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APPLICATION OF MATHIEU FUNCTIONS AND THE POINT MATCHING METHOD TO ELLIPTIC CONDUCTORS

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Introduction

Analysis of elliptic conductors, carrying a known total current, using the point matching method (PMM) in circular-cylinder coordinates [1] failed for the values of axes ratio $b/a < 0.5$, where a and b are the major and minor axes of the ellipse, respectively. This work shows that the use of elliptic-cylinder coordinates in conjunction with the point matching method overcomes this problem. This paper is a part of the work investigating the difficulties encountered in the point matching method, and is motivated by the fact that, despite its limitations, the method is still attractive as suggested by its use in recent papers [2]-[4].

Analysis

Fig. 1 shows the cross section of a long elliptic conductor with a total current I flowing in the $+z$ direction with a time dependence of $e^{j\omega t}$. The electric field in the conductor has only z-component, E_z. The wave equation that E_z has to satisfy can be solved in elliptic-cylinder coordinates (η, φ) to yield the expressions for the fields inside $(\eta < \eta_s)$ and outside $(\eta > \eta_s)$ the conductor:

$$E_z = \sum_{n=0}^{\infty} C_n \, ce_{2n}(\varphi) \, Ce_{2n}(\eta), \quad \eta < \eta_s \tag{1}$$

$$E_z = \frac{j\omega \, \mu \, I}{2\pi} \eta + \sum_{n=0}^{\infty} B_n \cos(2n\varphi) \, e^{-2n\eta}, \quad \eta > \eta_s \tag{2}$$

where $ce_{2n}(\varphi)$'s are even, periodic Mathieu functions of order $2n$, and $Ce_{2n}(\eta)$'s are the corresponding modified Mathieu functions. The azimuthal magnetic field, H_φ, is found in both regions using Maxwell's equations:

$$H_\varphi = \frac{1}{j\omega \, \mu \, \rho \sqrt{\cosh^2 \eta - \cos^2 \varphi}} \sum_{n=0}^{\infty} C_n \, ce_{2n}(\varphi) \, Ce'_{2n}(\eta) \;, \; \eta < \eta_s \tag{3}$$

$$H_\varphi = \frac{1}{j\omega \, \mu \, \rho \sqrt{\cosh^2 \eta - \cos^2 \varphi}} \left[\frac{j\omega \, \mu I}{2\pi} - \sum_{n=0}^{\infty} 2nB_n \cos(2n\varphi) \, e^{-2n\eta} \right], \eta > \eta_s \tag{4}$$

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where $\rho = \sqrt{a^2 - b^2}$. The unknown coefficients C_n's and B_n's are found by matching the tangential electric and magnetic fields, E_z, and H_φ, at selected points on the conductor's surface. Fig. 2 and Fig. 3 show the normalized magnitude and phase angle of E_z on the conductor's surface for various values of axes ratio. For comparison, Fig. 4 shows the erroneous result obtained using the point matching method in circular-cylinder coordinates for $b/a = 0.4$. All computations are carried out with $a/\delta = 5$ where δ is the skin depth. The distance d is measured from point A in the direction of the arrow AB (Fig. 1).

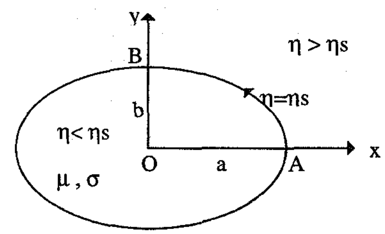

Fig.1. Cross section of an elliptic conductor.

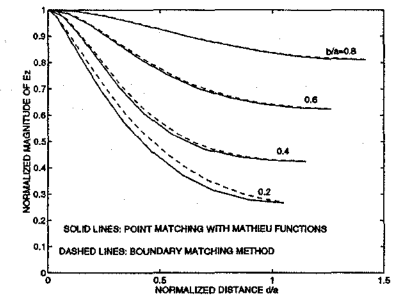

Fig. 2. Normalized magnitude of E_z.

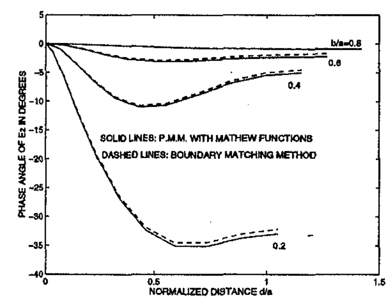

Fig. 3. Phase angle of E_z.

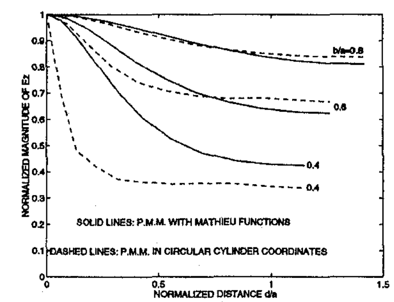

Fig. 4. Erroneous results obtained using PMM in circular-cylinder coordinates.

References

[1] A. M. Hussein and P. P. Biringer, J. Appl. Phys., vol. 60, 3356, (1986).
[2] A. Boag, Y. Leviatan, A. Boag, IEEE Trans. Ant. Propag. AP-41 926, (1993).
[3] H. Na and H. Kim, IEEE Trans. Ant. Propag. AP-43, 426 (1995).
[4] J. M. Tranquilla and H. M. Al-Rizzo, IEEE Trans. Ant. Propag. AP-43, 63, (1995).