ		0.00		0.00		20.00		10.00		0.00		0.00
87		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
88		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
90		Y		I		0.00		0.00		41.07		20.54		0.00		0.00
92		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
94		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
95		Y		PQ		0.00		0.00		20.00		10.00		0.00		0.00
96		Y		PQ		0.00		0.00		20.00		10.00		0.00		0.00
98		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
99		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
100		Y		Z		0.00		0.00		0.00		0.00		42.68		21.34
102		Y		PQ		0.00		0.00		0.00		0.00		20.00		10.00
103		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
104		Y		PQ		0.00		0.00		0.00		0.00		40.00		20.00
106		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
107		Y		PQ		0.00		0.00		40.00		20.00		0.00		0.00
109		Y		PQ		40.00		20.00		0.00		0.00		0.00		0.00
111		Y		PQ		20.00		10.00		0.00		0.00		0.00		0.00
112		Y		I		20.49		10.24		0.00		0.00		0.00		0.00
113		Y		Z		41.80		20.90		0.00		0.00		0.00		0.00
114		Y		PQ		20.00		10.00		0.00		0.00		0.00		0.00

#### SHUNT CAPACITORS

83		Y		Z		0.00		-217.91		0.00		-215.33		0.00		-215.99
88		Y		Z		0.00		-53.52		0.00		0.00		0.00		0.00
90		Y		Z		0.00		0.00		0.00		-52.72		0.00		0.00
92		Y		Z		0.00		0.00		0.00		0.00		-53.83		

#### 5. Voltage Regulators Setting

REGULATOR ID.		NODE A		NODE B		PT RATIO		CT RATE		BAND WIDTH		PHASES		CONNECTION
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Reg-1		150		149		20.00		700.00		2.00		A-B-C		AG-BG-CG
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PHASE		R-VOLT		X-VOLT		VOLTAGE LEVEL		RELAY VOLTAGE		TAP
-------	--	--------	--	--------	--	---------------	--	---------------	--	-----

A		3.000		7.500		120.000		120.466		7
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REGULATOR ID.		NODE A		NODE B		PT RATIO		CT RATE		BAND WIDTH		PHASES		CONNECTION
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Reg-2		9		14		20.00		50.00		2.00		A		AG
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PHASE		R-VOLT		X-VOLT		VOLTAGE LEVEL		RELAY VOLTAGE		TAP
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A		0.400		0.400		120.000		120.765		-1
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REGULATOR ID.		NODE A		NODE B		PT RATIO		CT RATE		BAND WIDTH		PHASES		CONNECTION
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Reg-3		25		26		20.00		50.00		1.00		A-C		AG-CG
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PHASE		R-VOLT		X-VOLT		VOLTAGE LEVEL		RELAY VOLTAGE		TAP
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A		0.400		0.400		120.000		119.567		0
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C		0.400		0.400		120.000		120.254		-1
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REGULATOR ID.		NODE A		NODE B		PT RATIO		CT RATE		BAND WIDTH		PHASES		CONNECTION
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Reg-4		160		67		20.00		300.00		2.00		A-B-C		AG-BG-CG
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PHASE		R-VOLT		X-VOLT		VOLTAGE LEVEL		RELAY VOLTAGE		TAP
-------	--	--------	--	--------	--	---------------	--	---------------	--	-----

A		0.600		1.300		124.000		124.112		8
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B		1.400		2.600		124.000		123.205		1
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C		0.200		1.400		124.000		124.430		5
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### 5.2.5 4-node Grounded Y-Grounded Y, step down transformer, unbalance loading

#### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	7199.5579<	0.00	7199.5579<-120.00	120.00	7199.5579<	120.00	LG
102	7163.7234<	-0.14	7110.4857<-120.18	119.26	7082.0359<	119.26	LG
103	2305.4926<	-2.26	2254.6574<-123.62	114.79	2202.8175<	114.79	LG
104	2174.9706<	-4.12	1929.8144<-126.80	102.85	1832.6956<	102.85	LG

#### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 101						
To Node 102   230.06< -35.91   345.72<-152.64   455.05< 84.65						
<hr/>						
Node 102						
From Node 101   230.07< -35.91   345.73<-152.64   455.06< 84.65						
To Node 103   230.07< -35.91   345.73<-152.64   455.06< 84.65						
<hr/>						
Node 103						
From Node 102   689.66< -35.91   1036.37<-152.64   1364.10< 84.65						
To Node 104   689.66< -35.91   1036.37<-152.64   1364.10< 84.65						
<hr/>						
Node 104						
From Node 103   689.66< -35.91   1036.37<-152.64   1364.10< 84.65						
<hr/>						

#### 3.Losses (KW)

	TOTAL	PHASE A	PHASE B	PHASE C
NODE 101 To NODE 102	111.349	28.273	36.647	46.428
NODE 102 To NODE 103	98.366	13.718	30.979	53.669
NODE 103 To NODE 104	1250.710	317.578	411.635	521.497
TOTAL	359.570	479.261	621.595	

TOTAL FEEDER POWER LOSS= 1460.4255

#### 4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A	P	Q	PHASE B	P	Q	PHASE C	P	Q
<hr/>											
104	Y	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62			

### 5.2.6 4-node Grounded Y-Grounded Y, step up transformer, unbalance loading

#### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	7199.5579<	0.00	7199.5579<-120.00	120.00	7199.5579<	120.00	LG
102	7160.9192<	-0.09	7120.2011<-120.25	119.25	7127.5413<	119.25	LG
103	13839.7057<	-2.14	13663.0365<-123.29	13655.4876<	13615.2765<	115.14	LG
104	13815.7718<	-2.17	13614.5457<-123.38			114.88	LG

#### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 101						
To Node 102   216.72< -33.94   293.29<-149.21   366.62< 96.70						
<hr/>						
Node 102						
From Node 101   216.74< -33.94   293.30<-149.21   366.62< 96.70						
To Node 103   216.74< -33.94   293.30<-149.21   366.62< 96.70						
<hr/>						
Node 103						
From Node 102   108.54< -33.94   146.88<-149.21   183.61< 96.70						
To Node 104   108.54< -33.94   146.88<-149.21   183.61< 96.70						
<hr/>						
Node 104						
From Node 103   108.57< -33.96   146.90<-149.23   183.62< 96.68						
<hr/>						

#### 3.Losses (KW)

	TOTAL	PHASE A	PHASE B	PHASE C
NODE 101 To NODE 102	78.435	21.037	26.024	31.374
NODE 102 To NODE 103	69.304	12.174	22.294	34.836
NODE 103 To NODE 104	24.593	6.596	8.160	9.837

TOTAL		39.808		56.478		76.047			
TOTAL FEEDER POWER LOSS= 172.3326									
4. Updated Loads (KW/KVAR)									
NODE	CONFIG	TYPE	PHASE A	PHASE B	PHASE C				
			P	Q	P	Q			
<hr/> SPOT LOADS <hr/>									
104	I	Y	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62

### 5.2.7 4-node Δ-Grounded Y, step down transformer, unbalance loading

#### 1. Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
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101	12470.0000<	30.00	12470.0000<	-90.00	12470.0000<	150.00	LL
102	12350.1995<	29.60	12313.6887<	-90.39	12332.7108<	149.75	LL
103	2291.3960<	-32.40	2261.1401<-153.79	2213.1897<	85.15	LG	
104	2158.1090<	-34.24	1935.7283<-157.01	1848.3358<	73.36	LG	

#### 2. Current Flow (AMP)

MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
-----	-------	-----	-------	-----	-------

Node 101	To Node 102	285.58<	-27.58	402.95<-149.60	349.15<	74.31
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Node 102	From Node 101	285.59<	-27.58	402.96<-149.60	349.16<	74.31
	To Node 103	285.59<	-27.58	402.96<-149.60	349.16<	74.31

Node 103	From Node 102	695.05<	-66.03	1033.20<	177.15	1352.56<	55.17
	To Node 104	695.05<	-66.03	1033.20<	177.15	1352.56<	55.17

Node 104	From Node 103	695.05<	-66.03	1033.20<	177.15	1352.56<	55.17
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#### 3. Losses (KW)

TOTAL	PHASE A	PHASE B	PHASE C
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NODE 101 To NODE 102	82.024	22.660	32.027	27.337
NODE 102 To NODE 103	97.489	13.934	30.790	52.765
NODE 103 To NODE 104	1239.527	316.433	408.393	514.701
TOTAL		353.027	471.210	594.803

TOTAL FEEDER POWER LOSS= 1419.0398

#### 4. Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A	PHASE B	PHASE C	
			P	Q	P	Q

SPOT LOADS						
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104	I	Y	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62
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### 5.2.8 4-node Δ-Grounded Y, step up transformer, unbalance loading

#### 1. Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
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101	12470.0000<	30.00	12470.0000<	-90.00	12470.0000<	150.00	LL
102	12363.7353<	29.78	12390.6204<	-90.45	12333.2437<	149.55	LL
103	13789.3996<	27.72	13731.8604<	-93.48	13645.4748<	145.44	LG
104	13765.4532<	27.69	13683.4514<	-93.57	13605.1406<	145.18	LG

#### 2. Current Flow (AMP)

MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
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Node 101	To Node 102	309.19<	-35.17	249.50<-146.46	319.12<	98.07
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Node 102	From Node 101	309.20<	-35.17	249.51<-146.46	319.12<	98.07
	To Node 103	309.20<	-35.17	249.51<-146.46	319.12<	98.07

Node 103	From Node 102	108.94<	-4.08	146.14<-119.40	183.74<	127.00
	To Node 104	108.94<	-4.08	146.14<-119.40	183.74<	127.00

-----  
Node 104  
From Node 103 | 108.97< -4.10 | 146.16<-119.41 | 183.75< 126.99  
-----

3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 101	To NODE 102	58.225	20.456	16.590	21.178
NODE 102	To NODE 103	69.222	12.264	22.070	34.888
NODE 103	To NODE 104	24.563	6.603	8.122	9.838
TOTAL		39.322	46.783	65.904	

TOTAL FEEDER POWER LOSS= 152.0090

4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C	
			P	Q	P	Q	P	Q
SPOT LOADS								
104	Y	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62

### 5.2.9 4-node $\Delta$ - $\Delta$ , step down transformer, unbalance loading

1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	12470.0000<	30.00	12470.0000<	-90.00	12470.0000<	150.00	LL
102	12341.0188<	29.81	12370.2718<	-90.48	12301.7864<	149.55	LL
103	3901.7464<	27.20	3972.4669<	-93.91	3871.4002<	145.74	LL
104	3430.6528<	24.27	3647.4292<-100.36	3293.7634<	138.61	LL	

2.Current Flow (AMP)

MAG ANGLE   MAG ANGLE   MAG ANGLE							
Node 101							
To Node 102   361.65< -41.03   283.47<-153.03   366.47< 93.15							
Node 102							
From Node 101   361.67< -41.03   283.48<-153.04   366.48< 93.15							
To Node 103   361.67< -41.03   283.48<-153.04   366.48< 93.15							
Node 103							
From Node 102   1084.13< -41.03   849.76<-153.04   1098.56< 93.15							
To Node 104   1084.13< -41.03   849.76<-153.04   1098.56< 93.15							
Node 104							
From Node 103   1084.14< -41.03   849.76<-153.04   1098.56< 93.15							

3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 101	To NODE 102	77.454	27.631	21.785	28.038
NODE 102	To NODE 103	268.606	101.700	62.481	104.425
NODE 103	To NODE 104	869.992	310.367	244.696	314.930
TOTAL		439.699	328.961	447.393	

TOTAL FEEDER POWER LOSS= 1216.0529

4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C	
			P	Q	P	Q	P	Q
SPOT LOADS								
104	D	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62

### 5.2.10 4-node $\Delta$ - $\Delta$ , step up transformer, unbalance loading

1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	12470.0000<	30.00	12470.0000<	-90.00	12470.0000<	150.00	LL
102	12362.4885<	29.78	12392.1736<	-90.45	12334.3720<	149.55	LL
103	23676.2053<	27.25	24060.6921<	-93.57	23573.3629<	146.03	LL
104	23610.3530<	27.17	24015.6885<	-93.73	23492.7399<	145.86	LL

2.Current Flow (AMP)

	MAG	ANGLE		MAG	ANGLE		MAG	ANGLE
<hr/>								
Node 101								
<hr/>								
To Node 102	312.19< -34.83	248.02<-147.16		316.42< 98.70				
<hr/>								
Node 102								
From Node 101	312.20< -34.83	248.02<-147.17		316.42< 98.70				
To Node 103	312.20< -34.83	248.02<-147.17		316.42< 98.70				
<hr/>								
Node 103								
From Node 102	156.35< -34.83	124.21<-147.17		158.47< 98.70				
To Node 104	156.35< -34.83	124.21<-147.17		158.47< 98.70				
<hr/>								
Node 104								
From Node 103	156.38< -34.84	124.23<-147.19		158.47< 98.69				
<hr/>								

3.Losses (KW)

	TOTAL		PHASE A		PHASE B		PHASE C	
<hr/>								
NODE 101 To NODE 102	58.092		20.650		16.483		20.958	
NODE 102 To NODE 103	201.459		75.784		47.829		77.846	
NODE 103 To NODE 104	18.215		6.475		5.168		6.571	
TOTAL	102.910		69.480		105.376			

TOTAL FEEDER POWER LOSS= 277.7658

4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C		
			P		Q		P		Q
<hr/>									
			SPOT LOADS						

104 | D | PQ | 1275.00 | 790.17 | 1800.00 | 871.78 | 2375.00 | 780.62

### 5.2.11 4-node Grounded Y-Δ, step down transformer, unbalance loading

1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
<hr/>							
101	7199.5579< 0.00	7199.5579<-120.00	7199.5579< 120.00		LG		
102	7111.1412< -0.20	7143.6290<-120.43	7111.1125< 119.54		LG		
103	3896.2732< -2.82	3972.1042<-123.83	3875.0534< 115.70		LL		
104	3425.3972< -5.76	3646.3344<-130.28	3297.6665< 108.58		LL		

2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 101						
<hr/>						
To Node 102	309.77< -41.69	315.53<-145.18		387.18< 85.89		
<hr/>						
Node 102						
From Node 101	309.79< -41.69	315.54<-145.19		387.19< 85.89		
To Node 103	309.79< -41.69	315.54<-145.19		387.19< 85.89		
<hr/>						
Node 103						
From Node 102	1083.86< -71.03	849.88< 176.98	1098.66< 63.14			
To Node 104	1083.86< -71.03	849.88< 176.98	1098.66< 63.14			
<hr/>						
Node 104						
From Node 103	1083.86< -71.03	849.88< 176.97	1098.66< 63.14			
<hr/>						

3.Losses (KW)

	TOTAL		PHASE A		PHASE B		PHASE C
<hr/>							
NODE 101 To NODE 102	101.212		31.229		32.239		37.743
NODE 102 To NODE 103	268.591		101.649		62.498		104.444
NODE 103 To NODE 104	869.941		310.272		244.717		314.952
TOTAL	443.150		339.455		457.139		

TOTAL FEEDER POWER LOSS= 1239.7439

4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C		
			P		Q		P		Q
<hr/>									
			SPOT LOADS						

104 | D | PQ | 1275.00 | 790.17 | 1800.00 | 871.78 | 2375.00 | 780.62

### 5.2.12 4-node Grounded Y-Δ, step up transformer, unbalance loading

#### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	7199.5579<	0.00	7199.5579<-120.00	7199.5579< 120.00		LG	
102	7119.7665<	-0.39	7146.8850<-120.27	7149.6874< 119.55		LG	
103	23703.7856<	57.22	24040.6316<-63.60	23576.7495< 176.10		LL	
104	23637.9558<	57.14	23995.6762<-63.75	23496.0843< 175.94		LL	

#### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 101						
<hr/>						
To Node 102	333.42<	-28.20	269.49<-155.37	274.29< 100.26		
<hr/>						
Node 102						
From Node 101	333.43<	-28.21	269.50<-155.37	274.30< 100.26		
To Node 103	333.43<	-28.21	269.50<-155.37	274.30< 100.26		
<hr/>						
Node 103						
From Node 102	156.34<	-4.80	124.22<-117.16	158.42< 128.72		
To Node 104	156.34<	-4.80	124.22<-117.16	158.42< 128.72		
<hr/>						
Node 104						
From Node 103	156.37<	-4.82	124.24<-117.18	158.43< 128.70		
<hr/>						

#### 3.Losses (KW)

	TOTAL	PHASE A	PHASE B	PHASE C
NODE 101 To NODE 102	75.802	27.932	23.907	23.962
NODE 102 To NODE 103	201.410	75.773	47.833	77.803
NODE 103 To NODE 104	18.210	6.474	5.168	6.568
TOTAL	110.180	76.909	108.333	

TOTAL FEEDER POWER LOSS= 295.4217

#### 4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C	
			P	Q	P	Q	P	Q
SPOT LOADS								
104	D	PQ	1275.00	790.17	1800.00	871.78	2375.00	780.62

### 5.2.13 4-node Ungrounded Y-Δ, step down transformer, unbalance loading

#### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	7199.5579<	0.00	7199.5579<-120.00	7199.5579< 120.00		LG	
102	7110.6705<	-0.19	7142.6758<-120.44	7112.5395< 119.54		LG	
103	3896.2939<	-2.83	3972.0766<-123.03	3875.0682< 115.70		LL	
104	3425.4306<	-5.76	3646.2590<-130.28	3297.6872< 108.58		LL	

#### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
<hr/>						
Node 101						
<hr/>						
To Node 102	309.78<	-41.69	315.53<-145.19	387.16< 85.89		
<hr/>						
Node 102						
From Node 101	309.80<	-41.69	315.53<-145.19	387.17< 85.89		
To Node 103	309.80<	-41.69	315.53<-145.19	387.17< 85.89		
<hr/>						
Node 103						
From Node 102	1083.81<	-71.03	849.93< 176.98	1098.66< 63.14		
To Node 104	1083.81<	-71.03	849.93< 176.98	1098.66< 63.14		
<hr/>						
Node 104						
From Node 103	1083.81<	-71.03	849.93< 176.98	1098.66< 63.14		
<hr/>						

#### 3.Losses (KW)

	TOTAL	PHASE A	PHASE B	PHASE C
NODE 101 To NODE 102	77.447	23.593	24.009	29.844
NODE 102 To NODE 103	268.589	101.639	62.506	104.444
NODE 103 To NODE 104	869.937	310.256	244.729	314.952
TOTAL	435.488	331.244	449.241	

TOTAL FEEDER POWER LOSS= 1215.9731

4. Updated Loads (KW/KVAR)																
NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C							
					P		Q		P		Q					
SPOT LOADS																
104		D		PQ		1275.00		790.17		1800.00		871.78		2375.00		780.62

### 5.2.14 4-node Ungrounded Y-Δ, step up transformer, unbalance loading

1.Voltage Profile											
NODE		MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG	
101		7199.5579<	0.00	7199.5579<-120.00		7199.5579< 120.00		LG			
102		7119.3537<	-0.37	7145.7734<-120.28		7151.2157< 119.54		LG			
103		123703.8879<	57.22	124040.7198<-63.60		123576.8567< 176.10		LL			
104		123638.0669<	57.14	123995.7679<-63.75		123496.1956< 175.94		LL			

2.Current Flow (AMP)											
		MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE		
Node 101											
To Node 102			333.39<-28.21		269.45<-155.37		274.28< 100.27				
Node 102											
From Node 101			333.40<-28.21		269.46<-155.37		274.29< 100.26				
To Node 103			333.40<-28.21		269.46<-155.37		274.29< 100.26				
Node 103											
From Node 102			156.32<-4.80		124.20<-117.16		158.41< 128.72				
To Node 104			156.32<-4.80		124.20<-117.16		158.41< 128.72				
Node 104											
From Node 103			156.35<-4.82		124.22<-117.18		158.42< 128.70				

3.Losses (KW)													
		TOTAL	PHASE A	PHASE B	PHASE C								
Node 101	To Node 102		58.068		22.233		17.765		18.069				
Node 102	To Node 103		201.376		75.757		47.824		77.795				
Node 103	To Node 104		18.207		6.473		5.167		6.568				
										TOTAL			
											104.463		
												70.756	
													102.432

TOTAL FEEDER POWER LOSS= 277.6508

4. Updated Loads (KW/KVAR)																
NODE	CONFIG	TYPE	PHASE A			PHASE B			PHASE C							
					P		Q		P		Q					
SPOT LOADS																
104		D		PQ		1275.00		790.17		1800.00		871.78		2375.00		780.62

### 5.2.15 4-node Open grounded Y-open Δ, step down transformer, unbalance loading

1.Voltage Profile											
NODE		MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG	
101		7199.5579<	0.00	7199.5579<-120.00		7199.5579< 120.00		LG			
102		6951.8801<	0.70	7171.5163<-122.00		7312.6802< 120.54		LG			
103		3632.2966<	0.06	4121.3419<-127.56		3450.0657< 108.94		LL			
104		3306.7885<	-1.47	3906.5594<-131.89		3073.0811< 103.11		LL			

2.Current Flow (AMP)											
		MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE		
Node 101											
To Node 102			424.78<-73.82		440.30<-118.52		0.02<-153.55				
Node 102											
From Node 101			424.80<-73.82		440.30<-118.52		0.00< 0.00				
To Node 103			424.80<-73.82		440.30<-118.52		0.00< 0.00				
Node 103											
From Node 102			735.18<-73.82		569.91< 176.32		762.01< 61.48				
To Node 104			735.18<-73.82		569.91< 176.32		762.01< 61.48				

Node 104  
From Node 103 | 735.19< -73.82 | 569.91< 176.32 | 762.01< 61.48

### 3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 101	To NODE 102	109.745	42.726	44.925	22.094
NODE 102	To NODE 103	125.115	46.769	28.104	50.243
NODE 103	To NODE 104	405.237	143.556	112.305	149.375
TOTAL		233.050	185.334	221.712	

TOTAL FEEDER POWER LOSS= 640.0964

### 4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C	
			P	Q	P	Q	P	Q
SPOT LOADS								

104	D	PQ	850.00	526.78	1200.00	581.19	1583.33	520.42
-----	---	----	--------	--------	---------	--------	---------	--------

## 5.2.16 4-node Open grounded Y-open Δ, step up transformer, unbalance loading

### 1.Voltage Profile

NODE	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE	LL / LG
101	7199.5579<	0.00	7199.5579<-120.00	7199.5579< 120.00		LG	
102	7000.7918<	0.02	7206.6727<-121.31	7263.9994< 120.51		LG	
103	124762.7476<	54.99	122758.0346<-68.80	122455.4317< 177.61		LL	
104	124717.5537<	54.94	122730.1711<-68.91	122398.9243< 177.50		LL	

### 2.Current Flow (AMP)

	MAG	ANGLE	MAG	ANGLE	MAG	ANGLE
Node 101						
To Node 102	368.82< -52.60	295.41<-119.46	0.02<-153.56			
Node 102						
From Node 101	368.84< -52.60	295.41<-119.46	0.00< 0.00			
To Node 103	368.84< -52.60	295.41<-119.46	0.00< 0.00			
Node 103						
From Node 102	107.29< -5.54	85.41<-119.46	106.65< 127.40			
To Node 104	107.29< -5.54	85.41<-119.46	106.65< 127.40			
Node 104						
From Node 103	107.31< -5.55	85.43<-119.49	106.65< 127.38			

### 3.Losses (KW)

		TOTAL	PHASE A	PHASE B	PHASE C
NODE 101	To NODE 102	65.324	28.732	23.460	13.132
NODE 102	To NODE 103	93.556	35.682	22.616	35.258
NODE 103	To NODE 104	8.459	3.030	2.419	3.010
TOTAL		67.444	48.496	51.399	

TOTAL FEEDER POWER LOSS= 167.3394

### 4.Updated Loads (KW/KVAR)

NODE	CONFIG	TYPE	PHASE A		PHASE B		PHASE C	
			P	Q	P	Q	P	Q
SPOT LOADS								

104	D	PQ	850.00	526.78	1200.00	581.19	1583.33	520.42
-----	---	----	--------	--------	---------	--------	---------	--------

## 5.3 Calculate regulator tap numbers

Any setting of regulator taps, which regulates voltage of relay of regulator in desired bandwidth, can be an acceptable answer. This study aims at the setting which regulates the voltage of the relay as close as possible to reference voltage of relay. In this case, small changes in loads do not result in tap changing.

### 5.3.1 13-node feeder

#### Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	650	632	20.00	700.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	3.000	9.000	122.000	122.156	10		
B	3.000	9.000	122.000	121.823	7		
C	3.000	9.000	122.000	122.079	10		

### 5.3.2 34-node feeder

#### Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	814	850	120.00	100.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	2.700	1.600	122.000	121.835	14		
B	2.700	1.600	122.000	121.694	5		
C	2.700	1.600	122.000	121.861	6		
REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-2	852	832	120.00	100.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	2.500	1.500	124.000	124.032	12		
B	2.500	1.500	124.000	123.713	12		
C	2.500	1.500	124.000	124.001	12		

### 5.3.3 37-node feeder

#### Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	799	701	40.00	350.00	2.00	A-B-C	AB-CB
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
AB	1.500	3.000	122.000	121.741	7		
CB	1.500	3.000	122.000	122.240	4		

### 5.3.4 123-node feeder

#### 5.Voltage Regulators Setting

REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-1	150	149	20.00	700.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	3.000	7.500	120.000	119.701	6		
REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-2	9	14	20.00	50.00	2.00	A	AG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	0.400	0.400	120.000	120.007	-1		
REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-3	25	26	20.00	50.00	1.00	A-C	AG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	0.400	0.400	120.000	120.285	2		
C	0.400	0.400	120.000	120.247	0		
REGULATOR ID.	NODE A	NODE B	PT RATIO	CT RATE	BAND WIDTH	PHASES	CONNECTION
Reg-4	160	67	20.00	300.00	2.00	A-B-C	AG-BG-CG
PHASE	R-VOLT	X-VOLT	VOLTAGE LEVEL	RELAY VOLTAGE	TAP		
A	0.600	1.300	124.000	124.016	9		
B	1.400	2.600	124.000	124.025	3		
C	0.200	1.400	124.000	124.391	6		

### **Conclusion**

It is emphasized that conventional NR method is suitable for power flow studies of unbalanced three-phase distribution feeders. Accurate modeling of components of distribution feeder is vital, and it is explained. Formulation of three phase distribution system admittance matrix, effect of voltage regulators, and formulation of Jacobian matrix are three factors that are different from the transmission systems. The obtained Jacobian can be used for further studies of the distribution network. The proposed algorithm was tested on 123, 37, 34, 13, and 4 node IEEE test feeders. Comparing the results with published IEEE results, the capability of NR method for solving complex ill conditioned distribution systems can be reassured.

## References

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- [4] <http://www.ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html>
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**Appendix A**  
**Conductor Data**

Size	Stranding	Material	DIAM Inches	GMR Feet	RES $\Omega/\text{mile}$	Capacity Amps
1		ACSR	0.355	0.00418	1.38	200
1	7 STRD	Copper	0.328	0.00992	0.765	270
1	CLASS A	AA	0.328	0.00991	1.224	177
2	6/1	ACSR	0.316	0.00118	1.69	180
2	7 STRD	Copper	0.292	0.00883	0.964	230
2	7/1	ACSR	0.325	0.00504	1.65	180
2	AWG SLD	Copper	0.258	0.00836	0.945	220
2	CLASS A	AA	0.292	0.00883	1.541	156
3	6/1	ACSR	0.281	0.0043	2.07	160
3	AWG SLD	Copper	0.229	0.00745	1.192	190
4	6/1	ACSR	0.25	0.00437	2.57	140
4	7/1	ACSR	0.257	0.00452	2.55	140
4	AWG SLD	Copper	0.204	0.00663	1.503	170
4	CLASS A	AA	0.232	0.007	2.453	90
5	6/1	ACSR	0.223	0.00416	3.18	120
5	AWG SLD	Copper	0.1819	0.0059	1.895	140
6	6/1	ACSR	0.198	0.00394	3.98	100
6	AWG SLD	Copper	0.162	0.00526	2.39	120
6	CLASS A	AA	0.184	0.00555	3.903	65
7	AWG SLD	Copper	0.1443	0.00468	3.01	110
8	AWG SLD	Copper	0.1285	0.00416	3.8	90
9	AWG SLD	Copper	0.1144	0.00371	4.6758	80
10	AWG SLD	Copper	0.1019	0.00330	5.9026	75
12	AWG SLD	Copper	0.0808	0.00262	9.3747	40
14	AWG SLD	Copper	0.0641	0.00208	14.8722	20
16	AWG SLD	Copper	0.0508	0.00164	23.7262	10
18	AWG SLD	Copper	0.0403	0.00130	37.6726	5
19	AWG SLD	Copper	0.0359	0.00116	47.5103	4
20	AWG SLD	Copper	0.032	0.00103	59.684	3
22	AWG SLD	Copper	0.0253	0.00082	95.4835	2
24	AWG SLD	Copper	0.0201	0.00065	151.616	1
1/0		ACSR	0.398	0.00446	1.12	230
1/0	7 STRD	Copper	0.368	0.01113	0.607	310
1/0	CLASS A	AA	0.368	0.0111	0.97	202
2/0		ACSR	0.447	0.0051	0.895	270
2/0	7 STRD	Copper	0.414	0.01252	0.481	360
2/0	CLASS A	AA	0.414	0.0125	0.769	230
3/0	12 STRD	Copper	0.492	0.01559	0.382	420

*(Continued)*

### Conductor Data (continued)

Size	Stranding	Material	DIAM Inches	GMR Feet	RES $\Omega/\text{mile}$	Capacity Amps
3/0	6/1	ACSR	0.502	0.006	0.723	300
3/0	7 STRD	Copper	0.464	0.01404	0.382	420
3/0	CLASS A	AA	0.464	0.014	0.611	263
3/8	INCH STE	Steel	0.375	0.00001	4.3	150
4/0	12 STRD	Copper	0.552	0.0175	0.303	490
4/0	19 STRD	Copper	0.528	0.01668	0.303	480
4/0	6/1	ACSR	0.563	0.00814	0.592	340
4/0	7 STRD	Copper	0.522	0.01579	0.303	480
4/0	CLASS A	AA	0.522	0.0158	0.484	299
250,000	12 STRD	Copper	0.6	0.01902	0.257	540
250,000	19 STRD	Copper	0.574	0.01813	0.257	540
250,000	CON LAY	AA	0.567	0.0171	0.41	329
266,800	26/7	ACSR	0.642	0.0217	0.385	460
266,800	CLASS A	AA	0.586	0.0177	0.384	320
300,000	12 STRD	Copper	0.657	0.0208	0.215	610
300,000	19 STRD	Copper	0.629	0.01987	0.215	610
300,000	26/7	ACSR	0.68	0.023	0.342	490
300,000	30/7	ACSR	0.7	0.0241	0.342	500
300,000	CON LAY	AA	0.629	0.0198	0.342	350
336,400	26/7	ACSR	0.721	0.0244	0.306	530
336,400	30/7	ACSR	0.741	0.0255	0.306	530
336,400	CLASS A	AA	0.666	0.021	0.305	410
350,000	12 STRD	Copper	0.71	0.0225	0.1845	670
350,000	19 STRD	Copper	0.679	0.0214	0.1845	670
350,000	CON LAY	AA	0.679	0.0214	0.294	399
397,500	26/7	ACSR	0.783	0.0265	0.259	590
397,500	30/7	ACSR	0.806	0.0278	0.259	600
397,500	CLASS A	AA	0.724	0.0228	0.258	440
400,000	19 STRD	Copper	0.726	0.0229	0.1619	730
450,000	19 STRD	Copper	0.77	0.0243	0.1443	780
450,000	CON LAG	AA	0.77	0.0243	0.229	450
477,000	26/7	ACSR	0.858	0.029	0.216	670
477,000	30/7	ACSR	0.883	0.0304	0.216	670
477,000	CLASS A	AA	0.795	0.0254	0.216	510
500,000	19 STRD	Copper	0.811	0.0256	0.1303	840
500,000	37 STRD	Copper	0.814	0.026	0.1303	840
500,000	CON LAY	AA	0.813	0.026	0.206	483
556,500	26/7	ACSR	0.927	0.0313	0.1859	730
556,500	30/7	ACSR	0.953	0.0328	0.1859	730
556,500	CLASS A	AA	0.858	0.0275	0.186	560
600,000	37 STRD	Copper	0.891	0.0285	0.1095	940
600,000	CON LAY	AA	0.891	0.0285	0.172	520
605,000	26/7	ACSR	0.966	0.0327	0.172	760
605,000	54/7	ACSR	0.953	0.0321	0.1775	750
636,000	27/7	ACSR	0.99	0.0335	0.1618	780

Conductor Data (continued)

Size	Stranding	Material	DIAM Inches	GMR Feet	RES $\Omega/\text{mile}$	Capacity Amps
666,600	54/7	ACSR	1	0.0337	0.1601	800
700,000	37 STRD	Copper	0.963	0.0308	0.0947	1040
700,000	CON LAY	AA	0.963	0.0308	0.148	580
715,500	26/7	ACSR	1.051	0.0355	0.1442	840
715,500	30/19	ACSR	1.081	0.0372	0.1442	840
715,500	54/7	ACSR	1.036	0.0349	0.1482	830
715,500	CLASS A	AA	0.974	0.0312	0.145	680
750,000	37 STRD	AA	0.997	0.0319	0.0888	1090
750,000	CON LAY	AA	0.997	0.0319	0.139	602
795,000	26/7	ACSR	1.108	0.0375	0.1288	900
795,000	30/19	ACSR	1.14	0.0393	0.1288	910
795,000	54/7	ACSR	1.093	0.0368	0.1378	900
795,000	CLASS A	AA	1.026	0.0328	0.131	720

**Appendix B**  
**Concentric Neutral 15 kV Cable**

Conductor Size AWG or kcmil	Diameter over Insulation Inches	Diameter over Screen Inches	Outside Diameter Inches	Copper Neutral No. x AWG	Ampacity UG Duct Amps
<i>Full Neutral</i>					
2(7x)	0.78	0.85	0.98	10 x 14	120
1(19x)	0.81	0.89	1.02	13 x 14	135
1/0(19x)	0.85	0.93	1.06	16 x 14	155
2/0(19x)	0.90	0.97	1.13	13 x 12	175
3/0(19x)	0.95	1.02	1.18	16 x 12	200
4/0(19x)	1.01	1.08	1.28	13 x 10	230
250(37x)	1.06	1.16	1.37	16 x 10	255
350(37x)	1.17	1.27	1.47	20 x 10	300
<i>1/3 Neutral</i>					
2(7x)	0.78	0.85	0.98	6 x 14	135
1(19x)	0.81	0.89	1.02	6 x 14	155
1/0(19x)	0.85	0.93	1.06	6 x 14	175
2/0(19x)	0.90	0.97	1.10	7 x 14	200
3/0(19x)	0.95	1.02	1.15	9 x 14	230
4/0(19x)	1.01	1.08	1.21	11 x 14	240
250(37x)	1.06	1.16	1.29	13 x 14	260
350(37x)	1.17	1.27	1.39	18 x 14	320
500(37x)	1.29	1.39	1.56	16 x 12	385
750(61x)	1.49	1.59	1.79	15 x 10	470
1000(61x)	1.64	1.77	1.98	20 x 10	550

**Tape-Shielded 15 kV Cable**  
Tape Thickness = 5 mils

Conductor Size AWG or kcmil	Diameter over Insulation Inches	Diameter over Screen Inches	Jacket Thickness mils	Outside Diameter Inches	Ampacity in UG Duct Amps
1/0	0.82	0.88	80	1.06	165
2/0	0.87	0.93	80	1.10	190
3/0	0.91	0.97	80	1.16	215
4/0	0.96	1.02	80	1.21	245
250	1.01	1.08	80	1.27	270
350	1.11	1.18	80	1.37	330
500	1.22	1.30	80	1.49	400
750	1.40	1.48	110	1.73	490
1000	1.56	1.66	110	1.91	565